Capacity Analysis of Holographic MIMO Channels with Practical Constraints

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Abstract—Holographic Multiple-Input and Multiple-Output (MIMO) is envisioned as a promising technology to realize unprecedented spectral efficiency by integrating a large number of antennas into a compact space. Most research on holographic MIMO is based on isotropic scattering environments, and the antenna gain is assumed to be unlimited by deployment space. However, the channel might not satisfy isotropic scattering because of generalized angle distributions, and the antenna gain is limited by the array aperture in reality. In this letter, we aim to analyze the holographic MIMO channel capacity under practical angle distribution and array aperture constraints. First, we calculate the spectral density for generalized angle distributions by introducing a wavenumber domain-based method. And then, the capacity under generalized angle distributions is analyzed and two different aperture schemes are considered. Finally, numerical results show that the capacity is obviously affected by angle distribution at high signal-to-noise ratio (SNR) but hardly affected at low SNR, and the capacity will not increase infinitely with antenna density due to the array aperture constraint.

Index Terms—Holographic MIMO, channel capacity, angle distribution, wavenumber domain, array aperture.

I. INTRODUCTION

With the increasing demand for data transmission, higher spectral efficiency technologies attract great interest. Holographic Multiple-Input and Multiple-Output (MIMO) is expected to realize incredible spectral efficiency based on the excellent performance of massive MIMO technology [1]. Compared with traditional massive MIMO systems, holographic MIMO systems are equipped with denser (possibly infinite) antennas in a limited space [2].

By using the channel degrees of freedom (DoFs), massive MIMO technology can significantly improve the channel capacity [3], [4]. With dense antenna deployment, holographic MIMO technology has the potential to further improve the channel capacity [5]. To clarify the performance improvement

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brought by holographic MIMO, it is necessary to analyze the channel model and capacity of the holographic MIMO system.

In [2], a holographic MIMO small-scale fading model in Fourier plane-wave spectral representation is proposed. And then, the channel model in Fourier expansion form and the method of generating non-isotropic channels are given in [6], [7]. Based on the channel model proposed in [2], the authors of [8] analyzed the upper limit of freedom of holographic MIMO in the wavenumber domain. In [9], the spatial correlation matrices of holographic MIMO systems with different antenna spacing are compared. To analyze the performance of the continuous source and destination, the authors of [10] derive the capacity bound for parallel linear source and destination based on electromagnetic information theory.

As discussed, existing works are mainly based on ideal assumptions. However, the angle distribution and array aperture constrain the capacity in reality. Therefore, this letter aims to analyze the realistic capacity of holographic MIMO channels with practical angle distribution and array aperture constraints. First, we use the transformation of coordinates in [2] to calculate the spectral density in the wavenumber domain for generalized angle distribution. Then, mathematical expressions for the capacity of holographic MIMO channels are presented. Finally, simulation results show the effects of angle spread, signal-to-noise ratio (SNR), propagation scenarios, array aperture, and antenna spacing on channel capacity.

Notations: Fonts a, a, and A represent scalars, vectors, and matrices, respectively. $n \sim \mathcal{N}_{\mathbb{C}}(0, \sigma^2)$ stands for a circularly-symmetric complex-Gaussian random variable with variance σ^2 . $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$ are three orthonormal vectors. \mathbf{A}^H denotes Hermitian of A. $\mathbb{E}\{\cdot\}$ is the expectation operator. $\lambda_i(\mathbf{A})$, rank(A), and tr (A) represent i-th sorted eigenvalue, rank, and trace of A ,respectively. diag (a) denotes the diagonal matrix with elements from a.

II. HOLOGRAPHIC MIMO CHANNEL MODELING

A. Channel Model in Wavenumber Domain

Consider a holographic MIMO communication system, where both the receiver (RX) and the transmitter (TX) are z-oriented planar arrays with a large number of elements arranged below half wavelength. RX and TX span the rectangular regions of xy-dimensions $L_{R,x}$, $L_{R,y}$ and $L_{S,x}$, $L_{S,y}$, respectively. The system model can be given as

$$\mathbf{y} = \sqrt{\rho G_t G_r \mathbf{H} \mathbf{x} + \mathbf{n}},\tag{1}$$

where $\mathbf{y} \in \mathbb{C}^{N_R}$ and $\mathbf{x} \in \mathbb{C}^{N_S}$ denote the received and transmitted signal vectors, respectively. $\mathbf{n} \in \mathbb{C}^{N_R}$ accounts

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for thermal noise that is distributed as $\mathbf{n} \sim \mathcal{N}_{\mathbb{C}} (\mathbf{0}, \sigma^2 \mathbf{I}_{N_R})$. $\mathbf{H} \in \mathbb{C}^{N_R \times N_S}$ is the small-scale fading channel matrix. G_t and G_r are the antenna gain of TX and RX, respectively, and ρ is the SNR.

