Energy- and Spectral- Efficiency Tradeoff for D2D-Multicasts in Underlay Cellular Networks

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Abstract

Underlay in-band device-to-device (D2D) multicast communication, where the same content is disseminated via direct links in a group, has the potential to improve the spectral and energy efficiencies of cellular networks. However, most of the existing approaches for this problem only address either spectral efficiency (SE) or energy efficiency (EE). We study the trade-off between SE and EE in a single cell D2D integrated cellular network, where multiple D2D multicast groups (MGs) may share the uplink channel with multiple cellular users (CUs). We explore SE-EE trade-off for this problem by formulating the EE maximization problem with constraint on SE and maximum available transmission power. A power allocation algorithm is proposed to solve this problem and its efficacy is demonstrated via extensive numerical simulations. The trade-off between SE and EE as a function of density of D2D MGs, and maximum transmission power of a MG is characterized.

Index Terms

Multicasting, Device-to-Device communication, energy-and-spectral efficiency trade-off, LTE-A networks.

I. INTRODUCTION

Supporting ever increasing number of mobile users with data-hungry applications, running on battery-limited devices, is posing a daunting challenge to telecommunications community. Underlay device-to-device (D2D) communication, which allows physically proximate mobile users to directly communicate with each other by reusing the spectrum and without going through the base station, holds promise to help us tackle this challenge [1]. In a cellular network, underlay D2D communication offers opportunities for spectrum reuse and spatial diversity which may lead to enhanced coverage, higher throughput, and robust communication in the network [2]. Further, for applications such as weather forecasting, live streaming, or file distribution, which may require the same chunk of data distributed to geographically proximate group of users, D2D multicasting may provide better utilization of network resources compared to D2D unicast or Base Station (BS) based multicast. However, extensive deployment of underlay D2D multicast in a network may cause severe co-channel interference due to spectrum reuse and rapid battery depletion of the multicasting D2D nodes due to higher transmit power to mitigate co- channel interference and data-forwarding.

In cellular networks, two metrics namely spectrum efficiency (SE) and energy efficiency (EE) characterize how efficiently the spectrum and energy resources, respectively, are used. However, often conflicting nature of these metrics may not allow for simultaneous maximization of both in a network. There exists an extensive body of work that explores SE-EE trade-off in cellular networks [3], [4] and some work that explore the nature of this trade-off for D2D communication [5], [6]. However, currently no systematic study of the nature of such trade-off for multiple D2D multicasts in underlay cellular networks exists. To the best of our knowledge, this letter provides the first such study.

Using stochastic geometry, we formulate a resource allocation problem that maximizes the EE of multiple D2D-multicasts in underlay cellular networks with constraints on SE and maximum transmission power. The formulated problem is non-convex, and is solved using the proposed heuristic gradient power allocation algorithm. We establish the trade-off between EE and SE with various network parameters such

as density of D2D multicast transmitters, and maximum transmission power of MGs through numerical simulations.

Organization: Section II introduces the system model. Section III and IV introduce the problem formulation and the optimal power allocation algorithms, respectively. Performance evaluation is in Section V. Finally, Section VI concludes the paper.

II. SYSTEM MODEL

A D2D-integrated cellular network is considered, where multiple D2D-multicast groups (MGs) may share the uplink channel with multiple cellular users (CUs). Let $\mathcal{K}=\{1,2,\ldots,K\}$ denote the total number of orthogonal channels that can be shared by CUs and D2D MGs. Sharing of uplink channels is assumed instead of downlink because of asymmetric uplink and downlink traffic loads [7], and also as the eNB (evolved Node Base station) can handle interference effectively. The spatial distribution of CUs and MGs on the k^{th} channel is modeled as homogeneous Poisson Point Process $\Pi_{c,k}$ with density λ_c^k , and $\Pi_{g,k}$ with density λ_g^k , respectively, in the Euclidean plane \mathbb{R}^2 . The proposed system model is an abstraction of a system where a single cell is divided into small cells, and multiple CUs may share a single channel.

