Computing Science Group

Robust Localization in Cluttered Environments with NLOS Propagation

Sarfraz Nawaz
Niki Trigoni

CS-RR-09-16

Oxford University Computing Laboratory
Wolfson Building, Parks Road, Oxford, OX1 3QD

Robust Localization in Cluttered Environments with NLOS Propagation

Sarfraz Nawaz
Niki Trigoni
Oxford University Computing Laboratory
Wolfson Building, Parks Road
Oxford OX1 3QD
Sarfraz.Nawaz@comlab.ox.ac.uk
Niki.Trigoni@comlab.ox.ac.uk

***Abstract*—In this paper, we propose a robust multilateration algorithm for localizing sensor nodes in cluttered environments where the estimated distances between an unlocalized node and reference nodes with known coordinates may contain large errors due to non line of sight signal propagation. We show that the traditional least squares multilateration is severely affected even if one of the measured distances is erroneous whereas our approach functions properly even if half of the measured distances contain large errors due to non line of sight signals. Our algorithm is independent of the physical layer used to perform ranging and does not require the identification of direct and reflected signals or any prior information about the statistical properties of measurement errors or characterization of the environment where the sensor nodes are deployed.**

I. Introduction

A majority of sensor network applications rely on the assumption that each sensor node can determine its location with in the physical space where it has been deployed. For example, in a monitoring application each sensor node reports the sensed data along with its coordinates to a base-station where these are used for analysis or presentation. Some of other applications include locating objects or personnel in a building and target tracking. The availability of location information at each individual node also allows the sensor network to run completely distributed and localized algorithms, for example, geographic routing [1], data aggregation [2] and smart query processing [3] etc

One of the localization techniques involves the measurement of distances between the unlocalized node and reference points with known coordinates. These reference points could be part of a fixed infrastructure or other sensor nodes that have already calculated their own coordinates. These reference points are generally referred as anchor nodes. The distances between the unlocalized node and anchor nodes are generally estimated by measuring time of flight (ToF) of an acoustic [4], [5] or a radio [6] signal. These measured distances and anchor node coordinates are then used to calculate the coordinates of the unlocalized node.

When the sensor nodes are deployed in a cluttered environment, for example, inside an ordinary office building, the ranging signals used for distance measurement can reflect and bounce off multiple surfaces before arriving at the receiver. This makes it difficult for the receiver to estimate the true distance accurately either due to interference between the signals travelling along multiple paths or due to the absence of direct clear line of sight between the transmitter and the receiver [6]. This introduces a large positive error in some of the estimated distances depending on the relative position of the transmitter receiver pair, the environment and the physical characteristics of the signals used. The accuracy of the calculated node coordinates is severely affected due to the presence of these erroneous distance estimates. Thus accurately localizing sensor nodes in cluttered environments is a challenging problem and is the main focus of this work. Following are the primary contributions of this work,

- We show that the presence of even a single erroneous distance measurement due to non line of sight signals severely degrades the accuracy of traditional localization algorithms.
- We present a novel localization algorithm that is robust against the distance measurement errors arising from non line of sight signals.
- Our approach does not require the identification of line of sight and non line of sight signals and is thus independent of the physical layer used to perform ranging.
- Our approach does not require any prior information about the statistical properties of non line of sight measurement errors and thus can be used in ad-hoc deployments without collecting any measurement data.

The rest of this paper is organized as follows. Section II outlines the related work. Section III discusses the traditional least squares based localization and demonstrates its shortcomings when used in cluttered environments. Section IV presents our robust multilateration algorithm and an analysis of its robustness. Section V compares our approach with least squares based localization algorithms through simulations. Section VI presents results from a small experimental testbed. Finally section VII concludes this paper.

II. Related Work

Venkatesh and Buehrer [7] propose a linear programming based algorithm for localizing a node in a cluttered indoor environment using distance measurements from both direct and non line of sight UWB radio signals. However, they assume that it is possible to distinguish between the direct and non line of sight measurements from the characteristics of the received UWB radio signals at the physical layer [8]. Therefore, their approach is restricted to the UWB radio technology. Our algorithm on the other hand, does not require the identification of direct and non line of sight signals and is thus completely independent of the physical layer used to perform range measurements.

Guven et al. [9] propose a weighted least squares algorithm for localization in cluttered environments. They also use a non line of sight identification technique based on channel characteristics and assign smaller weights to measurements coming from signals that are identified as non line of sight. However, the channel characteristics depend on the environment in which the sensor nodes are deployed and thus require measurement and data collection campaigns to build a channel model [10] for a specific environment.

There is also some research literature available that deals with localization in cellular networks in the presence of multipath and non line of sight signals. Wylie and Holtzman [11] propose to identify non line of sight range measurements from a moving transmitter to a set of fixed base stations by comparing the standard deviation of a series of measurements with a threshold. These non line of sight ranges are then corrected by employing the knowledge of actual measurement noise and then used with least squares to determine the coordinates of the transmitter. However, this approach assumes that the transmitter is moving and thus the variation of obstructions between the transmitter and non line of sight base station leads to a larger standard deviation for a series of range measurements. Thus this approach cannot be used with stationary nodes.

Chen [12] propose an algorithm for localizing mobile phones in the presence of non line of sight range measurements. Their approach does not require the identification of non line of sight measurements but it depends on a heuristic that the sum of squared residuals of a least squares estimate can be used as an indicator of the accuracy of calculated node coordinates. They apply least squares multilateration on all possible combinations of the distance measurements and then calculate the final node coordinates as a weighted combination of these intermediate estimates where the weights depend on the sum of squared residual values of each estimate. Therefore, the computational complexity of this approach grows exponentially with the number of distance measurements. Dulman et al. [13] has also shown that the sum of squared residuals cannot be used as a measure of localization accuracy.

