Channel Access Method Classification For Cognitive Radio Applications

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Abstract—Motivated by improved detection and prediction of temporal holes, we propose a two stage algorithm to classify the channel access method used by a primary network. The first stage extends an existing fourth-order cumulant-based modulation classifier to distinguish between TDMA, OFDMA, and CDMA. The second stage proposes a novel collision detector using the sample variance of the same cumulant to detect contentionbased channel access methods. Our proposed method is blind and independent of the received SNR. Simulations show that our classification of TDMA, OFDMA, and CDMA is robust to network load while detection of contention outperforms existing methods.

Index Terms—Automatic signal classification, channel access method, collision detection.

I. INTRODUCTION

➤ OGNITIVE radios (CRs) require radio scene analysis in • order to achieve a more efficient utilization of the scarce radio spectrum [1]. Various works that dealt with the radioscene analysis problem have mainly focused on the binary hypothesis problem of detecting the presence or absence of spectrum opportunities, i.e., spectrum holes, through various spectrum sensing methods. Various methods proposed for quickly detecting spectrum holes and scheduling CR transmissions efficiently require knowing the medium access protocol used by the primary user (PU) network. In particular, schemes have been proposed if the PU network uses a time slotted method [2], [3], orthogonal frequency division multiple access (OFDMA) [4], [5], code division multiple access (CDMA) [6], [7], or contention-based methods [8], [9], [10]. Hence, we are motivated to propose a method to classify the channel access method used by the PU network.

A. Related Work

There has been little work on identifying the channel access method used by the primary network. Existing works fall into two categories: identification of particular standards or identification of class of standards. First, work such as [11] and [12] aim to determine the specific standard used based on detailed knowledge of PHY/MAC characteristics such as packet structure and preamble format. [13] uses supervised learning of the frame lengths and inter-arrival times to distinguish between members of the 802.11 family. The second category, which we will be focusing on, identify the class of standards, i.e., OFDMA for 802.11n and CDMA for CDMA2000. The only existing work that we know of in this category is [14]. The authors of [14] employ a Support Vector Machine (SVM) approach to classify between TDMA, carrier sense multiple access with collision avoidance (CSMA/CA), slotted ALOHA, and pure ALOHA networks. However, such a supervised learning approach has limitations in unknown fading channels due to the lack of labeled data. Further, CDMA and OFDMA systems are not addressed in [14].

B. Contributions

The two key contributions of this work are as follows. First, we extend an existing fourth-order cumulant-based modulation type classifier [15] to distinguish between TDMA, CDMA, and OFDMA. Second, we propose a novel method for detecting collusins using the sample variance of the same cumulant and thus, detect contention-based channel access methods.

The rest of the paper is organized as follows. The system model and notation is described in Section II. Our proposed method is described in Section III. Simulation results and comparison with existing work is provided in Section IV. Finally, the paper is concluded in Section V.

II. SYSTEM MODEL

We consider a system consisting of a single sensing node receiving signals from a network of N_{total} PUs communicating amongst themselves. Let \mathcal{U} be the index set of the PUs. We model the signal transmitted by the *i*th PU as

$$x_i(n) = a_i(n)s_i(n) \tag{1}$$

where *n* is the time index, $a_i(n) = 1$ if the *i*th PU is transmitting at time *n* and 0 otherwise, and $s_i(n) \in \mathbb{C}$ is the signal transmitted by the PU, if active. Both the activity $a_i(n)$ and the signal $s_i(n)$ depend on the channel access method used by the PU network.

For both TDMA and contention-based channel access methods, we assume that $s_i(n)$ is a single carrier signal with linear memory-less modulation, such as QAM. TDMA enforces orthogonality in time, i.e., $\sum_{i=1}^{N_{\text{total}}} a_i(n) \le 1$, while contentionbased schemes do not, i.e., $\sum_{i=1}^{N_{\text{total}}} a_i(n) \in \mathbb{N} \cup \{0\}$.

For OFDMA, $s_i(n)$ is an OFDM modulated signal with N_{sc} subcarriers and a cyclic prefix of length N_p . Of these N_{sc} subcarriers, subset S_i are assigned to the *i*th PU.