Let $|\mathcal{U}_g|$ be the number of receivers in the g^{th} MG ($|\mathcal{U}_g|=1$ corresponds to unicast communication.) We consider the variable number of receivers in every MG and assume that they always have data demands. The transmission powers of CU and D2D MG transmitter (D2D-Tx) on the k^{th} channel are denoted by $p_{c,k}$ and $p_{g,k}$, respectively. In addition, total transmission power of CUs and D2D MGs is P_C and P_G , i.e.

$$\sum_{k=1}^{|\mathcal{K}|} p_{c,k} = P_C, \qquad (1a), \qquad \sum_{k=1}^{|\mathcal{K}|} p_{g,k} \le P_G \qquad (1b)$$

For analysis, a reference receiver at the origin of cell (eNB for cellular and a typical D2D receiver for D2D communication) is considered. The radio propagation channel gain between the i^{th} transmitter and the j^{th} receiver, denoted as $h_{i,j}$, is assumed to be Rayleigh faded, and independently and identically exponentially distributed with unit mean, i.e. $h_{i,j} \sim \exp(1)$. Therefore, the received power at the j^{th} receiver is $p_j = p_i h_{i,j} d_{i,j}^{-\alpha}$, whereas $d_{i,j}$ denotes the distance between the i^{th} and the j^{th} node, and α is the path-loss exponent.

As we are considering the scenarios where a channel is shared by multiple CUs and multiple MGs, thus, the r^{th} ($r \in \mathcal{U}_g$) D2D-Rx experiences interference from co-channel CUs and other D2D MG-Tx. Therefore, the signal-to-interference-plus-noise-ratio (SINR) at the r^{th} ($r \in \mathcal{U}_g$) D2D receiver on the k^{th} channel is

$$\gamma_{g,r}^{k} = \frac{p_{g,k} h_{g,r,k} d_{g,r}^{-\alpha}}{\sum_{c \in \Pi_{c,k}} p_{c,k} h_{c,r,k} d_{c,r}^{-\alpha} + \sum_{g' \in \Pi_{g,k}} p_{g',k} h_{g',r,k} d_{g',r}^{-\alpha} + N_0},$$
(2)

As the system model is interference limited, therefore, thermal noise N_0 can be omitted, and hence (2) becomes

$$\gamma_{g,r}^{k} = \frac{h_{g,r,k} d_{g,r}^{-\alpha}}{I_{c,r,k} + I_{g',g,k}},\tag{3}$$

where $I_{c,r,k} = \sum_{c \in \Pi_{c,k}} \frac{p_{c,k}}{p_{g,k}} h_{c,r,k} d_{j,r,k}^{-\alpha}$ and $I_{g',g,k} = \sum_{g' \in \Pi_{g,k}} \frac{p_{g,k}}{p_{g',k}} h_{g',r,k} d_{g',r,k}^{-\alpha}$. In a MG, transmission rate is decided by channel conditions of the worst user [8], so, SIR, and the corresponding date rate, respectively, are

$$\gamma_g^k = \min_{r \in \mathcal{U}_g} \left(\frac{h_{g,r,k} d_{g,r}^{-\alpha}}{I_{c,r,k} + I_{g',g,k}} \right), \quad R_g^k = \log_2 \left(1 + \gamma_g^k \right)$$
 (4)

An outage event for a MG g occurs if the aggregate rate of the g^{th} group falls below its target rate, R_g^{th} . Therefore, the outage probability of the g^{th} MG is given by the following lemma.

Lemma 1: The outage probability of a D2D MG communicating on the k^{th} shared channel is

$$\Pr\left(R_g^k < R_g^{\text{th}}\right) = 1 - \exp\left\{-\chi_{g,k} \left[\lambda_c^k \left(\frac{p_{c,k}}{p_{g,k}}\right)^{\frac{2}{\alpha}} + \lambda_g^k\right]\right\},\tag{5}$$

where $\chi_{g,k}=\pi d_{g,r}^2\Gamma\left(1+\frac{2}{\alpha}\right)\Gamma\left(1-\frac{2}{\alpha}\right)\left(2^{\left(R_g^{\text{th}}/|\mathcal{U}_g|\right)}-1\right)^{2/\alpha}$, $\Gamma(x)=\int_0^\infty t^{x-1}e^{-t}dt$ denotes complete gamma function.

