NOMA Design with Power-Outage Tradeoff for Two-User Systems

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Abstract-This letter proposes a modified non-orthogonal multiple-access (NOMA) scheme for systems with a multiantenna base station (BS) and two single-antenna users, where NOMA transmissions are conducted only when the absolute correlation coefficient (CC) between the user channels exceeds a threshold and the BS uses matched-filter (MF) precoding along the user with the stronger average channel gain. We derive the average minimal transmit power to guarantee the signal-tointerference-plus-noise-ratio (SINR) levels of both users. Our results show that the average minimal power grows logarithmically in the reciprocal of the CC threshold and a non-zero threshold is necessary for the modified NOMA scheme to have finite average minimal transmit power. Further, for the massive MIMO scenario, we derive the scaling laws of the average transmit power and outage probability with respect to the antenna numbers, as well as their tradeoff law. Simulation results are shown to validate our theoretical results.

Index Terms—Non-orthogonal multiple access (NOMA), minimal transmit power, channel correlation, power scaling law, outage probability.

I. INTRODUCTION

N ON-ORTHOGONAL multiple-access (NOMA) is a potential technique for the next generation mobile communications. With superposition coding at the transmitting base station (BS) and successive interference cancellation (SIC) at the receiving users [1], NOMA can provide improved spectral efficiency via the extra degree-of-freedom in the power domain compared to traditional orthogonal multiple-access (OMA) schemes [2].

Early works on NOMA considered the single-input-singleoutput (SISO) case, where the BS is equipped with a singleantenna, e.g., [3]-[5]. For NOMA with multiple uniformly distributed users, in [3], expressions were derived for both the sum-rate and the outage probability of each individual user. It was shown that NOMA has higher sum-rate than OMA; while for the outage probability, the choices of user rates and power coefficients are critical. In [4], the sum-rate superiority of NOMA to OMA was shown for the two-user cluster case and fixed power allocation. The significance of user-pairing based on channel norm difference was also demonstrated. Further, the outage probability of the stronger user was analyzed given quality-of-service (QoS) guarantee of the weak user. In [5], for multiple users, the sum-rate optimization over power allocation with fairness consideration were studied for both OMA and NOMA systems and the optimized sum-rate of NOMA was shown to be higher.

There are also results on multi-input-multi-output (MIMO) NOMA, where the BS is equipped with multiple antennas, e.g., [6]–[8]. The work in [6] proved that for 2-user case, an upper bound of the sum-rate in OMA systems also serves as a lower bound of the sum-rate in NOMA systems when applying the same precoding and postcoding to OMA and NOMA. The work was generalized to multiple-user case in [7]. In [8], for a massive MIMO scenario, the sum-rate achieved by NOMA with two-user clusters was studied and compared to the multi-user beamforming scheme. A hybrid of NOMA and multi-user beamforming scheme was also proposed for sum-rate improvement.

Different from existing works, we study the power consumption and the tradeoff between power consumption and outage probability for MIMO-NOMA systems with a twouser cluster and matched-filter (MF) precoding according to the channel of the stronger user. Considering the significance of channel correlation coefficient (CC) [8], we propose a modified correlation-based NOMA (CB-NOMA) scheme, where NOMA is used only when the absolute CC of the user channels is above a threshold. A tight approximation is derived for the average minimal transmit power of CB-NOMA to guarantee QoS for both users. The result reveals the behaviour of the average minimum transmit power with respect to the correlation-threshold and shows the superiority of CB-NOMA to the original NOMA in power saving. Further, for the massive MIMO scenario, the scaling laws of the power consumption and outage probability are derived to guide the threshold design.

II. NOMA SCHEME WITH CORRELATION THRESHOLD

We consider the downlink transmissions in a multi-user system with an *M*-antenna BS and 2 single-antenna users. The channel vector for User *k* can be modeled as: $g_k = \sqrt{\beta_k} h_k$, for k = 1, 2, where β_k is the large-scale fading coefficient and $h_k \in \mathbb{C}^{M \times 1}$ represents the small-scale fading effect with independent and identically distributed (i.i.d.) elements following the Rayleigh distribution, i.e., $h_k \sim \mathcal{CN}(0, I_M)$ for k = 1, 2. Let ρ be the absolute CC between h_1 and h_2 , which is given by:

$$\rho \triangleq \frac{|\boldsymbol{h}_1^H \boldsymbol{h}_2|}{\|\boldsymbol{h}_1\| \|\boldsymbol{h}_2\|}.$$
(1)

In other words, $\rho = |\cos \theta|$, where θ is the angle between the two channel vectors. We assume perfect CSI at the transmitter. Without loss of generality, we assume that $\beta_1 \ge \beta_2$.