For CDMA, the *i*th PU's data stream $d_i(n)$ is spread using its code $c_i(n)$ of length L_c :

$$s_i(n) = c_i (n \mod L_c) d_i(n).$$
⁽²⁾

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For both OFDMA and CDMA, we assume that all PUs are transmitting simultaneously, i.e., $a_i(n) = 1$. Though this appears to be a strong assumption, squelching the received signal and normalization of the test statistic ensure that it does not impact the performance of our algorithm.

We denote the tuple of channel access method and modulation type used by \mathcal{M} . Non-contention based channel access methods covered by this work are listed in Table II and collected in the set \mathfrak{M} .

The packet arrival rate at the PUs determines the rate of collisions in a contention-based scheme. For simplicity, we assume the following conditions: 1) all packets have the same length, 2) a packet is generated from each PU according to a Poisson process with rate λ_i , 3) packets that collide are not retransmitted. As a result, the aggregate messages to the channel will also be a Poisson arrival process with parameter $G = \sum_{i=1}^{N} \lambda_i$. This is referred to as offered load.

The downconverted signal received at the sensing node is

$$r(n) = \sum_{i=1}^{N_{\text{total}}} h_i(n) x_i(n) + v(n)$$
(3)

where $v(n) \sim CN(0, \sigma_v^2)$ is white Gaussian noise with variance σ_v^2 and $h_i(n)$ includes the PU's transmit power, the fading channel from the PU to the sensing node, and path loss with exponent γ . Let y(n) be the squelched received signal.

III. IDENTIFYING CHANNEL ACCESS METHOD

TDMA, OFDMA, and CDMA have distinguishable features due to their signal structure viz., modulation used in TDMA, the effect of the inverse fourier transform in OFDMA, and the pseudonoise sequence used by users in CDMA. Contentionbased methods are distinguished by the probability of collisions between users. We now show that these properties can be distinguished using the sample mean and sample variance of the normalized fourth-order cumulant of the squelched signal.

A. Normalized 4th-Order Cumulant C_{42} and its Properties

Cumulants of multiple random variables are particular polynomial combinations of their joint higher order moments [16]. We define *k*th-order cumulants C_{ki} , for $i \in \{0, ..., k\}$, of the zero mean complex signal y(n) as the cumulant of k - i repeated copies of y(n) and *i* repeated copies of $y^*(n)$. Also, let $M_{ki} \triangleq E[y^{k-i}(n)(y^*(n))^i]$.

Our proposed test statistic requires estimating the cumulants of frames of J samples each. The unbiased maximum likelihood estimators for the cumulants of the fth frame are denoted by $\hat{C}_{ki}(f)$ and are computed by formulae derived in [15]. Similar to [15], the cumulant estimates are normalized by the signal power $\hat{C}_{21}(f) - \sigma_v^2$:

$$\tilde{C}_{42}(f) = \hat{C}_{42}(f) \left(\hat{C}_{21}(f) - \sigma_{\nu}^2 \right)^{-2}.$$
(4)

Note that C_{21} of a signal is the signal power and does not depend on the modulation type. We denote the *u*'th user's signal's C_{ki} by $C_{ki,u}$. If the value depends on the modulation type or channel access method, it is denoted by $C_{ki,u}(\mathcal{M})$.

 TABLE I

 SAMPLE MEAN AND VARIANCE OF C_{42} ESTIMATED FROM J SAMPLES FOR

 UNIT POWER NOISE-LESS SIGNALS

Constellation	C_{42}	$J \operatorname{var}(\hat{C}_{42})$ from [15] ¹	$J \operatorname{var}(\hat{C}_{42})$
BPSK	-2.0000	36.00	0.00
PAM(4)	-1.3600	34.72	10.24
PAM(8)	-1.2381	32.27	9.98
PAM(16)	-1.2094	31.67	9.90
PAM(32)	-1.2024	31.52	9.88
PAM(64)	-1.2006	31.49	9.88
PAM(∞)	-1.2000	31.47	9.87
$PSK(\geq 4)$	-1.0000	12.00	0.00
V32	-0.6900	9.70	1.42
V29	-0.5816	8.75	1.77
QAM(4,4)	-0.6800	9.54	1.38
QAM(8,8)	-0.6191	8.82	1.39
QAM(16,16)	-0.6047	8.65	1.39
QAM(32,32)	-0.6012	8.61	1.39
QAM(∞)	-0.6000	8.59	1.39
BPSK-OFDM	0	-	~8
QPSK-OFDM	0	-	~4

Since the \tilde{C}_{42} is normalized by signal power, its mean value is independent of any attenuation due to flat-fading [15].

