Blockwise Phase Rotation-Aided Analog Transmit Beamforming for 5G mmWave Systems

Md. Abdul Latif Sarker, Igbafe Orikumhi, Dong Seog Han and Sunwoo Kim

Abstract—In this letter, we propose a blockwise phase rotation-aided analog transmit beamforming (BPR-ATB) scheme to improve the spectral efficiency and the bit-error-rate (BER) performance in millimeter wave (mmWave) communication systems. Due to the phase angle optimization issues of the conventional analog beamforming, we design the BPR-ATB for reducing the rotated beamspace of the equivalent channel and improving the minimum Euclidean distance. To verify the effectiveness of the proposed BPR-ATB scheme, we employ an Alamouti coding technique at the transmitter and evaluate the bit-error-rate performance for mmWave multiple-input and single-output systems. The simulation results show that the proposed BPR-ATB scheme outperforms the conventional discrete Fourier transform-based ATB scheme.

Index Terms—5G-millimeter-wave systems, blockwise phase rotation, Alamouti coding, spectral, and BER performance.

I. INTRODUCTION

THE millimeter-wave (mmWave) technology plays a major role in the fifth-generation (5G) wireless communications owing to the large bandwidth [1] and spectral efficiency [2], [3]. The mmWave technology operates in the 30 to 300 GHz band [1], [2], hence the large spectral resource in contrast with microwave technologies operating in the sub 6 GHz band [4]. Typically, mmWave system requires massive antenna arrays, which are equipped with the base station (BS) for achieving a highly directive beamforming [5]. For deployment of this system, the leading barriers are the hardware limitations, the channel sparsity, the free-space path loss, beamforming construction, and phase angle optimization.

The sparse nature of the channel and the discrete Fourier transform-based analog beamforming (DFT-ATB) schemes have been investigated in [1], [3], [5]–[9]. The authors designed a joint antenna selection based transmit beamforming in [7]. A phase control DFT based hybrid precoding scheme is presented in [8], [9]. Particularly, the traditional analog beamforming incurs a quantization error in communication systems owing to their low minimum Euclidean distance [4], [8], [10]. In addition, the conventional DFT-ATB scheme shows a 'beam squint' challenge with a wideband channel [6], [11]. The 'beam squint' leads a higher channel spreading factor due to the structural leakage of the conventional analog beamforming. To get a betterminimum Euclidean distance of

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analog precoding, the authors proposed a Golden-Hadamard (GH) based precoding in [12]. Although the GH scheme achieved a remarkable bit-error-rate (BER) performance in microwave systems, the scheme shows a phase angle optimization problem in highly directive wireless systems due to their wide rotated beamspace. Hence, we design a blockwise phase rotation-aided analog transmit beamforming (BPR-ATB) for mmWave communication systems.

In this article, we propose a BPR-ATB scheme to minimize the rotated beamspace of the equivalent channel and improving the minimum Euclidean distance of traditional analog beamforming such as the DFT-ATB scheme. To this end, we seek to obtain an efficient rotated beamspace of the equivalent channel and improve the minimum Euclidean distance. We first run back [12, eq. (9)] and then design a BPR-ATB scheme to get the effective rotated beamspace and generate a satisfactory spectral efficiency of the mmWave communications. After that, we implement the proposed BPR-ATB scheme with Alamouti code and set a power factor-based parameter κ in the BER performance metric. Finally, we show the superiority of the proposed BPR-AB scheme over the DFT-ATB scheme in terms of a downlink mmWave multiple-input and single-output (MISO) systems through computer simulations.

II. CHANNEL AND SIGNAL MODELS

We consider a downlink mmWave MISO system with N_t transmit antennas and a single antenna receiver. Then the received signal vector $\mathbf{y} \in \mathbb{C}^{1 \times T}$ can be modeled as

$$\mathbf{y} = \sqrt{\frac{PN_t}{L}} \mathbf{h}^H \mathbf{X} + \mathbf{z},\tag{1}$$

where P denotes the transmit power, L is the number of paths, \mathbf{X} is the $N_t \times T$ space-time codeword matrix, T is the number of time slot, and $\mathbf{z} \sim \mathcal{CN}(0,1)$ is additive white Gaussian noise vector with zero-mean and unit variance. The narrowband mmWave channel $\mathbf{h} \in \mathbb{C}^{N_t \times 1}$ with L propagation paths [1], [5], that is