Proof: Please refer to Appendix A

Similarly, the SIR of a CU transmitting on the k^{th} channel is

$$\gamma_c^k = \frac{p_{c,k} h_{c,b} d_{c,b}^{-\alpha}}{\sum_{c \in \Pi_{c,k}} p_{c,k} h_{c',b,k} d_{c',b}^{-\alpha} + \sum_{g \in \Pi_{g,k}} p_{g,k} h_{g,b,k} d_{g,b}^{-\alpha}}$$
(6)

with the corresponding outage probability given by the following lemma.

Lemma 2: The outage probability of a CU on the k^{th} channel, can be expressed as

$$\Pr\left(R_c^k < R_c^{\text{th}}\right) = 1 - \exp\left\{-\chi_{c,k} \left[\lambda_c^k + \lambda_g^k \left(\frac{p_{g,k}}{p_{c,k}}\right)^{\frac{2}{\alpha}}\right]\right\}$$
(7)

where R_c^{th} denotes the date rate threshold of CU and $\chi_c^k = \pi d_{c,b}^2 \Gamma\left(1 + \frac{2}{\alpha}\right) \Gamma\left(1 - \frac{2}{\alpha}\right) \left(2^{\left(R_c^{\text{th}}\right)} - 1\right)^{2/\alpha}$.

Proof: Similar to the proof of Lemma 1.

These lemmas allow us to infer that

- The outage probability of D2D MGs increases with increase in MG geographical spread, $d_{g,r}$, because channel fading becomes too severe with increasing distance.
- The outage probability of D2D MGs decreases with decrease in densities of CUs and D2D MGs, this is due to mitigation of co-channel interference.
- The outage of D2D MGs increases with increase in $p_{c,k}$, because it creates more interference to D2D transmission. While, higher $p_{c,k}$ decrease the outage of CUs.

III. OPTIMAL RESOURCE ALLOCATION FOR D2D- MULTICASTS IN UNDERLAY CELLULAR COMMUNICATION

The average throughput, SE_g^k , of D2D mulicast communication on the k^{th} channel, can be expressed as [10]:

$$SE_g^k = |u_g| \lambda_g^k \log_2 \left(1 + \gamma_g^k \right) \Pr \left(R_g^k \ge R_g^{th} \right)$$
(8)

The EE is defined as the ratio of average throughput to the power consumption [5]. As in [10], the power consumption of D2D MGs which are communicating on the k^{th} channel is $\lambda_g^k p_{g,k}$. Therefore, the total energy efficiency of D2D multicast underlay network is

$$EE_g = \sum_{k=1}^{|\mathcal{K}|} EE_g^k = \sum_{k=1}^{|\mathcal{K}|} SE_g^k / (\lambda_g^k p_{g,k})$$

$$(9)$$

To ensure the high data rate to both CUs and D2D users, thresholds on outage probabilities of both the D2D users and CUs are put as follows:

$$\Pr\left(R_c^k < R_c^{\text{th}}\right) \le \Theta_c, \quad \text{and} \quad \Pr\left(R_q^k < R_q^{\text{th}}\right) \le \Theta_g,$$
 (10)

where Θ_c and Θ_g denote the outage thresholds for cellular and D2D MGs transmissions, respectively. The transmit power in the k^{th} channel should be less than the allowed upper bound for that channel, denoted as $p_{a,k}^{\text{up}}$. Thus, we have

$$0 \le p_{g,k} \le p_{g,k}^{\text{up}} \tag{11}$$

From (5), (7), and (10), feasible power region is

$$p_{g,k} \ge p_{c,k} \left(\frac{-\ln\left(1 - \Theta_g\right)}{\lambda_c^k \chi_{g,k}} - \frac{\lambda_g^k}{\lambda_c^k} \right)^{-\frac{\alpha}{2}}, \tag{12}$$