The additivity of the unnormalized C_{42} [16, Theorem 2.3.1(vi)] can be used to express the mean and variance of the estimated \tilde{C}_{42} when U PUs are transmitting simultaneously:

$$E\left[\tilde{C}_{42,U}(f)|\mathcal{M}\right] = \frac{\sum_{u \in U} C_{21,u}^2 C_{42,u}(\mathcal{M})}{\left(\sum_{u \in U} C_{21,u}\right)^2}$$
(5)
$$\operatorname{var}\left[\tilde{C}_{42,U}(f)|\mathcal{M}\right] = \frac{\sum_{u \in U} C_{21,u}^4 \operatorname{var}\left[\tilde{C}_{42,u}(f)|\mathcal{M}\right]}{\left(\sum_{u \in U} C_{21,u}\right)^4} + \frac{\sigma_{\nu}^8 \operatorname{var}\left[\tilde{C}_{42,\nu}(f)\right]}{\left(\sum_{u \in U} C_{21,u}\right)^4}.$$
(6)

Expressions to compute $C_{42,u}(\mathcal{M})$ and var $[\tilde{C}_{42,u}(f)|\mathcal{M}]$ are provided in the Appendix while values for noise-less signals are listed in Table I as computed from (5) and (6). In the interest of readability, the left hand side of (5) and (6) does not explicitly mention the conditional dependence of the mean and variance on the signal powers and noise variance.

B. Proposed Method

Since collisions would modify test statistics that depend on the signal structure, we use a 2 stage algorithm that classifies between TDMA, CDMA, and OFDMA first and then detect whether there are collisions in the second stage. The first stage is a multihypothesis test using the sample mean of $\tilde{C}_{42}(f)$ to classify as TDMA, CDMA, or OFDMA. This is an extension of the modulation classification method proposed in [15]. The second stage proposes a novel binary hypothesis test to detect collisions by thresholding the sample variance of $\tilde{C}_{42}(f)$.

We divide the squelched received signal $\{y(n)\}_{n \in \{1,...,JF\}}$ into *F* frames of *J* samples each. As proposed in [15], The sample mean *W* of the normalized C_{42} can be used to identify the modulation type of the received signal. We now extend it

¹Derivation of var[\hat{C}_{42}] in [15] missed an O(1/J) term. Details provided in the appendix.

to identify TDMA, OFDMA, and CDMA. In particular, we classify the received signal as being one of the classes listed in Table II. Let $\tilde{C}_{42,U}(f)$ be the estimated normalized C_{42} if $U \subseteq \mathcal{U}(f)$ indexed PUs were transmitting in frame f. Let $\mathcal{U}(f)$ denote the set of PUs transmitting simultaneously in frame f. Then, the measured $\tilde{C}_{42}(f)$ can be written as:

$$\tilde{C}_{42}(f) = \sum_{U \subseteq \mathcal{U}} \mathbb{1}_{\{U = \mathcal{U}(f)\}} \tilde{C}_{42,U}(f).$$

$$\tag{7}$$

Since $\tilde{C}_{42}(f)$ has a Gaussian distribution due to the central limit theorem, (7) implies that $\tilde{C}_{42}(f)$ has a Gaussian mixture distribution with mean and variance given by

$$\mathbb{E}\left[\tilde{C}_{42}(f)\big|\mathcal{M}\right] = \sum_{U \subseteq \mathcal{U}} P\left(U = \mathcal{U}(f)\right) \mathbb{E}\left[\tilde{C}_{42,U}(f)\big|\mathcal{M}\right] \quad (8)$$

$$\operatorname{var}\left[\tilde{C}_{42}(f)\middle|\mathcal{M}\right] = \sum_{U \subseteq \mathcal{U}} P\left(U = \mathcal{U}(f)\right) \left\{\operatorname{var}\left[\tilde{C}_{42,U}(f)\middle|\mathcal{M}\right] + \left(\operatorname{E}\left[\tilde{C}_{42,U}(f)\middle|\mathcal{M}\right] - \operatorname{E}\left[\tilde{C}_{42}(f)\middle|\mathcal{M}\right]\right)^{2}\right\}.$$
(9)