Qi and Kobayashi [14] derive a Cramer Rao lower bound for localization in a cluttered environment where both direct and reflected signals are present. They show that if no prior information about the statistical properties of the non line of sight distance estimates is available, then these measurements provide no new information that can be used to reduce the bound on the localization error. In a later work [15], they show that if statistical information like probability distribution of the non line of sight measurements is available, then such measurements can be used to lower the localization error in cluttered environments. However, this information can only be collected through measurement campaigns in the specified environment and processed off line before localization can be performed. This makes this approach cumbersome and unsuitable for ad-hoc deployments. On the other hand, if no prior information is available for non line of sight distance estimates, then these measurements must be detected and filtered out before calculating the location coordinates using least squares estimation because it is extremely susceptible to large measurement errors.

III. Least Squares Multilateration

In this section, we outline least squares multilateration and analyze its performance in the presence of non line of sight distance estimates. Let us suppose that there are m fixed anchor nodes with coordinates (x_i, y_i) where $i = 1, 2, \ldots, m$. A non anchor node that wishes to determine its coordinates, estimates its distance to three or more anchor nodes. Let us suppose that d_i is the estimated distance to anchor node i. If (x, y) are coordinates of the non-anchor node, then we can write a system of equations as,

$$\begin{aligned} (x_1 - x)^2 + (y_1 - y)^2 &= d_1^2 \\ (x_2 - x)^2 + (y_2 - y)^2 &= d_2^2 \\ &\vdots \\ (x_m - x)^2 + (y_m - y)^2 &= d_m^2 \end{aligned} \tag{1}$$

The only unknowns in the above system of equations are the coordinates x and y of the unlocalized node. These can be determined by solving a problem that is known as *least squares* and is given as

$$\hat{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{argmin}} \sum_{i=1}^{m} r_i(\mathbf{x})^2 \tag{2}$$

where $\mathbf{x} = [x, y]^T$, $\hat{\mathbf{x}}$ is a vector of estimated coordinates and $r_i(\mathbf{x})$ is a residual function given as,

$$r_i(\mathbf{x}) = \left\{ (x_i - x)^2 + (y_i - y)^2 \right\} - d_i^2 \quad i = 1, 2, \ldots, m \tag{3}$$

This residual function $r_i(\mathbf{x})$ is a nonlinear function of x and y. Therefore, the problem given in (2) is an unconstrained nonlinear optimization problem and is generally known as *nonlinear least squares*. It can be solved by using any of the Newton type optimization algorithms [16]. These are iterative algorithms and require a starting point $\mathbf{x_0} = [x_0, y_0]^T$ which is then gradually improved in each iteration until a local minimum of the above defined objective function is found.

The system of nonlinear equations given in Eq. (1) can be linearized by subtracting one of the equations from the remaining $m - 1$ equations. If we subtract the last equation from the others, this results in the following linear system,

$$
\begin{aligned}
(x_1 - x_m)x + (y_1 - y_m)y &= b_1 \\
(x_2 - x_m)x + (y_2 - y_m)y &= b_2 \\
&\vdots \\
(x_{m-1} - x_m)x + (y_{m-1} - y_m)y &= b_{m-1}
\end{aligned} \tag{4}
$$

where,

$$
b_i = \frac{1}{2}\left\{x_i^2 - x_m^2 + y_i^2 - y_m^2 + d_m^2 - d_i^2\right\} \tag{5}
$$

In matrix notation, the linear system given in Eq. (4) can be expressed as,

$$
\mathbf{A}\mathbf{x} = \mathbf{b} \tag{6}
$$

where,

$$
\mathbf{A} = \begin{bmatrix} x_1 - x_m & y_1 - y_m \\ x_2 - x_m & y_2 - y_m \\ \vdots & \vdots \\ x_{m-1} - x_m & y_{m-1} - y_m \end{bmatrix} \tag{7}
$$

and

$$
\mathbf{b} = \frac{1}{2}\begin{bmatrix} x_1^2 - x_m^2 + y_i^2 - y_m^2 + d_m^2 - d_i^2 \\ x_2^2 - x_m^2 + y_i^2 - y_m^2 + d_m^2 - d_i^2 \\ \vdots \\ x_{m-1}^2 - x_m^2 + y_i^2 - y_m^2 + d_m^2 - d_i^2 \end{bmatrix} \tag{8}
$$

The system of linear equations given in Eq. (4) can be solved for x and y by using the least squares approach given in Eq. (2) with the following residual function,

$$
r_i(\mathbf{x}) = (x_i - x_m)\,x + (y_i - y_m)\,y - b_i \quad i = 1, \ldots, m-1 \tag{9}
$$

When this residual function which is linear in the unknowns x and y is used, the problem expressed in Eq. (2) is known as *linear least squares* and has a closed form solution given as,

$$
\hat{\mathbf{x}} = \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{b} \tag{10}
$$

Least squares is the most popular algorithm for estimating parameters from multiple noisy measurements. It is also the oldest technique dating back to the nineteenth century when it was first published by Legendre in 1805. Gauss later claimed that he had been using this approach since 1795. Stigler [17] presents an interesting account of this priority dispute. Guass, however, is credited with developing a probabilistic justification of least squares and showing that the least square estimate of parameters is optimal when the measurement errors are identical and independently distributed (iid) with a normal distribution with zero mean. However, the major shortcoming of least squares is its sensitivity to *outliers*. When using least squares estimation, even if a single measurement contains an error that is significantly different from others, the estimated parameters are severely affected. In the context of localization in cluttered environments this means that even if a single distance between the unlocalized node and an anchor node is overestimated due to multipath reflections or the absence of direct line of sight, the position estimate calculated using least squares will be inaccurate. In order to demonstrate this, we set up a simulation where 10 anchor nodes are deployed randomly in a 10×10 unit area. A single unlocalized node measures distance d_i to an anchor node i such that

