

Repositório ISCTE-IUL

Deposited in *Repositório ISCTE-IUL*:

2020-12-14

Deposited version:

Accepted Version

Peer-review status of attached file:

Peer-reviewed

Citation for published item:

Souto, N. & Correia, A. (2020). Frequency domain equalization for single and multiuser generalized spatial modulation systems in time dispersive channels. *IEEE Wireless Communications Letters*. 9 (3), 316-320

Further information on publisher's website:

10.1109/LWC.2019.2953270

Publisher's copyright statement:

This is the peer reviewed version of the following article: Souto, N. & Correia, A. (2020). Frequency domain equalization for single and multiuser generalized spatial modulation systems in time dispersive channels. *IEEE Wireless Communications Letters*. 9 (3), 316-320, which has been published in final form at <https://dx.doi.org/10.1109/LWC.2019.2953270>. This article may be used for non-commercial purposes in accordance with the Publisher's Terms and Conditions for self-archiving.

Use policy

Creative Commons CC BY 4.0

The full-text may be used and/or reproduced, and given to third parties in any format or medium, without prior permission or charge, for personal research or study, educational, or not-for-profit purposes provided that:

- a full bibliographic reference is made to the original source
- a link is made to the metadata record in the Repository
- the full-text is not changed in any way

The full-text must not be sold in any format or medium without the formal permission of the copyright holders.

Frequency Domain Equalization for Single and Multiuser Generalized Spatial Modulation Systems in Time Dispersive Channels

Nuno Souto, *Senior Member, IEEE*, and Américo Correia, *Senior Member, IEEE*

Abstract—In this letter, a low-complexity iterative detector with frequency domain equalization is proposed for generalized spatial modulation (GSM) aided single carrier (SC) transmissions operating in frequency selective channels. The detector comprises three main separate tasks namely, multiple-input multiple-output (MIMO) equalization, active antenna detection per user and symbol wise demodulation. This approach makes the detector suitable for a broad range of MIMO configurations, which includes single-user and multiuser scenarios, as well as arbitrary signal constellations. Simulation results show that the receiver can cope with the intersymbol interference induced by severe time dispersive channels and operate in difficult underdetermined scenarios.

Index Terms—Generalized spatial modulations (GSM), large scale MIMO (LS-MIMO), Single carrier with frequency domain equalization (SC-FDE).

N. Souto and A. Correia are with the ISCTE-University Institute of Lisbon and Instituto de Telecomunicações, 1649-026 Lisboa, Portugal (e-mail: nuno.souto@lx.it.pt, américo.correia@iscte-iul.pt).

This work was funded by FCT/MEC through national funds and co-funded by FEDER – PT2020 partnership agreement under the project UID/EEA/50008/2019 and grant SFRH/BSAB/142993/2018.

I. INTRODUCTION

Generalized spatial modulation (GSM) [1]-[2] is a multiple-input multiple-output (MIMO) scheme that offers a tradeoff between the high spectral efficiency (SE) of full spatial multiplexing MIMO and the low complexity of the single radio frequency (RF) transmitter chain of spatial modulations (SMs) [3]. GSM relies on the use of multiple RF chains in order to support transmission over multiple active antenna elements (AEs). The information is mapped onto a transmit antenna combination (TAC) and on the modulated symbols, thus increasing the SE. Due to the transmission of multiple streams, GSM detection is more complex than SM. While the same symbols can be transmitted on all active AEs [1], in this letter we are concerned with the higher SE approach where a different symbol is sent on each active AE [2].

A lot of research efforts have focused on SM and GSM schemes operating in flat fading channels [1]-[6]. However, in broadband systems the channel is often severely time dispersive and leads to high intersymbol interference (ISI) levels. Although orthogonal frequency division multiplexing (OFDM) is very popular for frequency selective environments, the combination with SM sacrifices most of its benefits [7]. A better suited alternative is single carrier (SC) transmission which can potentially avoid SM-OFDM limitations while also providing higher frequency diversity. Motivated by this, the combination of SC with SM [7]-[9] and GSM [10]-[14] recently started to attract substantial research efforts. Regarding GSM-SC, which is the focus of this letter, a compressed-sensing (CS) based approach was studied in [10], but since it is applied in the time domain it can result in high complexity. A low complexity detection scheme was proposed in [11] which can achieve good performances in several scenarios. It was designed for zero-padded SC (ZP-SC) systems and has a complexity that grows directly with the GSM constellation size, making its application in large-scale systems difficult. In [12], several tree search algorithms for ZP-SC were proposed and evaluated. They can also achieve good performances but rely on the search over the whole GSM set making them impractical for large-scale systems. In [13] several time-domain turbo equalization detectors were proposed but were designed specifically for application to ZP-SC systems. Regarding cyclic-prefixed (CP) aided SC transmissions, most of the research has been restricted to SM only [7][9], where the special structure of CP-

SC is exploited in order to implement part of the processing in the frequency domain. In [14], message passing (MP) based algorithms were proposed for multiuser (MU) MIMO with GSM. It was shown that it could be extended to CP-SC but the complexity can become very high.