To save transmit power and achieve a balance between transmit power and the QoS performance, we propose a

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modified CB-NOMA scheme in which the BS transmits to both users with a common time-frequency resource block as well as common beamformer only when ρ , the absolute correlation coefficient, exceeds a threshold ρ_{th} . Otherwise the BS keeps silent to save power. The design of ρ_{th} and its effects on the performance will be analyzed in later sections. If $\rho_{th} = 0$, the scheme is the same as the original NOMA.

When the BS serves both users, the BS data symbols are superposition-coded as [2]:

$$s = \sqrt{P_1}s_1 + \sqrt{P_2}s_2,$$
 (2)

where $s_1, s_2 \sim C\mathcal{N}(0, 1)$ are the data symbols for the users and P_1, P_2 are the power allocated to the users, respectively. The total BS transmit power is thus $P = P_1 + P_2$.

Let b be the BS beamformer. Thus, the transmit vector from the BS is bs and the signal received by users is given by:

$$y_k = \boldsymbol{b}^H \boldsymbol{g}_k \boldsymbol{s} + \boldsymbol{n}_k, \quad k = 1, 2, \tag{3}$$

where $n_k \sim C\mathcal{N}(0,1)$ is received noise. Since $\beta_1 \geq \beta_2$, User 1 has is the stronger user with a higher average channel gain. According to the principle of successive interference cancellation (SIC), User 2 decodes s_2 from y_2 treating s_1 as noise. User 1 firstly decodes s_2 from y_1 treating s_1 as noise, then cancels the component of s_2 from y_1 , and after that decodes s_1 without interference. The signal-to-interferenceplus-noise-ratios (SINRs) for User k to decode s_l are thus given by:

$$\operatorname{SINR}_{1,s_1} = P_1 \beta_1 \left| \boldsymbol{b}^H \boldsymbol{h}_1 \right|^2, \tag{4}$$

SINR_{k,s2} =
$$\frac{P_2 \beta_k |\boldsymbol{b}^H \boldsymbol{h}_k|}{1 + P_1 \beta_1 |\boldsymbol{b}^H \boldsymbol{h}_k|^2}, \ k = 1, 2.$$
 (5)

It is noteworthy that the decoding order of our CB-NOMA scheme depends on user path-loss coefficients only, equivalently, depends on the *average* channel gains. This is different from the NOMA schemes in [6] and [7], where the decoding order depends on the *instantaneous* channel gains, i.e., depends on both β_k 's and \mathbf{h}_k 's. The latter requires extra communications between the BS and the users about the instantaneous CSI.

III. AVERAGE MINIMAL TRANSMIT POWER WITH SINR GUARANTEE

In this section, the average minimal transmit power for the CB-NOMA scheme to guarantee QoS of both users are studied. Denote γ_1 and γ_2 as the SINR requirements for the users. Since s_2 needs to be decoded successfully by both users while s_1 only needs to be decoded by User 1, to guarantee the QoS requirements, we need:

$$\min\left(\mathrm{SINR}_{1,s_2}, \mathrm{SINR}_{2,s_2}\right) \ge \gamma_2,\tag{6}$$

and
$$\operatorname{SINR}_{1,s_1} \ge \gamma_1$$
 (7)

By using $P_1 = P - P_2$ and (5) into (6), we have

$$P_{2} \geq \frac{\gamma_{2}}{1 + \gamma_{2}} \left(\frac{1}{\min\left(\beta_{1} \left| \boldsymbol{b}^{H} \boldsymbol{h}_{1} \right|^{2}, \beta_{2} \left| \boldsymbol{b}^{H} \boldsymbol{h}_{2} \right|^{2}\right)} + P \right).$$
(8)