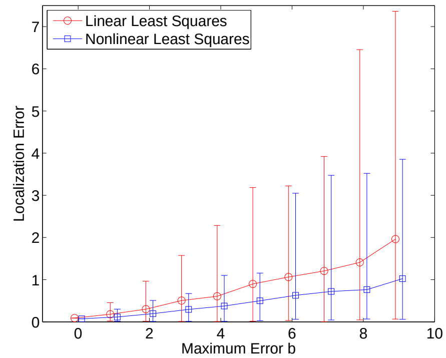


Fig. 1: Overestimated distances offset the coordinates determined by least squares algorithms.

$$
d_i = \bar{d}_i + e_i \tag{11}
$$

where $\bar{d}_i$ is the true distance and e_i are identical and independently distributed (iid) measurement errors with a normal distribution $\mathcal{N}(0,\sigma)$. We randomly select one distance estimate and also add an error drawn from a uniform distribution $\mathcal{U}(a,b)$ with $a = 0$, $b > 0$ to it to emulate the overestimation of distance due to multipath or the absence of direct line of sight between the unlocalized node and the anchor node. These distance estimates and the anchor coordinates are then used to determine the coordinates of the unlocalized node using both the linear and nonlinear least square approaches discussed above. Fig. (1) shows the results of this simulation where the horizontal axis is the maximum b of the uniform distribution $\mathcal{U}(a,b)$ and the vertical axis is the localization error. Each point on the graph is an average of 100 runs of simulation and the bars indicate minimum and maximum values. It shows that even a single non line of sight distance measurement can offset the position estimate calculated using both linear and nonlinear least squares and the localization error increases as the non line of sight error is increased.

We can use the closed form solution of linear least squares to explain this behaviour. Let us suppose that $\bar{\mathbf{b}}$ is a vector formed by using true distances $\bar{d}_i$, then the true position of the unlocalized node is

$$
\bar{\mathbf{x}} = \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T\bar{\mathbf{b}} \tag{12}
$$

and the localization error is given as,

$$
\|\mathbf{x} - \bar{\mathbf{x}}\| \leq \|\mathbf{F}\|\|\epsilon\| \tag{13}
$$

where $\|.\|$ is the l_2 norm and

$$\mathbf{F} = \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T \tag{14}$$

$$\epsilon = \mathbf{b} - \bar{\mathbf{b}} \tag{15}$$

ϵ is a vector of random variables with,

$$\epsilon_i = \left(\bar{d_m}e_m + \frac{1}{2}e_m^2\right) - \left(\bar{d}_i e_i + \frac{1}{2}e_i^2\right) \quad i = 1, 2, \ldots, m-1 \tag{16}$$

This shows that the vector norm $\|\epsilon\|$ and thus the localization error bound given in Eq. (13) increases even if a single measurement error e_i is large. Nonlinear least squares also exhibits a similar in the presence of large measurement errors as shown in fig. (1). Linear least squares results in a larger localization error as compared to nonlinear least squares because the linearization of equation system given in Eq. (1) introduces error e_m in all of the remaining $m-1$ measurements as shown in Eq. (16).

IV. Robust Multilateration

In the previous section, we outlined the least squares algorithm for calculating coordinates of an unlocalized node and showed that even a single distance measurement with large error offsets the calculated coordinates. The least squares algorithm estimates the coordinates by minimizing the sum of squared residuals as shown in Eq. (2). Since the residuals r_i are squared, the ones corresponding to the measurements with large errors become relatively large and thus offset the estimated coordinates. This problem can be avoided by minimizing an objective function where the residuals for large errors do not become even larger. One such objective function is the sum of absolute values of the residuals,

$$\hat{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{argmin}} \sum_{i=1}^{m} |r_i(\mathbf{x})| \tag{17}$$

Thus an estimate of the node coordinates $\hat{\mathbf{x}}$ that is not affected by large distance measurement errors can be obtained by solving the optimization problem given in Eq. (17). We refer to this as *Robust Multilateration*.

In order to demonstrate the robustness of this approach, we analyse a one dimensional localization problem. Let us suppose that we want to determine a one dimensional coordinate x of an unlocalized node with the help of anchor nodes located at x_i and the measured distances d_i between the anchor nodes and the unlocalized node as shown in fig. (2). Then for each anchor node i we have,

$$x_i - x = d_i \quad i = 1, 2, \cdots, m \tag{18}$$

For this problem, Eq. (17) becomes

$$\hat{x} = \underset{x}{\operatorname{argmin}} \sum_{i=1}^{m} |c_i - x| \tag{19}$$

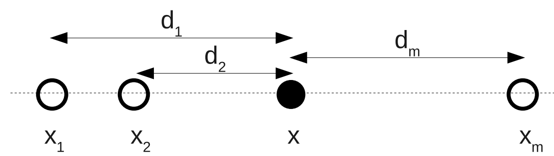


Fig. 2: One dimensional localization

where $c_i = x_i - d_i$. This function is piecewise linear and convex, and thus the minimizer $\hat{x}$ can be determined by setting the derivative $f'(x)$ to zero. The derivative $f'(x)$ where it exists is given as,

$$f'(x) = \sum_{i=1}^{m} \operatorname{sgn}(c_i - x) \tag{20}$$

where

$$\operatorname{sgn}(z) = \begin{cases} 1 & z > 0 \\ 0 & z = 0 \\ -1 & z < 0 \end{cases} \tag{21}$$

This shows that the minimizer $\hat{x}$ of the objective function given in Eq. (19) is a quantity that is larger than half of the c_i and smaller than the other half or in other words it is the *median* of c_i values. Since the median is not affected by outliers, the estimated coordinate $\hat{x}$ is robust to large measurement errors in d_i. For this one dimensional problem, upto 50% of the measured distances can be erroneous with out affecting the estimated coordinate $\hat{x}$.