Further, assuming that the same number, say K, of PUs transmit at any time, then (8) and (9) can be simplified to

$$\mathbb{E}\left[\tilde{C}_{42}(f)\middle|\mathcal{M}\right] = C_{42,U'}(\mathcal{M}) \text{ and}$$
(10)
$$\operatorname{var}\left[\tilde{C}_{42}(f)\middle|\mathcal{M}\right] = \sum_{\substack{U \in \mathcal{U} \\ |U| = K}} P(U = \mathcal{U}(f)) \operatorname{var}\left[\tilde{C}_{42,U}(f)\middle|\mathcal{M}\right]$$
(11)

where $U' \subseteq \mathcal{U}$ and |U'| = K. For TDMA, K = 1, while $K = N_{\text{total}}$ for CDMA and OFDMA. Since this simplification is not possible for contention-based channel access methods, we separate the contention detection to the second stage.

The first stage of our algorithm uses (10), (11), and the sample mean W of $\tilde{C}_{42}(f)$ to find the most likely class $\hat{\mathcal{M}}$ amongst those listed in Table II.

$$\hat{\mathcal{M}} = \underset{\mathcal{M}' \in \mathfrak{M}}{\arg \max} P\left(C_{42} = W | \mathcal{M}'\right)$$
(12)

For the second stage of our algorithm, detection of contention, we use (9) to note that varying number of simultaneously transmitting PUs increases the sample variance:

$$\operatorname{Var}\left[\tilde{C}_{42}(f)\middle|\mathcal{M}\notin\mathfrak{M}\right]>\operatorname{Var}\left[\tilde{C}_{42}(f)\middle|\mathcal{M}\in\mathfrak{M}\right].$$

We use this fact to estimate our confidence in the inference $\hat{\mathcal{M}}$. We define $P_{C|T} \in (0, 1)$ as the maximum probability of detecting collisions for non-contention based medium access control protocols. We compute the second moment of $\tilde{C}_{42}(f)$ around the theoretically expected $C_{42}(\hat{\mathcal{M}})$:

$$\hat{\varsigma}^2 = \frac{1}{F} \sum_{f=1}^{F} \left(\tilde{C}_{42}(f) - C_{42}\left(\hat{\mathcal{M}} \right) \right)^2.$$
(13)

If the correct class has been detected, then $\hat{\varsigma}^2$ is the sample variance of $\tilde{C}_{42}(f)$ and by Cochran's Theorem [17], the sample variance is distributed as

$$\hat{\varsigma}^2 \sim F^{-1} \operatorname{var} \left[\tilde{C}_{42}(f) \middle| \mathcal{M} \right] \chi_F^2 \tag{14}$$

where χ_F^2 is a χ^2 distribution with *F* degrees. We choose a threshold τ such that $P_{C|T} > P(\varsigma^2 > \tau | \mathcal{M} \notin \mathcal{H}_C)$:

$$\tau = F^{-1} \operatorname{var} \left[\tilde{C}_{42}(f) \middle| \hat{\mathcal{M}} \right] \chi_F^{-2} \left(1 - P_{C|T} \right).$$
(15)

TABLE II LIST OF CHANNEL ACCESS METHODS AND MODULATIONS IN \mathfrak{M} in (12).

Class Label	Channel Access Method	Modulation Type & Levels
M1		BPSK
M2		4/8/16/32/64-PSK
M3-M7	IDMA	4/8/16/32/64-PAM
M8-M10		16/64/256-QAM
M11	OEDMA	4-QAM
M12	OFDWA	16-QAM
M13		BPSK
M14	CDMA	4-QAM
M15		16-QAM

If $\hat{\varsigma}^2 < \tau$ then we declare the inference $\hat{\mathcal{M}}$ from (12) to be correct. If not, then we infer that the channel access method is contention-based.

IV. RESULTS AND COMPARISONS

The theoretical distributions of both the proposed test statistics have been described in the previous section. Since our proposed algorithm consists of a multi-hypothesis test, it is not possible to derive a closed form expression for the classification accuracy. Hence, in this section, we use simulations to study the performance of our proposed channel access method classification algorithm.