By using (8) into (4) and $P_1 = P - P_2$, it can be shown that

$$\operatorname{SINR}_{1,s_{1}} \leq \frac{1}{1+\gamma_{2}} P\beta_{1} \left| \boldsymbol{b}^{H} \boldsymbol{h}_{1} \right|^{2} - \frac{\gamma_{2}}{1+\gamma_{2}} \cdot \frac{\beta_{1} \left| \boldsymbol{b}^{H} \boldsymbol{h}_{1} \right|^{2}}{\min \left(\beta_{1} \left| \boldsymbol{b}^{H} \boldsymbol{h}_{1} \right|^{2}, \beta_{2} \left| \boldsymbol{b}^{H} \boldsymbol{h}_{2} \right|^{2} \right)}.$$
⁽⁹⁾

Finally, substituting (9) into (7), we can obtain

$$P \ge P_{\min} \triangleq \frac{\gamma_1(1+\gamma_2)}{\beta_1 \left| \boldsymbol{b}^H \boldsymbol{h}_1 \right|^2} + \frac{\gamma_2}{\min\left(\beta_1 \left| \boldsymbol{b}^H \boldsymbol{h}_1 \right|^2, \beta_2 \left| \boldsymbol{b}^H \boldsymbol{h}_2 \right|^2\right)},\tag{10}$$

with equality if and only if (8) takes equality. P_{\min} defined in (10) is the achievable lower bound on the instantaneous transmit power to guarantee the SINR constraints for both users. The condition in (10) is only a necessary condition since γ_1 and γ_2 may not be achieved even when $P \ge P_{\min}$ with a non-optimal power allocation. On the other hand, if the optimum power allocation is used, the condition is both necessary and sufficient, that is, γ_1 and γ_2 are guaranteed for the two users if and only if $P \ge P_{\min}$.

We consider the matched filter (MF) beamforming based on channel vector of the stronger user, i.e.,

$$b = \frac{h_1}{\|h_1\|}.$$
 (11)

The minimal transmit power can be rewritten as:

$$P_{\min} = \frac{\gamma_1(1+\gamma_2)}{\beta_1 \|\boldsymbol{h}_1\|^2} + \frac{\gamma_2}{\min(\beta_1 \|\boldsymbol{h}_1\|^2, \beta_2 \|\boldsymbol{h}_2\|^2 \rho^2)}.$$
 (12)

The following theorem is proved for the average value of P_{\min} . *Theorem 1:* Define

$$\tilde{P}_{lo} \triangleq \frac{\gamma_1(1+\gamma_2)}{(M-1)\beta_1} + \frac{\gamma_2}{(M-1)\beta_2\rho_{th}^2}F(1,1;M;1-\rho_{th}^{-2}),\tag{13}$$

where $F(\cdot, \cdot; \cdot; \cdot)$ is the hypergeometric function and ρ_{th} is the CC threshold. The average minimal transmit power for CB-NOMA to guarantee SINR levels of both users, γ_1 and γ_2 , has the following lower and upper bounds:

$$\tilde{P}_{lo} \le \mathbb{E}\left[P_{\min}\right] \le \tilde{P}_{lo}\left(1 + \min\left\{\frac{\beta_2}{\beta_1}, \frac{\gamma_2}{\gamma_1}\right\}\right).$$
(14)

Proof See Appendix A.

Theorem 1 provides a lower and an upper bound on the minimal average transit power. It is the foundation of our analysis on the average minimal transmit power. In what follows, we provide several corollaries based on Theorem 1 for more insights.

Corollary 1: $\lim_{\rho_{th}\to 0} \mathbb{E}[P_{\min}] = \infty$ and $\mathbb{E}[P_{\min}] < \infty$ for any $\rho_{th} > 0, \gamma_1 > 0, \gamma_2 > 0$.

This result means that if $\rho_{th} = 0$, i.e., no constraint on the CC for using NOMA, any user SINR constraints cannot be guaranteed with finite average transmit power. This is problematic in energy efficiency. (19) shows that the unbounded average power is caused by the scenario when ρ_{th} is in the vicinity of zero, i.e., the two channel vectors are close-to-orthogonal. Naturally, a beam cannot serve the user with

channel vector orthogonal to the beam, (which is the weaker user as the beamformer is based on the channel of the stronger user). Thus the SINR constraint of that user $\gamma_2 > 0$ can never be achieved. This motivates the proposed CB-NOMA scheme with a non-zero threshold on the channel correlation coefficient.

Corollary 2: When $\gamma_1 \gg \gamma_2$ or $\beta_1 \gg \beta_2$, the average minimal power of CB-NOMA can be tightly approximated as P_{lo} , i.e., $\mathbb{E}[P_{\min}] \approx P_{lo}$.