$$p_{g,k} \le p_{c,k} \left(\frac{-\ln\left(1 - \Theta_c\right)}{\lambda_q^k \chi_{c,k}} - \frac{\lambda_c^k}{\lambda_q^k} \right)^{\frac{\alpha}{2}}$$

$$\tag{13}$$

Let $p_{g,k}^{\text{low}} = p_{c,k} \left(\frac{-\ln(1-\Theta_g)}{\lambda_c^k \chi_{g,k}} - \frac{\lambda_g^k}{\lambda_c^k} \right)^{-\frac{\alpha}{2}}$ and $p_{g,k}^{\text{high}} = p_{c,k} \left(\frac{-\ln(1-\Theta_c)}{\lambda_g^k \chi_{c,k}} - \frac{\lambda_c^k}{\lambda_g^k} \right)^{\frac{\alpha}{2}}$. Therefore, infimum $p_{g,k}^{\text{inf}} = \max\{0, p_{g,k}^{\text{low}}\}$ and supremum $p_{g,k}^{\text{sup}} = \min\{p_{g,k}^{\text{up}}, p_{g,k}^{\text{high}}\}$ denote the upper and lower bound, respectively. With these tranformations, the EE optimization problem with constraints on SE and maximum transmission power is formulated as:

$$\mathbf{P}: \max_{p_{g,k}} EE_g = \sum_{k=1}^{|\mathcal{K}|} EE_g^k$$
s.t. $p_{g,k}^{\inf} \le p_{g,k} \le p_{g,k}^{\sup}$, and $\sum_{k=1}^{|\mathcal{K}|} p_{g,k} \le P_G$ (14)

The problem **P** is non-convex as we prove in Appendix B. From second constraint, one may infer that, as the CUs are primary users, so to maintain their priority over MG users, they are assumed to transmit at full power, i.e. P_c . While D2D MG users are secondary users, and interference creators for cellular transmissions. Thus, they are assumed to transmit at lower powers, so that their sum power does not overshoot the threshold P_G .

IV. POWER ALLOCATION FOR OPTIMAL RESOURCE ALLOCATION

The objective function in optimization problem \mathbf{P} is a summation of $|\mathcal{K}|$ functions. Let $p_{g,k}^*$ denote the global maximal point where single function EE_g^k achieves its maximum, and if the sum power constraint (1b) is removed, then $p_{g,k}$ are mutually independent. The EE_g achieves the maximum value when every EE_g^k achieves its maximum. Intuitively, finding power that maximizes the EE_g^k , is easier than solving **P** as a whole. The power $p_{g,k}^*$ that maximizes the individual EE_g^k can be found using the following lemma.

Lemma 3: The value $p_{g,k}^*$ that maximizes the EE_g^k is

$$p_{g,k}^* = \begin{cases} p_{g,k}^{\sup}, & p_{g,k}^{\sup} \le \left(\frac{2Z_k^g}{\alpha}\right)^{\alpha/2} \\ \left(\frac{2Z_k^g}{\alpha}\right)^{\alpha/2}, & p_{g,k}^{\inf} \le \left(\frac{2Z_k^g}{\alpha}\right)^{\alpha/2} \le p_{g,k}^{\sup} \\ p_{g,k}^{\inf}, & p_{g,k}^{\inf} \ge \left(\frac{2Z_k^g}{\alpha}\right)^{\alpha/2} \end{cases}$$

where $Z_{k}^{g}=\chi_{g}^{k}\lambda_{c}^{k}\left(p_{c,k}\right)^{\frac{2}{\alpha}}$ Proof: Please refer to Appendix C