Compared to single-user (SU) scenarios, receivers for MU GSM systems face the major challenge of having to detect several GSM symbols simultaneously while still maintaining a reduced complexity. This problem is aggravated by the fact that when considering all the users, the large number of total transmit antennas present can often make the system large-scale underdetermined. The challenge becomes even greater when the receiver has to mitigate the high ISI resulting from the operation of SC systems in multipath channels. Against this background, the main contributions of this letter are summarized as follows:

- We develop an iterative detector for CP-aided GSM-SC systems which separates the tasks of MIMO equalization, active AE detection per user and symbol wise demodulation. This splitting is accomplished through the alternating direction method of the multipliers (ADMM), which was previously applied in a simpler form in [6], within the context of SU GSM transmissions in flat fading channels. In this letter the detector is designed in order to cope with the more challenging MU scenarios and ISI inducing frequency selective channels.
- Through proper formulation of the MU GSM-SC detection problem, which uses a mixture of frequency domain and time domain variables, MIMO equalization is accomplished in the frequency domain while the GSM symbol decision is kept on the time domain and performed independently for each user. This allows the complexity to remain low even in large scale systems and highly dispersive channels.
- Numerical results show that the receiver is suitable for arbitrary SU and MU MIMO configurations with arbitrary signal modulations, it can cope with high ISI inducing channels and operate in difficult underdetermined scenarios.

Notation: $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and conjugate transpose of a matrix/vector, \otimes symbolizes the Kronecker product, $\|\cdot\|_2$ is the 2-norm of a vector, $\text{supp}(\cdot)$ returns the set of indices of nonzero elements in \mathbf{x}

(i.e., the support of \mathbf{x}), $\text{diag}(\cdot)$ represents a diagonal matrix, $\lfloor \cdot \rfloor$ is the floor function, $\binom{N}{k}$ denotes the number of combinations of N symbols taken k at a time, \mathbf{I}_n is the $n \times n$ identity matrix and $I_{\mathcal{D}}(\mathbf{v})$ is the indicator function which returns 0 if $\mathbf{v} \in \mathcal{D}$ and $+\infty$ otherwise.

II. SYSTEM MODEL AND PROBLEM STATEMENT

Let us consider a SC system where a base station with N_{rx} receiver antennas serves N_u users. Each user is equipped with N_{tx} transmitter antennas with only N_a active AEs at any given time. This allows a total of $N_{comb} = 2^{\lfloor \log_2 \binom{N_{tx}}{N_a} \rfloor}$ TACs available per user. Every active AE transmits a different M -QAM modulated symbol resulting in a total of $\lfloor \log_2 \binom{N_{tx}}{N_a} \rfloor + N_a \log_2 M$ bits mapped to each GSM symbol. A frequency selective channel with L resolvable paths is assumed for each pair of transmitter-receiver antennas. We consider that the system operates with N -sized blocks employing a CP with length N_{CP} ($N_{CP} \geq L - 1$), and that the channel is time invariant during a block. The GSM signal vector transmitted by user p ($p=0, \dots, N_u - 1$) during channel use t ($t = N_{CP}, \dots, N - 1$) can then be expressed as

$$\mathbf{s}_t^p = \left[\underbrace{\dots, 0, s_{t,0}^p, 0, \dots, 0, s_{t,N_a-1}^p, 0, \dots}_{N_{tx}} \right]^T \quad (1)$$

with $s_{t,j}^p \in \mathcal{A}$ ($j=0, \dots, N_a-1$) and \mathcal{A} denoting the M -sized complex valued constellation set. The received signal vector in the time domain can be written as

$$\mathbf{y}_t = \sum_{i=0}^{L-1} \mathbf{\Omega}^i \mathbf{s}_{t-i} + \mathbf{n}_t, \quad (2)$$

where $\mathbf{y}_t \in \mathbb{C}^{N_{rx} \times 1}$, $\mathbf{s}_t = \left[\mathbf{s}_t^{0T} \dots \mathbf{s}_t^{N_u-1T} \right]^T$ and $\mathbf{n}_t \in \mathbb{C}^{N_{rx} \times 1}$ is the vector containing independent zero-mean circularly symmetric Gaussian noise samples with covariance $2\sigma^2 \mathbf{I}_{N_{rx}}$. Matrix $\mathbf{\Omega}^i \in \mathbb{C}^{N_{rx} \times N_u N_{tx}}$ contains all the channel coefficients of the i^{th} tap and is defined as $\mathbf{\Omega}^i = \left[\mathbf{\Omega}^{i,0} \dots \mathbf{\Omega}^{i,N_u-1} \right]$, where

$$\mathbf{\Omega}^{i,p} = \begin{bmatrix} h_{1,1}^{i,p} & \cdots & h_{1,N_{rx}}^{i,p} \\ \vdots & \ddots & \vdots \\ h_{N_{rx},1}^{i,p} & \cdots & h_{N_{rx},N_{rx}}^{i,p} \end{bmatrix} \quad (3)$$

and $h_{r,u}^{i,p}$ represents the complex-valued channel gain between transmit antenna u of user p and receive antenna r .