When using the least squares approach for the same problem, then we have

$$\hat{x} = \underset{x}{\operatorname{argmin}} \sum_{i=1}^{m} (c_i - x)^2 \tag{22}$$

The derivative of the objective function $f'(x)$ is given as,

$$f'(x) = -2\sum_{i=1}^{m} (c_i - x) \tag{23}$$

which when set to zero provides a closed form solution for the node coordinate $\hat{x}$ as

$$\hat{x} = \frac{1}{m}\sum_{i=1}^{m} c_i \tag{24}$$

Thus the estimated node coordinate $\hat{x}$ generated by least squares algorithm for this one dimensional problem is the *mean* of c_i values. Since the mean is not resilient to outliers, thus even a single erroneous distance estimate will offset the calculated position when using least squares multilateration.

This analysis shows that our multilateration approach which corresponds to the median of observed data is robust and can withstand erroneous distance measurements. This result is also applicable when localizing a node in more than one dimension, although the analysis becomes intractable due to nonlinearity. When calculating the coordinates in higher dimensions, the nonlinear localization problem cannot be transformed into a linear one due to the presence of large errors. For example, if

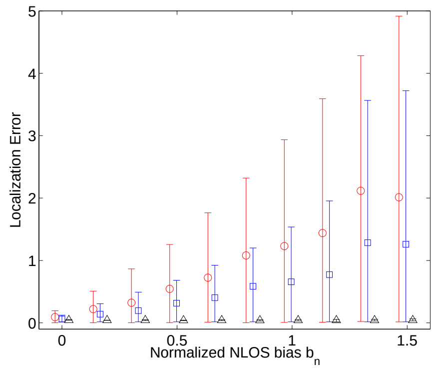
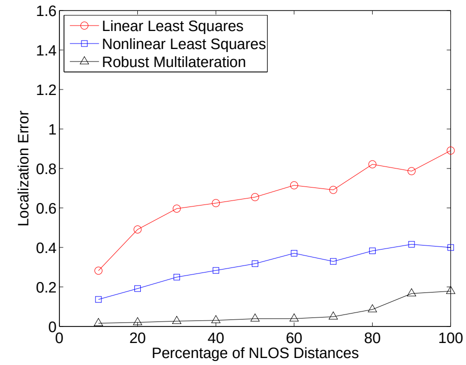
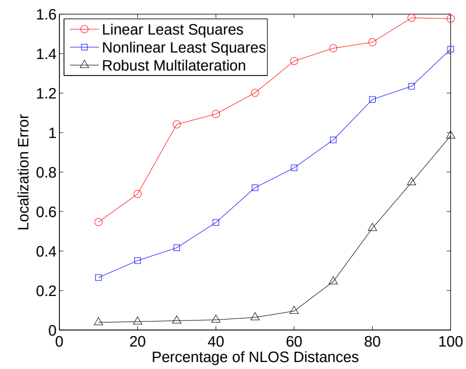


Fig. 3: Increasing the NLOS error



(a) Normalized NLOS bias $b_n = 0.4$



(b) Normalized NLOS bias $b_n = 0.7$

Fig. 4: Increasing the percentage of NLOS distances

a large measurement error due to non line of sight signals is present in the last equation in (1), then linearizing it to (4) will corrupt all the remaining measurements as shown in Eq. (16). Therefore, we have to use the residual function $r_i(\mathbf{x})$ given in Eq. (3) and thus the problem presented in Eq. (17) becomes a nonlinear optimization problem with an objective function that is only piecewise differentiable. Gonin and Money [18] present a detailed overview of numerical algorithms that can be used to solve optimization problems of this type. These are iterative algorithms and require an initial starting point. Estimated coordinates from the linear least squares algorithm can be used as the starting point for these algorithms.

The main advantage of our approach is that all the distance estimates are used directly as input to the algorithm without having to identify and filter out any erroneous distance measurements coming from non line of sight signals. Therefore, it is not dependent on underlying physical layer and can be used with any of the ranging technologies. Also no prior information about the statistical properties of the non line of sight measurements is required. Therefore, it can be easily used for localizing nodes in all types of cluttered environments without performing any measurement and data collection campaigns. The algorithm is robust against non line of sight distance measurement errors and is able to recover a good position estimate even in the presence of large errors where the coordinates generated by least squares are severely affected.

V. Simulations

In this section, we present some simulation results that demonstrate that our robust multilateration algorithm performs better than traditional least squares multilateration when some of the distance estimates contain large errors due to non line of sight signals. We model each measured distance d_i between the unlocalized node and the anchor node i as,

$$d_i = \bar{d}_i + e_i + l_i \tag{25}$$

where $\bar{d}_i$ is the true distance, e_i is zero mean Gaussian measurement error,

$$e_i \sim \mathcal{N}(0, \sigma) \tag{26}$$

where σ is the standard deviation of the Gaussian distribution and l_i is an error that is introduced due to non line of sight propagation such that

$$l_i \sim \mathcal{U}(a, b) \tag{27}$$

where $a = 0$ and $b > 0$ are the minimum and maximum values of the uniform distribution. For direct line of sight distance measurements $l_i = 0$.