A. Simulation System

Consider *N* PUs communicating by a TDMA, CDMA, OFDMA, or contention-based channel access method. The signals transmitted by these users are as described in Section II. We assume a Rayleigh flat fading channel between the PUs and our sensor node. The received SNR of each individual user is exponentially distributed [18] and we vary the average SNR. The offered load by the system is varied from 0.1 to 1. Our metric is the probability of correctly classifying channel access methods. We have chosen the parameter $P_{C|T}$ as 0.05. We present results averaged over the classes listed in Table II.

We also implemented the SVM-based classifier proposed by Hu et al. in [14] to distinguish between TDMA and slotted ALOHA. The SVM is trained using features of received energy, idle time, and busy time of the channel. In order to simulate a blind scenario, we trained the SVM with about 60,000 realizations consisting of 50 realizations from each modulation type, SNR, number of users, and traffic load.

B. Effect of Traffic Load

The traffic load does not affect the classification of TDMA, CDMA, and OFDMA because we squelch the input signal. Fig. 1 shows this classification accuracy as a function of the normalized load. However, the number of collisions in a contention-based method increase with load and causes the classification accuracy of contention-based channel access method to increase with normalized load.

SNR affects the variance of $\tilde{C}_{42}(f)$ for all classes but its mean is affected only for contention-based channel access methods. Therefore, low SNR affects the classification of contention-based methods more than that of the other classes.

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Fig. 1. Probability of correctly identifying the channel access method for TDMA, OFDMA, and CDMA and comparison with [14] for detection of TDMA.



Fig. 2. Probability of correctly identifying channel access methods as number of frames are increased. System: J = 500, 5dB SNR at 0.5 normalized load.

Furthermore, in case a few "hidden" PUs have very low SNRs due to, say, distance or deep fading channels, the classification accuracy will reduce for all classes and the detection of contention will suffer the most. However, if the remaining PUs have higher SNR, then the classification accuracy will increase.

Further, Figs. 1(a) and 1(b) compares with the SVM based classifier proposed in [14] which considers only TDMA and contention-based schemes. Since [14] separates only two classes, note that our proposed algorithm has significantly higher combined classification accuracy. The SVM proposed in [14] tends to classify all signals as contention-based at low loads possibly because channel idle and busy times depend more on the load than contention for the channel. However, further study is required to improve our algorithm's classification of contention-based schemes at low SNRs.

C. Number of Frames

Fig. 2 shows the classification accuracy as the number of frames *F* is increased. Increasing *F* reduces the sample variance of \tilde{C}_{42} which increases the probability of correctly classifying TDMA, CDMA, and OFDMA. Increasing *F* also increases the probability of observing a collision in contentionbased methods. So, the probability of correctly classifying contention-based methods increases. For comparison, note that Fig. 1 shows results for F = 200.

D. Computational Complexity

Computing cumulant estimates from *F* frames of *J* samples each requires O(FJ) operations. Computing the statistics for each class in \mathcal{M} requires $O(|\mathcal{M}|)$ operations. Hence, the first stage of the algorithm requires O(FJ) operations. By reusing values computed in the first stage, computing $\hat{\varsigma}^2$ and τ for the second stage requires O(F) operations. Thus, our algorithm requires O(FJ) computations dominated by the first stage.

V. CONCLUSION

In this article, we have presented a new algorithm to identify the channel access method utilized by a primary network. Our methods are not restricted to any specific standards. We extended a cumulant-based modulation type classification technique to differentiate between OFDMA, CDMA, and TDMA. We proposed a novel collision detection method using the sample variance of the cumulant estimator and, thus, identify contention-based channel access methods such as CSMA. These test statistics were chosen to make our methods robust to channel fading and size of the PU network.

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APPENDIX

STATISTICS OF SAMPLE ESTIMATES OF CUMULANTS

Consider *J* samples of a complex signal r(n) as given by (3). Define $y(n) = r(n) - \frac{1}{J} \sum_{n=1}^{J} r(n)$ so as to obtain a zero-mean form of the received signal. We wish to derive the mean and variance of the estimator $\tilde{C}_{42,y}$ where the subscript *y* indicates that the cumulant is computed from the signal y(n).

