

# Federated Learning with Multichannel ALOHA

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**Abstract**—In this paper, we study federated learning in a cellular system with a base station (BS) and a large number of users with local data sets. We show that multichannel random access can provide a better performance than sequential polling when some users are unable to compute local updates (due to other tasks) or in dormant state. In addition, for better aggregation in federated learning, the access probabilities of users can be optimized for given local updates. To this end, we formulate an optimization problem and show that a distributed approach can be used within federated learning to adaptively decide the access probabilities.

**Index Terms**—Federated Learning; Multichannel ALOHA

## I. INTRODUCTION

Federated learning [1]–[3] has been extensively studied as a distributed machine learning approach with data privacy. In federated learning, mobile phones or devices keep their data sets and exchange a parameter vector to be optimized in a certain learning problem (with data sets that are kept at devices or users). Throughout the paper, we interchangeably use devices and users.

Since each user uploads its local update to a server and the server sends the aggregated update back to users, in cellular setting, we can assume that all the exchanges are carried out through base stations (BSs). In [4]–[6], federated learning in a cell is considered, where all the users are located in a cell and communicate with a BS. Under this setting, in [4], [6], the notion of over-the-air computation [7], [8] is adopted for aggregation when analog versions of local updates are transmitted, where the BS receives a noisy version of the aggregated update. In [5], fading channels are taken into account.

In this paper, we consider the setting that all the users are located in a cell and a BS is to communicate with them as in [4]–[6]. However, we do not consider the notion of over-the-air computation. For efficient uploading with a limited system bandwidth, we consider multichannel random access (e.g., multichannel ALOHA) [9], [10]. In most cases for federated learning, it is assumed that all the users are able to upload their local updates at each iteration, and sequential polling with multiple access channels can allow to upload more local updates at the cost of wider bandwidth. However, in practice, some users (e.g., mobile phones) might be busy for other tasks or are in dormant state. As a result, although they are asked to upload, no local updates from them are available, which results in waste of channels. This motivates us to use multichannel random access, i.e., multichannel ALOHA, for more efficient uploading than polling in federated learning.

Furthermore, for effective aggregation with a small number of local updates, the access probability of each user in multichannel ALOHA can be optimized. To this end, we formulate an optimization problem and find the solution with a distributed implementation method in conjunction with federated learning.

## II. SYSTEM MODEL

Suppose that a federated learning system (FLS) consists of  $K$  mobile devices and one BS in a cell as in [4]–[6]. As mentioned earlier, we use mobile devices and user interchangeably. In FLS, each user has its data set  $\mathcal{D}_k = \{\mathbf{x}_k, y_k\}$ , where  $\mathbf{x}_k$  and  $y_k$  represent the input and output of user  $k$ , respectively [1], and there is a parameter or weight vector  $\mathbf{w}$  associated with the following optimization problem:  $\min_{\mathbf{w}} \frac{1}{K} \sum_{k=1}^K f_k(\mathbf{w})$ , where  $f_k(\mathbf{w})$  denotes the loss function at user  $k$ , which has in general the following form:  $f_k(\mathbf{w}) = \ell(\mathbf{w}, \mathbf{x}_k, y_k)$ . For example, the loss function for linear regression is given by

$$\ell(\mathbf{w}, \mathbf{x}_k, y_k) = \frac{1}{2} |\mathbf{x}_k^T \mathbf{w} - y_k|^2. \quad (1)$$

In FLS, the users do not upload their data set to the BS, but send their local updates for given weight vector through iterations. Let  $\mathbf{w}(t)$  denote the weight vector at iteration  $t$ , where  $t$  is the index for the iteration. Then, user  $k$  can find its local update with  $\mathcal{D}_k$  as follows:

$$\mathbf{w}_k(t+1) \leftarrow \text{LocalUpdate}(\mathbf{x}_k, y_k, \mathbf{w}(t)), \quad (2)$$

where the local update depends on the loss function. For the loss function at user  $k$  in (1), the local update becomes  $\mathbf{w}(t) - h_k(\mathbf{x}_k^T \mathbf{w}(t) - y_k)\mathbf{x}_k$  with step size  $h_k > 0$  for the gradient descent (GD) algorithm. Here,  $\mathbf{w}_k(t)$  and  $\mathbf{w}(t)$  represent the weight vector  $\mathbf{w}$  at user  $k$  and BS at iteration  $t$ , respectively (the subscript  $t$  is used for the user index). Once all the users send their local updates to the BS, the BS is able to update the weight vector as follows:

$$\mathbf{w}(t+1) = \frac{1}{K} \sum_{k=1}^K \mathbf{w}_k(t+1), \quad (3)$$

which is referred to as aggregation.

