Unknown Inputs Observer Applied to an Alternating Activated Sludge Process

Imen Manaa [1] and Faouzi M'Sahli [1]

Abstract— This work deals with the design of an unknown inputs observer for MIMO nonlinear hybrid systems. It consists in a switching between different observer to a simultaneous estimate of the unavailable states and inputs. A nonlinear reduced model of an alternating activated sludge process, which is decomposed on an aerobic phase and an anoxic phase, is presented. They are the results of a nitrification and de-nitrification phases. The principal goal of the observer is directed to cure the problem of unavailable input of the influent ammonia concentration and non-measured states of every sub-model. Simulation results verify the effectiveness of the proposed hybrid observers.

Index Terms— Nonlinear hybrid system, Unknown inputs observer, Activated sludge process.

I. INTRODUCTION

Most of complex systems can be modeled as a class of hybrid systems which involves both continuous and discrete dynamics. These types of systems have been the subject of intense research [1]. Because of their generality, collecting severe observer synthesis procedures, to derive a control laws, is often difficult. In recent years, several methods were proposed in the linear case but there is always a lack in a switched nonlinear systems theory. In this work, we are interested in a class of hybrid system that constitute a set of a continuous subsystem and a switching rule that arranges the commutation between them [2].

In the linear case, different approaches are proposed in the literature. In [3], a design of a linear Luenberger observers and a stabilizing gains calculation using a Linear Matrix Inequality method are considered. An estimation structure for a class of piecewise system has been introduced supposing that the switching time has to be known [4,5]. In [6], two types of linear observers , based on the prediction errors, are designed. The observability problem, for a linear hybrid system, has been treated in [7].

Recent results of researches are obtained for the synthesis of observers for nonlinear hybrid systems. For example, in [8] and to estimate continuous and discrete states, a step by step sliding mode observation is employed. Another high order sliding mode observer for autonomous switching nonlinear systems with jumps is addressed in [9]. A delay approach, based on a new matrix conditions derived to ensure the convergence of the hybrid observer with both linear and nonlinear correction terms, is discussed in [10].

As well, in a relatively recent works, the unknown input observers has received more attention. Such observers allow the simultaneously estimate of the unavailable states and the unknown inputs. This has become a necessity in many engineering applications. A current developments on unknown inputs observers are discussed for linear hybrid systems in [11, 12]. But until now, a nonlinear hybrid case is not clearly defined by the researchers.

The practical contribution of the present paper is to extend a nonlinear observer design defined in [13] to a switched method. In fact, a full order high gain observer for the simultaneous estimation of the non-measured states and the unknown inputs are defined for hybrid systems. This approach does not need the output differentiation and it only assumes that the dynamics of these inputs are bounded without making any hypothesis on how these inputs vary. The presented algorithm is applied to a realistic model of an activated sludge (AS) waste-water treatment process (WWTP) defined in [14].

The considered system is one of the most interesting area of research to conserve water resources and to preserve water quality rejected in the nature. It consists in a biological nitrogen (N) removal which is important to be reduced in order to avoid the environment and the human body problems that N causes. There are different configurations of the AS. Earlier, a separated tanks configurations are considered but, in recent years, a single tank treatment is developed. Due to the complexity of his representation, a reduced AS model (RASM) is described in [14]. Indeed, a classical Luenberger observer is applied for the resulting continuous nonlinear models after linearization. This linear observer allow the states estimation. The principal goal of the present hybrid nonlinear observer is directed to cure the problem of unavailable parameters and states of the activated sludge process. It's to improve the quality estimation using an unknown inputs observer for the hybrid nonlinear control. The remainder of this paper is organized as follows. In section II, a description of the functioning of the alternating activated sludge process and his model are given. Section III is devoted to introduce the problem formulation and so the class of the nonlinear hybrid system considered for the AS model. One details the state and the input estimation algorithm where the transition conditions are considered in some hypothesis. Then, the simulation results are shown in order to illustrate the performance of the designed hybrid observer. Finally, some comments and conclusions are given.