$$\mathbf{h} = \sum_{l=1}^{L} \alpha_l \mathbf{a}(\theta_l), \tag{2}$$

where α_l is the complex gain of the l-th path, θ_l represents the angle of departure of the l-th path, $\mathbf{a}(\theta_l)$ denotes the transmit steering vector of the l-th path, which is given by

$$\mathbf{a}(\theta_l) = [1, e^{j\frac{2\pi d}{\lambda}\sin\theta_l}, ..., e^{j\frac{(N_t - 1)\pi d}{\lambda}\sin\theta_l}]^T, \tag{3}$$

the wavelength, $\lambda = c/f_c$, c is the speed of light, f_c is the carrier frequency, and $d = \lambda/2$ is the antenna spacing.

III. BLOCKWISE PHASE ROTATION-AIDED ANALOG TRANSMIT BEAMFORMING (BPR-ATB) SCHEME

A. BPR-ATB matrix design

Due to the phase angle optimization and beam squint issues in the high dimensional ATB scheme, we first run back [12, eq. (9)] and then we design the Golden Hadamard based BPR-ATB scheme in this section. Let the number of total transmit antennas $N_t = 2^q$ and $\xi = n\{(1+n)^q - (1-n)^q\}/2^q$ where $q = \log_2 N_t$ and n denotes the root of the geometric number. Consider the space-time codeword matrix \mathbf{X} as

$$\mathbf{X} = \mathbf{F}_{2q} \mathbf{S},\tag{4}$$

where S is a $2^{q-1} \times T$ orthogonal space time block code matrix, \mathbf{F}_{2^q} be a propose $2^q \times 2^{q-1}$ BPR-ATB matrix constructed by 2^{q-1} columns of a $2^q \times 2^q$ recursive Golden-Hadamard matrices as follows

$$\mathbf{F}_{2^{q}} = \frac{g}{\sqrt{\xi}} \begin{bmatrix} \mathbf{W}_{2^{q-1}} \mathbf{A}_{2^{q-1}} & \mathbf{W}_{2^{q-1}} \mathbf{B}_{2^{q-1}} \\ \mathbf{W}_{2^{q-1}} \mathbf{B}_{2^{q-1}} & -\mathbf{W}_{2^{q-1}} \mathbf{A}_{2^{q-1}} \end{bmatrix}, \quad (5)$$

where g denotes the golden number [12], [13], $\mathbf{W}_{2^{q-1}}$ is a $2^{q-1} \times 2^{q-1}$ block Hadamard matrix, $\mathbf{A}_{2^{q-1}} = \mathrm{diag}\{e^{j\phi_{\nu_1}}\}$ and $\mathbf{B}_{2^{q-1}} = \mathrm{diag}\{e^{j\phi_{\nu_2}}\}$ are the $2^{q-1} \times 2^{q-1}$ block-diagonal phase rotation matrix, ν_1 and ν_2 are the block-order of $\mathbf{A}_{2^{q-1}}$ and $\mathbf{B}_{2^{q-1}}$. By substituting (4) in (1), the system can achieve a spectral efficiency for MISO system given as

$$R = \log_2 \left\{ 1 + \frac{P}{\sigma^2} \mathbf{h}^H \mathbf{F}_{2^q} \mathbf{F}_{2^q}^H \mathbf{h} \right\}. \tag{6}$$

B. Problem formulation

Particularly, the phase rotation on the transmitted signals is effectively equivalent to rotating the phases of the corresponding channel coefficients. It should be noted that, the

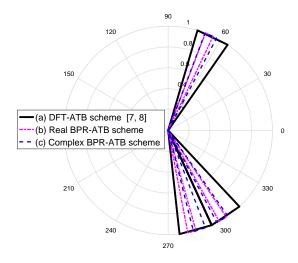


Fig. 1. The rose diagram of the rotated beamspace for the effective equivalent mmWave channel (a) the conventional DFT-ATB scheme [7], [8] in solid lines, (b) and (c) the effective equivalent mmWave channel with the proposed BPR-ATB scheme in doted lines.