The result can be obtained directly from (14). The application scenario for NOMA is to serve both users at the same time-frequency block when one user has a significantly stronger channel [2] and a large difference on large scale fading coefficients, i.e., $\beta_1 \gg \beta_2$ is beneficial to NOMA systems [4]. Given the difference in the average channel gains, it is also reasonable to expect a significantly better service to the stronger user, i.e., $\gamma_1 \gg \gamma_2$. For these scenarios, Corollary 2 provides a tight approximation on the average minimal transmit power for any SINR constraints.

The approximation P_{lo} given in (13) shows the behaviour of the average minimal transmit power with respect to the SINR constraints and other network parameters. For example, \tilde{P}_{lo} increases with γ_1 and γ_2 while decreases with ρ_{th} , β_1 , β_2 and M. To further explore the behavior of $\mathbb{E}[P_{\min}]$ with respect to M and ρ_{th} , we introduce the following asymptotic result.

Corollary 3: When $\gamma_1 \gg \gamma_2$ or $\beta_1 \gg \beta_2$, for any fixed M, when $\rho_{th} \rightarrow 0$,

$$\mathbb{E}[P_{\min}] \approx \frac{\gamma_1(1+\gamma_2)}{(M-1)\beta_1} - \frac{\gamma_2}{\beta_2} \frac{\ln \rho_{th}^2 + \psi(M-1) + C}{\left(1 - \rho_{th}^2\right)^{M-1}}, \quad (15)$$

where $\psi(\cdot)$ is the di-gamma function and $C \approx 0.5772$ is the Euler-Mascheroni constant.

Proof See Appendix B.

(15) provides a closed-form expression for the average power when ρ_{th} is close to zero. It shows that the average power to guarantee SINR constraints increases as $\ln(1/\rho_{th}^2)$ for small threshold.

Next, we consider massive MIMO where $M \gg 1$ and study the asymptotic behaviour with respect to the scaling of both M and ρ_{th} jointly. Both the average transmit power and the outage probability are considered to see the tradeoff. With the proposed CB-NOMA, the users are in outage if and only if the BS is silent, i.e., the absolute CC is smaller than the threshold, the outage probability is thus,

$$P_{\text{out}} = \mathbb{P}\left[\rho < \rho_{th}\right] = 1 - \left(1 - \rho_{th}^2\right)^{M-1}, \qquad (16)$$

which is an increase function of M and ρ_{th} .

(16) shows that with a fixed M, the outage probability increases as ρ_{th} increases; at the same time, the power consumption decreases. Thus we can adjust the balance between power consumption and outage performance via the design of ρ_{th} . Notice that with a fixed ρ_{th} value, the outage probability increases as M increases, though the power consumption reduces. Thus for massive MIMO systems, a threshold design where ρ_{th} decreases with M is desirable. For this matter, we have the following results.



Fig. 1. Average minimal power versus ρ_{th}^2 where M = 8 and 16, $\beta_1 = 0$ dB, $\beta_2 = -10 \text{dB}, \ \gamma_1 = 10 \text{dB}, \ \gamma_2 = 0 \text{dB}.$

Corollary 4: When $M \to \infty$ and $\rho_{th}^2 = \lambda/M^{\tau}$ for a constant $\lambda > 0$, the following results on the average transmit power and the outage probability can be obtained:

- when $\tau > 1$, $P_{\text{out}} \to 0$ and $P_{lo} \to \infty$;
- when $\tau > 1$, $P_{\text{out}} \to 1$ and $\tilde{P}_{lo} \to 0$; when $\tau = 1$, $P_{\text{out}} \to 1 e^{-\lambda}$ and $\tilde{P}_{lo} \to \frac{\gamma_2}{\beta_2} e^{\lambda} E_1(\lambda)$,

where $E_1(\cdot)$ is the exponential integral function.

Proof See Appendix C.

Corollary 4 shows the limits of $\mathbb{E}[P_{\min}]$ and P_{out} . The two performance measures naturally compete with each other since a higher outage probability means less transmissions and less power consumption realized with a higher ρ_{th} . The most interesting threshold design is when $\tau = 1$, meaning that the square of threshold decreases linearly in M, i.e., $\rho_{th}^2 = \lambda/M$ for a fixed λ . In this case, both $\mathbb{E}[P_{\min}]$ and P_{out} have nontrivial bounded limits and by adjusting the value of λ , we can achieve a continuous tradeoff curve for the power consumption and outage probability. Another observation is that the limits are independent of γ_1 and β_1 , the two parameters of the stronger user. The outage probability limit is also independent of the parameters of the weaker user, while the average power consumption depends on γ_2 and β_2 , meaning that the power consumption of CB-NOMA in a massive MIMO scenario is dominated by the weaker user.