For an electromagnetic wave in arbitrary direction, its azimuth ϕ and elevation θ can be mapped to the wavenumber domain, i.e.,

$$\begin{cases} k_x = k \sin \theta \cos \phi \\ k_y = k \sin \theta \sin \phi \\ k_z = k \cos \theta \end{cases}$$
(2)

where k is the wavenumber, k_x , k_y and k_z denote the wavenumber in x, y and z directions, respectively. Then the channel matrix **H** can be approximated as the superposition of the channel responses in the wavenumber domain [6], i.e.,

$$\mathbf{H} \approx \sqrt{N_R N_S} \mathbf{\Phi}_r \mathbf{H}_a \mathbf{\Phi}_s^H, \qquad (3)$$

where

• $\mathbf{\Phi}_r \in \mathbb{C}^{N_r \times n_R}$ collects column vectors $\phi_r(l_x, l_y) \in \mathbb{C}^{N_r \times 1}$ with n_R being the cardinality of the steering vector at RX. The *i*-th element of $\phi_r(l_x, l_y)$ is $[a_r(l_x, l_y, \mathbf{r})]_i$, which can be expressed as

$$[a_r (l_x, l_y, \mathbf{r})]_i = \frac{1}{\sqrt{N_r}} e^{-j \left(\frac{2\pi}{L_{R,x}} l_x r_{x_i} + \frac{2\pi}{L_{R,y}} l_y r_{y_i} + \gamma_s (l_x, l_y) r_{z_i}\right)},$$
(4)

• $\Phi_s \in \mathbb{C}^{N_s \times n_s}$ collects column vectors $\phi_s(m_x, m_y) \in \mathbb{C}^{N_s \times 1}$ with n_s being the cardinality of the steering vector at TX. The *j*-th element of $\phi_s(m_x, m_y)$ is $[a_s(m_x, m_y, \mathbf{s})]_i$, which can be expressed as

$$[a_{s}(m_{x}, m_{y}, \mathbf{s})]_{j}$$

$$= \frac{1}{\sqrt{N_{s}}} e^{-j\left(\frac{2\pi}{L_{S,x}}m_{x}s_{x_{j}} + \frac{2\pi}{L_{S,y}}m_{y}s_{y_{j}} + \gamma_{s}(m_{x}, m_{y})s_{z_{j}}\right)},$$
(5)

• $\mathbf{H}_a \in \mathbb{C}^{n_R \times n_S}$ is the angular random matrix collecting $H_a(\ell_x, \ell_y, m_x, m_y) \sim \mathcal{N}_{\mathbb{C}}(0, \sigma^2(\ell_x, \ell_y, m_x, m_y))$, which is the angular response that maps source direction onto receive direction. $\sigma^2(\ell_x, \ell_y, m_x, m_y)$ represents the energy distribution of the channel in the wavenumber domain $\Omega_R(\ell_x, \ell_y)$ and $\Omega_S(m_x, m_y)$, i.e.,

$$\left\{ \begin{bmatrix} \frac{2\pi m_x}{L_{S,x}}, \frac{2\pi (m_x+1)}{L_{S,x}} \end{bmatrix} \times \begin{bmatrix} \frac{2\pi m_y}{L_{S,y}}, \frac{2\pi (m_y+1)}{L_{S,y}} \end{bmatrix} \right\}, \\ \begin{bmatrix} \frac{2\pi \ell_x}{L_{R,x}}, \frac{2\pi (\ell_x+1)}{L_{R,x}} \end{bmatrix} \times \begin{bmatrix} \frac{2\pi \ell_y}{L_{R,y}}, \frac{2\pi (\ell_y+1)}{L_{R,y}} \end{bmatrix} \right\}.$$
(6)

Under the assumption that scattering is separable [6], the scattering decouples and $\sigma^2(\ell_x, \ell_y, m_x, m_y)$ becomes

$$\sigma^2\left(\ell_x, \ell_y, m_x, m_y\right) = \sigma_S^2\left(m_x, m_y\right) \sigma_R^2\left(\ell_x, \ell_y\right).$$
(7)