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II. Modeling of the Alternating Activated Sludge Process

A. Process description

The process considered in this work is presented in fig.1. It is a real small size treatment plants which is designed for less than 10.000 (p.e.) (population equivalent). It consists on unique aeration tank $(V_{aer} = 0.03m^3)$ equipped with aeration surface which provide oxygen in the reactor to create nitrification and de-nitrification conditions. The settler is a tank where the biomass is either recirculated to the aeration tank. The earlier nonlinear models as the ASM1 are not very attractive due to the high complex schemes systems. So, we are interested on her reduced order models, based on some biochemical considerations, describing the non linear behavior of the process developed in [14]. We thus distinguish an aerobic phase and an anoxic phase, using state variables related to four substrate concentrations.
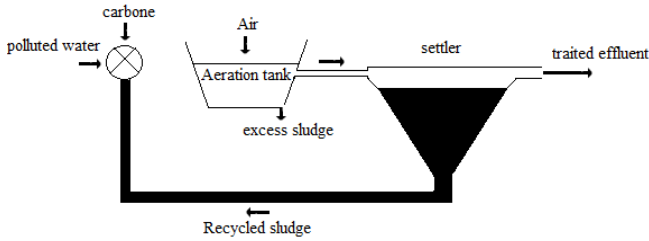

Fig. 1.   The activated process model.

The first phase is an aeration period, air is injected in large quantities in the reactor to convert the pollution as ammonium nitrate (aerobic phase). Then, the aeration is stopped and a carbon source is external optionally added at the mixer to remove the nitrate into a nitrogen form (anoxic phase).

B. Process Model

The reduced resulting process consists of four state variables with an alternating phase operation due to the values of the oxygen transfer coefficient kla between zero and a high values.
The nonlinear model differential equations of the process can be stated as [14]:

$$
\begin{cases}
\dot{S}_S = D_S S_{Sin} + D_c S_{Sc} - (D_S + D_c)S_S - \frac{1}{Y_H}(\tilde{\beta}_1 + \tilde{\beta}_2) \\
\quad + \tilde{\beta}_7 \\[2mm]
\dot{S}_{NO_3} = -(D_S + D_c)S_{NO_3} - \frac{1-Y_H}{2.86Y_H}\tilde{\beta}_2 + \tilde{\beta}_3 \\[2mm]
\dot{S}_{NH_4} = D_S S_{NH_4 in} - (D_S + D_c)S_{NH_4} \\
\quad - i_{NBM}(\tilde{\beta}_1 + \tilde{\beta}_2) - \tilde{\beta}_3 + \tilde{\beta}_6 \\[2mm]
\dot{S}_{O_2} = -(D_S + D_c)S_{O_2} + kla(S_{O_2 sat} - S_{O_2}) - \frac{1-Y_H}{Y_H}\tilde{\beta}_1 \\
\quad - 4.57\tilde{\beta}_3
\end{cases}
\tag{1}
$$

S_s, S_{NO_3}, S_{NH_4} and S_{O_2} are defined as a readily Biodegradable substrate, the nitrate, the ammonia and the dissolved oxygen concentrations, respectively. $S_{i_{in}}$ are the concentration of the input variables of the system. In fact, $S_{S_{in}}$ is the influent biodegradable substrate and $S_{NH_4 in}$ is the influent ammonia concentration. S_{S_c} is an oxygenous carbone source. D_s and D_c are the rates of dilution defined as Q_s/V_{aer} and Q_c/V_{aer}, respectively.
$\tilde{\beta}_i$ are the simplified form of the process kinetics β_i of the standard mode ASM1. λ_i are a specific parameters to the reduced nonlinear model.

$$
\begin{aligned}
\tilde{\beta}_1 &= \lambda_1 S_S \frac{S_{O_2}}{S_{O_2} + K_{O_2 H}} \\
\tilde{\beta}_2 &= \lambda_1 S_S \frac{S_{NO_3}}{S_{NO_3} + K_{NO_3}} \frac{K_{O_2 H}}{S_{O_2} + K_{O_2 H}} \\
\tilde{\beta}_3 &= \lambda_2 \frac{S_{NH_4}}{S_{NH_4} + K_{NH_4 AUT}} \frac{S_{O_2}}{S_{O_2} + K_{O_2 AUT}} \\
\tilde{\beta}_6 &= \lambda_3 \\
\tilde{\beta}_7 &= \lambda_4 \left(\frac{S_{O_2}}{S_{O_2} + K_{O_2 H}} + \eta_{NO_3 h} \frac{S_{NO_3}}{S_{NO_3} + K_{NO_3}} \frac{K_{O_2 H}}{S_{O_2} + K_{O_2 H}} \right)
\end{aligned}
\tag{2}
$$