IV. NUMERICAL RESULT

In this section, simulation results are demonstrated to show the performance of the proposed CB-NOMA scheme, as well as to verify the the accuracy of our analytical results.

In Fig. 1, the average minimal power of the CB-NOMA scheme to guarantee the SINR requirements for the two users is shown as a function of ρ_{th}^2 , where $\beta_1 = 0$ dB, $\beta_2 = -10$ dB, $\gamma_1 = 10 \mathrm{dB}, \ \gamma_2 = 0 \mathrm{dB}.$ It can be seen that the average minimal power decreases with ρ_{th}^2 . Further, the figure also shows that P_{lo} is an accurate approximation for all parameter values, while the result in (15) is tight for small ρ_{th}^2 , e.g., when $\rho_{th}^2 < 0.1$. We can also see that the average minimal transmit



Fig. 2. Average minimal power versus M where $\rho_{th}^2 = 0.02$ and 0.005, $\beta_1 = 0$ dB, $\beta_2 = -10$ dB, $\gamma_1 = 10$ dB, $\gamma_2 = 0$ dB.



Fig. 3. Average minimal power versus M where $\rho_{th}^2 = 1/M^{\tau}$, $\beta_1 = 0$ dB, $\beta_2 = -10$ dB, $\gamma_1 = 10$ dB, $\gamma_2 = 0$ dB

power increases linearly in $\log(1/\rho_{th}^2)$ as ρ_{th}^2 approaches 0, which verifies the asymptotic behaviour in Corollary 3.

Fig. 2 depicts the average minimal power versus M with the same values of $\gamma_1, \gamma_2, \beta_1, \beta_2$ as before. The accuracy of the approximations in Corollary (13) and (15) is verified again. The average minimal transmit power decreases as the number of antennas M increases.

Fig. 3 shows the average minimal power and outage probability versus M where $\rho_{th}^2 = 1/M^{\tau}$, while other parameter values are the same as before. When $\tau = 1$, the average minimal power decreases with M and converges to a positive constant, which matches perfectly with the asymptotic result; when $\tau = 0.5 < 1$, it decreases with M and approaches to 0; when $\tau = 2 > 1$, it increases unbounded with M, which validate the results in Corollary 4. And the behavior of the outage probability also matches the results in Corollary 4.

V. CONCLUSION

A modified NOMA scheme based on the channel correlation coefficient is proposed in this work for systems with multipleantennas at the BS and two single-antenna users. We derived the average minimal transmit power with QoS guarantee for both users and proved that a positive channel correlation threshold is required for finite average transmit power. The behaviour of the average transmit power with respect to the threshold and the BS antenna number is shown. Moreover, to balance the outage probability and average transmit power in massive MIMO systems, we proposed to design the threshold as a decreasing function with the number of BS antennas. The scaling laws of the outage probability and average minimal power as well as their tradeoff law are derived based on the threshold design.

APPENDIX A Proof of Theorem 1

Since h_1 and h_2 are independent each following $\mathcal{CN}(\underline{0}, \mathbf{I})$, $\|h_1\|^2$ and $\|h_2\|^2$ are independent following the Gamma distribution with shape parameter M and scale parameter 1; ρ^2 follows the Beta distribution with parameters 1 and M-1 [9]; and $\|h_1\|^2, \|h_2\|^2, \rho^2$ are mutually independent.