For the first set of simulations, we randomly deploy 10 anchor nodes in a 10×10 unit area and an unlocalized node is placed roughly in the middle of this region. One randomly chosen distance estimate between the unlocalized node and an anchor node is set as a non line of sight distance and all the remaining estimates are line of sight. These estimated distances and anchor node coordinates are then used to localize the node using both least squares and robust multilateration. In all the simulations, unless otherwise mentioned, the measurement errors e_i have $\sigma = 0.1$. Fig. (3) shows the results of this simulation where the vertical axis is the localization error and the horizontal axis is the normalized non line of sight bias b_n. The normalized NLOS bias is the ratio of maximum non line of sight error b and the maximum of all the true distances $\bar{d}_{max}$. Each point on the curves is an average of 100 runs of simulation and the bars indicate minimum and maximum values. This shows that the

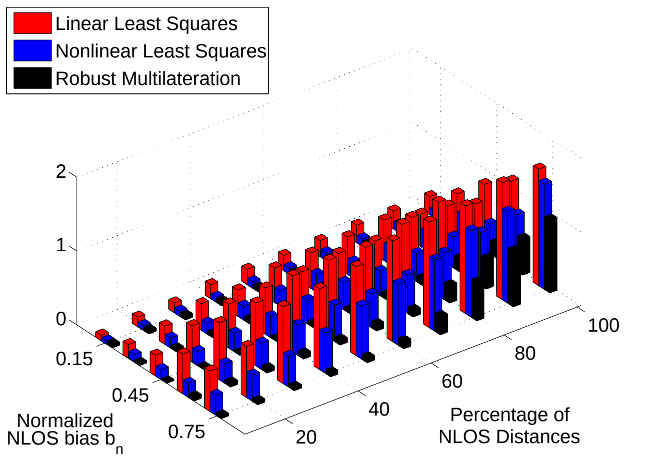


Fig. 5: Localization error for a range of NLOS bias and percentage of measurements corrupted with NLOS errors

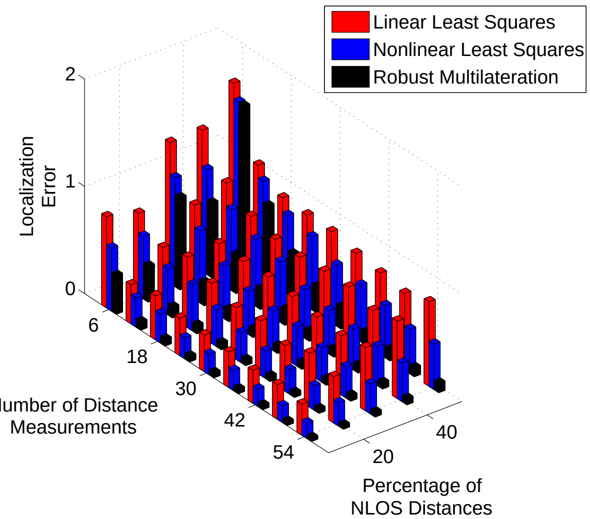


Fig. 6: Localization error for a range of total number of anchor nodes and percentage of NLOS range measurements.

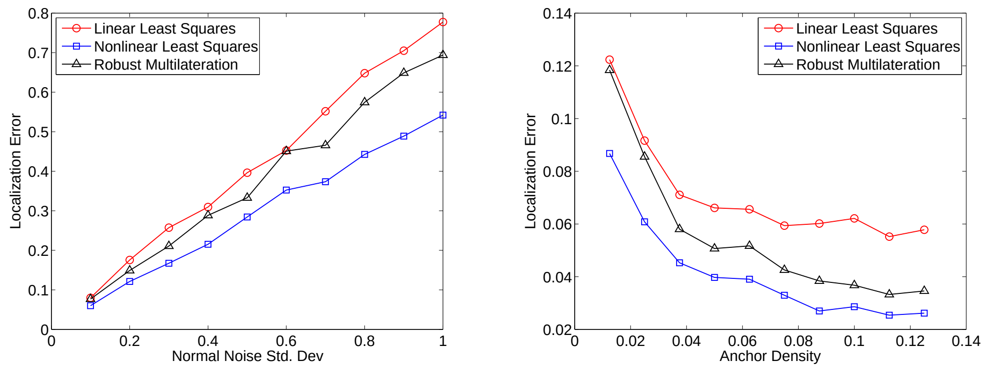
performance of least squares multilateration deteriorates even if one of the distance estimate is erroneous due to non line of sight signals and the localization error increases as the non line of sight error in the distance estimate is increased. Our approach, on the other hand, is not affected by the non line of sight distance measurement.

For the second set of simulations, we randomly deploy 50 anchor nodes in a 10×10 unit area and again an unlocalized node is placed roughly in the middle of the region. In each simulation run, a fixed percentage of estimated distances are randomly chosen to be non line of sight. The coordinates of the unlocalized node are then calculated using all the distance measurements and the anchor node coordinates. Fig. (4) shows the results of this simulation for two different values of normalized NLOS bias b_n. This shows that for any given NLOS error, the error in the node coordinates computed from least squares increases as the number of NLOS distances is increased. The localization error of our robust algorithm starts to increase only after the percentage of NLOS distances increases beyond a certain point. When the non line of sight error is small, our approach can withstand a larger percentage of non line of sight measurements. As the non line of sight bias is increased, the point where the localization error of our algorithm starts to increase, starts to come down to 50% NLOS distances. Fig. (4b) shows that even when half of the estimated distances contain NLOS errors that could be as large as 70% of the largest true distance between the unlocalized node and the anchor nodes, our algorithm still results in localization error that is smaller than traditional least squares based localization approaches.