Then the angular random matrix $\mathbf{H}_a \in \mathbb{C}^{n_R \times n_S}$ can be obtained as [6]

$$\mathbf{H}_{a} = \operatorname{diag}\left(\boldsymbol{\sigma}_{R}\right) \mathbf{W} \operatorname{diag}\left(\boldsymbol{\sigma}_{S}\right), \qquad (8)$$

where $\boldsymbol{\sigma}_{R} \in \mathbb{R}^{n_{R}}_{+}$ and $\boldsymbol{\sigma}_{S} \in \mathbb{R}^{n_{S}}_{+}$ collect $\{\sigma_{R}(\ell_{x}, \ell_{y})\}$ and $\{\sigma_{S}(m_{x}, m_{y})\}$, respectively, and $\mathbf{W} \sim \mathcal{CN}(0, \mathbf{I}_{n_{r}n_{s}})$.

B. Numerical Solutions for Spectral Density in Wavenumber Domain

The following is the calculation method about $\sigma_R(\ell_x, \ell_y)$, which also applies to $\sigma_S(m_x, m_y)$. We first let the polar coordinates $k_R = \frac{\sqrt{k_x^2 + k_y^2}}{k}$ and $\phi_R = \phi$. The Jacobian of polar coordinates is given as

$$\mathbf{J}\left(k_{R},\phi_{R}\right) = \begin{bmatrix} \frac{\partial\theta}{\partial k_{R}} & \frac{\partial\theta}{\partial \phi_{R}} \\ \frac{\partial\phi}{\partial k_{R}} & \frac{\partial\phi}{\partial \phi_{R}} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{1-k_{R}^{2}}} & 0 \\ 0 & 1 \end{bmatrix}.$$
 (9)

Then $\sigma_R^2(\ell_x, \ell_y)$ can be calculated by the angle distribution function $f(\theta_R, \phi_R)$ as

$$\sigma_R^2(l_x, l_y) = \iint_{\Omega_R(l_x, l_y)} f(\theta_R, \phi_R) d\theta_R d\phi_R$$

=
$$\iint_{\Omega_R(l_x, l_y)} f(\arcsin(k_R), \phi_R) \frac{1}{\sqrt{1 - k_R^2}} dk_R d\phi_R.$$

(10)



Fig. 1. Three regions divided from the integral range of $\sigma_R^2(l_x, l_y)$ in fourth quadrant when $|\ell_x| < |\ell_y|$.

The integral range $\Omega_R(\ell_x, \ell_y)$ can be divided into two or three regions under (k_R, ϕ_R) domain. The number of divided regions depends on whether the region is adjacent to the coordinate axis, and the numerical relationship between $|\ell_x|$ and $|\ell_y|$ will affect the division method [2]. We take the fourth quadrant and $|\ell_x| < |\ell_y|$ for example as shown in Fig.1, the integral region $\Omega_R(\ell_x, \ell_y)$ can be divided into three regions as [2]

$$\left\{ \left(\phi_R, k_R\right) \mid 2\pi - \arctan \frac{|c|}{a} \le \phi_R \le 2\pi - \arctan \frac{|d|}{a} \right\},$$
$$\min(1, \frac{a}{\cos(2\pi - \phi)}) \le k_R \le \min(1, \frac{|c|}{\sin(2\pi - \phi)}) \right\},$$
$$\left\{ \left(\phi_R, k_R\right) \mid 2\pi - \arctan \frac{|d|}{a} \le \phi_R \le 2\pi - \arctan \frac{|c|}{b} \right\},$$
$$\min(1, \frac{|d|}{\sin(2\pi - \phi)}) \le k_R \le \min(1, \frac{|c|}{\sin(2\pi - \phi)}) \right\},$$
$$\left\{ \left(\phi_R, k_R\right) \mid 2\pi - \arctan \frac{|c|}{b} \le \phi_R \le 2\pi - \arctan \frac{|d|}{b} \right\},$$
$$\min(1, \frac{|d|}{\sin(2\pi - \phi)}) \le k_R \le \min(1, \frac{b}{\cos(2\pi - \phi)}) \right\}.$$

3

Referring to the standard 3D MIMO channel model [11], wrapped Gaussian (WG) and truncated Laplacian (TL) are used to model the azimuth angle distribution and the elevation angle distribution, respectively, and they are independent of each other. The corresponding curves are plotted in Fig.2. Therefore, the angle distribution function $f(\theta, \phi)$ can be expressed in the following form.