Dropping the CP, we can concatenate the received vectors as $\mathbf{y} = [\mathbf{y}_0^T \dots \mathbf{y}_{N-1}^T]^T$ and write

$$\mathbf{y} = \mathbf{\Omega}\mathbf{s} + \mathbf{n}, \quad (4)$$

where, $\mathbf{s} = [\mathbf{s}_0^T \dots \mathbf{s}_{N-1}^T]^T$, $\mathbf{n} = [\mathbf{n}_0^T \dots \mathbf{n}_{N-1}^T]^T$ and

$$\mathbf{\Omega} = \begin{bmatrix} \mathbf{\Omega}^0 & \mathbf{0} & \cdots & \mathbf{\Omega}^{L-1} & \cdots & \mathbf{\Omega}^1 \\ \vdots & \mathbf{\Omega}^0 & \ddots & \vdots & \ddots & \vdots \\ \mathbf{\Omega}^{L-1} & \vdots & \ddots & \mathbf{0} & & \mathbf{\Omega}^{L-1} \\ \mathbf{0} & \mathbf{\Omega}^{L-1} & & \mathbf{\Omega}^0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{\Omega}^{L-1} & \cdots & \mathbf{\Omega}^0 \end{bmatrix}. \quad (5)$$

The block circulant structure of the channel matrix $\mathbf{\Omega} \in \mathbb{C}^{NN_{rx} \times NN_{rx}}$ allows it to be factorized as

$$\mathbf{\Omega} = (\mathbf{F}^H \otimes \mathbf{I}_{N_{rx}}) \mathbf{H} (\mathbf{F} \otimes \mathbf{I}_{N_{rx}}), \quad (6)$$

where \mathbf{F} represents the unitary $N \times N$ discrete Fourier transform (DFT) matrix. \mathbf{H} is a block diagonal matrix,

i.e., $\mathbf{H} = \text{diag}(\mathbf{H}_0, \dots, \mathbf{H}_{N-1})$ with $\mathbf{H}_k = [\mathbf{H}_k^0 \cdots \mathbf{H}_k^{N_u-1}]$,

$$\mathbf{H}_k^p = \begin{bmatrix} \sum_{i=0}^{L-1} h_{1,1}^{i,p} \omega^{ki} & \cdots & \sum_{i=0}^{L-1} h_{1,N_{rx}}^{i,p} \omega^{ki} \\ \vdots & \ddots & \vdots \\ \sum_{i=0}^{L-1} h_{N_{rx},1}^{i,p} \omega^{ki} & \cdots & \sum_{i=0}^{L-1} h_{N_{rx},N_{rx}}^{i,p} \omega^{ki} \end{bmatrix}, \quad k = 0, \dots, N-1, \quad (7)$$

and ω denoting a N^{th} primitive root of unity. The received block can then be expressed in the frequency domain as

$$\mathbf{Y} = (\mathbf{F} \otimes \mathbf{I}_{N_{rx}}) \mathbf{y} = \mathbf{H}\mathbf{S} + \mathbf{N}. \quad (8)$$

with $\mathbf{S} = (\mathbf{F} \otimes \mathbf{I}_{N_u N_{rx}}) \mathbf{s}$ and $\mathbf{N} = (\mathbf{F} \otimes \mathbf{I}_{N_{rx}}) \mathbf{n}$. The maximum likelihood detection (MLD) problem for the described system model can be formulated in the frequency domain as

$$\min_{\mathbf{s}} f(\mathbf{s}) = \|\mathbf{Y} - \mathbf{H}\mathbf{S}\|_2^2 \quad (9)$$

$$\text{subject to } \mathbf{S} = (\mathbf{F} \otimes \mathbf{I}_{N_u N_{rx}}) \mathbf{s} \quad (10)$$

$$\mathbf{s} \in \mathcal{A}_0^{NN_u N_{rx}} \quad (11)$$

$$\text{supp}(\mathbf{s}_t^p) \in \mathbb{S}, \quad t = 0, \dots, N-1, \quad p = 0, \dots, N_u-1, \quad (12)$$

where $\mathcal{A}_0 \stackrel{\text{def}}{=} \mathcal{A} \cup \{0\}$ and \mathbb{S} denotes the set of valid TACs. Due to constraints (11) and (12), finding the exact solution requires a computational complexity that grows exponentially with the problem size making it most often impractical.

III. FREQUENCY DOMAIN GSM DETECTOR

In this section, we apply a generalized version of ADMM [14] as a heuristic to provide good quality solutions with reduced complexity for the MLD problem. Firstly we encode constraints (11) and (12) into (9) and rewrite the problem using a mixture of frequency domain and time domain variables as

$$\min_{\mathbf{S}, \mathbf{x}, \mathbf{z}} \|\mathbf{Y} - \mathbf{H}\mathbf{S}\|_2^2 + \sum_{t=0}^{N-1} \sum_{p=0}^{N_u-1} I_{\mathbb{S}}(\mathbf{x}_t^p) + I_{\mathcal{A}_0^{NN_u N_{rx}}}(\mathbf{z}) \quad (13)$$

$$\text{subject to } \mathbf{S} = (\mathbf{F} \otimes \mathbf{I}_{N_u N_{rx}}) \mathbf{x}, \quad (14)$$

$$\mathbf{S} = (\mathbf{F} \otimes \mathbf{I}_{N_u N_{rx}}) \mathbf{z}, \quad (15)$$

where $\mathbf{x} = [\mathbf{x}_0^T \dots \mathbf{x}_{N-1}^T]^T$ with $\mathbf{x}_t = [\mathbf{x}_t^0{}^T \dots \mathbf{x}_t^{N_u-1}{}^T]^T$. The first term in (13) concerns the channel equalization, the summation in the second term refers to the active antenna detection for each time instant and individual user while the third term deals with the individual alphabet symbols (including 0) on each antenna. The three terms use **three different** variables, \mathbf{S} , \mathbf{x} and \mathbf{z} which are related through constraints (14) and (15). With these auxiliary variables, the objective function becomes separable over the three terms and

will allow us to split the main problem. The augmented Lagrangian function (ALF) can be written as