Fig. (5) shows the mean localization error for a range of NLOS normalized bias and percentage of NLOS measurements where each bar is an average of 100 runs of simulations. This shows that our approach is robust against NLOS measurement errors for the entire range of normalized NLOS bias when the number of NLOS measurements is less than or equal to half of the total number of distance measurements.

In the next set of simulations, we randomly deploy an increasing number of anchor nodes in a 10×10 unit area and choose a fixed percentage of distance measurements to an unlocalized node in the middle of this region to be non line of sight. For these simulations, we fix the normalized NLOS bias $b_n = 0.5$. Fig. (6) shows the mean localization error of the calculated coordinates for a range of anchor nodes and the percentage of NLOS measurements. Each bar in the figure is an average of 100 runs of simulations. These results show that the localization performance of our approach deteriorates when the total number of range measurements is very small and half or more of these measurements are non line of sight. As we increase the number of total measurements or decrease the percentage of NLOS measurements, our algorithm becomes more robust.

In order to observe the behaviour of our algorithm when all the measured distances are due to line of sight signals, we performed two more simulations. For the first set of simulations, we deploy 10 anchor nodes in a 20×20 unit area and an unlocalized node in the middle of this region. We assume that all the estimated distances d_i are due to direct line of sight signals with $l_i = 0$ and contain only measurement noise e_i. Fig. (7a) shows the results of this simulation where the vertical axis is the localization error and the horizontal axis is the standard deviation σ of the zero mean Gaussian measurement errors e_i. For the second set of simulations, we varied the number of anchor nodes deployed in a 10×10 unit area. Fig. (7b) shows the result of this simulation where the vertical axis is mean localization error and the horizontal axis anchor density. These shows that when all the measured distances are from line of sight signals, the localization accuracy of our approach falls between linear and nonlinear least squares.



(a) Increasing measurement noise

(b) Increasing anchor density

Fig. 7: Localization error with all LOS range measurements containing only Gaussian measurement errors

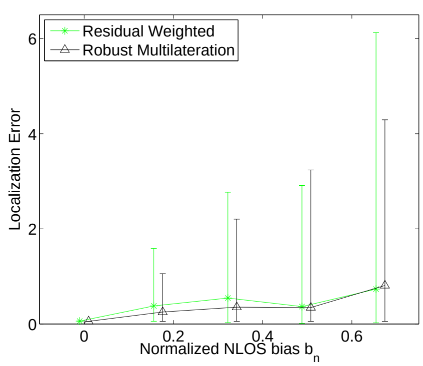


Fig. 8: Comparison of robust multiateration and residual weighted algorithms.

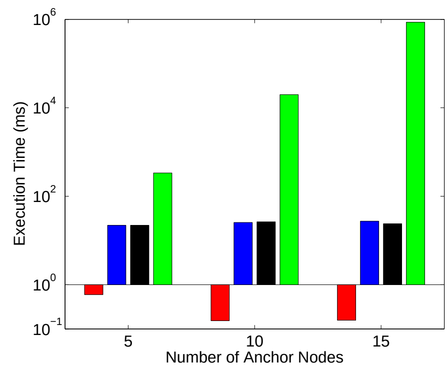
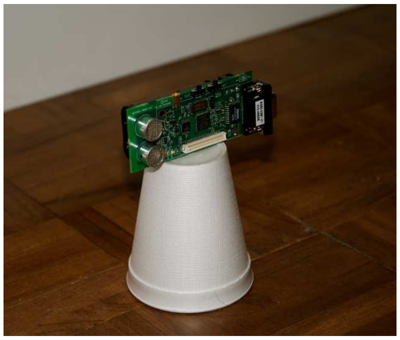


Fig. 9: Execution times of various localization algorithms.

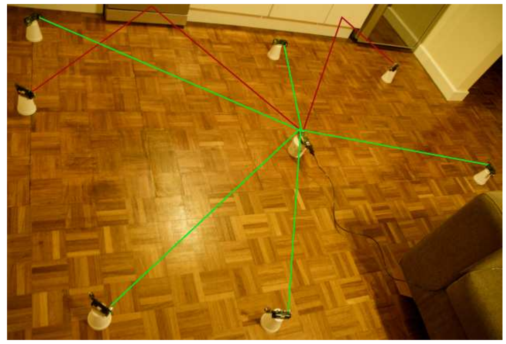
We also compare the performance of our algorithm against the residual weighted approach described by Chen [12]. We randomly deploy 10 anchor nodes in a 10×10 unit area and an unlocalized node in the middle of this region. Three of these anchor nodes are assumed to be non line of sight and the non line of sight error is gradually increased. Fig. (8) shows the results of this simulation where the vertical axis is the localization error and the horizontal axis is normalized NLOS bias b_n. Each point on the curves is an average of 100 runs of simulation and the bars indicate minimum and maximum values. It shows that our approach results in a smaller localization error as compared to the residual weighted algorithm. Another advantage of our approach is its lower computational complexity. This is evident from the execution times as shown in fig. (9). The linear least squares algorithm has the smallest execution time because it has a closed form solution. The nonlinear least squares and our algorithm are iterative approaches and have a similar execution time. The execution time of residual weighted algorithm, on the other hand, is extremely large because it applies nonlinear least squares on all possible combinations of anchor nodes.