III. Design of the Unknown Inputs Observer for the Process

A. Problem statement

The switched process can be putting under the following form:

$$
\begin{cases}
\dot{x}(t) = f_g(x(t), u(t), v(t)) \\
y_g = \bar{C}x = x^1
\end{cases}
\tag{3}
$$

This form is detailed as follows [13]:

$$
x = \begin{pmatrix} x^1 \\ \vdots \\ x^q \end{pmatrix}; f_g(x,u,v) = \begin{pmatrix} f_g^1(x^1, x^2, u, v) \\ f_g^2(x^1, x^2, x^3, u, v) \\ \vdots \\ f_g^{q-1}(x, u, v) \\ f_g^q(x, u, v) \end{pmatrix};
$$

$$
\bar{C} = \left[I_{n_1}, 0_{n_1 \times n_2}, 0_{n_1 \times n_3}, \dots, 0_{n_1 \times n_q} \right]
$$

where x, y and w are the system states, the measurable output system and the inputs respectively. $x(t) \in \Re^n$, $x^k \in \Re^{n_k}$, $k = 1, ..., q$ and $p = n_1 \geq n_2 \geq ... \geq n_q$, $\sum_{k=1}^{q} n_k = n$. The input $w = (u,v) \in W$ the set of bounded absolutely continuous functions with bounded derivatives from $\Re^+$ into W a compact set of $\Re^s$. $u \in U \subset \Re^{s-m}$ is the known input subvector and $v \in V \subset \Re^m$ is unknown. With I_{n_1} is the identity matrix; $0_{n_1 \times n_j}$ the null matrix with $j = 2, ..., n_q$ and $f_g(x, u, v) \in \Re^n$ with $f_g^k(x, u, v) \in \Re^{n_k}$.

Based on (3), we have to select the adequate subsystem which can describe the dynamic behavior of the nonlinear hybrid system. In fact, the mode location $g \in \nabla = \{1, 2\}$ is fixed to identify the active subsystem (aerobic/anoxic) at every instant of time and to have acceptable switching strategies according some conditions for the states of the activated sludge process. If the subsystem is specified and every subsystem is considered to be uniformly observable one can conclude the whole observability of the presented system.

The reduced AS model must assume some adoption hypothesis for an unknown input observer:

H1. The states $x(t)$, the input $u(t)$ and the unknown inputs v are bounded, $x(t) \in X$, $u(t) \in U$ and $v \in V$ where $X \subset \Re^n$, $U \subset \Re^{s-m}$ and $V \subset \Re^m$ are compacts sets.

H2. There exist $\alpha_{f,g}$, $\beta_{f,g} > 0$ such that $\forall x \in \Re^n$, $\forall k \in \{1,...,q-1\}$, $\forall (u,v) \in U \times V$;

$$\alpha_{f,g}^2 I_{n_{k+1}} \leq \left(\frac{\partial f_g^k}{\partial x^{k+1}}(x,u,v) \right)^T \frac{\partial f_g^k}{\partial x^{k+1}}(x,u,v) \leq \beta_{f,g}^2 I_{n_{k+1}}$$
(4)

One also assumes that for $1 \leq k \leq q-1$, for all $(u,v) \in U \times V$, $x^{k+1} \longmapsto f_g^k(u,v,x^k,...,x^k,x^{k+1})$ from $\Re^{n_{k+1}}$ into $\Re^{n_k}$ is one to one.

