

Scheduling Distributed Resources in Heterogeneous Private Clouds

G. Kesidis, Y. Shan, A. Jain, B. Urgaonkar

J. Khamse-Ashari and I. Lambadaris

School of EECS, PSU, State College, PA

SCE Dept, Carleton Univ., Ottawa, Canada

{gik2,yxs182,axj182,uu1}@psu.edu

{jalalkhamseashari,ioannis}@sce.carleton.ca

Abstract

We first consider the static problem of allocating resources to (*i.e.*, scheduling) multiple distributed application frameworks, possibly with different priorities and server preferences, in a private cloud with heterogeneous servers. Several fair scheduling mechanisms have been proposed for this purpose. We extend prior results on max-min and proportional fair scheduling to this constrained multiresource and multiserver case for generic fair scheduling criteria. The task efficiencies (a metric related to proportional fairness) of max-min fair allocations found by progressive filling are compared by illustrative examples. They show that “server specific” fairness criteria and those that are based on residual (unreserved) resources are more efficient.

I. INTRODUCTION AND BACKGROUND

We consider a cloud provider that needs to run multiple software applications on its IT infrastructure. These applications may be distributed and are also called frameworks or workloads in the literature. The cloud provider’s infrastructure consists of multiple servers connected by a network. A server may be a physical machine or virtual machine (*e.g.*, an instance or a container). A server is also referred to as a worker or a slave in some popular resource management solutions. Each framework desires multiple IT resources (CPU, memory, network bandwidth, *etc.*) for each of its “tasks.” A task is a framework-specific basic unit of work that must be placed within a single server at a given time (*e.g.*, it is useless for a task to be allocated CPU from one server and memory from another). The provider’s challenge then is to determine who should get how many resources from which servers. Our interest is in a *private* cloud setting wherein notions of fairness have often been used as the basis for this resource allocation problem. In a public setting, on the other hand, the provider’s goal is typically to maximize its profit.

What are meaningful notions of fairness for such *multi-resource and multi-server* settings? This question has received much attention in the recent past. Proposed fair schedulers include Dominant Resource Fairness (DRF) [12] extended to multiple servers¹, Task Share Fairness (TSF) [31], Per Server Dominant Share Fairness (PS-DSF) [18], [16], [17], among others, *e.g.*, [5]. DRF is resource based, whereas TSF and “containerized” DRF [11] are task based². In the following, we additionally consider variants of these schedulers that employ current residual (unreserved) capacities of the servers in the fairness criteria (somewhat similar to “best fit” variants [32]).

Background on existing approaches and their assumptions: Typically static problem formulations are considered under a variety of simplifying assumptions on framework behavior that we discuss below:

This research was supported in part by NSF CNS 1526133, NSF CNS 1717571 and a Cisco Systems URP gift.

¹DRF was originally defined for a single server in [12]. The multiple-server version, called DRFH in [32], [11], is also commonly called just DRF as done in Apache Mesos [13] and as we do herein also.

²Containerized DRF has a “sharing-incentive” property not possessed by DRF, and TSF possesses “strategy-proofness” and “envy-freeness” properties which are not possessed by containerized DRF [31]. Unlike DRF and TSF, PS-DSF is not necessarily Pareto optimal but is “bottleneck” fair. These properties are not addressed herein

- It is assumed that frameworks congest all the available servers. That is, it is assumed that there is sufficient work to completely occupy at least one resource in every server.
- It is also assumed that the frameworks' required resources (presumably to achieve certain performance needs) are well characterized, *e.g.*, [9], [19], [21], [3], [24], [33], [7], [1], [20].
- Frameworks are assumed to have linearly elastic resource demands in the following sense. Each task has a known requirement $d_{n,r}$ for the resource type r . Therefore, if $x_{n,i}$ were the number of tasks of framework n placed on server i , the framework would consume $x_{n,i}d_{n,r}$ amount of resource r on server i .
- $x_{n,i}$ may take on non-negative real values rather than being restricted to be non-negative integer valued³.
- Finally, frameworks may have different service priorities and server preference constraints (as in *e.g.*, service-quality constraints [34] or cache-affinity constraints), see also [31].

Note that in some settings, a goal is to minimize the number of servers to accommodate workloads with finite needs, again as in multidimensional bin-packing problems [4], [6], [8]. Such problem formulations are typically motivated by the desire to economize on energy. However, frequently cycling power to (booting up) servers may result in software errors and there are energy spikes associated with boot-up resulting in increased electricity costs [10]. We are not interested in such settings herein.

Typically in existing papers, max-min fairness with respect to a proposed fairness criteria is specified assuming the aforementioned congested regime under the following (linear) capacity constraints:

$$\forall i, r, \quad \sum_n x_{n,i} d_{n,r} \leq c_{i,r}, \quad (1)$$

where $c_{i,r}$ is the amount of available resource r in server i for the instances under consideration⁴. Additionally, there may be placement constraints, $\delta_{n,i} \in \{0, 1\}$, whereby $x_{n,i} > 0 \Rightarrow \delta_{n,i} = 1$. Max-min fair allocation may be expressed as the solution of a constrained centralized optimization problem. Alternatively, max-min fairness with respect to the proposed fairness criteria may be approximated by a greedy, iterative “progressive filling” allocation. The latter approach is often preferred because of the benefits this offers for online implementations. Moreover, progressive filling arguments can be used to establish other potentially desirable fairness properties of schedulers defined for private clouds⁵.