Now, two cases arise, Case 1: when $\sum_{k=1}^{|\mathcal{K}|} p_{g,k}^* \leq P_G$ Then, the optimum power that is allocated is $p_{g,k} = p_{g,k}^*, \forall k = 1, \dots, K$, and the maximum value of

energy efficiency is $EE_g = \sum_{k=1}^{|\mathcal{K}|} EE_g^k(p_{g,k}^*)$. Case 2: when $\sum_{k=1}^{|\mathcal{K}|} p_{g,k}^* > P_G$. Then, set $p_{g,k} = p_{g,k}^*, \forall k = 1, \ldots, K$, and update the value of $p_{g,k}$, such that power constraint (1b) is maintained, while causing the least reduction in EE_g . Let $p_{g,k}^{inst}$ be the instant value, having initial value of $p_{g,k}^{inst} = p_{g,k}^*$. Let d denotes the difference between maximum available power and sum of assigned power. δ and n denote the step size and number of steps, respectively, with $\delta = d/n$. Parameter n controls the balance between computational effort and the performance, its value is assigned in accordance with convergence rate the error-tolerance requirements. As we are adjusting the power value which gives maximum value of the function, therefore, the function value decreases with

reducing power, i.e, $EE_g^k\left(p_{g,k}^{inst}-\delta\right) < EE_g^k\left(p_{g,k}^{inst}\right)$. To satisfy the equality constraint (1b) while having the least reduction in EE_g , we need to adjust that $p_{g,j}, (j \in \mathcal{K})$ for which EE_g^j decreases the least after decreasing the instantaneous transmit power.

$$j = \underset{|\mathcal{K}|}{\operatorname{argmin}} |EE_g^k \left(p_{g,k}^{inst} \right) - EE_g^k \left(p_{g,k}^{inst} - \delta \right) |$$

$$p_{g,j}^{inst} = p_{g,j}^{inst} - \delta$$

$$(15)$$

$$p_{q,j}^{inst} = p_{q,j}^{inst} - \delta \tag{16}$$

Iterating this process at least n times, leads to the sum power constraint (1b) be met, and a near-to-optimal solution to (14) is achieved. The formal description is given in Algorithm 1. The computation complexity of the proposed algorithm is $\mathcal{O}(n)$.

Algorithm 1: The proposed power allocation algorithm

```
Input: K, n, P_G, P_C, \epsilon, Output: EE_q
  1 begin
                Find p_{g,k}^* from Lemma 3, and assign p_{g,k}=p_{g,k}^* d =P_G-\sum_{k=1}^K p_{g,k} if d \leq 0 then
  2
  3
                         \overline{EE}_g = \sum_{k=1}^K EE_g^k \left( p_{q,k}^* \right)
  5
  6
  7
                          \begin{aligned} & \textbf{while} \ |P_G - \sum_{k=1}^K p_{g,k}| \geq \epsilon \ \textbf{do} \\ & | \ \ j = \underset{left}{\operatorname{argmin}} |EE_g^k \left( p_{g,k}^{inst} \right) - EE_g^k \left( p_{g,k}^{inst} - \delta \right) | \end{aligned}
  8
                               \begin{array}{l} \text{if } p_{g,j} + \delta > p_{g,j}^{\sup} \quad or \quad p_{g,j} - \delta < p_{g,j}^{\inf} \text{ then} \\ \mid \quad |\mathcal{K}| = |\mathcal{K}| \cap j, \end{array}
10
11
12
                                      p_{g,j} = p_{g,j} - \delta, and \operatorname{der}_j = |EE_g^{j'}(p_{g,j})|
13
                return EE_g = \sum_{k=1}^K EE_g^k\left(p_{g,k}\right)
14
```

V. SIMULATION RESULTS

To explore the existence of trade-off between SE and EE for multiple D2D multicasts in underlay cellular networks and exploit it to design optimal resource allocation schemes, we carried out extensive numerical simulations. Some of the simulation parameters are as follows. The CU density $\lambda_c^k =$ $[1e^{-4}, 1e^{-5}, 1e^{-4}, 1e^{-4}, 1e^{-4}]$ and D2D density $\lambda_g^k = [1e^{-3}, 1e^{-4}, 1e^{-3}, 1e^{-3}, 1e^{-3}]$ are considered, respectively. The CU density and D2D density are randomly chosen values. We used the following parameter values: $\alpha = 3$, $|u_g| = 3$, $p_{g,k}^{\text{up}} = 15 \text{dBm}$, $P_G = 25 \text{dBm}$, $P_{c,k} = 26 \text{dBm}$, $\Theta_c = 0.1$, and $\Theta_g = 0.1$. The value of alpha has an impact on EE and SE, however, that is insignificant.