H3. The output x^1 can be portioned as follows:
$x^1 = \begin{pmatrix} x_1^1 \\ x_2^1 \end{pmatrix}$ with $x_1^1 \in \Re^{m_1}$, $x_2^1 \in \Re^{p-m_1}$ and $m \leq m_1 < p$, we can deduce then the following partition f_g^1 can be partioned as: $f_g^1(x^1,x^2,u,v) = \begin{pmatrix} f_{1,g}^1(x^1,x^2,u,v) \\ f_{2,g}^1(x^1,x^2,u,v) \end{pmatrix}$ that has to satisfy the following two conditions:

• There exist $\alpha_{v,g}, \beta_{v,g} > 0$ such that $\forall x \in X$, $\forall (u,v) \in U \times V$:

$$\alpha_{v,g}^2 I_m \leq \left(\frac{\partial f_{1,g}^1}{\partial v}(x^1,x^2,u,v) \right)^T \frac{\partial f_{1,g}^1}{\partial v}(x^1,x^2,u,v) \leq \beta_{v,g}^2 I_m$$
(5)

One also assumes that for all $(u,x^1,x^2) \in U \times \Re^{n_1+n_2}$, $v \longmapsto f_1^1(u,v,x^1,x^2)$ from $\Re^m$ into $\Re^{m_1}$ is one to one.

• $Rank \begin{pmatrix} \frac{\partial f_{1,g}^1}{\partial x^2}(x^1,x^2,u,v) & \frac{\partial f_{1,g}^1}{\partial v}(x^1,x^2,u,v) \\ \frac{\partial f_{2,g}^1}{\partial x^2}(x^1,x^2,u,v) & \frac{\partial f_{2,g}^1}{\partial v}(x^1,x^2,u,v) \end{pmatrix} = (n_2 + m) = 2$ for all $(x^1,x^2) \in \Re^{n_1+n_2}$.

H4. The time derivative of the unknown input $v(t)$ of the system is a completely unknown function, $\varepsilon(t)$, which is uniformly bounded, considering that $\sup_{t \geq 0} \|\varepsilon(t)\| \leq \beta_\varepsilon$ where $\beta_\varepsilon > 0$, is a real unknown number.

To succeed the observer design, we shall make some simple notations change:

$$\tilde{x} = \begin{pmatrix} \tilde{x}^1 \\ \tilde{x}^2 \end{pmatrix}; \tilde{x}^1 = \begin{pmatrix} x_1^1 \\ v \end{pmatrix}; \tilde{x}^2 = \begin{pmatrix} x_2^1 \\ x^2 \\ \vdots \\ x^q \end{pmatrix};$$

$$C = diag(C_1,C_2); C_1 = [\ I_{m_1} \quad 0\ ]; C_2 = [\ I_{p-m_1} \quad 0\ ];$$

where C_2 is $(p-m_1) \times q(p-m_1)$ rectangular matrix.

$$\tilde{f}_g(\tilde{x},u) = \begin{pmatrix} \tilde{f}_g^1(\tilde{x},u) \\ \tilde{f}_g^2(\tilde{x},u) \end{pmatrix}; \tilde{f}_g^1(\tilde{x},u) = \begin{pmatrix} f_{1,g}^1(x^1,x^2,u,v) \\ 0 \end{pmatrix};$$

$$\tilde{f}_g^2(u,\tilde{x}) = \begin{pmatrix} f_{2,g}^1(x^1,x^2,u,v) \\ f_g^2(x^1,x^2,x^3,u,v) \\ \vdots \\ f_g^q(x,u,v) \end{pmatrix}.$$

System (3) can be then rewritten as follows:

$$\begin{cases} \dot{\tilde{x}}^1(t) = \tilde{f}_g^1(u(t),\tilde{x}(t)) \\ \dot{\tilde{x}}^2(t) = \tilde{f}_g^2(u(t),\tilde{x}(t)) \\ y_g = C\tilde{x} = \begin{pmatrix} x_1^1 \\ x_2^1 \end{pmatrix} \end{cases}$$
(6)

The above system can be written in condensed form:

$$\begin{cases} \dot{\tilde{x}}(t) = \tilde{f}_g(\tilde{x}(t),u(t)) \\ y_g = C\tilde{x} \end{cases}$$
(7)