Instead of max-min fairness, the cloud may admit and place instances so as to maximize, *e.g.*, total weighted tasking objective,

$$\sum_n \phi_n \sum_i x_{n,i} \quad (2)$$

subject to (1), where $\phi_n > 0$ is the priority of application framework n . In this paper, we relate this task efficiency objective to “proportional” fairness.

In Sections II and III, for generic fairness criteria, we generalize to multiple resources the static optimization problems of *e.g.*, [2], [23], [15] whose solutions correspond to max-min fairness and proportional fairness, respectively. In Section IV, a simple, greedy, iterative method intended to achieve max-min fairness called progressive filling is described. Progressive filling is important for online implementation. In Section V, the performance evaluation objectives of the following two sections are discussed: task efficiency (related to proportional fairness) and overall execution time. In Section VI, illustrative numerical examples are used to compare the task efficiencies of different schedulers, including variants using residual/unreserved server

³With x integer valued, such problems belong to the class of combinatorial-optimization multidimensional bin-packing problems, *e.g.*, [4], [6], [8], which are NP-hard. They have been extensively studied, including relaxations to simplified problems that yield approximately optimal solutions, *e.g.*, by Integer Linear Programs solved by iterated/online means.

⁴Note that if $x_{n,i} = 0$ then workload type n is not assigned to server i .

⁵Again, Pareto optimality, sharing incentive, strategy proofness, bottleneck fairness, and envy freeness [12] - properties that are not addressed herein.

resource capacities specified herein. In [26], we give the results of an online experimental study using our implementations of different schedulers on Spark and Mesos [22], [27] for benchmark workloads considering an execution-time performance metric. The paper concludes with a summary in Section VII and a brief discussion of future work (regarding scheduling in public clouds).

Our mathematical notation is given in Table I.

Symbol	Definition
i	server index
n	user/framework index
r	resource type index
ρ	index of the dominant resource
ϕ_n	weight/priority of user n
$x_{n,i}$	the number of tasks or workload intensity
$d_{n,r}$	per-task resource requirement
$c_{i,r}$	the total available resource amounts
$B_{n,i,r}$	$= d_{n,r}/c_{i,r}$
$\delta_{n,i}$	server preference indicator
N_i	the set of users that can run on server i
R_i	fully booked resources of server i under x
U_n, K_n, M_n	allocation-fairness scores

TABLE I
MATHEMATICAL NOTATION.

II. MAX-MIN FAIRNESS

To generalize previous results on max-min fairness (*e.g.*, [2], [12], [15]) to multiple resource types on multiple servers, consider the following general-purpose fairness criterion for framework n ,

$$U_n = \frac{1}{\phi_n} \sum_i u_{n,i} x_{n,i}, \quad (3)$$

for scalars $u_{n,i} > 0$ and priorities $\phi_n > 0$ (specific examples of fairness criteria are given below). In addition, consider the service-preference sets

$$N_i = \{n \mid \delta_{n,i} = 1\} \quad \text{where} \quad x_{n,i} > 0 \Rightarrow \delta_{n,i} > 0. \quad (4)$$

Relaxing the allocations $\{x_{n,i}\}$ to be real valued, consider strictly concave and increasing g with $g(0) = 0$, and define the optimization problem

$$\max_x \sum_n \phi_n g(U_n) \quad (5)$$

such that (here restating (1))

$$\forall i, r, \quad \sum_{n \in N_i} x_{n,i} B_{n,i,r} \leq 1 \quad \text{and} \quad \forall n, i \quad x_{n,i} \geq 0, \quad (6)$$

where

$$B_{n,i,r} := \frac{d_{n,r}}{c_{i,r}}. \quad (7)$$

Note that the objective is continuous and strictly concave and the domain given by (6) (equivalently (1)) is compact. So, simply by Weierstrass's Extreme Value Theorem, there exists a unique maximum.

Regarding fully booked resources in server i under allocations $x = \{x_{n,i}\}$, also let

$$R_i := \{(x, r) \mid \sum_{n \in N_i} x_{n,i} B_{n,i,r} = 1\}.$$

For the following definition, assume that $\forall n, i, r, B_{n,i,r} > 0$.

Definition 1: A feasible allocation $\{x_{n,i}\}$ satisfying (6) is said to be U -Max-Min Fair (MMF) if:

$$U_\ell > U_m, x_{m,i} > 0, \& \exists r \text{ s.t. } \sum_{n \in N_i} x_{n,i} B_{n,i,r} = 1$$

implies that $x_{\ell,i} = 0$.

