Simulation-Based Evaluation of Predictive Tracking for Sorting Bulk Materials

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***Abstract*— Multitarget tracking problems arise in many real-world applications. The performance of the utilized algorithm strongly depends both on how the data association problem is handled and on the suitability of the motion models employed. Especially the motion models can be hard to validate. Previously, we have proposed to use multitarget tracking to improve optical belt sorters. In this paper, we evaluate both the suitability of our model and the tracking and then of our entire system incorporating the image processing component via the use of highly realistic numerical simulations. We first assess the model using noise-free measurements generated by the simulation and then evaluate the entire system by using synthetically generated image data.**

I. Introduction

Bulk materials are ubiquitous in many industrial branches. A significant share of all energy produced worldwide—estimates are up to 10% [1]—is spent on processing and handling of bulk materials. Efficiently sorting bulk materials early in the supply chain can help to reduce energy consumption and costs. Furthermore, the scope of goods that can be handled like classical bulk materials expands as reliable sorting for new goods such as pharmaceutical and chemical substances becomes feasible as sorting technology advances. Sensor-based sorters based on, e.g., imaging technology are more versatile compared with traditional sorters such as trommel screens, magnet separators, and flotation cells. Even in cases in which classical separators are applicable, sensor-based sorters are sometimes integrated as an additional processing step as they are compact and can allow dry instead of wet sorting [2]. Moreover, sensor-based sorters can sort based on a combination of properties that would require more than one of the classical sorters. While multiple sensors can be combined in sensor-based sorters, sophisticated calibrations are necessary [3] in current optical belt sorters.

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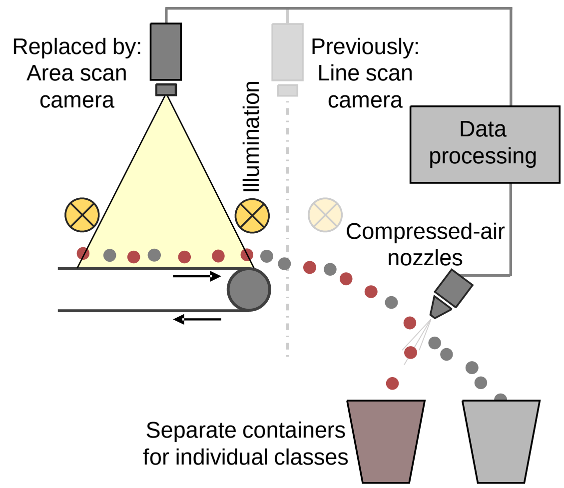


Fig. 1: Sketch of an optical belt sorter with the necessary modifications for the predictive tracking approach.

A major challenge in sensor-based sorting is the separation process as it is (unlike in many classical sorters) a distinct processing step following the classification. In this work, we put our focus on commonly used optical belt sorters. For these sorters, the bulk material travels over a belt or a slide before it enters a flight phase. In the flight phase, the particles pass an array of compressed-air nozzles aligned orthogonal to the transport direction. Bursts of air from these nozzles alter the flight path of specific particles, causing them to land in a different container than the particles that fly unobstructed.

In today's industrial machinery, it is common to utilize the observation used for classification purposes also for the localization of the particles. Due to delays between observation and separation, it is necessary to predict the particles' future positions. To be able to improve these predictions, we have proposed to use an area scan camera instead of a line scan camera as illustrated in Fig. 1. Based on this extension, we have introduced a concept called predictive tracking [4], [5], in which we keep track of each particle's motions to be able to accurately predict its future movements. Using image processing, the acquired image data is turned into position measurements. As there are typically multiple particles in the field of view concurrently, we need to use multitarget tracking algorithms. Using a simple yet efficient approach, we obtained promising results.

Our previous evaluation based on real image data presented in [4] relied on artificially introducing a prediction phase by discarding a part of each image. By comparing the predictions based on part of the image data with the best possible estimates using all image data available, we gave an assessment of the accuracy of the predictions generated by our model in comparison with a prediction straight in the transport direction. The latter is the implicit assumption used by current belt sorters without predictive tracking. In this paper, we evaluate the predictive tracking approach using simulations that provide a ground truth to evaluate against. First, we evaluate the motion models by using them with true instead of estimated parameters, which is regarded in more detail in [6]. Second, we evaluate the tracking approach without any imprecision introduced by the image processing and third, when incorporating the image processing component.

In the remainder of this paper, we first describe the advantages of the predictive tracking approach and lay out a simple algorithm. Then, we give a brief description of the simulation methodology. Afterward, we present our evaluation based on the numerical simulations and provide a conclusion.

II. Predictive Tracking

In this section, we first outline which new possibilities arise from using the predictive tracking approach. Afterward, we give details on our current algorithmic implementation.