We begin by studying the unbiased estimator for the (unnormalized) $\hat{C}_{42,y}$ cumulant:

$$\hat{C}_{42,y} = M_{42,y} - |\hat{C}_{20,y}|^2 - 2\hat{C}_{21,y}^2.$$
(16)

Since these terms are correlated, we have

$$\operatorname{var}[\hat{C}_{42,y}] = \operatorname{var}[\hat{M}_{42,y}] + \operatorname{var}[|\hat{M}_{20,y}|^2] + 4\operatorname{var}[\hat{M}_{21,y}^2] - 2\operatorname{cov}[\hat{M}_{42,y}, |\hat{M}_{20,y}|^2] - 4\operatorname{cov}[\hat{M}_{42,y}, \hat{M}_{21,y}^2] + 4\operatorname{cov}[|\hat{M}_{20,y}|^2, \hat{M}_{21,y}^2].$$
(17)

From the Appendix in [15] we have

$$\operatorname{var}[\hat{M}_{42,y}] = \frac{1}{J} (M_{84,y} - M_{42,y}^2)$$
(18)

The asymptotic analysis of both $\operatorname{var}[\hat{M}_{21,y}^2]$ and $\operatorname{cov}[\hat{M}_{42,y}, \hat{M}_{21,y}^2]$ as derived in [15] are incorrect since they fail to take into account some O(1/J) terms as

shown in the following derivation. Using the definition $\alpha \triangleq M_{42,y} - M_{21,y}^2$,

$$\operatorname{var}[\hat{M}_{21,y}^{2}] = \frac{(J-1)(J-2)(J-3)}{J^{3}} M_{21,y}^{4} + \frac{6(J-1)(J-2)}{J^{3}} M_{21,y}^{2} M_{42,y}^{2} - \left(M_{21,y}^{2} + \frac{\alpha}{J}\right)^{2} + O(1/J^{2}) = \left(1 - \frac{6}{J}\right) M_{21,y}^{4} + \frac{6}{J} M_{21,y}^{2} M_{42,y} - \left(M_{21,y}^{2} + \frac{\alpha}{J}\right)^{2} + O(1/J^{2}) \approx \frac{4}{J} M_{21,y}^{2} (M_{42,y} - M_{21,y}^{2})$$
(19)

The error is in failing to take into account the $-\frac{6}{J}M_{21,y}^4$ part of the first term. A similar derivation also gives the corrected expression

$$\operatorname{cov}[\hat{M}_{42,y}, \hat{M}_{21,y}^2] = \frac{2}{J} M_{21,y} (M_{63,y} - M_{42,y} M_{21,y})$$
(20)

Following similar derivations we can find the rest of the terms in (17) as

$$\operatorname{var}[|\hat{M}_{20,y}|^{2}] = \frac{2}{J} M_{42,y} |M_{20,y}|^{2} - \frac{4}{J} |M_{20,y}|^{4} + \frac{2}{J} \operatorname{Re}\{M_{40,y} M_{20,y}^{*2}\}$$
(21)

$$\operatorname{cov}[\hat{M}_{42,y}, |\hat{M}_{20,y}|^2] = \frac{2}{J} \operatorname{Re}\{M_{62,y}M_{20,y}^*\} - \frac{2}{J}M_{42,y}|M_{20,y}|^2$$
(22)

$$\operatorname{cov}[|\hat{M}_{20,y}|^2, \hat{M}_{21,y}^2] = \frac{4}{J} M_{21,y} \operatorname{Re}\{M_{41,y} M_{20,y}^*\} - \frac{4}{J} M_{21,y}^2 |M_{20,y}|^2$$
(23)

Substituting (18)–(23) into (17) we find the general expression for the asymptotic variance of the $C_{42,y}$ estimate as follows:

$$J \operatorname{var}[\hat{C}_{42,y}] \approx M_{84,y} - M_{42,y}^{2} + 8M_{21,y} \left[2M_{21,y}(M_{42,y} - M_{21,y}^{2} - |M_{20,y}|^{2}) + 2 \operatorname{Re}\{M_{41,y}M_{20,y}^{*}\} - M_{63,y} + M_{21,y}M_{42,y} \right] + 2 \operatorname{Re}\{M_{20,y}^{*}(M_{40,y}M_{20,y}^{*} - 2M_{62,y})\} + 2|M_{20,y}|^{2}(3M_{42,y} - 2|M_{20,y}|^{2}).$$
(24)