1) Aerobic phase: We can define for aeration phase of the alternating sludge process the following elements:

$$x = \begin{pmatrix} x^1 \\ x^2 \\ x^3 \end{pmatrix}; f(x,u,v) = \begin{pmatrix} f^1(x^1,x^2,u,v) \\ f^2(x^1,x^2,x^3,u,v) \\ f^3(x,u,v) \end{pmatrix};$$

$$\bar{C} = [I_2, 0_{2\times 1}, 0_{2\times 1}]$$

In the aerobic phase, one considers $g = 1$, $x = (\ S_{NH_4} \quad S_{NO_3} \quad S_{O_2} \quad S_S\ )^T \in \Re^n$, the states $x(t) \in \Re^4$, $x^k \in \Re^{n_k}$, $k = 1,...,3$, $p = n_1 \geq n_2 \geq n_3$, $n_1 = 2$, $n_2 = 1$, $n_3 = 1$, $\sum_{k=1}^{3} n_k = 4$, the outputs $y(t) = (\ S_{NH_4} \quad S_{NO_3}\ )^T \in \Re^p$.

2) Anoxic phase: In this case, the oxygen is absent and the coefficient of transfer of the oxygen (kla) is equal to zero.
We can so define the mode location $g = 2$, the states $x = (\ S_{NH_4} \quad S_{NO_3} \quad S_S\ )^T \in \Re^n$, the states $x(t) \in \Re^3$, $x^k \in \Re^{n_k}$, $k = 1,...,2$, $n_1 = 2$, $n_2 = 1$, $\sum_{k=1}^{2} n_k = 3$, the outputs $y(t) = (\ S_{NH_4} \quad S_{NO_3}\ )^T \in \Re^p$.

$$x = \begin{pmatrix} x^1 \\ x^2 \end{pmatrix}; f(x,u,v) = \begin{pmatrix} f^1(x^1,x^2,u,v) \\ f^2(x^1,x^2,x^3,u,v) \end{pmatrix};$$

$$\bar{C} = [I_2, 0_{2\times 1}]$$

B. Observer design

In this section, we will be interested to an estimation observer which allow recovery of the unavailable states and inputs for the nonlinear hybrid AS process. In fact, for every subsystem and on switching between different values of the gain to achieve better parameters and states estimation and to guarantee smooth error convergence, an observer for system (3) can be written in the following form:

$$\dot{\hat{\tilde{x}}}(t) = \tilde{f}_g(\hat{\tilde{x}}(t),u(t)) - (\Lambda_g(x,u,v))^+\Delta^{-1}K(C\hat{\tilde{x}} - y) \quad (8)$$

where

• $(\Lambda_g(x,u,v))^+$ is the left inverse of diagonal bloc matrix with $\Lambda_g = blockdiag(\Lambda_{1,g},\Lambda_{2,g})$;

$$\Lambda_{1,g} = diag(I_{m_1}, \frac{\partial f_{1,g}^1}{\partial v}(x^1(t), x^2(t), u(t), v(t))$$

$$\Lambda_{2,g} = diag(I_{p-m_1}, \frac{\partial \bar{f}_g^1}{\partial x^2}(x,u), ...,$$

$$\frac{\partial \bar{f}_g^1}{\partial x^2}(x,u) \prod_{k=1}^{q-1} \frac{\partial \bar{f}_g^k}{\partial x^{k+1}}(x,u,v))$$

- $K = diag(K_1, K_2)$;

$$K_1 = \begin{pmatrix} 2I_{m_1} \\ I_{m_1} \end{pmatrix}; K_2 = \begin{pmatrix} C_q^1 I_{p-m1} \\ C_q^2 I_{p-m1} \\ \vdots \\ C_q^q I_{p-m1} \end{pmatrix}$$

with $C_q^k = \frac{q!}{k!(q-k)!}$ for $1 \le k \le q$.