$$\begin{aligned}
L_{\mathbf{P}_x, \mathbf{P}_z}(\mathbf{S}, \mathbf{z}, \mathbf{x}, \mathbf{U}, \mathbf{W}) &= \|\mathbf{Y} - \mathbf{H}\mathbf{S}\|_2^2 + \sum_{t=0}^{N-1} \sum_{p=0}^{N_u-1} I_{\mathbb{S}}(\mathbf{x}_t^p) + I_{\mathcal{A}_0^{NN_u N_x}}(\mathbf{z}) \\
&+ \left(\mathbf{S} - (\mathbf{F} \otimes \mathbf{I}_{N_u N_x})\mathbf{x} + \mathbf{U}\right)^H \mathbf{P}_x \left(\mathbf{S} - (\mathbf{F} \otimes \mathbf{I}_{N_u N_x})\mathbf{x} + \mathbf{U}\right) \\
&+ \left(\mathbf{S} - (\mathbf{F} \otimes \mathbf{I}_{N_u N_x})\mathbf{z} + \mathbf{W}\right)^H \mathbf{P}_z \left(\mathbf{S} - (\mathbf{F} \otimes \mathbf{I}_{N_u N_x})\mathbf{z} + \mathbf{W}\right) - \mathbf{U}^H \mathbf{P}_x \mathbf{U} - \mathbf{W}^H \mathbf{P}_z \mathbf{W}. \quad (16)
\end{aligned}$$

where $\mathbf{U}, \mathbf{W} \in \mathbb{C}^{NN_u N_x \times 1}$ are the scaled dual variables associated to the equality constraints (14) and (15) while $\mathbf{P}_x = \text{diag}(\rho_0^x, \dots, \rho_{NN_u N_x - 1}^x)$ and $\mathbf{P}_z = \text{diag}(\rho_0^z, \dots, \rho_{NN_u N_x - 1}^z)$, with $\rho_i^x, \rho_i^z \in [0, +\infty[$, denote the penalty matrices. The gradient ascent method is then applied to the dual problem [15] resulting in the following sequence of iterative steps.

- *Step 1: Minimization of the ALF over \mathbf{S} .* The frequency domain estimate at iteration $q+1$ can be obtained from $\nabla_{\mathbf{S}^H} L_{\mathbf{P}_x, \mathbf{P}_z}(\mathbf{S}, \mathbf{z}, \mathbf{x}, \mathbf{U}, \mathbf{W}) = 0$ which, exploiting the block diagonal structure of \mathbf{H} , results in

$$\mathbf{S}_k^{(q+1)} = \left(\mathbf{H}_k^H \mathbf{H}_k + \mathbf{P}_{x,k} + \mathbf{P}_{z,k}\right)^{-1} \left(\mathbf{H}_k^H \mathbf{Y}_k + \mathbf{P}_{x,k} \left(\mathbf{X}_k^{(q)} - \mathbf{U}_k^{(q)}\right) + \mathbf{P}_{z,k} \left(\mathbf{Z}_k^{(q)} - \mathbf{W}_k^{(q)}\right)\right), \quad k = 0, \dots, N-1. \quad (17)$$

$\mathbf{S}_k^{(q+1)}$, \mathbf{Y}_k , $\mathbf{X}_k^{(q)}$, $\mathbf{Z}_k^{(q)}$, $\mathbf{U}_k^{(q)}$, $\mathbf{W}_k^{(q)}$, $\mathbf{P}_{x,k}$ and $\mathbf{P}_{z,k}$ represent slices of $\mathbf{S}^{(q+1)}$, \mathbf{Y} , $\mathbf{X}^{(q)}$, $\mathbf{Z}^{(q)}$, $\mathbf{U}^{(q)}$, $\mathbf{W}^{(q)}$, \mathbf{P}_x and \mathbf{P}_z matching the k^{th} frequency. \mathbf{X} and \mathbf{Z} are the frequency domain representations of \mathbf{x} and \mathbf{z} , i.e., $\mathbf{X} = (\mathbf{F} \otimes \mathbf{I}_{N_u N_x})\mathbf{x}$ and $\mathbf{Z} = (\mathbf{F} \otimes \mathbf{I}_{N_u N_x})\mathbf{z}$.

- *Step 2: Minimization of the ALF over \mathbf{x} .* Since the summation in (16) is written using the indicator function applied to each individual user and time instance this step reduces to

$$\mathbf{x}_t^{p(q+1)} = \Pi_{\mathbb{S}} \left(\mathbf{r}_t^{p(q+1)}\right), \quad t = 0, \dots, N-1, \quad p = 0, \dots, N_u-1, \quad (18)$$

where $\mathbf{r}^{(q+1)} = \left(\left(\mathbf{F}^H \otimes \mathbf{I}_{N_u N_x}\right)\left(\mathbf{S}^{(q+1)} + \mathbf{U}^{(q)}\right)\right)$ and $\Pi_{\mathbb{S}}(\cdot)$ denotes the projection onto \mathbb{S} .

- *Step 3: Minimization of the ALF over \mathbf{z} .* In this case we get

$$\mathbf{z}^{(q+1)} = \Pi_{\mathcal{A}_0^{NN_u N_x}} \left(\left(\mathbf{F}^H \otimes \mathbf{I}_{N_u N_x}\right)\left(\mathbf{S}^{(q+1)} + \mathbf{W}^{(q)}\right)\right), \quad (19)$$

where $\Pi_{\mathcal{A}_0^{NN_u N_{rx}}}$ denotes the projection onto $\mathcal{A}_0^{NN_u N_{rx}}$ which can be implemented as a simple rounding of each component to the closest element in \mathcal{A}_0 .