$$f(\theta,\phi) = \frac{Q_l}{\sqrt{2}\sigma_l} e^{-\frac{\sqrt{2}|\theta-\theta_0|}{\sigma_l}} \cdot \frac{Q_g}{\sqrt{2\pi}\sigma_g} e^{-\frac{(\phi-\phi_0)^2}{2\sigma_g^2}},\qquad(11)$$

where Q_l and Q_g are the normalization factors, ϕ_0 and θ_0 are central angle of the azimuth and elevation, σ_g and σ_l are variance of the azimuth and elevation. We only consider the wave propagation towards -z axis at TX and RX, so the elevation $\theta \in [0, \pi/2]$ and the azimuth $\phi \in [\phi_0 - \pi, \phi_0 + \pi]$.



Fig. 2. Probability density functions (PDFs) of WG and TL distributions.

III. PERFORMANCE ANALYSIS

We now use the channel model in Section II to analyze the capacity of the holographic MIMO system. Since Φ_r and Φ_s in (3) are semi-unitary matrices [6], the holographic MIMO communication systems can be equivalently expressed as

$$\mathbf{y}_a = \sqrt{\rho G_t G_r N_S N_R} \mathbf{H}_a \mathbf{x}_a + \mathbf{n}_a, \tag{12}$$

where $\mathbf{y}_a = \mathbf{\Phi}_r^{\mathrm{H}} \mathbf{y} \in \mathbb{C}^{n_R}$ and $\mathbf{x}_a = \mathbf{\Phi}_s^{\mathrm{H}} \mathbf{x} \in \mathbb{C}^{n_S}$ denote the received and transmitted signal vectors in the wavenumber domain, respectively. $\mathbf{n}_a = \mathbf{\Phi}_r^{\mathrm{H}} \mathbf{n} \in \mathbb{C}^{n_R}$ is the angular noise vector distributed as $\mathbf{n}_a \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \mathbf{I}_{n_R})$.

A. Capacity under Generalized Angle Distributions

In the case of unknown channel state information at the transmitter, the capacity can be obtained as [6]

$$C = \sum_{i=1}^{\operatorname{rank}(\mathbf{H}_a)} \mathbb{E}\left\{\log_2\left(1 + \frac{\rho G_t G_r N_S N_R}{n_S} \times \lambda_i(\mathbf{H}_a \mathbf{H}_a^{\mathrm{H}})\right)\right\},\tag{13}$$

where $\lambda_i(\mathbf{H}_a \mathbf{H}_a^{\mathrm{H}})$ is *i*-th sorted eigenvalue of $\mathbf{H}_a \mathbf{H}_a^{\mathrm{H}}$, and $n_S \approx \frac{\pi L_{S,x} L_{S,y}}{\lambda^2}$ denotes the cardinality of the steering vector at TX [8].

By substituting (8) into (13), the channel capacity with generalized angle distributions can be expressed as

$$C = \sum_{i=1}^{\operatorname{rank}(\mathbf{A})} \mathbb{E} \Big\{ \log_2 \Big(1 + \frac{\rho G_t G_r N_S N_R}{n_S} \times \lambda_i (\operatorname{diag} \left(\boldsymbol{\sigma}_R \odot \boldsymbol{\sigma}_R \right) \mathbf{W} \mathbf{W}^{\mathrm{H}} \operatorname{diag} \left(\boldsymbol{\sigma}_S \odot \boldsymbol{\sigma}_S \right)) \Big) \Big\},$$
(14)

where the elements of σ_R and σ_S can be obtained by (10).

B. Capacity for Special Cases

It can be seen from (13) that the channel capacity depends on the eigenvalue distribution of $\mathbf{H}_{a}\mathbf{H}_{a}^{\mathrm{H}}$ when the antenna configuration is fixed. According to Jensen's inequality [12], the channel capacity reaches the upper bound when the eigenvalues are uniformly distributed, which corresponds to isotropic scattering. Assuming that the total power gain is 1, i.e., tr ($\mathbf{H}_{a}\mathbf{H}_{a}^{\mathrm{H}}$) = 1, the upper bound is

$$C_{upper} = n_M \log_2 \left(1 + \frac{\rho G_t G_r N_S N_R}{n_S n_M} \right), \qquad (15)$$

where n_M is $\min(n_S, n_R)$.