- $\Delta = diag(\Delta_1, \Delta_2)$;

$$\Delta_1(\theta_{1,g}) = diag(\frac{1}{\theta_{1,g}^{q-1}} I_{m_1}, \frac{1}{\theta_{1,g}^{2(q-1)}} I_{m_1})$$

$$\Delta_2(\theta_{2,g}) = diag(\frac{1}{\theta_{2,g}} I_{(p-m_1)}, \frac{1}{\theta_{2,g}^2} I_{(p-m_1)}, ...,$$

$$\frac{1}{\theta_{2,g}^q} I_{(p-m_1)})$$

with $\bar{f}_g^1(x^1, x^2, u) = f_{2,g}^1(x^1, x^2, u, v(x^1, x^2, u))$

Otherwise, the developed observer form is detailed as follows:

$$\begin{cases} \dot{\hat{x}}_1^1(t) = f_{1,g}^1(x^1(t), \hat{x}^2(t), u(t), \hat{v}(t)) - 2\theta_{1,g}^{q-1}(\hat{x}_1^1 - x_1^1) \\ \dot{\hat{v}} = -\theta_{1,g}^{2(q-1)} \frac{\partial f_{1,g}^1}{\partial v}(x^1(t), \hat{x}^2(t), u(t), \hat{v}(t))(\hat{x}_1^1 - x_1^1) \end{cases}$$
(9)

$$\begin{cases} \begin{pmatrix} \dot{\hat{x}}_2^1(t) \\ \dot{\hat{x}}^2(t) \\ \vdots \\ \dot{\hat{x}}^q(t) \end{pmatrix} = \begin{pmatrix} f_{2,g}^1(x^1, \hat{x}^2, u, \hat{v}) \\ f_g^2(x^1, \hat{x}^2, \hat{x}^3, u, \hat{v}) \\ \vdots \\ f_g^q(x^1, \hat{x}^2, ..., \hat{x}^q, u, \hat{v}) \end{pmatrix} \end{cases}$$
(10)

$$-\Lambda_{2,g}(x^1, \hat{x}^2, ..., \hat{x}^q, u, \hat{v})\Delta_2^{-1}(\theta_{2,g})K_2(C_2\hat{x}_2^1 - y_2)$$

Remark1. During the transient period, a chattering phenomena can appear on the continuous time observer. To overcome this problem a procedure of reinitialisation of the real and estimated states can be applied for every subsystem.

Th1. If the nonlinear hybrid system described by (3) satisfy hypothesis (H1.)-(H4.), then, the system (7) is an unknown input observer with an exponential convergence for relatively high values of the design parameters for all subsystem.

H5. $\exists \ \tau_{min} \ge 0$ defined by:

$$\tau_{i+1} - \tau_i \ge \tau_{min}; i = 1, ..., N.$$

with $(\tau_{i+1} - \tau_i)$ define the duration between two switches.

Remark2. Hypothesis (H5.) means that we exclude the zenon phenomenon. The duration between two switches is supposed to be sufficiently large so that the unknown inputs observer guarantee the fast convergence to the real system before a new switch.

Satisfying the Theorem and a good choice of τ_{min} can guarantee the exponential convergence of the hybrid observer. Hypothesis (H5.) allow then the convergence of the observer.

Th2. If the nonlinear hybrid system described by (3) satisfy the Theorem (Th1.) and the hypothesis (H5.), then, the system (7) is an unknown inputs observer with an exponential convergence for relatively high values of the design parameters to a global system (3).

IV. SIMULATION RESULTS

In order to inspect the performances of the hybrid unknown input observation technique, we make a digital simulation of the activated sludge process considering the changes in the coefficient of oxygen transfer kLa due to the alternating presence and absence of the air. We can distinguish an aerobic phase with $(kla \ne 0)$ and an anoxic phase with $(kla = 0)$. Some identified and initialized values are also given on table1 and a complete scheme of the proposed process and the hybrid observer is depicted in figure 2.
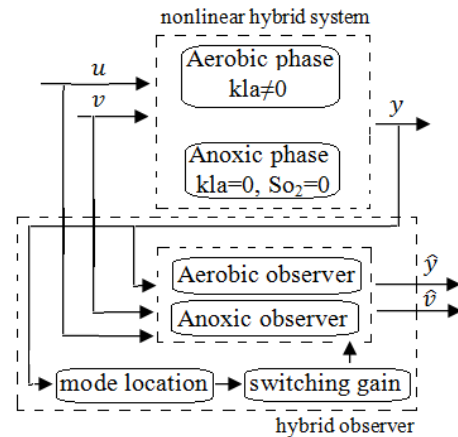