• *Step 4: Dual variable update.* The update of the dual variables is accomplished through

$$\mathbf{U}^{(q+1)} = \mathbf{U}^{(q)} + \mathbf{S}^{(q+1)} - \mathbf{X}^{(q+1)}, \quad (20)$$

$$\mathbf{W}^{(q+1)} = \mathbf{W}^{(q)} + \mathbf{S}^{(q+1)} - \mathbf{Z}^{(q+1)}. \quad (21)$$

Algorithm 1 summarizes all the required steps, with $\hat{\mathbf{s}}$ denoting the final estimate and Q the maximum number of iterations. In lines 11-14, I is the support of $\mathbf{x}^{(q+1)}$, \bar{I} is the respective complement (i.e., $\bar{I} = \{1, \dots, NN_u N_{rx}\} \setminus I$), and $\hat{\mathbf{s}}_I$ ($\hat{\mathbf{s}}_I^{candidate}$) is the reduced $N_a N_u N \times 1$ vector containing the nonzero elements of $\hat{\mathbf{s}}$ ($\hat{\mathbf{s}}^{candidate}$) given by the support I . For initialization of the algorithm we can perform a random selection of a vector \mathbf{s} with elements constrained within the constellation limits, followed by the projection over \mathbb{S} and $\mathcal{A}_0^{NN_u N_{rx}}$ (using (18) and (19)) in order to obtain \mathbf{x}^0 and \mathbf{z}^0 . \mathbf{U}^0 and \mathbf{W}^0 can be set as 0. Alternatively, we can apply the same procedure but starting with an initial vector \mathbf{s} computed using the MMSE-FDE (as in [7]). The penalty coefficients, ρ_i^x and ρ_i^z , are used as tuning parameters for achieving the best performance for a specific problem setting. Regarding the implementation of the algorithm, the matrix multiplications $(\mathbf{F} \otimes \mathbf{I}_{N_u N_{rx}})$ and $(\mathbf{F}^H \otimes \mathbf{I}_{N_u N_{rx}})$, which are required in lines 6, 7 and 14, can be efficiently performed through $N_u N_{rx}$ fast Fourier transforms (FFTs). The matrix inverses in (17) only need to be computed in the first iteration of the algorithm. Taking this into account, it can be seen that the complexity in real-valued floating point operations (flops) per subcarrier is

$$\begin{aligned} C_{GSM-FDE-ADMM} = & Q \left[8N_u^2 N_{rx}^2 + 3N_u N_{rx} + 2N_u N_{comb} (N_a - 1) \right. \\ & \left. + 20N_u N_{rx} \log_2 N + 40N_u N_{rx} / N \right] + 4N_u^3 N_{rx}^3 + N_u^2 N_{rx}^2 (4N_{rx} + 7) \\ & + N_u N_{rx} (12N_{rx} - 1) + N_{rx} (4 \log_2 N - 6 + 8/N), \end{aligned} \quad (22)$$

Algorithm 1: Proposed Multiuser ADMM-based Frequency Domain Equalizer for GSM-MIMO (MU-ADMM-FDE)

- 1: **Input:** $\mathbf{Y}, \mathbf{H}, \mathbf{U}^0, \mathbf{W}^0, \mathbf{x}^0, \mathbf{z}^0, \mathbf{P}_x, \mathbf{P}_z, Q$
 - 2: $f_{best} = \infty$.
 - 3: $\mathbf{X}^0 \leftarrow (\mathbf{F} \otimes \mathbf{I}_{N_u N_{rx}}) \mathbf{x}^0, \mathbf{Z}^0 \leftarrow (\mathbf{F} \otimes \mathbf{I}_{N_u N_{rx}}) \mathbf{z}^0$.
 - 4: **for** $q=0,1,\dots,Q-1$ **do**
 - 5: Compute $\mathbf{S}_k^{(q+1)}$ using (17) for all frequencies $k=0,1,\dots,N-1$.
 - 6: Obtain $\mathbf{x}_t^{p(q+1)}$ for all symbol positions and users with projection (18).
 - 7: Obtain $\mathbf{z}^{(q+1)}$ with projection (19).
 - 8: $I \leftarrow \text{supp}(\mathbf{x}^{(q+1)})$
 - 9: $\hat{\mathbf{s}}_{\bar{I}}^{candidate} \leftarrow 0, \hat{\mathbf{s}}_I^{candidate} \leftarrow \prod_{\mathcal{A}^{N_a N_u N}} (\mathbf{s}_I^{(q+1)})$.
 - 10: **If** $f(\hat{\mathbf{s}}^{candidate}) < f_{best}$ **then**
 - 11: $\hat{\mathbf{s}}_{\bar{I}} \leftarrow 0, \hat{\mathbf{s}}_I \leftarrow \hat{\mathbf{s}}_I^{candidate}$.
 - 12: $f_{best} = f(\hat{\mathbf{s}}^{candidate})$.
 - 13: **end if**
 - 14: $\mathbf{X}^{(q+1)} \leftarrow (\mathbf{F} \otimes \mathbf{I}_{N_u N_{rx}}) \mathbf{x}^{(q+1)}, \mathbf{Z}^{(q+1)} \leftarrow (\mathbf{F} \otimes \mathbf{I}_{N_u N_{rx}}) \mathbf{z}^{(q+1)}$
 - 15: $\mathbf{U}^{(q+1)} = \mathbf{U}^{(q)} + \mathbf{S}^{(q+1)} - \mathbf{X}^{(q+1)}$.
 - 16: $\mathbf{W}^{(q+1)} = \mathbf{W}^{(q)} + \mathbf{S}^{(q+1)} - \mathbf{Z}^{(q+1)}$
 - 17: **end for**
 - 18: **Output:** $\hat{\mathbf{s}}$.
-