A. Advantages of the Predictive Tracking Approach

The initial motivation for predictive tracking is the observation that particles of some bulk materials do not move straight in the transport direction of the belt, as is implicitly assumed by optical belt sorters that only localize each particle at one point in time. Previously, strategies mainly focused on adjusting the hardware of the sorter in ways that adapt the particles' motions to the old, implicit model. One strategy is to use a longer belt to allow the bulk material to settle on the belt and fully adapt to the velocity of the belt. However, increasing the length of the belt always induces additional costs. The adaptation process can further be supported by using a fluted belt, which comes at the cost of making the belt harder to clean. Furthermore, there are bulk materials for which both strategies do not suffice and for which the precision of the predictions of the old, implicit model does not enable reliable sorting using current optical belt sorters.

Besides the advantage regarding the separation process, there is a multitude of possibilities to improve the classification that performing tracking gives rise to. For one, more useful data for classification purposes can be obtained directly by using multiple observations of each particle. Multiple perspectives from each particle can be obtained—either by simply observing it travel along the belt or, e.g., by using vibrations to induce rotations. Using multiple observations is particularly useful for finding damages or other features that might be at an unknown point on the surface of a particle. Furthermore, the lighting can be changed as particles travel along the belt to collect additional features to derive classification decisions with higher reliability.

Moreover, incorporating data stemming from additional imaging sensors is facilitated. The pattern of estimated or predicted particle positions can be matched across sensors to fuse the visual information about the individual particles. Furthermore, the pattern could also be used to derive a temporal calibration of the sensors.

One additional concept to improve the classification that we have obtained promising preliminary results for is using motion-based classification. For example, different classes of particles vary in regard to their motions orthogonal to the transport direction and in regard to how fast the particles adapt to the velocity of the belt. Additionally, if a slide is used in the feeding process, the velocities of the particles at the moment they are applied to the belt differ strongly. Incorporating the slide in the tracking process would allow us to obtain even more information about each particle's motion behavior to further improve the classification.

B. Current Algorithmic Implementation

To use standard algorithms for multitarget tracking under unknown associations, the image data has to be reduced to a set of position measurements using image processing techniques [7]. The first step in the image processing chain is determining the constant background to use as a reference by averaging over multiple frames. Then, connected component analysis is used to detect the particles. Each particle's centroid is then determined in pixel coordinates. Additional information, such as about the shape or extent of the object can also be obtained using image processing and can be used to support the tracking[1].

By transforming a particle's centroid as obtained by the image processing into world coordinates and using this as a measurement of the particle's actual centroid, the estimate of the particle's position and velocity is updated. For this, a global association likelihood approach [8, Ch. 10.3] is pursued in which we choose the most likely association between the currently known particles and the measurements. When dealing with Gaussian distributions, the maximization of the product of the likelihoods can also be seen as a minimization of the sum of the Mahalanobis distances. In accordance to set distances such as the OSPA metric [9], individual associations beyond a certain Mahalanobis distance are not seen as valid associations but are rather viewed as the lack of a measurement of one target and another measurement of a potentially new target.

To reliably determine the appearance and disappearance of targets, we generate likelihoods using knowledge about the scenario at hand. As shown in Fig. 2, particles are more likely to appear in the part that is (in regard to the transport direction) at the beginning of the region observed. On the other hand, particles whose predictions lie behind the end of the observable region are considered likely to have disappeared.

We incorporate this knowledge into the matrix used to perform the association that is illustrated in Fig. 3. The likelihood that a measurement stems from a certain track

[1] E.g., if we predict that two particles with a certain surface area will collide, a detection of one larger particle at the prediction position of the collision indicates that the image processing has deemed the colliding particles to be one single particle.

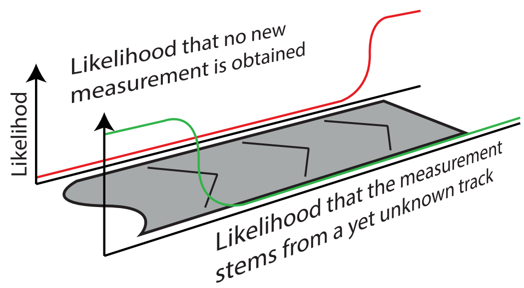


Fig. 2: Likelihood that a measurements stems from a new track or that a track is not observed anymore.

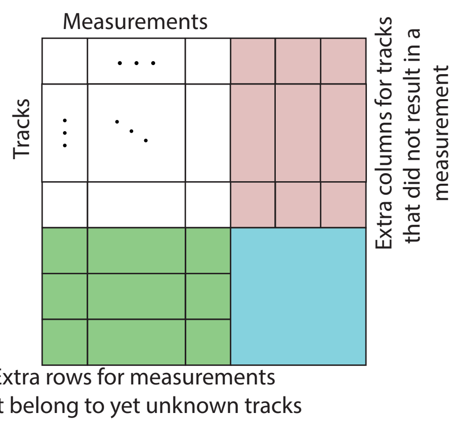


Fig. 3: Matrix used for deriving the association.