Now, y(n) is a noisy signal. We will rewrite (24) in terms of cumulants so that we can quantify the effect of noise using the additive property of cumulants. The following moment–

cumulant equivalence relations are easy to derive:

$$\begin{split} M_{84,y} = & C_{84,y} + 16C_{63,y}C_{21,y} + 12 \operatorname{Re}\{C_{64,y}C_{20,y}\} \\ & + 72C_{21,y}^2C_{42,y} + 18C_{42,y}^2 + 16|C_{41,y}|^2 \\ & + |C_{40,y}|^2 + 6 \operatorname{Re}\{C_{40,y}^*C_{20,y}^2\} \\ & + 96 \operatorname{Re}\{C_{41,y}^*C_{20,y}\}C_{21,y} + 36|C_{20,y}|^2C_{42,y} \\ & + 72|C_{20,y}|^2C_{21,y}^2 + 24C_{21,y}^4 + 9|C_{20,y}|^4 \quad (25) \\ M_{63,y} = & C_{63,y} + 6 \operatorname{Re}[C_{20,y}C_{43,y}] + 9|C_{20,y}|^2C_{21,y} + 6C_{21,y}^3 \\ & + 9C_{21,y}C_{42,y} \quad (26) \\ M_{42,y} = & C_{42,y} + |C_{20,y}|^2 + 2C_{21,y}^2 \\ M_{40,y} = & C_{40,y} + 3C_{20,y}^2 \\ M_{21,y} = & C_{21,y}. \quad (27) \end{split}$$

Gaussian noise has all the relevant cumulants zero except for $C_{21,\nu} = \sigma_{\nu}^2$. By slight abuse of notation, let $C_{ki,x}$ be the C_{ki} cumulant of the noiseless signal component of y(n). Then, except for $C_{21,y}$, we can rewrite all the relevant cumulants as $C_{ki,y} = C_{ki,x}$. $C_{21,y}$ can be rewritten as $C_{21,y} = C_{21,x} + \sigma_{\nu}^2$. Using these relations and (25)-(27), we can rewrite (24) in terms of cumulants.

A. Single User Signals

If it is known that the received signal r(n) consists of a single user's signal, i.e., no collisions have occurred and it is not a CDMA or OFDMA signal, then we can use the modulation type M of the signal to describe var $[\tilde{C}_{42,y}]$. We do this by assuming that the normalizing factor $(\hat{C}_{21,y} - \sigma_v^2)^2$ is perfectly estimated. Then, after normalization, $C_{ki,x}$ are replaced by $C_{ki}(M)$ where $C_{ki}(M)$ is the C_{ki} cumulant of a unit power signal having modulation M. After normalization, $C_{21,y}$ would be replaced by $C_{21,y}/(C_{21,y} - \sigma_v^2)$. Using these relations and (25)-(27), we can rewrite (24) in terms of cumulants for signals modulated by real constellations as:

$$J \operatorname{var}[\tilde{C}_{42,y}] = C_{84}(M) + 4C_{63}(M) \left[\frac{C_{21,y}}{C_{21,y} - \sigma_{\nu}^{2}}\right] + 12 \operatorname{Re}\{C_{62}^{*}(M)C_{20}(M)\} + 17C_{42}(M)^{2} - 8 \left[\frac{C_{21,y}}{C_{21,y} - \sigma_{\nu}^{2}}\right]^{2} C_{42}(M) + 34|C_{20}(M)|^{2}C_{42}(M) + 16|C_{41}(M)|^{2} + 24 \operatorname{Re}\{C_{41}(M)^{*}C_{20}(M)\} \left[\frac{C_{21,y}}{C_{21,y} - \sigma_{\nu}^{2}}\right] + |C_{40}(M)|^{2} + 6 \operatorname{Re}\{C_{40}(M)^{*}C_{20}(M)^{2}\} + 24 \left[\frac{C_{21,y}}{C_{21,y} - \sigma_{\nu}^{2}}\right]^{4}$$
(28)

where we use the fact that real constellations have all real moments, i.e., $M_{20} = M_{21}$, $M_{40} = M_{41} = M_{42}$, and $M_{62} = M_{63}$.