Fig. 1 depicts the behavior of EE with respect to required SE and the available D2D transmission power. It can be observed that for a given power level, with increase in SE requirement (1 to 10 bps/Hz), EE first increases then starts to decrease. This is because, for lower SE requirements, eNB tries to support many MGs per channel until outage thresholds are not violated. This leads to increase in sum rate and consequently, an increase in EE. While, for higher requirement of SE, eNB reduces the number of MG transmitters sharing a channel until outage probability constrains are not fulfilled. Therefore, sum rate decreases, consequently EE decreases. Similarly, increasing the transmission power of MG transmitter for fixed SE, higher data rate may be supported for small range D2D communication, therefore, the sum rate increases initially with power consumption. However, after some power threshold, MG transmitters start

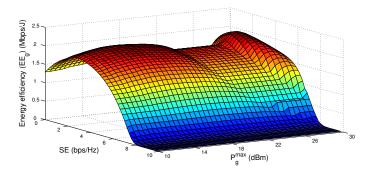


Fig. 1. EE as a function of SE and D2D users transmitter power

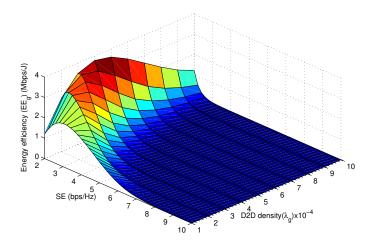


Fig. 2. EE as a function of SE and D2D users density

causing co-channel interference to CUs, and thus, CU outage probability starts increasing. To compensate this, eNB reduces the number of MGs to fulfil CU outage probability thresholds. Therefore, decrease in sum rate and consequently decrease in EE occurs. Fig. 2 depicts EE as a function of SE and D2D density. It can be observed that, for a given SE requirement, with increasing D2D density, EE first increases and then decreases. This is because, adding D2D users to the network (i.e increasing λ_g), results in an exponential increase in the average sum rate, and consequent increase in EE. However, in high density, mutual interference starts increasing, and that limits the average sum rate per channel, leading to a decrease in EE.

VI. CONCLUSION

For underlay D2D multicast in cellular networks, we addressed the energy efficiency and spectral efficiency trade-off. We assumed that multiple D2D-multicast group may share the channel with multiple CUs. Exact expression of average sum-rate and its relation with energy consumption is derived by utilizing the stochastic geometry. An energy optimization problem is formulated, having constraints on maximum power, and outage data rate. Our results showed that EE has different behavior with available power and spectral requirement. With increasing power, SE improves while EE initially increases then decreases. Similarly, with increase in D2D MG density, EE initially increases and then decreases. Indeed, for the EE, there is an optimal value of SE requirement that can be supported, which results in the maximal value of EE for each value of transmitter power of MG.