Note that if instead $x_{\ell,i} > 0$ in this definition, then $x_{\ell,i}$ can be reduced and $x_{m,i}$ increased to reduce $U_\ell - U_m$. Also, if $\{x_{n,i}\}$ is U -MMF and $x_{m,i}, x_{\ell,i} > 0$ for some server i then $U_m = U_\ell$. Quantization (containerization) issues associated with workload resource demands are considered in [11].

Under multi-server DRF [12], [32], frameworks n are selected using criterion

$$M_n = \frac{1}{\phi_n} x_n \max_r \frac{d_{n,r}}{\sum_j c_{j,r}}, \quad (8)$$

where $x_n = \sum_i x_{n,i}$. That is, under multi-server DRF,

$$\forall i, u_{n,i} = \max_r \frac{d_{n,r}}{\sum_j c_{j,r}}. \quad (9)$$

The server-specific PS-DSF criterion can be written as

$$K_{n,j} = \frac{\sum_i x_{n,i} d_{n,\rho(n,j)}}{\phi_n c_{j,\rho(n,j)}} = \frac{B_{n,j,\rho(n,j)} x_n}{\phi_n}, \quad (10)$$

where ρ is such that

$$B_{n,j,\rho(n,j)} := \max_r B_{n,j,r} \text{ when } \delta_{n,j} = 1. \quad (11)$$

Max-min fairness according to the joint framework-server criterion $K_{n,j}$ is considered in [18], [16], [17]. Here define

$$\begin{aligned} K_n &= \sum_i K_{n,i} \delta_{n,i} = \frac{1}{\phi_n} x_n \sum_i B_{n,i,\rho(n,i)} \delta_{n,i} \\ &= \frac{1}{\phi_n} x_n \sum_i \max_r \frac{d_{n,r}}{c_{i,r}} \delta_{n,i} \end{aligned} \quad (12)$$

So, under PS-DSF,

$$\forall i \in N_i, u_{n,i} = \max_r \frac{d_{n,r}}{c_{i,r}}. \quad (13)$$

Proposition 1: A solution $x = \{x_{n,i}\}$ of the optimization (5) s.t. (6) has at least one resource r fully booked in each server i . In addition, there is a unique U -MMF solution if also:

$$\exists j \text{ s.t. } \delta_{m,j} = 1 = \delta_{\ell,j} \Rightarrow \forall r, d_{m,r} = d_{\ell,r}, N_m = N_\ell, \text{ and } \forall i, u_{m,i} = u_{\ell,i}. \quad (14)$$

Proof: See Appendix A. The proof is an adaptation of that in [2], [15] for a single resource type.

Considering (9) and (13), $\forall r, d_{m,r} = d_{\ell,r}$ implies $\forall i, u_{m,i} = u_{\ell,i}$. So, (14) is satisfied for both DRF and PS-DSF when only frameworks with the same resource demands share the same set of servers.

For task-based allocations (integer-valued x), max-min fairness can be approximated by a greedy incremental optimization known as progressive filling, see [2], [12], [26].

III. PROPORTIONAL FAIRNESS

For weighted proportional fairness, consider the objective

$$\max_x \sum_n \phi_n g_a(x_n), \quad (15)$$

i.e., without dividing by ϕ_n in the argument of g_a [23]. For parameter $a > 0$ specifically take

$$g_a(X) = \begin{cases} \log(X) & \text{if } a = 1 \\ (1-a)^{-1} X^{1-a} & \text{else} \end{cases}$$

i.e., $g'_a(X) = 1/X^a$, again see [23]. Obviously, in the case of $a = 1$ ($g = \log$), whether the factor ϕ is in the argument of g is immaterial.

The following generalizes Lemma 2 of [23] on Proportional Fairness. See also the proportional-fairness/efficiency trade-off framework of [14] for a single server.

Proposition 2: A solution x^* of the optimization (15) s.t. (6) is uniquely (weighted) (ϕ, a) x -proportional fair, i.e., for any other feasible solution x ,

$$\Phi(x, x^*) := \sum_n \phi_n \frac{x_n - x_n^*}{(x_n^*)^a} \leq 0. \quad (16)$$

Proof: See Appendix B.

From the proof, $\{x_n^* = \sum_i x_{n,i}^*\}_n$ is unique though $x^* = \{x_{n,i}^*\}_{n,i}$ may not be. We can normalize $\hat{\phi}_n := \phi_n / \sum_k \phi_k$ and when $a = 1$ write (16) as

$$\sum_n \hat{\phi}_n \frac{x_n}{x_n^*} \leq 1.$$

A possible definition of the *efficiency* of a feasible allocation is (2) corresponding to $a = 0$,

$$\sum_n \phi_n \sum_i x_{n,i} = \sum_n \phi_n x_n, \quad (17)$$

i.e., the weighted total number of tasks scheduled. So, the optimization of Proposition 2 with $a = 1$ gives an allocation x^* that is related to a task efficient allocation. Clearly, x^* satisfying (16) for all other allocations x with $a = 1$ does not necessarily maximize (17). This issue is analogous to estimating the mean of the ratio of positive random variables $E(X/X^*)$ using the ratio of the means EX/EX^* , see e.g. p. 351 of [28] or (11) of [25]. For simplicity in the following, we use (17) instead of (16).