Fig. 2. Hybrid observer scheme

This section is dedicated to illustrate the performances of the observer proposed in the previous part. Note that the inputs of every subsystem are $w(t) = \begin{pmatrix} S_{S_c} & S_{S_{in}} & S_{NH_{4in}} \end{pmatrix}^T \in \mathfrak{R}^s$. $u(t) = \begin{pmatrix} u_1 & u_2 \end{pmatrix}^T = \begin{pmatrix} S_{S_c} & S_{S_{in}} \end{pmatrix}^T = \begin{pmatrix} 16000 & 183.6 \end{pmatrix}^T$ are the known

input and $v(t) = (S_{NH_{4in}}) = 31.56 gm^{-3}$ is the unknwon input.

The mode location g is known and it is fixed considering some conditions on the S_{NO_3} concentration. In this case, the results are taken without reinitialisation of the states during the transient time. The initial conditions of the real and estimated states are $x(0) = [5.3 \quad 0.34 \quad 0 \quad 10]^T$ and $\hat{x}(0) = [5.3 \quad 0.34 \quad 10^{-4} \quad 9]^T$, respectively. The initial conditions of the unknown input are $\hat{v}(0) = 0$.

The optimal values of the gain are chosen as $\theta_{1,1} = 80$, $\theta_{2,1} = 70$, $\theta_{1,2} = 20$ and $\theta_{2,2} = 10$. This choice insures fast convergence and good speed. Firstly, the results of real and estimated four states are drawn in figures 4,5,6 and 7. Besides, the obtained estimation of the non-measured input for all subsystems is depicted in figure 3 which demonstrate there convergence to the true value. The Simulation of the hybrid nonlinear unknown input observer (HNUIO) shows the effectiveness of such observer to estimate both the continuous states and input of the system so as to give a satisfactory convergence when the transient value g is known. The estimation evolution gives a satisfactory convergence of such observer. Secondly, to demonstrate the robustness of such observer, one can apply a measurement noise for the outputs of the system. The estimation of the S_s concentration and the unknown input are depicted in the figure 8 and 9. The results can be considered acceptable as they show fast convergence in all subsystems and a respectable behaviour as well.
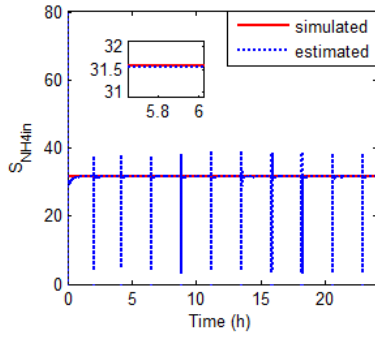

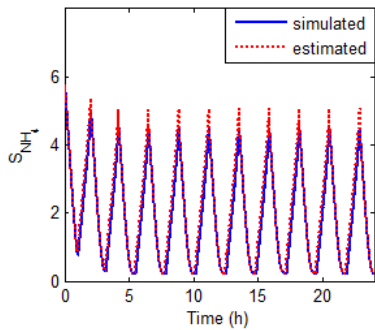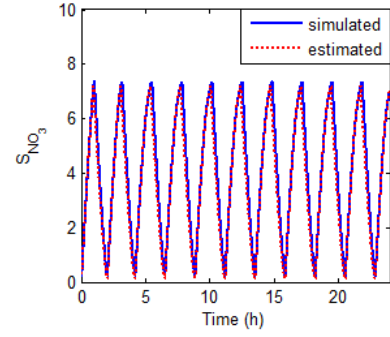Fig. 5. The estimation evolution of NO_3 concentration.

Fig. 6. The estimation evolution of O_2 concentration.

Fig. 3. The estimation evolution of unknown input concentration.

Fig. 7. The estimation evolution of S concentration.