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APPENDIX A PROOF OF LEMMA 1

$$\begin{split} &\Pr\left(R_g^k < R_g^{\text{th}}\right) = 1 - \Pr\left(R_g^k \ge R_g^{\text{th}}\right) \forall g \in \lambda_g^k \\ &= 1 - \Pr\left(\gamma_g^k \ge 2^{\frac{R_g^{\text{th}}}{|\mathcal{U}g|}} - 1\right) = 1 - \Pr\left(\frac{h_{g,r,k}d_{g,r}^{-\alpha}}{I_{c,r,k} + I_{g',g,k}} \ge 2^{\frac{R_g^{\text{th}}}{|\mathcal{U}g|}} - 1\right) \\ &= 1 - \Pr\left[h_{g,r,k} \ge 2^{\frac{R_g^{\text{th}}}{|\mathcal{U}g|-1}} d_{g,r}^{\alpha} \left(I_{c,r,k} + I_{g',g,k}\right)\right] \\ &= 1 - \left\{E\left(\prod_{j \in \Pi_{c,k}} \exp\left(-\left(2^{\frac{R_g^{\text{th}}}{|\mathcal{U}g|}} - 1\right) d_{g,r}^{\alpha} \frac{p_{c,k}}{p_{g,k}} h_{c,r,k} d_{c,g}^{-\alpha}\right)\right) \\ &\times E\left(\prod_{g \in \Pi_{g,k}} \exp\left(-\left(2^{\frac{R_g^{\text{th}}}{|\mathcal{U}g|}} - 1\right) d_{g,r}^{\alpha} \frac{p_{g,k}}{p_{g',k}} h_{g',g,k} d_{g',g}^{-\alpha}\right)\right)\right\} \\ &= 1 - \mathcal{L}_{I_{c,r,k}\left(h_{c,r,k}\right)} \left(T_{g,k} d_{g,r}^{\alpha}\right) \mathcal{L}_{I_{g',g,k}\left(h_{g',g,k}\right)} \left(T_{g,k} d_{g,r}^{\alpha}\right), \end{split}$$

where $T_{g,k}=\left(2^{rac{R_g^{ ext{th}}}{|\mathcal{U}_g|}}-1
ight)$. According to definition of Laplace transform, we have

$$\begin{split} \mathcal{L}_{I_{c,r,k}\left(h_{c,r,k}\right)}\left(T_{g,k}d_{g,r}^{\alpha}\right) &= \exp\left[-\lambda_{g}^{k}\left(\frac{p_{c,k}}{p_{g,k}}\right)^{\frac{2}{\alpha}}\pi T_{g,k}^{\frac{2}{\alpha}}d_{g,r}^{2}\Gamma\left(1+\frac{2}{\alpha}\right)\Gamma\left(1-\frac{2}{\alpha}\right)\right] \\ \mathcal{L}_{I_{g',g,k}}\left(h_{g',g,k}\right)\left(T_{g,k}d_{g,r}^{\alpha}\right) &= \exp\left[-\lambda_{g}^{k}\int_{0}^{\infty}E\left(h_{g',g,k}\right)\left(1-e^{-T_{g,k}d_{g,r}^{\alpha}r^{-\alpha}}\right)dr\right] \\ &= \exp\left[-\lambda_{g}^{k}\left(\frac{p_{g,k}}{p_{g',k}}\right)^{\frac{2}{\alpha}}\pi T_{g,k}^{\frac{2}{\alpha}}d_{g,r}^{2}\Gamma\left(1+\frac{2}{\alpha}\right)\Gamma\left(1-\frac{2}{\alpha}\right)\right] \end{split}$$

APPENDIX B

PROOF THAT THE FORMULATED PROBLEM P IS NON-CONVEX

Let
$$Y_k^g = |u_g|\log_2\left(1 + \gamma_g^k\right) \exp\left(-\chi_g^k \lambda_g^k\right)$$
 and $Z_k^g = \chi_g^k \lambda_c^k \left(p_{c,k}\right)^{\frac{2}{\alpha}}$, then we can write EE_g^k as
$$EE_g^k \triangleq \left(Y_k^g/p_{g,k}\right) \exp\left(-Z_k^g \left(\frac{1}{p_{g,k}}\right)^{\frac{2}{\alpha}}\right)$$

Taking double derivative of EE_q^k with respect to $p_{g,k}$.

$$\frac{d^2\left(EE_g^k\right)}{dp_{g,k}^2} = 2Y_k^g \exp\left(-Z_k^g \left(\frac{1}{p_{g,k}}\right)\right)^{2/\alpha} \left(\frac{1}{p_{g,k}}\right)^{3+4/\alpha}$$
$$\times \left(p_{g,k}^{4/\alpha} - \frac{Z_k^g}{\alpha} \left(\frac{2}{\alpha} + 3\right) p_{g,k}^{2/\alpha} + \frac{2(Z_k^g)^2}{\alpha^2}\right)$$