Note that the priority ϕ_n of framework n could factor its resource footprint $\{d_{n,r}\}_r$. Alternatively, the resource footprints of the frameworks can be explicitly incorporated into the main optimization objective via a fairness criterion. The proof of the following corollary is just as that of Proposition 2. Recall that the generic fairness criterion U_n (3) is a linear combination of $\{x_{n,i}\}_i$.

Corollary 1: A solution x^* of the optimization problem

$$\max_x \sum_n \phi_n \log(U_n) \quad \text{s.t.} \quad (6)$$

is uniquely $(\phi, 1)$ U -proportional fair, *i.e.*, for any other feasible x ,

$$\sum_n \phi_n \frac{U_n - U_n^*}{U_n^*} \leq 0.$$

Again, optimal $\{U_n^*\}$ would be unique but $x^* = \{x_{n,i}^*\}_{n,i}$ may not be.

Recall for DRF and PS-DSF, the K_n (12) and M_n (8), respectively, are proportional to x_n . Thus, using $U_n = K_n$ or $U_n = M_n$ in Corollary 1 reduces to the result of Proposition 2 when $a = 1$.

IV. PROGRESSIVE FILLING TO APPROXIMATE MAX-MIN FAIR ALLOCATION

In the following evaluation studies, resources are incrementally (taskwise) allocated to frameworks n with the intention to approximate max-min fairness (with respect to the fairness criterion used). The approach is greedy: simply, the framework n with smallest fairness criterion U_n (or $U_{n,i}$), based on *existing* allocations $\{x_{n,i}\}_{n,i}$, will be allocated a resource increment $\{\varepsilon d_{n,i}\}_i$ for small⁶ $\varepsilon > 0$. If a framework's resource demands cannot be accommodated with available resources, the framework with the next smallest fairness criterion will be allocated by this *progressive filling* approach [2], [12]. The choice of server from which to allocate can be random, *e.g.*, as for the Mesos default task-level progressive filling for DRF, see [26]. Alternatively, the framework and server can be jointly chosen (*e.g.*, using PS-DSF).

Note how progressive filling can operate in the presence of churn in the set of active frameworks, where in asynchronous fashion, new frameworks could be initiated or a framework would release all of its resources once its computations are completed, see [26]. In the following we assess the efficiencies of max-min fair approximations by progressive filling according to different schedulers.

Because there is no resource revocation, a problem occurs when, say, servers are booked so that there are insufficient spare resources to allocate for a task of a just initiated framework (particularly a higher priority one). Thus, new frameworks may need to wait for sufficient resources to be released (by the termination of other frameworks). Alternatively, *all* existing frameworks could be reallocated whenever any new framework initiates or any existing framework terminates. Though within a server such reallocations are commonplace in a private setting, the effect of such “live” reallocations may be that tasks need to be terminated and reassigned to other servers (or live migrated). The following illustrative numerical examples allocate a single initial framework batch (without framework churn). In the following emulation study for equal priority workloads and framework churn, we work with the default progressive-filling mechanism in Mesos wherein existing frameworks are not adjusted upon framework churn.

V. EVALUATION OBJECTIVES: TASK EFFICIENCY OF MAX-MIN FAIR ALLOCATIONS

In the following, though we aim for max-min fairness with progressive filling, we are also interested in the proportional fairness achieved. We compare the efficiency (17) of the allocations achieved by progressive filling for examples with heterogeneous workloads and servers. In the performance evaluation of our Mesos implementations, efficiency is defined by overall execution time.

Though PS-DSF allocations achieved by progressive filling may not be Pareto optimal, we show that they are more efficient, even in some of our Mesos experiments where servers are (at least initially) selected at random.

In the following, for brevity, we consider only cases with frameworks of equal priority ($\forall n, n', \phi_n = \phi_{n'}$) and without server-preference constraints (*i.e.*, $\delta_{n,i} \equiv 1$).

⁶Typically $\varepsilon = 1$ when allocations x are measured in “tasks”.

$\begin{matrix} (n,i) \\ \text{sched.} \end{matrix}$	(1,1)	(1,2)	(2,1)	(2,2)	total
DRF [12], [32]	6.55	4.69	4.69	6.55	22.48
TSF [31]	6.5	4.7	4.7	6.5	22.4
RRR-PS-DSF	19.44	1.15	1.07	19.42	41.08
BF-DRF [32]	20	2	0	19	41
PS-DSF [17]	19	0	2	20	41
rPS-DSF	19	2	2	19	42

TABLE II

WORKLOAD ALLOCATIONS $x_{n,i}$ FOR DIFFERENT SCHEDULERS UNDER PROGRESSIVE FILLING FOR ILLUSTRATIVE EXAMPLE WITH PARAMETERS (18) AND (19). AVERAGED VALUES OVER 200 TRIALS REPORTED FOR THE FIRST THREE SCHEDULERS OPERATING UNDER RRR SERVER SELECTION.