VI. Experiment

In order to validate our simulation results, we have run an experiment using MIT Cricket motes [5]. These motes can measure distances among each other using a radio and an ultrasound signal. A transmitter simultaneously transmits a radio and an ultrasound pulse and the receiver estimates the distance to the transmitter using the time difference of arrival between the two signals. Fig. (10b) shows the experimental set up where eight Cricket motes are deployed. One mote acts as a receiver and is connected to a computer for data logging purposes. Other motes act as transmitters and continuously transmit radio and ultrasound pulses. The transmitted ultrasound pulses travel in the form of a narrow beam and can be reflected off solid plane surfaces. We use this fact and place two motes in such a manner that their transmitted signals bounce off a wall and hence form non line of sight distances to the receiver. In fig. (10b), these are shown with solid red lines and direct distances are shown with solid green lines. Using the received signals, the receiver estimates distances to all the transmitting motes. The errors between the true distances and the measured distances for all the transmitters are shown in fig. (11a). The receiver calculates its coordinates using theses measured distances and the coordinates of transmitting motes



(a) Cricket Mote



(b) Experiment set up

Fig. 10: A small network of MIT Cricket motes where a single node is localized in the presence of range measurements from both line of sight and non line of sight ultrasound signals.

with linear least squares, nonlinear least squares and robust multilateration. Fig. (11b) shows the localization results where the red circles indicate the position estimates generated from linear least squares, blue squares indicate position coordinates generated from nonlinear least squares and black triangles show the coordinates calculated from robust multilateration. These results show that robust multilateration generates location estimates that are quite close to the true position despite the fact that two of distance estimates are non line of sight and thus contain large measurement error. However, when the same measurements are used as inputs to linear and nonlinear least squares, the calculated position estimates contain significantly larger error as compared to those calculated with robust multilateration.

Fig. (11c) shows the localization results when one of the two transmitters forming non line of sight distances is switched off. Thus there are five direct line of sight distance estimates and only one distance measurement coming from non line of sight reflected signals. This shows that even a single erroneous distance measurement can introduce significant error in the location estimate computed from linear and nonlinear least squares algorithms. Robust multilateration, on the other hand, computes position estimates that are quite close to the true location of the receiver. We then switch off the transmitter providing the only remaining non line of sight distance estimate. The localization results for this case are shown in fig. (11d). These results show that the linear and nonlinear least square algorithms work well only if all the distance estimates are from direct line of sight signals and do not contain any large errors. However, robust multilateration calculates an accurate position estimate even in the presence of erroneous distance measurements. Fig. (12) shows the mean localization error for all the algorithms in all three cases i.e. two non line of sight distances, one non line of sight distance and all direct distance measurements. Although, we have presented our results from a specific hardware testbed, our algorithm is not restricted to a specific hardware or signals used for estimating distance. It can be used with any of the ranging technologies whether it is based on acoustics, ultrasound or any of the radio signals.

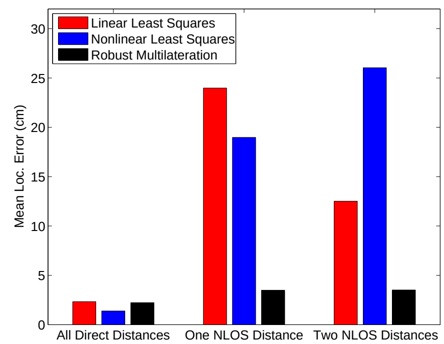


Fig. 12: Localization error

VII. Conclusion

In this paper, we presented a robust algorithm for localizing sensor nodes in cluttered environments where non line of sight signal propagation can introduce large distance measurement errors. We presented results from both simulations and a real experiment and showed that our algorithm can localize sensor nodes with much higher accuracy as compared to traditional least squares localization approaches even when a significant percentage of the measured distances contain large errors due to non line of sight signals.

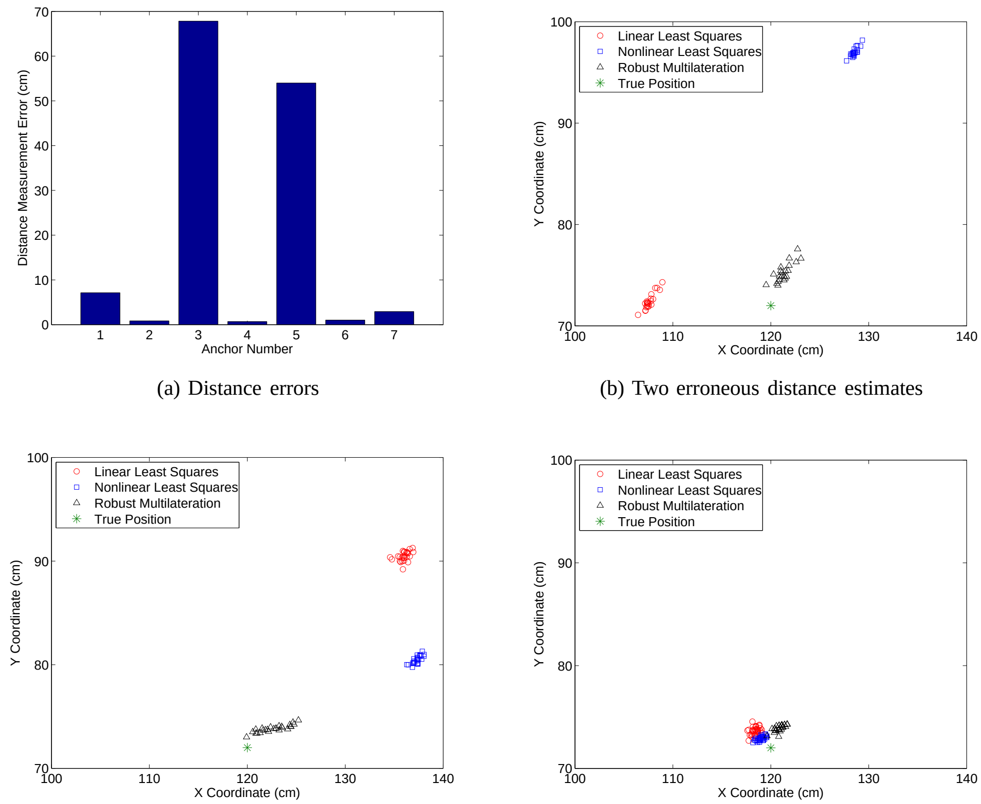
VIII. Acknowledgements

This work is supported by the EPSRC grant EP/F064209/1 on Actuated Acoustic Sensor Networks for Industrial Processes (AASN4IP).