corresponding to a complexity order of $O(N_u^3 N_{tx}^3 + N_u^2 N_{tx}^2 N_{rx} + N_u N_{comb} N_a + N_u N_{tx} \log_2 N)$ which is similar to the MMSE based FDE receiver from [7]. Compared with the MP-GSM receiver [14] and the CS-ZF approach from [10], which have complexity orders of $O(N N_u N_{rx} N_{comb} M^{N_a})$ and $O(N_u^2 N_{tx}^2 N_{rx} N^2 + N_{rx}^3 N^2)$ (assuming the use of an interior-point method as in [17]), the complexity of the proposed MU-FD-ADMM tends to grow much slower with the size of the problem setting.

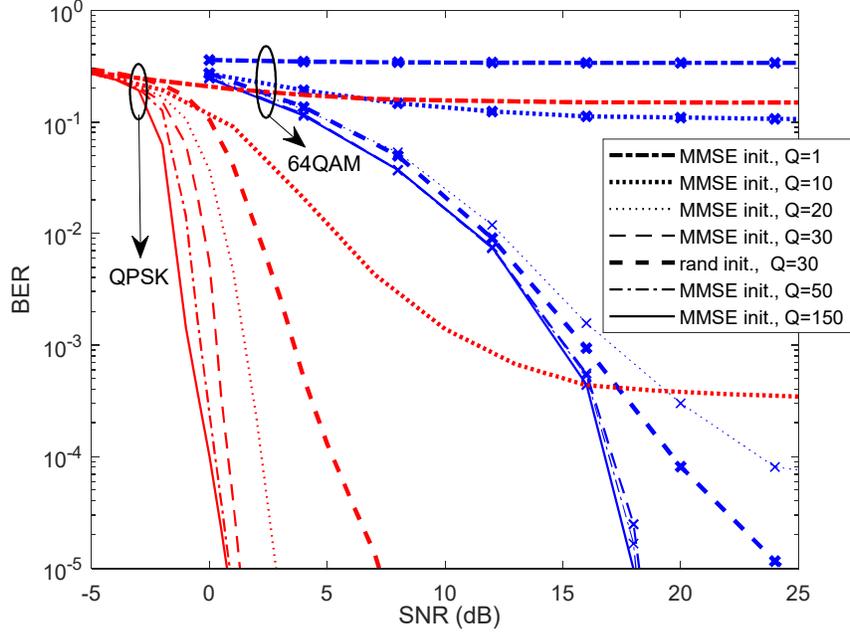


Fig. 1. BER performance of a multiuser SC-GSM-MIMO system with $N_u=12$, $N_{tx}=7$, $N_a=2$, $N_{rx}=42$ and different configurations of the MU-ADMM-FDE.

IV. NUMERICAL RESULTS

In this section, we evaluate the performance of the proposed detector using Monte Carlo simulations. An uncoded MU SC system with $N=128$, a block duration of $67\mu\text{s}$ and a CP with $16.7\mu\text{s}$ was considered. The adopted channel model was the Extended Typical Urban model (ETU) [18] (similar conclusions could be drawn for other severely time-dispersive channels). All the channel coefficients were independently drawn according to a zero-mean complex Gaussian distribution. Randomly selected modulated symbols were transmitted on the active AEs with $E[|s_i|^2] = 1$.

Fig. 1 plots the bit error rate (BER) as a function of the signal to noise ratio (SNR) per user and receive antenna, for a MU scenario with $N_u=12$, $N_{tx}=7$, $N_a=2$ and $N_{rx}=42$ and different configurations of the MU-ADMM-FDE. Curves for QSPK and 64-QAM modulations are shown, which correspond to SEs of 8 and 16 bits per channel use (bpcu). The penalty parameters values were $\rho_i^x = \rho_i^z = 28$. It can be observed that both the initialization and the number of iterations influence the behavior of the detector. For example, starting