$\begin{matrix} (n,i) \\ \text{sched.} \end{matrix}$	(1,1)	(1,2)	(2,1)	(2,2)
DRF [12], [32]	2.31	0.46	0.46	2.31
TSF [31]	2.29	0.46	0.46	2.29
RRR-PS-DSF	0.59	0.99	1	0.49

TABLE III

SAMPLE STANDARD DEVIATION OF ALLOCATIONS $x_{n,i}$ FOR DIFFERENT SCHEDULERS UNDER RRR SERVER SELECTION WITH. AVERAGED VALUES OVER 200 TRIALS REPORTED.

VI. ILLUSTRATIVE NUMERICAL STUDY OF FAIR SCHEDULING BY PROGRESSIVE FILLING

In this section, we consider the following typical example of our numerical study with two heterogeneous distributed application frameworks ($n = 1, 2$) having resource demands per unit workload:

$$d_{1,1} = 5, d_{1,2} = 1, d_{2,1} = 1, d_{2,2} = 5; \quad (18)$$

and two heterogeneous servers ($i = 1, 2$) having two different resources with capacities:

$$c_{1,1} = 100, c_{1,2} = 30, c_{2,1} = 30, c_{2,2} = 100. \quad (19)$$

For DRF and TSF, the servers i are chosen in round-robin fashion, where the server order is randomly permuted in each round; DRF under such randomized round-robin (RRR) server selection is the default Mesos scheduler, cf. next section. One can also formulate PS-DSF under RRR wherein RRR selects the server and the PS-DSF criterion only selects the framework for that server. Frameworks n are chosen by progressive filling with integer-valued tasking (x), i.e., whole tasks are scheduled.

Numerical results for scheduled workloads for this illustrative example are given in Tables II & III, and unused resources are given in Tables IV and V. 200 trials were performed for DRF, TSF and PS-DSF under RRR server selection, so using Table III we can obtain confidence intervals for the averaged quantities given in Table II for schedulers under RRR. For example, the 95% confidence interval for task allocation of the first framework on the second server (i.e., $(n, i) = (1, 2)$) under TSF is

$$(6.5 - 2 \cdot 0.46/\sqrt{200}, 6.5 + 2 \cdot 0.46/\sqrt{200}) = (6.43, 6.57).$$

Note how PS-DSF's performance under RRR is comparable to when frameworks and servers are jointly selected [17], and with low variance in allocations. We also found that RRR-rPS-DSF performed just as rPS-DSF over 200 trials.

We found task efficiencies improve using *residual* forms of the fairness criterion. For example, the residual PS-DSF (rPS-DSF) criterion is

$$\tilde{K}_{n,j,x_j} = x_n \max_r \frac{d_{n,r}}{\phi_n(c_{j,r} - \sum_{n'} x_{n',j} d_{n',r})}$$

(i, r) sched.	(1,1)	(1,2)	(2,1)	(2,2)
DRF [32]	62.56	0	0	62.56
TSF [31]	62.8	0	0	62.8
RRR-PS-DSF	1.8	4.6	4.86	1.92
BF-DRF [32]	0	10	1	3
PS-DSF [17]	3	1	10	0
rPS-DSF	3	1	1	3

TABLE IV

UNUSED CAPACITIES $c_{i,r} - \sum_n x_{n,i} d_{i,r}$ FOR DIFFERENT SCHEDULERS UNDER PROGRESSIVE FILLING FOR ILLUSTRATIVE EXAMPLE WITH PARAMETERS (18) AND (19). AVERAGED VALUES OVER 200 TRIALS REPORTED UNDER RRR SERVER SELECTION.

(i, r) sched.	(1,1)	(1,2)	(2,1)	(2,2)
DRF [12], [32]	11.09	0	0	11.09
TSF [31]	10.99	0	0	10.99
RRR-PS-DSF	0.59	0.99	1	0.49

TABLE V

SAMPLE STANDARD DEVIATION OF UNUSED CAPACITIES $c_{i,r} - \sum_n x_{n,i} d_{i,r}$ FOR DIFFERENT SCHEDULERS UNDER RRR SERVER SELECTION OVER 200 TRIALS.

That is, this criterion makes scheduling decisions by progressive filling using *current residual* (unreserved) capacities based on the *current* allocations x . From Table II, we see the improvement is modest for the case of PS-DSF.

Improvements are also obtained by *best-fit* server selection. For example, best-fit DRF (BF-DRF) first selects framework n by DRF and then selects the server whose residual capacity most closely matches their resource demands $\{d_{n,r}\}_r$ [32].