References

[1] Y.-J. Kim, R. Govindan, B. Karp, and S. Shenker, "Geographic routing made practical," in *NSDI'05: Proceedings of the 2nd conference on Symposium on Networked Systems Design & Implementation*. Berkeley, CA, USA: USENIX Association, 2005, pp. 217–230.

[2] S. Madden, M. J. Franklin, J. M. Hellerstein, and W. Hong, "Tag: a tiny aggregation service for ad-hoc sensor networks," *SIGOPS Oper. Syst. Rev.*, vol. 36, no. SI, pp. 131–146, 2002.



(a) Distance errors

(b) Two erroneous distance estimates

(c) One erroneous distance estimate

(d) All direct distances

Fig. 11: Calculated coordinates of the receiver node in the testbed experiment.

[3] C. Intanagonwiwat, R. Govindan, D. Estrin, J. Heidemann, and F. Silva, "Directed diffusion for wireless sensor networking," *IEEE/ACM Trans. Netw.*, vol. 11, no. 1, pp. 2–16, 2003.

[4] J. Sallai, G. Balogh, M. Maroti, A. Ledeczi, and B. Kusy, "Acoustic ranging in resource-constrained sensor networks," in *ICWN '04*, Las Vegas, Nevada, USA, June 2004.

[5] N. B. Priyantha, A. Chakraborty, and H. Balakrishnan, "The cricket location-support system," in *MobiCom '00: Proceedings of the 6th annual international conference on Mobile computing and networking*. New York, NY, USA: ACM, 2000, pp. 32–43.

[6] S. Gezici, Z. Tian, G. Giannakis, H. Kobayashi, A. Molisch, H. Poor, and Z. Sahinoglu, "Localization via ultra-wideband radios: a look at positioning aspects for future sensor networks," *Signal Processing Magazine, IEEE*, vol. 22, no. 4, pp. 70–84, July 2005.

[7] S. Venkatesh and R. M. Buehrer, "A linear programming approach to nlos error mitigation in sensor networks," in *IPSN '06: Proceedings of the 5th international conference on Information processing in sensor networks*. New York, NY, USA: ACM, 2006, pp. 301–308.

[8] S. Venkatesh and R. Buehrer, "Non-line-of-sight identification in ultra-wideband systems based on received signal statistics," *Microwaves, Antennas & Propagation, IET*, vol. 1, no. 6, pp. 1120–1130, Dec. 2007.

[9] I. Guven, C.-C. Chong, F. Watanabe, and H. Inamura, "Nlos identification and weighted least-squares localization for uwb systems using multipath channel statistics," *EURASIP J. Adv. Signal Process*, vol. 2008, p. 36, 2008.

[10] A. Molisch, D. Cassioli, C.-C. Chong, S. Emami, A. Fort, B. Kannan, J. Karedal, J. Kunisch, H. Schantz, K. Siwiak, and M. Win, "A comprehensive standardized model for ultrawideband propagation channels," *Antennas and Propagation, IEEE Transactions on*, vol. 54, no. 11, pp. 3151–3166, Nov. 2006.

[11] M. Wylie and J. Holtzman, "The non-line of sight problem in mobile location estimation," in *Universal Personal Communications, 1996. Record., 1996 5th IEEE International Conference on*, vol. 2, Sep-2 Oct 1996, pp. 827–831 vol.2.

[12] P.-C. Chen, "A non-line-of-sight error mitigation algorithm in location estimation," in *Wireless Communications and Networking Conference, 1999. WCNC. 1999 IEEE*, 1999, pp. 316–320 vol.1.

[13] S. Dulman, P. Havinga, A. Baggio, and K. Langendoen, "Revisiting the cramer-rao bound for localization algorithms," in *DCOSS Work-in-Progress session*, T. V. K. Roemer and Y. Yu, Eds., Santorini, Greece, jun 2008, pp. 1–4.

[14] Y. Qi and H. Kobayashi, "Cramer-rao lower bound for geolocation in non-line-of-sight environment," in *Acoustics, Speech, and Signal Processing, 2002. Proceedings. (ICASSP '02). IEEE International Conference on*, vol. 3, 2002, pp. III–2473–III–2476 vol.3.

[15] ——, "On geolocation accuracy with prior information in non-line-of-sight environment," in *Vehicular Technology Conference, 2002. Proceedings. VTC 2002-Fall. 2002 IEEE 56th*, vol. 1, 2002, pp. 285–288 vol.1.

[16] J. E. Dennis, Jr. and R. B. Schnabel, *Numerical Methods for Unconstrained Optimization and Nonlinear Equations*. Society for Industrial and Applied Math, 1996.

[17] S. M. Stigler, "Gauss and the invention of least squares," *Annals of Statistics*, vol. 9, no. 3, pp. 465–474, 1981.

[18] R. Gonin and A. H. Money, *Nonlinear Lp-norm estimation*. New York, NY, USA: Marcel Dekker, Inc., 1989.