It is difficult to satisfy isotropic propagation in reality, so there is a gap between the upper bound and the channel capacity under generalized angle distributions. However, there is a special case for the gap. According to the equivalent infinitesimal $\ln(1+x) \sim x$, when SNR is very low, i.e., $\rho \rightarrow 0$, the capacity can be observed as

$$C_{low-snr} = \sum_{i=1}^{\operatorname{rank}(\mathbf{H}_{a})} \mathbb{E} \left\{ \log_{2} \left(1 + \frac{\rho G_{t} G_{r} N_{R} N_{S}}{n_{S}} \lambda_{i} \left(\mathbf{H}_{a} \mathbf{H}_{a}^{\mathrm{H}} \right) \right) \right\}$$

$$\approx \sum_{i=1}^{\operatorname{rank}(\mathbf{H}_{a})} \mathbb{E} \left\{ \frac{\rho G_{t} G_{r} N_{R} N_{S}}{n_{S} \ln 2} \lambda_{i} \left(\mathbf{H}_{a} \mathbf{H}_{a}^{\mathrm{H}} \right) \right\}$$

$$= \frac{\rho G_{t} G_{r} N_{R} N_{S}}{n_{S} \ln 2} \sum_{i=1}^{\operatorname{rank}(\mathbf{H}_{a})} \mathbb{E} \left\{ \lambda_{i} \left(\mathbf{H}_{a} \mathbf{H}_{a}^{\mathrm{H}} \right) \right\}$$

$$= \frac{\rho G_{t} G_{r} N_{R} N_{S}}{n_{S} \ln 2} \mathbb{E} \left\{ \operatorname{tr} \left(\mathbf{H}_{a} \mathbf{H}_{a}^{\mathrm{H}} \right) \right\}$$

$$= \frac{\rho G_{t} G_{r} N_{R} N_{S}}{n_{S} \ln 2}.$$
(16)

Thus the channel capacity will no longer depend on angle distributions and the gap will approximately disappear when SNR tends to zero.

C. Capacity for Different Aperture Schemes

A convenient implementation of holographic MIMO is discrete aperture scheme, in which the horizontal or vertical antenna spacing is usually below half of the wavelength. However, the spatial utilization of discrete aperture scheme is limited. To improve the performance with limited apertures, continuous aperture scheme is proposed, in which the antenna spacing d tends to zero [13]. Fig. 3 illustrates the characteristics and relationships of the two different aperture schemes.

In the discrete aperture scheme, the antenna gain of each patch antenna is a constant and can be calculated by





Fig. 3. Discrete and continuous aperture schemes.

$$G = \frac{4\pi\eta S}{\lambda^2},\tag{17}$$

where $\eta < 1$ is aperture efficiency, and S is antenna area, and λ is the operating wavelength [14]. Then the capacity of the holographic MIMO system in the discrete aperture scheme can be given by

$$C = \sum_{i=1}^{\operatorname{rank}(\mathbf{A})} \mathbb{E} \Big\{ \log_2 \Big(1 + \frac{16\pi^2 \rho \eta_t \eta_s A_R A_S N_R N_S}{n_S \lambda^4} \times \\ \lambda_i (\operatorname{diag} \left(\boldsymbol{\sigma}_R \odot \boldsymbol{\sigma}_R \right) \mathbf{W} \mathbf{W}^{\mathrm{H}} \operatorname{diag} \left(\boldsymbol{\sigma}_S \odot \boldsymbol{\sigma}_S \right)) \Big) \Big\},$$
(18)

where η_t and η_s denote the aperture efficiency at TX and RX, A_R and A_S denote the antenna area at TX and RX, respectively.

In the continuous aperture scheme, the area of each antenna at TX and RX is $A_S = \frac{L_{S,x}L_{S,y}}{N_S}$ and $A_R = \frac{L_{R,x}L_{R,y}}{N_R}$, respectively. Therefore, the capacity of the holographic MIMO system in the continuous aperture scheme can be given by

$$C = \sum_{i=1}^{\operatorname{rank}(\mathbf{A})} \mathbb{E} \Big\{ \log_2 \Big(1 + \frac{16\pi^2 \rho \eta_t \eta_s L_{S,x} L_{S,y} L_{r,x} L_{r,y}}{n_S \lambda^4} \times \lambda_i (\operatorname{diag} \left(\boldsymbol{\sigma}_R \odot \boldsymbol{\sigma}_R \right) \mathbf{W} \mathbf{W}^{\mathrm{H}} \operatorname{diag} \left(\boldsymbol{\sigma}_S \odot \boldsymbol{\sigma}_S \right)) \Big) \Big\}.$$
(19)

It can be seen that the capacity of continuous aperture scheme depends on the total area of TX and RX, and the capacity will not increase with more antennas in a limited space due to the constraint between the area and number of antenna.