VII. SUMMARY AND FUTURE WORK

For a private-cloud setting, we considered scheduling a group of heterogeneous, distributed frameworks to a group of heterogeneous servers. We extended two general results on max-min fairness and proportional fairness to this case for a static problem under generic scheduling criteria. Subsequently, we assessed the efficiency of approximate max-min fair allocations by progressive filling according to different fairness criteria. Illustrative examples in heterogeneous settings show that max-min fair PS-DSF and rPS-DSF scheduling, are superior to DRF in terms of task efficiency performance (a metric related to proportional fairness) and that the efficiency of these “server specific” schedulers did not significantly suffer from the use of randomized round-robin server selection. Task efficiency was also improved when either the “best fit” approach to selecting servers was used or the fairness criteria was modified to use current residual/unreserved resource capacities. We also open-source implemented oblivious (“coarse grained”) and workload-characterized (specified resource demands d) *online* prototypes of these schedulers on Mesos [22], [27], with the Mesos default/baseline being oblivious DRF. Using two different Spark workloads and heterogeneous servers, we showed that the schedulers were similarly ranked using the total execution time as the performance measure. Moreover, execution times could be shortened with workload characterization.

In future work, we will consider scheduling (admission control and placement) problems in a public cloud setting. To this end, note that similar objectives to those considered herein for a private-cloud setting, particularly (2), may be reinterpreted as overall revenue based on *bids* ϕ for virtual machines or containers with fixed resource allocations d . Also, as profit margins diminish in a maturing marketplace, one expects that public clouds will need to operate with greater resource efficiency. Note

that notions of fair scheduling and desirable properties of schedulers as defined in, *e.g.*, [12], [11], [30] may not be relevant to the public-cloud setting, where the expectation is that different customers/frameworks simply “get what they pay for.” Moreover, in a public cloud setting, what the customers do with their virtual machines/containers is arguably not the concern of the cloud operator so long as the customer complies with service level agreements. But, *e.g.*, notions of strategy proofness are important considerations in the design of auction [29] and spot-pricing mechanisms (where under spot price mechanisms, virtual machines or containers may be revoked).

REFERENCES

- [1] T. F. Abdelzaher, K. G. Shin, and N. Bhatti. Performance guarantees for web server end-systems: A control-theoretical approach. *IEEE Trans. Parallel Distrib. Syst.*, 13(1):80–96, 2002.
- [2] D. Bertsekas and R. Gallager. *Data Networks, 2nd Ed.* Prentice Hall, 1992.
- [3] A. Chandra, W. Gong, and P. Shenoy. Dynamic resource allocation for shared data centers using online measurements. In *Proceedings of the 2003 ACM SIGMETRICS International Conference on Measurement and Modeling of Computer Systems*, SIGMETRICS '03, 2003.
- [4] C. Chekuri and S. Khanna. On multi-dimensional packing problems. *SIAM Journal of Computing*, 33(4):837–851, 2004.
- [5] M. Chowdhury, Z. Liu, A. Ghodsi, and I. Stoica. HUG: Multi-resource fairness for correlated and elastic demands. In *Proc. USENIX NSDI*, March 2016.
- [6] H. Christensen, A. Khan, S. Pokutta, and P. Tetali. Multidimensional Bin Packing and Other Related Problems: A Survey. <https://people.math.gatech.edu/~tetali/PUBLIS/CKPT.pdf>, 2016.
- [7] I. Cohen, M. Goldszmidt, T. Kelly, J. Symons, and J. S. Chase. Correlating instrumentation data to system states: A building block for automated diagnosis and control. In *Proceedings of the 6th Conference on Symposium on Operating Systems Design & Implementation - Volume 6*, OSDI'04, 2004.
- [8] M. Cohen, V. Mirrokni, P. Keller, and M. Zadimoghaddam. Overcommitment in Cloud Services Bin packing with Chance Constraints. In *Proc. ACM SIGMETRICS*, Urbana-Campaign, IL, June 2017.
- [9] R. P. Doyle, J. S. Chase, O. M. Asad, W. Jin, and A. M. Vahdat. Model-based resource provisioning in a web service utility. In *Proceedings of the 4th Conference on USENIX Symposium on Internet Technologies and Systems - Volume 4*, USITS'03, 2003.
- [10] Duke utility bill tariff, 2012. <http://www.considerthecarolinas.com/pdfs/scschedulesopt.pdf>.
- [11] E. Friedman, A. Ghodsi, and C.-A. Psomas. Strategyproof allocation of discrete jobs on multiple machines. In *Proc. ACM Conf. on Economics and Computation*, 2014.
- [12] A. Ghodsi, M. Zaharia, B. Hindman, A. Konwinski, S. Shenker, and I. Stoica. Dominant resource fairness: Fair allocation of multiple resource types. In *Proc. USENIX NSDI*, 2011.
- [13] B. Hindman, A. Konwinski, M. Zaharia, A. Ghodsi, A. Joseph, R. Katz, S. Shenker, and I. Stoica. Mesos: A Platform for Fine-grained Resource Sharing in the Data Center. In *Proc. USENIX NSDI*, 2011.
- [14] C. Joe-Wong, S. Sen, T. Lan, and M. Chiang. Multi-resource allocation: Fairness-efficiency tradeoffs in a unifying framework. *IEEE/ACM Trans. Networking*, 21(6), Dec. 2013.
- [15] J. Khamse-Ashari, G. Kesidis, I. Lambadaris, B. Urgaonkar, and Y. Zhao. Constrained Max-Min Fair Scheduling of Variable-Length Packet-Flows to Multiple Servers. In *Proc. IEEE GLOBECOM*, Washington, DC, Dec. 2016.
- [16] J. Khamse-Ashari, I. Lambadaris, G. Kesidis, B. Urgaonkar, and Y. Zhao. An Efficient and Fair Multi-Resource Allocation Mechanism for Heterogeneous Servers. <http://arxiv.org/abs/1712.10114>, Dec. 2017.
- [17] J. Khamse-Ashari, I. Lambadaris, G. Kesidis, B. Urgaonkar, and Y. Zhao. An Efficient and Fair Multi-Resource Allocation Mechanism for Heterogeneous Servers. *IEEE Trans. Parallel and Distributed Systems (TPDS)*, May 2018.
- [18] J. Khamse-Ashari, I. Lambadaris, G. Kesidis, B. Urgaonkar, and Y. Zhao. Per-Server Dominant-Share Fairness (PS-DSF): A Multi-Resource Fair Allocation Mechanism for Heterogeneous Servers. <https://arxiv.org/abs/1611.00404>, Nov. 1, 2016.
- [19] R. Levy, J. Nagarajao, G. Pacifici, M. Spreitzer, A. Tantawi, and A. Youssef. Performance management for cluster based web services. In G. Goldszmidt and J. Schönwälder, editors, *Integrated Network Management VIII: Managing It All*, pages 247–261. Springer US, 2003.
- [20] C. Lu, T. F. Abdelzaher, J. A. Stankovic, and S. H. Son. A feedback control approach for guaranteeing relative delays in web servers. In *Proceedings of the Seventh Real-Time Technology and Applications Symposium*, RTAS '01, 2001.
- [21] D. A. Menasce. Web server software architectures. *IEEE Internet Computing*, 7(6):78–81, 2003.
- [22] Mesos multi-scheduler. <https://github.com/PSU-Cloud/mesos-ps/pull/1/files>.
- [23] J. Mo and J. Walrand. Fair end-to-end window-based congestion control. *IEEE/ACM Trans. Networking*, Vol. 8, No. 5:pp. 556–567, 2000.
- [24] M. N. Bennani and D. A. Menasce. Resource allocation for autonomic data centers using analytic performance models. In *Proceedings of the Second International Conference on Automatic Computing*, ICAC '05. IEEE Computer Society, 2005.
- [25] H. Seltman. Approximation of mean and variance of a ratio. <http://www.stat.cmu.edu/~hseltman/files/ratio.pdf>.

