GUEST EDITORS' INTRODUCTION

Tensor Computations—Part I

Adrian Feiguin, *Northeastern University, Boston, MA, 02115, USA*

Stefanos Kourtis, *Université de Sherbrooke, Sherbrooke, QC, J1K 2R1, Canada*

Tensorial representations of information (d-dimensional arrays of data) play an increasingly important role in contemporary science. Tensor-based data structures are by now widely adopted in diverse fields of science, including AI, operations research, biology, physics, and quantum information, to name but a few examples. In all of these fields, computational methods based on tensors enable breakthroughs by exploiting advantages over traditional tabular or matrix data structures. Software advances, in turn, inform hardware design for data processing in the exascale era. This special issue is motivated by these developments and aims to present a cross section of ongoing research on tensorial information processing methods, grouped under the umbrella term *tensor computations*.

The broader adoption of tensor data structures requires the development of sorely needed tools, algorithms, and efficient numerical techniques for their manipulation. Even when the data are sparse, performing matrix or vector operations natively in the tensor representation (that is, without casting data back into a potentially prohibitively large matrix or vector) may not be straightforward. This means that even common numerical tasks, such as decompositions (generalizations of the singular value decomposition to arbitrary rank tensors), marginalizations, inference, or sampling, may need to be reinvented so as to be efficiently applicable on data stored in tensorial structures. Efficient manipulation of tensor-based data structures, therefore, often requires inventing multilinear algebraic operations that are not straightforward applications of linear algebra. The design and development of such computational primitives constitute a central aspect of tensor computing.

The content of this special issue is dedicated to a number of topics of current interest in tensor computing communities. These topics are as follows: principled code development of tensor primitives for reproducible results, novel techniques for finding extrema in tensor train data structures, probabilistic tensor decomposition strategies, and tensor factorization as a tool for relation prediction. Part I of this special issue consists of contributions on the former two topics, while the latter two will be presented in a forthcoming issue.

Whereas standard linear algebra libraries (e.g., BLAS and LAPACK) that have been continually developed and optimized over the course of more than three decades have, by now, crystallized into mature software packages, implementations of basic multilinear algebra methods for tensor data are still evolving. At the same time, as these methods gain broader adoption, the code libraries that implement them are becoming increasingly elaborate. These factors give rise to versioning, compatibility, and result reproducibility considerations in the development and adoption of software. A lack of reproducibility in research, in particular, is recognized as an acute issue, often referred to as a "crisis" affecting many fields of science. The article "Tensorlab$^+$: A Case Study on Reproducibility in Tensor Research"[A1] presents a framework of best practices in code development and management for tensor algorithms. The article details the authors' efforts to incorporate reproducibility considerations in their implementations of tensor algorithms and represents a significant advance toward catalyzing the standardization of tensor primitives at an accelerated pace.

Tensor network data structures allow for sparse representations of high-dimensional data whose storage and manipulation may otherwise be intractable. On the other hand, certain data processing operations become less straightforward in a tensor network representation. The article "Tensor Extreme Estimation via Sampling: A New Approach for Determining Minimum/Maximum Element"[A2] describes methods for a fundamental operation on data represented in a commonly used tensor network architecture called tensor train, namely, finding extremal entries of the database. Through numerical experiments, the authors show that the proposed approaches scale favorably up to intermediate-size tensor networks. Finding extrema in data structures is a basic primitive that underlies

1521-9615 © 2024 IEEE
Digital Object Identifier 10.1109/MCSE.2024.3368956
Date of current version 12 April 2024.

APPENDIX: RELATED ARTICLES

A1. S. Hendrikx, R. Widdershoven, N. Vervliet, and L. De Lathauwer, "Tensorlab$^+$: A case study on reproducibility in tensor research," *Comput. Sci. Eng.*, vol. 25, no. 5, pp. 6–13, Sep./Oct. 2023, doi: 10.1109/MCSE.2023.3340434.

A2. A. Chertkov, G. Ryzhakov, G. Novikov, and I. Oseledets, "Tensor extrema estimation via sampling: A new approach for determining minimum/maximum elements," *Comput. Sci. Eng.*, vol. 25, no. 5, pp. 14–25, Sep./Oct. 2023, doi: 10.1109/MCSE.2023.3346208.

numerous higher level processing functions. As tensor trains are common in several fields ranging from physics to data science, the results of this work have the potential for wide applicability.

In recent years, lengthy review papers and articles have surveyed applications of tensor computations in various disciplines. Given the vast range of these applications, a comprehensive overview of this active research topic would likely be either too lengthy or too superficial to be instructive to readers. Instead, this special issue aims to cover a partial cross section of interesting directions within this highly active multidisciplinary field. We hope this special issue will provide a launchpad for further reading and an inspiration for further research in tensor computations.

ADRIAN FEIGUIN is an associate professor in the Department of Physics at Northeastern University, Boston, MA, 02115, USA. Contact him at feiguin@northeastern.edu.

STEFANOS KOURTIS is an assistant professor in the Department of Physics and Institut Quantique at the Université de Sherbrooke, Sherbrooke, QC, J1K 2R1, Canada. Contact him at stefanos.kourtis@usherbrooke.ca.