In this equation, the 1st and 2nd terms are greater than zero, therefore, we only focus on the last term. Let $\nu=p_{g,k}^{2/\alpha}$, then the last term can be written as

$$\mathcal{V} = \nu^2 - \frac{Z_k^g}{\alpha} \left(\frac{2}{\alpha} + 3 \right) \nu + \frac{2(Z_k^g)^2}{\alpha^2}$$

The solutions $\nu_{1,k}$ and $\nu_{2,k}$ can be found as

$$\nu_{i,k} = (Z_k^g/2\alpha^2) \left(2 + 3\alpha \mp \sqrt{\alpha^2 + 12\alpha + 4}\right), i \in \{1, 2\}$$

 $\mathcal V$ is positive on the interval $(0,\nu_{1,k})\cup(\nu_2,+\infty)$, and negative in interval $(\nu_{1,k},\nu_{2,k})$. Therefore, $d^2\big(EE_g^k\big)/dp_{g,k}^2$ is positive in interval $\Big(0,\nu_{1,k}^{\alpha/2}\Big)\cup\Big(\nu_2^{\alpha/2},+\infty\Big)$, and negative in interval $\Big(\nu_{1,k}^{\alpha/2},\nu_{2,k}^{\alpha/2}\Big)$. Therefore, EE_g^k is convex on interval $\Big(0,\nu_{1,k}^{\frac{\alpha}{2}}\Big)\cup\Big(\nu_{2,k}^{\frac{\alpha}{2}},+\infty\Big)$, but concave on the interval $\Big(\nu_{1,k}^{\alpha},\nu_{2,k}^{\alpha/2}\Big)$

APPENDIX C PROOF OF LEMMA 3

Take the derivative of EE_q^k with respect to $p_{g,k}$,

$$\frac{d\left(EE_g^k\right)}{dp_{g,k}} = \frac{Y_k^g}{p_{g,k}^2} \exp\left(-Z_k^g \left(\frac{1}{p_{g,k}^{2/\alpha}}\right)\right) \times \left(\frac{2Z_k^g}{\alpha} \left(\frac{1}{p_{g,k}^{2/\alpha}}\right) - 1\right)$$

From this equation, it can be observed that derivative of EE_g^k is positive if $p_{g,k}$ lies in interval $\left(0, \left(\frac{2Z_k^g}{\alpha}\right)^{\alpha/2}\right)$, and reaches the global maxima at $p_{g,k} = \left(\frac{2Z_k^g}{\alpha}\right)^{\alpha/2}$. Therefore, three feasible region exist, EE_g^k is increasing monotonically in $\left(0, \left(\frac{2Z_k^g}{\alpha}\right)^{\alpha/2}\right)$, decreasing monotonically in $\left(\left(\frac{2Z_k^g}{\alpha}\right)^{\alpha/2}\right)$, and having maximal point at $p_{g,k} = \left(\frac{2Z_k^g}{\alpha}\right)^{\alpha/2}$. If $p_{g,k}^{\sup} \leq \left(\frac{2Z_k^g}{\alpha}\right)^{\alpha/2}$, then EE_g^k is an increasing function, and, it reaches the maximum value when $p_{g,k}^* = p_{g,k}^{\sup}$. Second region is $p_{g,k}^{\inf} \leq \left(\frac{2Z_k^g}{\alpha}\right)^{\alpha/2} \leq p_{g,k}^{\sup}$, then the maximal point of EE_g^k is within feasible region. Therefore, optimal point is $p_{g,k}^* = \left(\frac{2Z_k^g}{\alpha}\right)^{\alpha/2}$. Third region is, if $p_{g,k}^{\inf} \geq \left(\frac{2Z_k^g}{\alpha}\right)^{\alpha/2}$, then EE_g^k decreases monotonically in the feasible region. Hence, the maximum value of $EE_{g,k}$ is achieved at $p_{g,k}^* = p_{g,k}^{\inf}$