- [26] Y. Shan, A. Jain, G. Kesidis, B. Urgaonkar, J. Khamse-Ashari, and I. Lambadaris. Online Scheduling of Spark Workloads with Mesos using Different Fair Allocation Algorithms. <https://arxiv.org/abs/1803.00922>, March 2, 2018.
- [27] Spark with HeMT. <https://github.com/PSU-Cloud/spark-hemt/pull/2/files>.
- [28] A. Stuart and K. Ord. *Kendall's Advanced Theory of Statistics*. Arnold, London, 6th edition, 1998.
- [29] Vickrey-Clarke-Groves auction. https://en.wikipedia.org/wiki/Vickrey-Clarke-Groves_auction.
- [30] W. Wang, B. Li, B. Liang, and J. Li. Towards multi-resource fair allocation with placement constraints. In *Proc. ACM SIGMETRICS*, Antibes, France, 2015.
- [31] W. Wang, B. Li, B. Liang, and J. Li. Multi-resource fair sharing for datacenter jobs with placement constraints. In *Proc. Supercomputing*, Salt Lake City, Utah, 2016.
- [32] W. Wang, B. Liang, and B. Li. Multi-resource fair allocation in heterogeneous cloud computing systems. *IEEE Transactions on Parallel and Distributed Systems*, 26(10):2822–2835, Oct. 2015.
- [33] W. Xu, P. Bodik, and D. Patterson. A flexible architecture for statistical learning and data mining from system log streams. In *Proceedings of Workshop on Temporal Data Mining: Algorithms, Theory and Applications at the Fourth IEEE International Conference on Data Mining*, Brighton, UK, 2004.
- [34] K.-K. Yap, T.-Y. Huang, Y. Yiakoumis, S. Chinchali, N. McKeown, and S. Katti. Scheduling packets over multiple interfaces while respecting user preferences. In *Proc. ACM CoNEXT*, Dec. 2013.