## III. UPLOADING VIA MULTIPLE CHANNELS

In FLS, local updating and aggregation at the BS in (2) and (3), respectively, are to be carried out iteratively. This iteration requires uploading the local weight vectors from  $K$  users. If  $K$  is large, the required time for uploading per iteration might be long. To shorten the uploading time, multiple channels can be used with a wider system bandwidth. In this section, we consider a random sampling approach to approximate for (3) with multiple channels and combine it with ALOHA.

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### A. Multichannel Random Access

The averaging in (3) requires all the  $K$  local updates. Provided that the system bandwidth is limited, all the local updates may not be available at each iteration. Thus, suppose that there are  $M$  parallel channels, where  $M \ll K$  so that  $M$  users can upload their local updates simultaneously at each iteration.

It is possible that the BS chooses a set of  $M$  users to upload their local updates at each iteration. Alternatively, a pre-determined user sequence can be used. In this case, it is not necessary for the BS to choose  $M$  users at each iteration. However, there are drawbacks. First, there can be users that cannot upload their local updates due to various reasons. For example, a user (or sensor) may not be available as it is in dormant state, or its local computation to find its update cannot be carried out as it is busy with some other tasks. Secondly, a user with negligible local update can be asked to upload its local update, which leads to a negligible impact on the aggregation in (3). To avoid the above drawbacks, we can consider multichannel ALOHA with the access probability that depends on the local update, which will be considered in Subsection III-B.

To address the first drawback, let  $p_{\text{comp}}$  denote the probability that a user is able to compute its local update. Thus, if  $M$  users are asked to upload their local updates by the BS, only  $p_{\text{comp}}M$  users on average are able to send their updates. As a result, polling with  $M$  channels may not be efficient if  $p_{\text{comp}}$  is not high (as  $(1 - p_{\text{comp}})M$  channels would be idle on average). To overcome this problem, we can consider multichannel ALOHA, where each user with local update can randomly choose one of  $M$  channels with a certain access probability and sends its local update.

Suppose that a user that can compute its local update can randomly choose one of  $M$  channels. We assume that the BS cannot receive any local updates if multiple users choose the same channel (due to packet collision). Let  $p$  denote the access probability, i.e., the probability that a user sends its local update. Clearly,  $p \leq p_{\text{comp}}$ . Then, the average number of local updates at the BS is given by

$$\eta = Kp \left(1 - \frac{p}{M}\right)^{K-1} \approx Kpe^{-\frac{pK}{M}} \leq Me^{-1}. \quad (4)$$

The maximum average number of local updates can be achieved if  $p = \frac{M}{K}$ . As a result, if  $p_{\text{comp}} \leq e^{-1}$  and  $M \leq e^{-1}K$ , we can see that multichannel ALOHA (with  $p = \frac{M}{K}$ ) can upload more local updates than polling on average for the aggregation in (3) as  $\eta \approx Me^{-1} \geq Mp_{\text{comp}}$ . Note that when multichannel ALOHA is used with  $M$  channels, the access probability becomes

$$p = \min \left\{ \frac{M}{K}, p_{\text{comp}} \right\}. \quad (5)$$

### B. Adaptive Access Probability based on Local Update

In this subsection, we address the second drawback. To this end, we need to allow that the access probability of each user, which was assumed to be the same for all users in the previous subsection, is now different and depends on its local update.