Similarly, for signals modulated by constellations having fourfold symmetry, such as QAM, we use $C_{20} = 0 = C_{21}$ to get

$$J \operatorname{var}[\tilde{C}_{42,y}] = C_{84}(M) + |C_{40}(M)|^{2} + 8 \left[\frac{C_{21,y}}{C_{21,y} - \sigma_{\nu}^{2}} \right] C_{63}(M) + 20 \left[\frac{C_{21,y}}{C_{21,y} - \sigma_{\nu}^{2}} \right]^{2} C_{42}(M) + 4 \left[\frac{C_{21,y}}{C_{21,y} - \sigma_{\nu}^{2}} \right]^{4} + 17 C_{42}(M)^{2}$$
(29)

This is the corrected form of [15, Eqns. 13].

Table I lists the statistics of the C_{42} estimation for unit power noise-less signals having different modulation types.

B. Statistics for OFDMA Signals

The use of fourth order cumulants for distinguishing OFDM signals from single carrier signals is proposed and analyzed in [19]. The moments for OFDM are found to be

$$M_{84} \approx 24$$
, $M_{63} \approx 6$, $M_{40} \approx 0$, $M_{42} \approx 2$, $M_{21} = 1$.

Using these moments in the corrected expression in (24) gives that $J \operatorname{var}[\hat{C}_{42}] \approx 4$ which coincidentally also matches the result derived in [19] from the incorrect expression of [15]. Note however that this result only applies for OFDM with subcarrier modulations that satisfy the fourfold symmetry such as QPSK. Using these moments the variance of the fourth order cumulant for a noisy OFDM signal can be found using (29).

C. Statistics for CDMA Signals

A CDMA signal which uses BPSK chips can be viewed as the sum of N_{total} BPSK signals given as

$$x(n) = \sum_{i=1}^{N_{\text{total}}} s_i(n)$$

where $s_i(n)$ is a BPSK formed by spreading the data to be transmitted with the particular code assigned to that user and is given by (2). Thus we can find the mean of the normalized fourth-order cumulant of a noiseless CDMA signal as $E[\hat{C}_{42}] = -2/N_{\text{total}}$ where we invoke the additivity property. In effect, the mean normalized fourth order cumulant approaches 0 as the number of users increases.

As for the variance of \hat{C}_{42} we can use the general expression in (24) once the moments are found. Due to the blind nature of our classification problem, we do not have knowledge of the true spreading code used by each user. As a result, the correlations from one chip to another within the same symbol period cannot be known. However, unlike the single carrier signals presented in this appendix, these correlations are clearly non-zero and are dependent on the codes used. To circumvent this issue we will assume that such correlations are negligible. Note that with this assumption we are treating CDMA signals to be similar to a sum of BPSK signals in which symbols from different symbol periods and different users are regarded as i.i.d. This is clearly an approximation, but we have found through simulations that the discrepancy is negligible in practice. With this assumption we can proceed to derive the moments of a sum of N_{total} BPSK signals to be

$$M_{42} = \binom{4}{2} \binom{N_{\text{total}}}{2} \frac{1}{N_{\text{total}}^2} + \frac{1}{N_{\text{total}}}$$

$$M_{63} = \binom{6}{2} \binom{4}{2} \binom{N_{\text{total}}}{3} \frac{1}{N_{\text{total}}^3} + \binom{6}{4} \binom{N_{\text{total}}}{2} \frac{2}{N_{\text{total}}^3}$$

$$+ \frac{1}{N_{\text{total}}^2}$$

$$M_{84} = \binom{8}{2} \binom{6}{2} \binom{4}{2} \binom{N_{\text{total}}}{4} \frac{1}{N_{\text{total}}^4} + \binom{8}{4} \binom{4}{2} \binom{N_{\text{total}}}{3} \frac{3}{J^4}$$

$$+ \binom{8}{6} + \frac{1}{2} \binom{8}{4} \binom{N_{\text{total}}}{2} \frac{2}{N_{\text{total}}^4} + \frac{1}{N_{\text{total}}^5}$$

The variance of \tilde{C}_{42} of the noisy signals can then be found through (27).