IV. NUMERICAL RESULTS

In this section, we numerically analyze the performance of holographic MIMO system based on the theoretical analysis in section III. According to the channel model standards 3GPP TR38.901 [15], three scenarios are selected to analyze the influence of different scenarios on the performance of the holographic MIMO system. Table I shows the default simulation parameters.

Fig. 4 depicts the performance of holographic MIMO versus angle spread. The array aperture for both TX and RX is set

Operating frequency	6 GHz		
Mean azimuth	90°		
Mean elevation	45°		
Antenna size	$\lambda/8$ $ imes$ $\lambda/8$		
Aperture efficiency	60%		
Scenarios (LOS)	UMa	UMi	RMa
Azimuth angle spread at transmitter σ_{sg}	14.0°	14.7°	7.9°
Elevation angle spread at transmitter σ_{sl}	0.3°	0.6°	0.1°
Azimuth angle spread at receiver σ_{rg}	65°	46°	33°
Elevation angle spread at receiver σ_{rl}	8.9°	4.4°	3.0°

as 15 $\lambda \times 15 \lambda$. The SNR is 30 dB and the antenna spacing is $\lambda/4$. We can find in Fig. 4 that when the elevation angle spread is fixed, the capacity will increase with the azimuth angle spread. When the azimuth angle spread is large enough, e.g., 80°, the capacity will increase slowly and still have a gap with the upper bound. Also, the capacity will increase fast with elevation angle spread when the elevation angle spread is small but increase slowly when the elevation angle spread is large. This is due to the mapping relationship between angle distribution and wavenumber distribution.



Fig. 4. Capacity in bit/s/Hz as a function of azimuth angle spread in degree under different elevation angle spread.

Fig. 5 compares the performance of holographic MIMO under different scenarios and array aperture. Both antenna spacing of TX and RX is $\lambda/4$. The array aperture is set to two different values, i.e., 15 $\lambda \times 15 \lambda$ and 30 $\lambda \times 30 \lambda$. It can be seen that the holographic MIMO system under UMi scenario obtains the largest channel capacity and under RMa scenario obtains the smallest channel capacity. This is because UMa and UMi scenarios have more diffuse angle distribution than RMa scenario. And the capacity gap is not obvious when the SNR is low, which consists with (16). Besides, the holographic MIMO system with 30 $\lambda \times 30 \lambda$ array obtains more capacity than 15 $\lambda \times 15 \lambda$ array under the same scenario. This is because the larger aperture brings greater angular resolution, which improves the multiplexing gain and hence improves the capacity.



Fig. 5. Capacity in bit/s/Hz as a function of SNR in dB under different scenarios and array aperture.

Fig. 6 depicts the effect of antenna spacing on channel capacity. We compare the capacity curves under different propagation scenarios. The array aperture for both TX and RX is set as 15 $\lambda \times 15 \lambda$. As illustrated in Fig. 6, when antenna spacing Δ is not smaller than $\lambda/8$, the channel capacity increases with the decrease of antenna spacing. This is because decreased antenna spacing results in a larger transmitting and receiving surface. However, when antenna spacing Δ is $\lambda/8$, the number of deployed antennas reaches an upper limit, and the aperture tends to be continuous. If further decreasing the antenna spacing, the area of each antenna should be reduced. In this case, the capacity is no longer increasing because the effect of antenna gain cancels out with the effect of antenna density. It should be noted that the aperture efficiency is assumed to be constant. In practice, aperture efficiency may change with the decrease in antenna size and therefore affects the capacity value, which needs to be verified by measurement.



Fig. 6. Capacity in bit/s/Hz as a function of normalized antenna spacing under different scenarios.

V. CONCLUSION

This letter analyzed the capacity of holographic MIMO channels with practical constraints. We focused on the capacity

constrained by generalized angle distribution and limited array aperture. Theoretical analysis and numerical results show that angle distribution and array aperture are the key factors affecting channel capacity. Due to the generalized angle distribution, the realistic capacity with sufficient angle spread cannot reach the upper bound. In addition, the angle distribution affects the capacity at high SNR more obviously than the capacity at low SNR. Besides, smaller antenna spacing with a limited aperture improves the capacity in the discrete aperture scheme but not improves the capacity in the continuous aperture scheme.

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