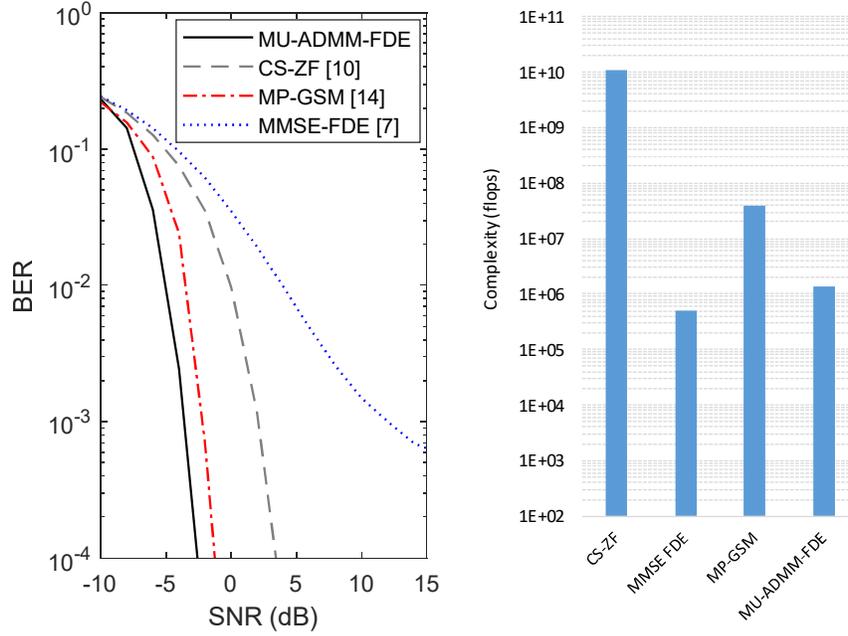


Fig. 2. BER performance of a multiuser SC-GSM-MIMO system with $N_u=10$, $N_{tx}=4$, $N_a=1$, $N_{rx}=32$, $N=64$, QPSK and different detectors.

the algorithm with the MMSE based initialization tends to achieve better performance than the random initialization with the same number of iterations. We can also see that we may need at least 30-50 iterations until the performance gains become small. While this may seem a large number, it is important to remember that the complexity cost per iteration is small. Furthermore, it is possible to reduce the number of iterations effectively used by adopting a stopping criterion based on the primal and dual residuals [15] or detecting stall conditions where variables do not show relevant change after several iterations.

In Fig. 2 we present the BER and complexity in real-valued flops of the MU-ADMM-FDE (with $Q=30$ iterations) and compare them against the MMSE-FDE [7], CS-ZF [10] and MP-GSM [14]. The setting corresponds to a system with a SE of 4 bpcu, where $N_u=10$, $N_{tx}=4$, $N_a=1$, $N_{rx}=42$ and the adopted modulation is QPSK. This scenario corresponds to an underdetermined system where, as expected, the MMSE-FDE has more difficulty to cope with. The other three detectors are capable of operating more reliably, with the MU-ADMM-FDE being the most effective in coping with the ISI induced by the channel and detecting the GSM symbols, providing gains over CS-ZF and MP-GSM of 6 dB and 1.3 dB at a BER of 10⁻⁴. It can also be seen

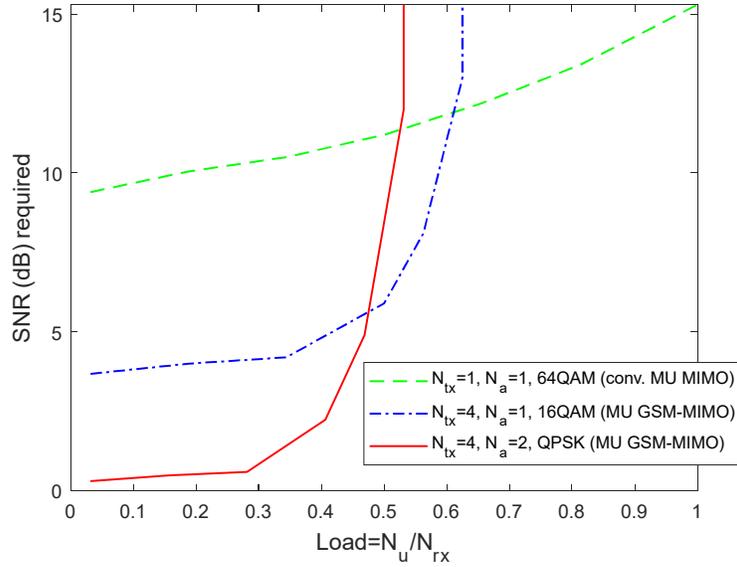


Fig. 3. SNR required for achieving a target BER of 10^{-4} versus loading factor (N_u/N_{rx}) considering $N_{rx}=32$ and 6 bpcu per user.

that the advantage is not only in terms of BER performance but also in terms of complexity as it is substantially lower than the complexity of CS-ZF and MP-GSM.

Fig. 3 illustrates the impact of changing the loading factor (defined as N_u/N_{rx}) on the SNR required to achieve a target BER of 10^{-4} when $N_{rx}=32$. Three different configurations with the same SE of 6 bpcu per user are considered. The proposed receiver is employed for all setups, including the case $N_{tx}=N_a=1$ (conventional MU MIMO). For low loads, the use of GSM has a clear performance advantage over the conventional MU system. For loads above 0.25, the GSM systems becomes underdetermined ($N_u N_{tx} > N_{rx}$) and, even though the proposed receiver is still able to perform well, the SNR degradation becomes sharper until the point where the BER of 10^{-4} becomes unreachable. In this high-load region, the conventional MU setup becomes a better performing solution (and it can be dealt with the same receiver). These results reveal that when there is a sufficient number of receiver antennas at the base station, increasing the spectral efficiency through the use of GSM might be a better strategy than increasing the modulation order as it tends to be “easier”, in terms of additional SNR required, to reliably infer the active transmit antennas than to detect the correct modulation symbols from a larger constellation alphabet. It is important to note that the

advantage of GSM schemes in terms of SNR is only possible as long as the number of receiver antennas is sufficiently higher than the existing sparsity ($N_u N_a$), otherwise reliable recovery of the sparse signal is not possible and that is why it is only achieved in the low loads regime.