conventional DFT-ATB scheme generates a satisfactory array gain with equivalent channel [7], [8], but this scheme suffers a phase angle optimization problem, which leads to a wide beamspace of the equivalent channel as shown in Fig. 1. Using by (6) and [7], the optimization problem can be formulated

$$\mathbf{f}_{2^{q}}^{opt} = \operatorname*{arg\,max}_{\mathbf{f}_{2^{q}}} R$$

$$s.t. \ \mathbf{f}_{2^{q}} \in \left\{ \mathcal{F}, 0 \right\}^{2^{q}},$$
and $\|\mathbf{f}_{2^{q}}\|_{0} = 2^{q-1},$

$$(7)$$

where \mathbf{f}_{2^q} denotes the vector of \mathbf{F}_{2^q} . We observe that the (7) maximizes the spectral efficiency but it has a non-convex objective function, which conducts a phase angle optimization problem. In addition, the spectral efficiency (6) is a correctly monotone enhancing function of beamforming gain $|\mathbf{hf}_{2^q}|$. To simplify this problem, we formulate the phase angle optimization as below:

Let h_{ν} be the ν -th element of \mathbf{h} and $\mathbf{f}_{2^q,\nu} \in \mathcal{F}$ be the effective analog beamforming vector. Thus, the phase angle optimization problem is given by

$$\varphi_{\nu}^{opt} = \arg\max_{\varphi_{\nu}} \left| \sum_{\nu=1}^{2^{q}} h_{\nu}^{*} \mathbf{f}_{2^{q}, \nu}(\varphi_{\nu}) \right|$$

$$s.t. \ \varphi_{\nu} \in \left\{ \frac{2\pi b}{2^{q}} \middle| b = 0, 1..., 2^{q} - 1 \right\},$$
and $\nu = 1, ..., 2^{q},$

$$(8)$$

where $\mathbf{f}_{\nu}(\varphi_{\nu}) = e^{j\varphi_{\nu}}/\sqrt{2^q}$. We observe the (8) is still leading an optimization problem due to the global phase angle, which generates an extensive beamspsce with a mmWave channel. As a result, the user suffers from a high computational burden to optimize the global phase angle. To overcome the phase angle optimization problem, we reformulate (8) and propose the BPR-ATB based algorithm in the Subsection C.

C. Proposed BPR-ATB based algorithm

Let $\varphi_{\nu} \in \{\phi_{\nu_1}, \phi_{\nu_2}\}$, where ϕ_{ν_1} and ϕ_{ν_2} are the block phase angle of $\mathbf{A}_{2^{q-1}}$ and $\mathbf{B}_{2^{q-1}}$. We set ϕ_{ν_1} and ϕ_{ν_2} in the designed transmit beamformer of \mathbf{F}_{2^q} . Then the optimal block phase angle is given by

$$\varphi_{\nu}(\phi_{\nu_{1}}^{opt}, \phi_{\nu_{2}}^{opt}) = \underset{\phi_{\nu_{1}}, \phi_{\nu_{2}}}{\operatorname{arg\,max}} \left| \sum_{\nu=1}^{2^{q}} h_{\nu}^{*} e^{j\phi_{\nu_{1}}} e^{j\phi_{\nu_{2}}} \right|$$

$$s.t. \ \phi_{\nu_{1}} \in \left\{ \frac{2\pi b_{1}}{2^{q-1}} \middle| b_{1} = 0, 1..., 2^{q-1} - 1 \right\},$$

$$\phi_{\nu_{2}} \in \left\{ \frac{2\pi b_{2}}{2^{q-1}} \middle| b_{2} = 2^{q-1}..., 2^{q} - 1 \right\},$$

$$\nu_{1} = \nu_{2} \text{ and } b_{1} \neq b_{2}.$$

$$(9)$$

Consider C as the set of indexes of useful antennas and $\nu \in \{S_{\nu_1}, S_{\nu_2}\}$, where S_{ν_1} and S_{ν_2} are the subset of beam indices. Based on (9), we demonstrate the proposed BPR-ATB scheme in **Algorithm 1**.