APPENDIX A: PROOF OF PROPOSITION 1

Define the Lagrangian to be maximized over x and over Lagrange multipliers $\lambda, \nu \geq 0$:

$$\begin{aligned} L = & \sum_n \phi_n g(U_n) + \sum_{i,r} \lambda_{i,r} (1 - \sum_{n \in N_i} x_{n,i} B_{n,i,r}) \\ & + \sum_{i,n \in N_i} \nu_{n,i} x_{n,i}. \end{aligned}$$

The first-order optimality condition,

$$\begin{aligned} & \forall i, n \in N_i, \delta_{n,i} = 1, \\ 0 = \frac{\partial L}{\partial x_{n,i}} = & u_{n,i} g'(U_n) - \sum_r \lambda_{i,r} B_{n,i,r} + \nu_{n,i}, \end{aligned} \quad (20)$$

and g strictly increasing imply

$$\forall i, n \in N_i, \sum_r \lambda_{i,r} B_{n,i,r} > \nu_{n,i} \geq 0. \quad (21)$$

So, $\forall i, \exists r$ s.t. $\lambda_{i,r} > 0$. Thus, complementary slackness is

$$\forall i, r, \lambda_{i,r} (1 - \sum_{n \in N_i} x_{n,i} B_{n,i,r}) = 0 \quad (22)$$

$$\Rightarrow \forall i, \exists r \text{ s.t. } \sum_{n \in N_i} x_{n,i} B_{n,i,r} = 1, \quad (23)$$

i.e., in every server i , one resource r (which may depend on i) is fully booked. So, the set of fully booked resources in server i under allocations $x = \{x_{n,i}\}$ can be characterized by $\{r \mid \lambda_{i,r} > 0\}$. Now by (20) and assumed strict concavity of g , uniquely

$$\begin{aligned} \forall i, n \in N_i, U_n = & (g')^{-1} \left(\sum_r \lambda_{i,r} \frac{B_{n,i,r}}{u_{n,i}} - \frac{\nu_{n,i}}{u_{n,i}} \right) \\ = & (g')^{-1} \left(\sum_{r: \lambda_{i,r} > 0} \lambda_{i,r} \frac{B_{n,i,r}}{u_{n,i}} - \frac{\nu_{n,i}}{u_{n,i}} \right). \end{aligned}$$

Now consider two frameworks m and ℓ and server i such that $x_{m,i} > 0$ and $\delta_{m,i} = 1 = \delta_{\ell,i}$. So, complementary slackness

$$\forall j, n \in N_j, \nu_{n,j} x_{n,j} = 0, \quad (24)$$

implies $\nu_{m,i} = 0$.

Because $(g')^{-1}$ is strictly decreasing (g strictly concave): if $\delta_{m,i} = 1 = \delta_{\ell,i}$ then

$$\begin{aligned} U_m &= (g')^{-1} \left(\sum_{r: \lambda_{i,r} > 0} \lambda_{i,r} \frac{B_{m,i,r}}{u_{m,i}} \right) \\ &\leq (g')^{-1} \left(\sum_{r: \lambda_{i,r} > 0} \lambda_{i,r} \frac{B_{\ell,i,r}}{u_{\ell,i}} - \frac{\nu_{\ell,i}}{u_{\ell,i}} \right) = U_{\ell}, \end{aligned}$$

where we have used assumption (14) which is sufficient for the inequality. Because of this and (23), a solution $x = \{x_{n,i}\}$ of the optimization (5) s.t. (6) is U -MMF.

APPENDIX B: PROOF OF PROPOSITION 2

The Lagrangian here is

$$\begin{aligned} L &= \sum_n \phi_n g_a(x_n) + \sum_{i,r} \lambda_{i,r} (1 - \sum_{n \in N_i} x_{n,i} B_{n,i,r}) \\ &\quad + \sum_{i,n \in N_i} \nu_{n,i} x_{n,i} \delta_{n,i} \end{aligned}$$

where, again, the Lagrange multipliers $\lambda, \nu \geq 0$. A first-order optimality condition is

$$\begin{aligned} \forall i, n \in N_i, \quad 0 &= \frac{\partial L}{\partial x_{n,i}}(x^*) \\ &= \phi_n g'_a(x_n^*) - \sum_r \lambda_{i,r} B_{n,i,r} + \nu_{n,i}. \end{aligned} \tag{25}$$

Multiplying (25) by $x_{n,i} - x_{n,i}^*$ and summing over i and $n \in N_i$ gives⁷

$$\begin{aligned} 0 &= \sum_n \phi_n g'_a(x_n^*) (x_n - x_n^*) + \sum_{i,n \in N_i} \nu_{n,i} (x_{n,i} - x_{n,i}^*) \\ &\quad - \sum_{i,r} \sum_{n \in N_i} \lambda_{i,r} B_{n,i,r} (x_{n,i} - x_{n,i}^*) \end{aligned}$$

where the first term is $\Phi(x, x^*)$ and recall the definition of N_i (4). Thus, by complementary slackness (22) and (24) (taking $x = x^*$ there in those equations),

$$\Phi(x, x^*) = \sum_{i,r: \lambda_{i,r} \neq 0} \lambda_{i,r} \left(\sum_{n \in N_i} x_{n,i} B_{n,i,r} - 1 \right) - \sum_{i,n \in N_i} \nu_{n,i} x_{n,i}.$$

Finally, no resource overbooking (6) implies $\Phi(x, x^*) \leq 0$.

⁷Simply use Fubini's theorem for the first term, $\sum_i \sum_{n \in N_i} \phi_n g'_a(x_n^*) (x_{n,i} - x_{n,i}^*) = \sum_n \sum_{i: \delta_{n,i}=1} \phi_n g'_a(x_n^*) (x_{n,i} - x_{n,i}^*)$