V. CONCLUSIONS

This letter presented a novel iterative detector for SU and MU SC-GSM transmissions in frequency-selective channels which accomplishes reduced complexity implementation through frequency domain equalization. Numerical simulations show that the proposed receiver can effectively cope with the ISI induced by severe time dispersive channels and operate in difficult underdetermined scenarios. The inherent splitting-based design of the algorithm allows it to easily deal with GSM based transmissions, which can be more attractive in low load scenarios, and switch to conventional MU detection whenever the load becomes high.

REFERENCES

- [1] A. Younis, N. Serafimovski, R. Mesleh, and H. Haas, "Generalised spatial modulation," in *Proc. Asilomar Conf. Signals, Syst. Comput.*, Pacific Grove, CA, USA, Nov. 2010, pp. 1498–1502.
- [2] J. Wang, S. Jia, and J. Song, "Generalised Spatial Modulation System with Multiple Active Transmit Antennas and Low Complexity Detection Scheme," *IEEE Trans. Wireless Commun.*, vol. 11, no. 4, pp. 1605-1615, Apr. 2012.
- [3] M. Di Renzo, H. Haas A. Ghayeb, S. Sugiura, and L. Hanzo, "Spatial modulation for generalized MIMO: Challenges, opportunities and implementation," *Proc. of the IEEE*, vol. 102, no. 1, pp. 56-103, Jan. 2014.
- [4] R. Rajashekar, K. V. S. Hari, and L. Hanzo, "Reduced-complexity ML detection and capacity-optimized training for spatial modulation systems," *IEEE Transactions on Communications*, vol. 62, no. 1, pp. 112–125, 2014.

- [5] S. Fan, Y. Xiao, L. Xiao, P. Yang, R. Shi and K. Deng, "Improved Layered Message Passing Algorithms for Large-Scale Generalized Spatial Modulation Systems," in *IEEE Wireless Communications Letters*, vol. 7, no. 1, pp. 66-69, Feb. 2018.
- [6] H. Lopes and N. Souto, "Iterative Signal Detection for Large-Scale GSM-MIMO Systems," in *IEEE Transactions on Vehicular Technology*, vol. 67, no. 8, pp. 7734-7738, Aug. 2018.
- [7] S. Sugiura and L. Hanzo, "Single-RF Spatial Modulation Requires Single-Carrier Transmission: Frequency-Domain Turbo Equalization for Dispersive Channels," in *IEEE Transactions on Vehicular Technology*, vol. 64, no. 10, pp. 4870-4875, Oct. 2015.
- [8] R. Rajashekar, K. V. S. Hari and L. Hanzo, "Transmit Antenna Subset Selection for Single and Multiuser Spatial Modulation Systems Operating in Frequency Selective Channels," in *IEEE Transactions on Vehicular Technology*, vol. 67, no. 7, pp. 6156-6169, July 2018.
- [9] Y. Zhao, Y. Xiao, P. Yang, B. Dong, R. Shi and K. Deng, "Generalized Approximate Message Passing Aided Frequency Domain Turbo Equalizer for Single-Carrier Spatial Modulation," in *IEEE Transactions on Vehicular Technology*, vol. 67, no. 4, pp. 3630-3634, April 2018.
- [10] Z. A. Shaikh, I. B. Collings, S. V. Hanly and P. Whiting, "Compressive sensing aided data detection for GSM systems in MIMO ISI wireless channels," 2015 IEEE International Conference on Communications (ICC), London, 2015, pp. 4582-4586.
- [11] L. Xiao, L. Dan, Y. Zhang, Y. Xiao, P. Yang and S. Li, "A Low-Complexity Detection Scheme for Generalized Spatial Modulation Aided Single Carrier Systems," in *IEEE Communications Letters*, vol. 19, no. 6, pp. 1069-1072, June 2015.
- [12] L. Xiao, P. Yang, Y. Zhao, Y. Xiao, J. Liu and S. Li, "Low-complexity tree search-based detection algorithms for generalized spatial modulation aided single carrier systems," 2016 IEEE International Conference on Communications (ICC), Kuala Lumpur, 2016, pp. 1-6.

- [13] L. Xiao, Y. Xia, Y. Zhao, P. Yang, M. Renzo, S. Li, W. Xiang, "Time-Domain Turbo Equalization for Single-Carrier Generalized Spatial Modulation," in *IEEE Transactions on Wireless Communications*, vol. 16, no. 9, pp. 5702-5716, Sept. 2017.
- [14] T. Lakshmi Narasimhan, P. Raviteja and A. Chockalingam, "Generalized Spatial Modulation in Large-Scale Multiuser MIMO Systems," in *IEEE Transactions on Wireless Communications*, vol. 14, no. 7, pp. 3764-3779, July 2015.
- [15] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, "Distributed optimization and statistical learning via the alternating direction method of multipliers," *Found. Trends Mach. Learn.*, vol. 3, no. 1, pp. 1–122, 2011.
- [16] S. Diamond, R. Takapoui and S. Boyd, "A General System for Heuristic Solution of Convex Problems over Nonconvex Sets," *Optimization Methods and Software*, vol. 33, no 1, pp. 165-193, 2018.
- [17] S. S. Chen, D. L. Donoho, and M. A. Saunders, "Atomic decomposition by basis pursuit," *SIAM Review*, vol. 43, no. 1, pp. 129-159, Feb. 2001.
- [18] "Evolved Universal Terrestrial Radio Access (E-UTRA); Base Station (BS) radio transmission and reception, " 3GPP TS 36.104 v15.3.0, June 2018.