Algorithm 1 Proposed BPR-ATB based Algorithm

- 1: **Input parameters:** b_1 , b_1 , b_2 q.
- 2: Output: $\mathbf{f}_{2^q,v}^{opt}$.
- 3: Obtain $\varphi_{\nu}(\phi_{\nu_1}^{opt}, \phi_{\nu_2}^{opt})$.
- 4: **Begin** $C := \{\nu\}, \ \nu \in \{S_{\nu_1}, S_{\nu_2}\}, \ \text{and} \ S_{\nu_1} = S_{\nu_2} = \emptyset.$
- 5: **for** $\nu_1 = 1 : 2^{q-1}$ **do**
- 6: Find

6: Find
$$\nu^{opt_1} = \arg\max_{\nu \in \mathcal{C}} \left| \sum_{\nu_1 \in \mathcal{S}_{\nu_1}} (h_{\nu_1}^* + h_{\nu}^* e^{j\phi_{\nu_2}}) e^{j\phi_{\nu_1}} \right|.$$
7: $\mathcal{S}_{\nu_1} := \mathcal{S}_{\nu_1} \bigcup \{\nu^{opt_1}\}.$
8: $\mathcal{C} := \mathcal{C} \mod \{\nu^{opt_1}\}.$

- 9: end for
- 10: **for** $\nu_2 = 1:2^{q-1}$ **do**
- 11: Find

$$\nu^{opt_2} = \arg\max_{\nu \in \mathcal{C}} \left| \sum_{\nu_2 \in \mathcal{S}_{\nu_2}} (h_{\nu_2}^* + h_{\nu}^* e^{j\phi_{\nu_1}}) e^{j\phi_{\nu_2}} \right|$$

- 12: $S_{\nu_2} := S_{\nu_2} \bigcup \{ \nu^{opt_2} \}$
- 13: $C := C \mod \{ \nu^{opt_2} \}.$
- 14: end for
- 15: Obtain $\mathbf{f}_{2^q,v}^{opt} = (ge^{j\phi_{\nu_1}^{opt}}e^{j\phi_{\nu_2}^{opt}})/\sqrt{\zeta}$ according to (9).

IV. SIMULATION RESULTS AND DISCUSSION

In this section, we compare the proposed BPR-ATB scheme against the conventional DFT-ATB scheme via computer simulations. To show the superiority of the proposed scheme, we employ a complex Alamouti coding technique at the transmitter. For example, if we use the k-th entry of a complex alamouti code with time slot T=2 in (4) where we consider T is equal to the number of radio frequency chain, then the codeword matrix X is given by

$$\mathbf{K} = \sqrt{\gamma_0 \kappa} \mathbf{F} \mathbf{S}_k$$

$$= \sqrt{\gamma_0 \kappa} \mathbf{F} \begin{bmatrix} s_{11} & -s_{21}^* \\ s_{21} & s_{11}^* \end{bmatrix}, \tag{10}$$

where S_k is the k-th entry of 2×2 complex Alamouti code [14], symbols s_{11} and s_{21} belongs to a quadrature amplitude modulation (QAM) constellation, $\kappa = |\mathbf{F}_{i,j}|^2$ is a power factor of the (i,j) element of analog beamforming **F**, and γ_0 is the received signal-to-noise power (SNR). The κ value depends on the structure of F matrix (see in Table I).

Consider S_k and S_l as the transmitted and decoded spacetime codewords, respectively, where $k \neq l$. The union bound on the bit-error-rate (BER) is formulated as [16]

$$BER \le \sum_{k \ne l} \frac{e(\mathbf{S}_k, \mathbf{S}_l)}{\log_2(M)} Q\left(\Xi_{k,l} \sqrt{\frac{\gamma_0 \kappa}{2}}\right), \tag{11}$$

TABLE I DIFFERENT POWER FACTOR OF RF PRECODING

Power factor of RF precoding, $\kappa = \mathbf{F}_{i,j} ^2$				
DFT [4], [8]	HA [15]	BPR		
		g-real value case	g-complex value case	
1	1	$(1+\sqrt{5})^2$	$(j+\sqrt{3})^2$	
$\overline{N_t}$	$\overline{N_t}$	4ξ	-4ξ	

where M denotes the constellation size, the operator $Q(\cdot)$ represents the Q-function [16], $\Xi_{k,l} = \|\mathbf{h}_{eq}^H \mathbf{e}_{k,l}\|_F$ where the operator $\|\cdot\|_F$ denotes a Frobenius norm, $\mathbf{e}_{k,l} = \mathbf{S}_k - \mathbf{S}_l$ represents an error matrix between the codewords S_k and S_l , $e(\mathbf{S}_k, \mathbf{S}_l)$ is the Hamming distance between the bit mappings corresponding to the vectors \mathbf{S}_k and \mathbf{S}_l . Invoking the Chernoff upper bound $Q(x) \leq e^{-x^2/2}$ in (11), the pairwise error probability can be upper bounded with an equivalent channel \mathbf{h}_{eq} as