In this subsection, we first formulate an optimization problem to approximate the aggregation in terms of the access probabilities of users (from the BS's perspective). Then, we show that each user can decide its access probability with its local update and (simple) feedback information from the BS. Let  $\mathbf{a} = \sum_{k=1}^K \mathbf{w}_k$ , which is the unnormalized aggregation. In addition, define  $\mathbf{u} = \sum_{k=1}^K \mathbf{w}_k \delta_k$ , where  $\delta_k \in \{0, 1\}$  becomes 1 if the BS receives the local update from user  $k$  and 0 otherwise. We assume that  $\delta_k$  is dependent on  $\mathbf{w}_k$ . It can be seen that  $\mathbf{u}$  is an approximation of  $\mathbf{a}$  for the aggregation in (3). To see the approximation error, we can consider the following conditional error norm:

$$\begin{aligned} \mathbb{E}[\|\mathbf{a} - \mathbf{u}\| \mid \mathcal{W}] &= \mathbb{E}[\|\sum_{k=1}^K \mathbf{w}_k(1 - \delta_k)\| \mid \mathcal{W}] \\ &\leq \sum_{k=1}^K a_k \mathbb{E}[1 - \delta_k \mid \mathbf{w}_k] \leq \sum_{k=1}^K a_k e^{-q_k}, \end{aligned} \quad (6)$$

where the first inequality is due to the triangle inequality and the second inequality is due to  $1 - x \leq e^{-x}$  for  $x \in (0, 1)$ , and  $a_k = \|\mathbf{w}_k\|$ . Here,  $q_k = \mathbb{E}[\delta_k \mid \mathbf{w}_k]$ , which is the probability that the BS receives the local update from user  $k$ .

Let  $p_k$  denote the probability that user  $k$  transmits its local update (or the access probability of user  $k$ ). Then, the probability that the BS successfully receives the local update from user  $k$ ,  $q_k$ , is given by

$$q_k = p_k \prod_{n \neq k} \left(1 - \frac{p_n}{M}\right) \leq p_k e^{-\sum_{n \neq k} \frac{p_n}{M}} \leq p_k e^{-\frac{P}{M}}, \quad (7)$$

where  $P = \sum_{k=1}^K p_k$ . Then, it can be shown that

$$Q = \sum_k q_k \leq \sum_k p_k e^{-\frac{P}{M}} = P e^{-\frac{P}{M}} \leq M e^{-1}. \quad (8)$$

In (8), the second inequality becomes the equality (i.e., the sum of the probabilities of successful uploadings,  $Q$ , can be maximized) when  $P = M$ . Thus, with  $P = M$ , from (7), we can have  $q_k \leq p_k e^{-1}$ . If  $P = M$ , the last inequality becomes equality in (8). Furthermore, since the bounds in (7) are tight when  $K$  is sufficiently large and  $p_n/M$  is sufficiently low, we will assume that

$$q_k = p_k e^{-1}. \quad (9)$$

From (6), (8), and (9),  $q_k$  can be decided to minimize the error bound as follows

$$\begin{aligned} &\min_{q_k} \sum_k a_k e^{-q_k} \\ &\text{subject to } \sum_k q_k = M e^{-1} \text{ and } q_k \in (0, e^{-1}), \forall k, \end{aligned} \quad (10)$$

which is a convex optimization problem. Note that the second constraint,  $q_k \in (0, e^{-1})$  is due to  $p_k \leq 1$  and (9). Then, the solution is given by

$$q_k^* = [\ln a_k - \ln \lambda]_0^{e^{-1}}, \quad (11)$$

where  $\lambda$  is a Lagrange multiplier and  $[x]_a^b$ , where  $a < b$ , is defined as

$$[x]_a^b = \begin{cases} x, & \text{if } a < x < b \\ a, & \text{if } x \leq a \\ b, & \text{if } x \geq b. \end{cases}$$

In (11), we can find  $\lambda$  to satisfy  $\sum_k q_k^* = Me^{-1}$ .

Alternatively,  $p_k$  can be obtained from (11) using (9), i.e.,  $p_k = q_k e$ , as follows:

$$p_k^* = [e \ln a_k - \psi]_0^1, \quad (12)$$

where  $\psi = e \ln \lambda$ . In addition, in (10), if a user cannot compute its local update, it needs to set  $a_k = 0$  so that  $p_k = 0$ .