From (12) we can find the following lower bound of P_{\min} :

$$P_{\min} \ge P_{lo} = \frac{\gamma_1(1+\gamma_2)}{\beta_1 \|\boldsymbol{h}_1\|^2} + \frac{\gamma_2}{\beta_2 \|\boldsymbol{h}_2\|^2 \rho^2}.$$
 (17)

Thus, by using the probability density functions (PDFs) of Gamma distribution and Beta distribution, the following can be derived:

$$\mathbb{E}\left[P_{\min}\right] \geq \iiint_{V} f_{\|\boldsymbol{h}_{1}\|^{2}}(x) f_{\|\boldsymbol{h}_{2}\|^{2}}(y) f_{\rho^{2}}(z) P_{\min,\log} dx dy dz$$

$$= \underbrace{\frac{\gamma_{1}(1+\gamma_{2})}{(M-1)\beta_{1}}}_{T_{0}} + \underbrace{\iiint_{V} f_{\|\boldsymbol{h}_{2}\|^{2}}(y) f_{\rho^{2}}(z) \frac{\gamma_{2}}{\beta_{2}yz} dx dy dz}_{T} \quad (18)$$

$$= T_{0} + \frac{\gamma_{2}}{(1-\rho_{th}^{2})^{M-1}\beta_{2}} \int_{\rho_{th}^{2}}^{1} \frac{(1-\rho^{2})^{M-2}}{\rho^{2}} d\rho^{2} = \tilde{P}_{lo}, (19)$$

where $V = \{(x, y, z) | x \in (0, \infty), y \in (0, \infty), z \in [\rho_{th}^2, 1]\}, f_X(\cdot)$ represents the PDF, and the last step can be obtained from the definition of hypergeometric function with some mathematical manipulations.

Next, we show the upper bound. First, define $V_1 \triangleq \{(x, y, z) | \beta_1 x < \beta_2 yz\}$ and $V_2 \triangleq \{(x, y, z) | \beta_1 x \ge \beta_2 yz\}$. By noticing that $V = V_1 \cup V_2$, we have from (18),

$$\mathbb{E}\left[P_{\min}\right] = T_0 + \underbrace{\iiint_{V_1} f_{\parallel \boldsymbol{h}_1 \parallel^2}(x) \frac{\gamma_2}{\beta_1 x} \mathrm{d}x \mathrm{d}y \mathrm{d}z}_{T_1} + \underbrace{\iiint_{V_2} f_{\parallel \boldsymbol{h}_2 \parallel^2}(y) f_{\rho^2}(z) \frac{\gamma_2}{\beta_2 y z} \mathrm{d}x \mathrm{d}y \mathrm{d}z}_{T_2}.$$
(20)

Since $V_2 \subseteq V$, we have $T_2 \leq T$ and thus:

$$T_0 + T_2 \le T_0 + T = \tilde{P}_{lo}.$$
 (21)

$$T_{1} \leq \frac{\gamma_{2}}{\gamma_{1}} \iiint_{V} f_{\parallel \boldsymbol{h}_{1} \parallel^{2}}(x) \frac{\gamma_{1}}{\beta_{1}x} \mathrm{d}x \mathrm{d}y \mathrm{d}z$$
$$\leq \frac{\gamma_{2}}{\gamma_{1}} \iiint_{V} f_{\parallel \boldsymbol{h}_{1} \parallel^{2}}(x) \frac{\gamma_{1}(1+\gamma_{2})}{\beta_{1}x} \mathrm{d}x \mathrm{d}y \mathrm{d}z = \frac{\gamma_{2}}{\gamma_{1}} T_{0}, (22)$$

and

$$T_{1} \leq \frac{\beta_{2}}{\beta_{1}} \iiint_{V} f_{\parallel \mathbf{h}_{2} \parallel^{2}}(y) \frac{\gamma_{2}}{\beta_{2} y} \mathrm{d}x \mathrm{d}y \mathrm{d}z$$
$$\leq \frac{\beta_{2}}{\beta_{1}} \iiint_{V} f_{\parallel \mathbf{h}_{2} \parallel^{2}}(y) f_{\rho^{2}}(z) \frac{\gamma_{2}}{\beta_{2} y z} \mathrm{d}x \mathrm{d}y \mathrm{d}z = \frac{\beta_{2}}{\beta_{1}} T.$$
(23)

From (22) and (23) we can obtain:

$$T_1 \le \min\left\{\frac{\beta_2}{\beta_1}, \frac{\gamma_2}{\gamma_1}\right\} (T_0 + T) = \min\left\{\frac{\beta_2}{\beta_1}, \frac{\gamma_2}{\gamma_1}\right\} \tilde{P}_{lo}.$$
 (24)

By combining (20), (21) and (24), the upper bound of $\mathbb{E}[P_{\min}]$ in (14) is proved.