$$P_r(\mathbf{S}_k \to \mathbf{S}_l | \mathbf{h}_{eq}) \le e^{-\frac{\gamma_0 \kappa \Xi_{k,l}^2}{4}}, \tag{12}$$

where $\mathbf{h}_{eq} = \mathbf{F}^H \mathbf{h}$, and $\Xi_{k,l}^2$ is the minimum Euclidean

$$\Xi_{k,l}^2 = \underset{\mathbf{S}_k \neq \mathbf{S}_l}{\arg \min} ||\mathbf{h}^H \mathbf{F}_{2^q} \mathbf{e}_{k,l}||_F,$$
(13)

where the property of the error matrix $\mathbf{e}_{k,l}\mathbf{e}_{k,l}^H=a\mathbf{I}$ for the orthogonal space-time block coding, a is a constant depending on the constellation [see in APPENDIX A].

Throughout the simulations, we assumed the parameters of Table II with different κ factor of Table I. In the case of mmWave system, we use a sparse geometric mmWave channel model [5] with L=3 paths and angles of departure uniformly distributed over $[-\pi/2, \pi/2]$. Since the exhaustive search method (8) obviously measures a high complexity, as a results, we designed a sub-optimal BPR-ATB algorithm in **Algorithm 1**. The total time complexity of the **Algorithm 1** is $\mathcal{O}(2^q\zeta)$, which is adoptable even if q is large and $2^q<\zeta$. We further consider the 2×2 block-diagonal rotation matrices with

TABLE II SIMULATION PARAMETERS

Total number of transmit antennas	$N_t = 4$
Total number of receive antenna	$N_r = 1$
Number of time slot	T=2
Channel path	L = 3
SNR	$\gamma_0 = 20 \text{ dB}$
Carrier frequency	$f_c = 60 \text{ GHz}$
Wavelength	$\lambda = 5 \text{ mm}$
Antenna spacing distance	$d = \lambda/2$
Modulation scheme	64 QAM

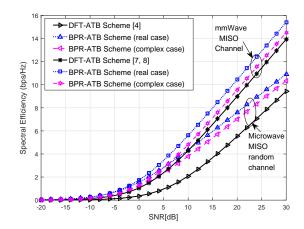


Fig. 2. Spectral Efficiency versus SNR ($N_t=4,N_{RF}=T=2,$ and $N_r = 1$ with different κ values of the analog transmit beamforming.

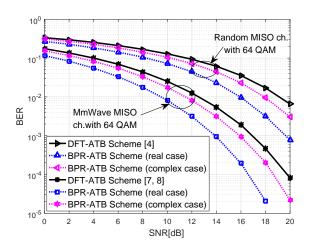


Fig. 3. BER performance of 64QAM modulated Alamouti coding with different κ values of analog transmit beamforming.

block phase angle $\phi_{\nu_1}=2\pi b_1/2^{q-1}$ and $\phi_{\nu_2}=2\pi b_2/2^{q-1}$ when $b_1\in\{0,1\}$ and $b_2\in\{2,3\}$ is applied in (9). We compute and set the parameter κ in the simulations such as $\kappa=0.25$ for both the DFT-ATB scheme [4] and the Hadamard based ATB scheme [15], $\kappa=0.52$ for the proposed real BRP-ATB scheme, and $\kappa=0.33$ for the proposed complex BRP-ATB scheme while q=2 is applied in Table I.

Fig. 2 illustrates the spectral efficiency comparisons of the proposed BPR-ATB scheme against the conventional DFT-ATB scheme. For the mmWave setup, the performance of the spectral efficiency is around 1.8 bits/s/Hz at 30 dB SNR values. The proposed BPR-ATB scheme also achieved good spectral performance with the traditional MISO random channel environment as shown in Fig. 2.