### C. Feedback Signal from the BS

Finding the solution of (10) requires all the  $a_k$ 's. Therefore, (10) cannot be carried out at each user. However, as in (12), if  $\psi$  is available at each user,  $p_k$  can be found.

Denote by  $\hat{P}_t$  an estimate of  $P$  at iteration  $t$ , which is  $\hat{P}_t = \sum_{k=1}^K s_{k,t}$ , where  $s_{k,t} \in \{0, 1\}$  represents the activity variable of user  $k$  at iteration  $t$  (i.e.,  $s_{k,t} = 1$  if user  $k$  transmits its local update at iteration  $t$ , otherwise  $s_{k,t} = 0$ ). Clearly,  $\hat{P}_t$  is seen as the total number of active users that send their local updates through  $M$  channels (regardless of collisions). Note that as shown in [10], it is possible to find the total number of active users in an existing machine-type communication (MTC) standard.

We consider a feedback signal,  $\psi$ , which is to be sent from the BS to users at the end of each iteration. From (8), using the dual ascent method [11],  $\psi$  can be adaptively decided to keep  $P = M$  close as follows:

$$\psi_{t+1} = \psi_t + \mu(\hat{P}_t - M), \quad (13)$$

where  $\psi_t$  represents the updated  $\psi$  at iteration  $t$  and  $\mu$  denotes the step size. At the end of iteration  $t$ , the BS can send  $\mathbf{w}(t+1)$  as well as  $\psi_{t+1}$  to all the users so that each one can not only find local updating, but also decide whether or not to transmit its local update according to (12).

Note that the access probability,  $p_k$ , is adaptively decided in (12) without knowing  $K$  and  $p_{\text{comp}}$ , which might be another advantage over polling when  $K$  and  $p_{\text{comp}}$  are not known to the BS.

## IV. SIMULATION RESULTS

In this section, the stochastic GD (SGD) algorithm is used in federated learning with the squared error loss function in (1). For simulations, at each user, we assume that  $\mathbf{x}_k$  of length  $L \times 1$  is an independent Gaussian random vector, i.e.,  $\mathbf{x}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$ . In addition, in each simulation run,  $\mathbf{w}$  is also generated as an independent Gaussian random vector, i.e.,  $\mathbf{w} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$ , while  $y_k = \mathbf{x}_k^T \mathbf{w}$  at user  $k$ ,  $k = 1, \dots, K$ .

Prior to presenting the main simulation results, we demonstrate the importance of choosing the users with significant local updates for uploading when the system bandwidth is limited. Suppose that only one user can upload at a time (i.e.,  $M = 1$ ). In this case, cyclic coordinate descent (CCD) as an example of SGD can be used as follows [1]:

$$\mathbf{w}(t+1) = \mathbf{w}(t) - \mu_1 (\mathbf{x}_{k(t)}^T \mathbf{w}(t) - y_{k(t)}) \mathbf{x}_{k(t)}, \quad (14)$$

where  $k(t)$  is the user index at iteration  $t$  and  $\mu_1$  is the step size. Here,  $k(t) = k \bmod t$  for CCD. For simplicity, we assume that  $p_{\text{comp}} = 1$ . For comparison, another approach where the user of the largest local update in terms of its norm is chosen

at each iteration is considered, where the index of the user that is to upload its local update at iteration  $t$  is chosen as

$$k(t) = \underset{k}{\operatorname{argmax}} \|(\mathbf{x}_{k(t)}^T \mathbf{w}(t) - y_{k(t)}) \mathbf{x}_{k(t)}\|. \quad (15)$$

In Fig. 1, the error norm,  $\|\mathbf{w}(t) - \mathbf{w}\|$ , with CCD and the uploading from the user corresponding to (15) (at each iteration) is shown as functions of the number of iterations when  $K = 100$ ,  $L = 10$ , and  $\mu_1 = 0.01$ . Clearly, it is shown that if the user of the largest local update (in terms of its norm) is chosen at each iteration as in (15), it can significantly improve the performance. Unfortunately, since the user corresponding to (15) is not known by the BS, the BS is not able to ask the user to upload its local update at each iteration. However, as discussed earlier, it is possible to take into account the norm of the local update when the access probability is decided with multichannel ALOHA, which might lead to performance improvement.