APPENDIX B PROOF OF COROLLARY 3

When $\gamma_1 \gg \gamma_2$ or $\beta_1 \gg \beta_2$, we have from Corollary 2 $\mathbb{E}[P_{\min}] \approx \tilde{P}_{lo}$ as defined in (13). In Appendix A, an alternative form of \tilde{P}_{lo} is derived in (19). The integral in (19) can be further calculated as follows:

$$\int_{\rho_{th}^{2}}^{1} \frac{\left(1-\rho^{2}\right)^{M-2}}{\rho^{2}} d\rho^{2}$$

$$= \sum_{k=0}^{M-2} (-1)^{k} \int_{\rho_{th}^{2}}^{1} \binom{M-2}{k} \rho^{2(k-1)} d\rho^{2}$$

$$= \sum_{k=1}^{M-2} \frac{(-1)^{k}}{k} \binom{M-2}{k} \left(1-\rho_{th}^{2k}\right) - \ln \rho_{th}^{2}$$

$$= -\psi(M-1) - C - \ln \rho_{th}^{2} + \mathcal{O}(\rho_{th}^{2}).$$

When $\rho_{th}^2 \to 0$, by ignoring the higher order terms of ρ_{th}^2 , (15) is obtained.

APPENDIX C Proof of Corollary 4

The limits of P_{out} in Corollary 4 can be obtained by:

$$\lim_{M \to \infty} \left(1 - \frac{\lambda}{M^{\tau}} \right)^{M-1} = \begin{cases} 0 & , \tau < 1 \\ e^{-\lambda} & , \tau = 1 \\ 1 & , \tau > 1. \end{cases}$$
(25)

For the average transmit power, we consider the alternative form of \tilde{P}_{lo} given by (19). When $\rho_{th}^2 = \frac{\lambda}{M^{\tau}}$, the limit of \tilde{P}_{lo} is given by:

$$\lim_{M \to \infty} \tilde{P}_{lo} = \lim_{M \to \infty} \left[\frac{\gamma_2}{\left(1 - \frac{\lambda}{M^{\tau}}\right)^{M-1} \beta_2} I \right], \qquad (26)$$

where

$$I \triangleq \int_{\frac{\lambda}{M^{\tau}}}^{1} \frac{\left(1-\rho^{2}\right)^{M}}{\rho^{2}} \mathrm{d}\rho^{2} = \frac{y=M\rho^{2}}{2} \int_{\lambda M^{1-\tau}}^{M} \frac{\left(1-\frac{y}{M}\right)^{M}}{y} \mathrm{d}y.$$
(27)

Now notice that $(1 + \frac{y}{M})^M < e^y < (1 + \frac{y}{M})^{M+1}$ for any $M \ge 1$, thus

$$1 > (1 - \frac{y^2}{M^2})^M = (1 - \frac{y}{M})^M [(1 + \frac{y}{M})^{M+1}]^{\frac{M}{M+1}} > \left(1 - \frac{y}{M}\right)^M e^{\frac{M}{M+1}y},$$
(28)

$$(1 - \frac{y^2}{M^2})^M = (1 - \frac{y}{M})^M (1 + \frac{y}{M})^M < (1 - \frac{y}{M})^M e^y.$$
(29)

These gives, for all $M \ge 1$,

$$\frac{(1-\frac{y^2}{M^2})^M}{y}e^{-y} \leqslant \frac{(1-\frac{y}{M})^M}{y} \leqslant \frac{1}{y}e^{-\frac{y}{2}}.$$
 (30)

Consequently, when $\tau < 1$, we have, as $M \to \infty$,

$$I \leqslant \int_{\lambda M^{1-r}}^{M} \frac{1}{y} e^{-\frac{y}{2}} \mathrm{d}y \leqslant \frac{1}{\lambda} M^{r-1} \int_{0}^{\infty} e^{-\frac{y}{2}} \mathrm{d}y \longrightarrow 0.$$
(31)

When $\tau > 1$,

$$I \ge \int_{\lambda M^{1-r}}^{1} \frac{(1-\frac{1}{M^2})^M}{y} e^{-y} \mathrm{d}y \longrightarrow \infty.$$
(32)

Finally when $\tau = 1$, by (30), and $(1 - \frac{y}{M})^M \longrightarrow e^{-y}$ for every y, we have

$$I \longrightarrow \int_{\lambda}^{\infty} \frac{e^{-y}}{y} \mathrm{d}y \tag{33}$$

through Lebesgue's dominated convergence theorem. Thus ends the proof.

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