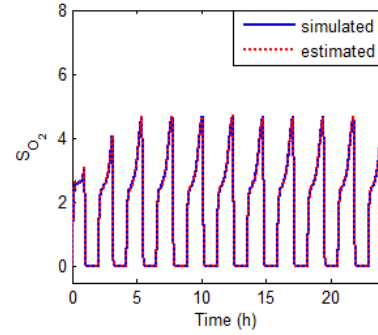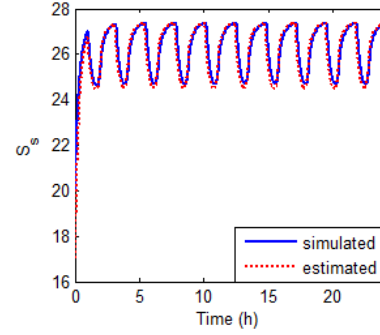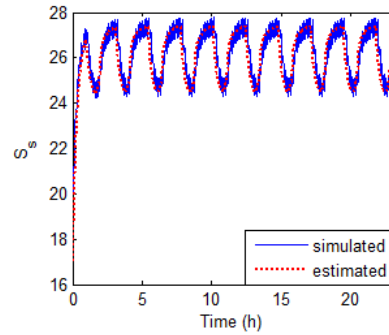Fig. 4. The estimation evolution of NH_4 concentration.

Fig. 8. The estimation evolution of S_s concentration with measurment noise.
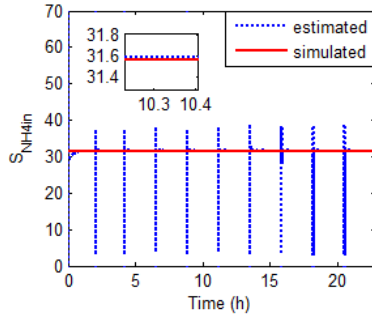
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Fig. 9. The estimation evolution of unknown input concentration with measurment noise.

V. CONCLUSION

An observer design is proposed for a class of nonlinear hybrid systems involving unknown inputs. The proposed observers allows the simultaneous estimation of the state and the unknown inputs considering that the mode location to be known. The proposed approach only assumes that the dynamics of these inputs are bounded without making any hypothesis on how these inputs vary. An application of such observer to a nonlinear reduced model of an activated sludge process, witch is considered as an hybrid system with a nitrification-denitrification phases, is presented. Performances of the nonlinear observer have been shown through the simulation results.

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VI. APPENDIX

TABLE I

PARAMETERS OF THE NONLINEAR REDUCED PROCESS.

Parameters	Values	Description
$Y_H(g.g^{-1})$	0.64	Coefficient of performance of heterotrophic biomass
$i_{NBM}(g.g^{-1})$	0.086	Mass of nitrogen contained in the concentrations of heterotrophic and autotrophic biomass
$S_{O_2sat}(g.m^{-3})$	9.5	Concentration of dissolved oxygen saturation
$K_{O_2H}(g.m^{-3})$	0.2	Coefficient of average saturation of oxygen for the heterotrophic biomass
$K_{NO_3}(g.m^{-3})$	0.5	Coefficient of average saturation of nitrate
$K_{NH_4AUT}(g.m^{-3})$	0.05	Coefficient of average saturation of ammonia for autotrophic biomass
$K_{O_2AUT}(g.m^{-3})$	0.4	Coefficient of average saturation of oxygen for the autotrophic biomass
$\lambda_1(g.m^{-3}.d^{-1})$	62.59	Growth rate of heterotrophic biomass
$\lambda_2(g.m^{-3}.d^{-1})$	187.37	Speed nitrate production by the autotrophic
$\lambda_3(g.m^{-3}.d^{-1})$	52.63	Speed of hydrolysis of slowly biodegradable substrate by the heterotrophic
$\lambda_4(g.m^{-3}.d^{-1})$	987.32	Ammonification of soluble organic nitrogen
$kla(d^{-1})$	225	the coefficient of oxygen transfer
η_{NO_3h}	0.31	Correction factor for the hydrolysis in anoxic phase
$Q_s(m^3.d^{-1})$	0.0343	the input flow
$Q_c(m^3.d^{-1})$	0.0005	the flow of the external carbon source