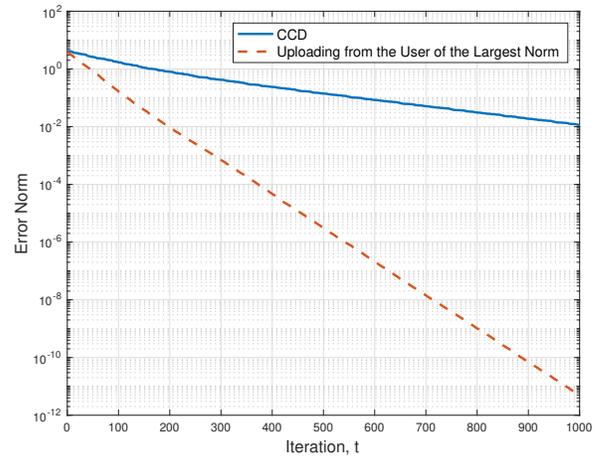


Fig. 1. Error norm,  $\|\mathbf{w}(t) - \mathbf{w}\|$ , with CCD and the uploading from the user corresponding to (15) (at each iteration) is shown as functions of iterations when  $K = 100$ ,  $L = 10$ , and  $\mu_1 = 0.01$ .

We now consider three different systems. One is based on polling with  $M$  channels (which can be seen as an SGD algorithm that does not take into account the significant of local updates (in terms of their norms) in choosing coordinates) and the other two systems are based on multichannel ALOHA. For convenience, Random Access 1 denotes the multichannel ALOHA system with an equal access probability of  $p$  in (5), while Random Access 2 represents the multichannel ALOHA system with the access probability in (12) and (13). In Fig. 2, we show the performance of three different systems for federated learning when  $K = 1000$ ,  $M = 10$ ,  $L = 10$ ,  $(\mu_1, \mu) = (0.01, 0.1)$ , and  $p_{\text{comp}} = 0.1$ . We can see that Random Access 2 outperforms the others.

In Fig. 3, the performance of three different systems for federated learning is shown in terms of  $M$  and  $p_{\text{comp}}$  when  $K = 1000$ ,  $L = 10$ ,  $(\mu_1, \mu) = (0.01, 0.1)$ , and the number of iterations is set to 100. It is shown in Fig. 3 (a), all the systems have improved performance as  $M$  increases. In Fig. 3 (b), it is shown that the performance of Random Access 1 is almost independent of  $p_{\text{comp}}$ , while its performance is worse than polling when  $p_{\text{comp}} > e^{-1}$  as expected. It is noteworthy that

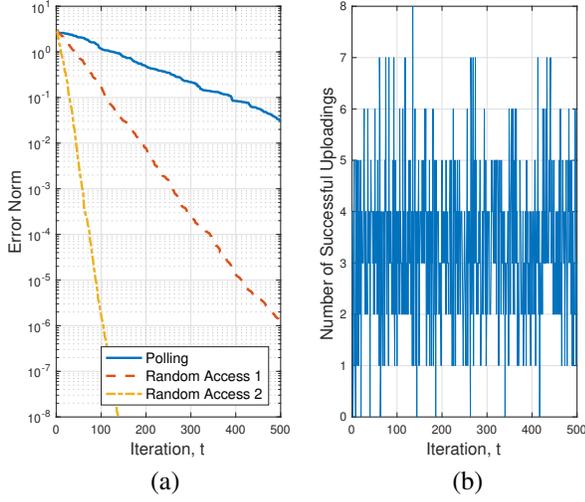


Fig. 2. Performance of three different systems for federated learning when  $K = 1000$ ,  $M = 10$ ,  $L = 10$ ,  $(\mu_1, \mu) = (0.01, 0.1)$ , and  $p_{\text{comp}} = 0.1$ : (a) Error norm,  $\|\mathbf{w}(t) - \mathbf{w}\|$ ; (b) Number of successfully uploadings (of Random Access 2).

the performance of Random Access 2 is degraded as  $p_{\text{comp}}$  increases, which is due to a convergence time to find  $\psi$  in (13).