Fig. 3 shows the BER performance of the 4×2 BPR-ATB scheme using 2×2 Alamouti coding and 64QAM constellations. We see that the DFT-ATB scheme achieves a worse spectral efficiency and BER performance in [4], [7], [8] for both mmWave and the familiar microwave MISO systems owing to their rotated wide beamspace (see in Fig.1) and a teeny κ factor (see in Table I). The proposed BPR-ATB scheme provides at least 2 dB more BER performance with the DFT-ATB scheme. Furthermore, both spectral efficiency and BER performance of a complex case of the proposed BPR-ATB scheme is slightly worsened than that of a real case of the proposed BPR-ATB scheme because of the reduction of the value of the Golden ratio.

V. CONCLUSIONS

We proposed a BPR-ATB scheme to minimize the rotated beamspace of the equivalent mmWave channel and improve the error performance of the systems. We verify the effectiveness of the proposed BPR-ATB scheme by computer simulation and compare the performance with the conventional DFT-ATB scheme. The traditional DFT-ATB scheme exhibited a worse spectral efficiency about 1.8 bits/s/Hz and BER performance difference of 2 dB when compared with the proposed BPR-ATB scheme. Hence, the proposed BPR-ATB scheme

can be extended further to the next generation multiple-input and multiple-output non-orthogonal multiple-access (MIMO-NOMA) systems, which will be explored in future studies.

APPENDIX A

From (11), we can measure as a general single-input and single-output case of BER as follows:

$$P_{e,b} = \int_0^\infty Q(a\sqrt{\gamma}) p_{\gamma}(\gamma) d\gamma$$
$$= \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \mathcal{M}_{\gamma} \left(\frac{-a^2}{2\sin^2\theta}\right) d\theta$$
(14)

where $\gamma = \kappa \gamma_0/2$. The Gaussian Q-fuction Q(x) is given by [16]

$$Q(x) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \exp\left(\frac{-x^2}{2\sin^2\theta}\right) d\theta. \tag{15}$$

It is noted that in (14), $\mathcal{M}_X(-s)=\int_0^\infty e^{-sx}p_X(x)dx$ is a moment generating function (MGF) of random variable X where $\mathcal{M}_\gamma(s)=\int_0^\infty e^{s\gamma}p_\gamma(\gamma)d\gamma$ is the Laplace transform of $p_\gamma(\gamma)$ with the exponent reversed sign. By using the Rayleigh channel, we can consider $p_\gamma(\gamma)=\exp(-\gamma/\overline{\gamma})/\overline{\gamma}$, where $\gamma\geq 0$ and $\overline{\gamma}$ idenotes the average SNR per bit. Hence, the Laplace transform is given by

$$\mathcal{M}_X(-s) = \frac{1}{1+s\overline{\gamma}}, \ s > 0. \tag{16}$$

Substituting (16) into (14) gives

$$P_{e,b}(a,\overline{\gamma}) = \frac{1}{2} - \frac{a}{2}\sqrt{\frac{\overline{\gamma}}{2 + a^2\overline{\gamma}}}.$$
 (17)

For the simplicity, (17) is derived for the binary phase shift keying (when a=1) modulation scheme, which can be straightforwardly extended to other modulation cases. For example, M-ary phase shift keying modulation scheme: we consider $a^2=2\sin^2(\pi/M)$ and applying (16) in (14), then we follow directly (17) and obtain the bit-error-rate probability for Rayleigh fading channel as

$$P_{e,b}(a,\overline{\gamma},M) = \frac{(M-1)}{M} - \frac{\sqrt{\mu}}{2} + \frac{(M-1)\sqrt{\mu}}{M} \tan^{-1}\left(\sqrt{\mu}\cot\frac{\pi}{M}\right),$$
(18)

where $\mu=(\overline{\gamma}\sin^2\frac{\pi}{M})/(1+\overline{\gamma}\sin^2\frac{\pi}{M})$. Similarly, using (14) and (16), we can measure the BER probability for M-ary QAM modulation as

$$P_{e,b}(a, \overline{\gamma}, M) = \frac{4\zeta}{\pi} \int_0^{\frac{\pi}{2}} \left(1 + \frac{3\overline{\gamma}}{2(M-1)\sin^2\theta} \right)^{-1} d\theta$$
$$-\frac{4\zeta^2}{\pi} \int_0^{\frac{\pi}{4}} \left(1 + \frac{3\overline{\gamma}}{2(M-1)\sin^2\theta} \right)^{-1} d\theta, \tag{19}$$

where
$$a^2 = 3/(M-1)$$
 and $\zeta = 1 - (1/\sqrt{M})$.

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