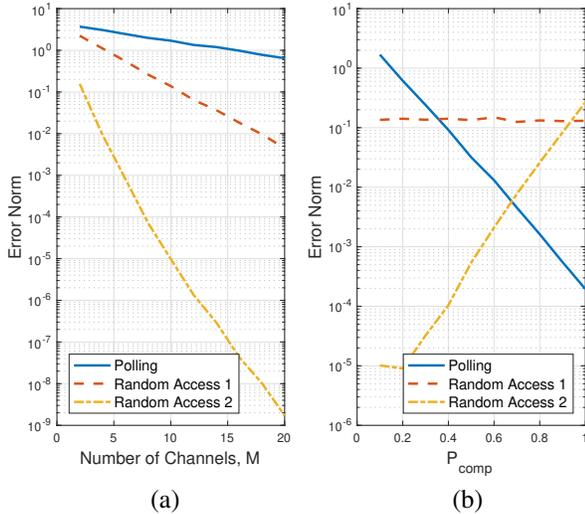


Fig. 3. Error norms of three different systems for federated learning when  $K = 1000$ ,  $L = 10$ ,  $(\mu_1, \mu) = (0.01, 0.1)$ , and the number of iterations is set to 100: (a) Error norm versus  $M$ ; (b) Error norm versus  $p_{\text{comp}}$ .

In Fig. 4 (a), we show the trajectory of error norms when  $K = 1000$ ,  $M = 10$ ,  $L = 10$ ,  $(\mu_1, \mu) = (0.01, 0.1)$ , and  $p_{\text{comp}} = 0.6$ . It is shown that Random Access 2 cannot upload local updates as  $p_k$  is too low for the first 50 iterations, as shown in Fig. 4 (b). Once  $\psi_t$  becomes low enough through the iteration in (13),  $p_k$  becomes sufficiently high to upload local updates and a better performance can be achieved with a sufficient number of iterations (say, more than 100 iterations).

## V. CONCLUSIONS

We studied federated learning within a cellular system and adopted multichannel ALOHA to upload local updates from

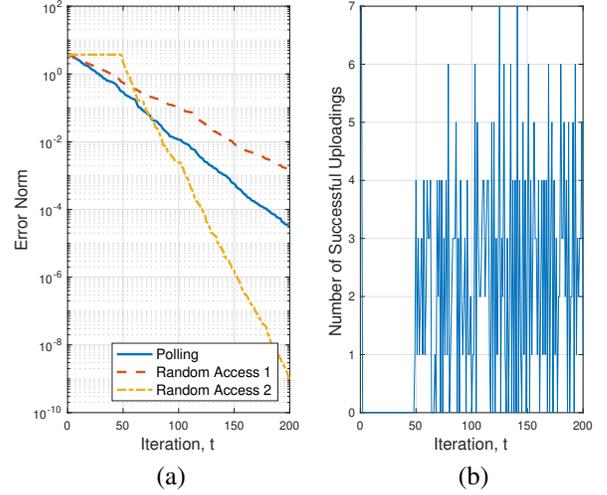


Fig. 4. Performance of three different systems for federated learning when  $K = 1000$ ,  $M = 10$ ,  $L = 10$ ,  $(\mu_1, \mu) = (0.01, 0.1)$ , and  $p_{\text{comp}} = 0.6$ : (a) Error norm,  $\|\mathbf{w}(t) - \mathbf{w}\|$ ; (b) Number of successfully uploadings (of Random Access 2).

a large number of users. It was shown that multichannel ALOHA can perform better than sequential polling when the probability that a user is able to upload its local update is less than  $e^{-1} \approx 0.3679$ . It was also demonstrated that the access probability can be optimized with the significant of local update at each user (which is measured by the norm of the local update) for better performance in terms of aggregation in federated learning. A distributed approach for optimizing access probability was also presented.

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