(based on its current prediction) constitutes the main part of the matrix. Additional rows are added and each is filled with the likelihood that the measurement stems from a yet unknown track. Moreover, extra columns are added and filled with the respective likelihood that the corresponding track is not observed again.

To find the association maximizing the product of the likelihoods, the logarithm is taken and the sign is inverted to transform the problem into an easier one of minimizing the sum. Based on this, standard linear assignment problem solvers can be used. In our application, it is crucial to use fast solvers such as LAPJV [10] or the auction algorithm [11]. The latter can be massively parallelized and is thus also suitable for implementations on GPUs [12]. Furthermore, fast gating methods [13, Ch. 4] can help reduce the computational burden in our application featuring real time constraints.

Once the most likely association is found, we use a standard Kalman filter with a constant velocity model to update the estimated state of each track and generate a prediction for the next time step. Both appearing and disappearing targets are common in our scenario. Missed detections also frequently occur, e.g., due to colliding targets. On real image data, reflections of the lighting can also pose a challenge. Furthermore, clutter measurements occur due to dirt on the belt. To deal with all of this adequately, we have implemented a track score approach [14, Ch. 6] which allows us to keep track of particles even if a measurement is missed and helps

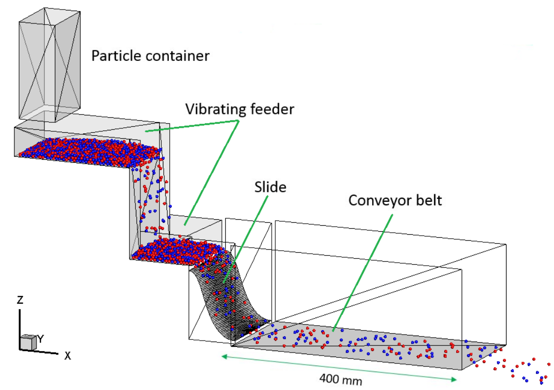


Fig. 4: Visualization of the simulated belt sorter.

to filter out measurements caused by dirt[2].

Due to delays in the system, a prediction as to when and where a particle will pass the separation mechanism has to be generated at a certain time before the particle reaches the separation mechanism. During that time, no new measurements can be taken into account anymore. In this phase, we currently merely perform prediction steps of the utilized Kalman filter, using the constant velocity model also used in the tracking phase. As we use the information acquired via the tracking for the purpose of generating a precise prediction for a future time step, we refer to our approach as predictive tracking. For simplicity, the prediction phase of the predictive tracking can be thought of as an area that does not need to be observed as measurements in this area cannot be used anymore. However, since merely the time of arrival at the separation mechanism is of importance, the actual physical extent of the prediction phase could be adapted for each individual particle according to the current estimate of the particle's velocity.

III. Simulation Approach

Our simulation is based on a small, experimental optical belt sorter that was specifically crafted for evaluating bulk material sorting processes. The belt sorter was designed using SolidWorks, a 3D-CAD tool, and was manufactured accordingly. The three-dimensional model used in the creation of the belt sorter was used as the basis for our simulation. All important parts of the optical belt sorter except for the separation mechanism were modeled, including not only the belt but also the vibrating feeder and the slide that are used to apply bulk material to the belt. The model of the sorter is shown in Fig. 4.

To simulate the particles' movements, we determined the relevant physical properties of the belt and particles experimentally [15]. Based on the three-dimensional model of the sorter, the particles' properties, and the specifications of vibrating feeder, we used the Discrete Element Method (DEM) to simulate the particles' movements. The DEM not only allows us to accurately model the interaction between the particles and the belt, but also particle–particle interactions as

[2]Unless the dirt always looks and moves like a particle, in which case it cannot be distinguished from an actual particle of the bulk material.

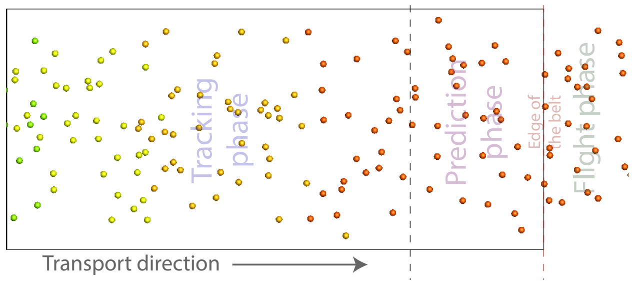


Fig. 5: One frame of the synthetic image data that was generated for the evaluation (some vertical blank space was cropped) with an illustration of the phases. The particles' colors describe their velocity from slow (green) to fast (red).

well as particle–wall interactions. Details of applying DEM-simulations to optical belt sorters are provided in [5], [6]. The separation process is not modeled yet, but Computational Fluid Dynamics will be employed in future work as done similarly recently in [16]. For our current evaluation, the separation process is not necessary as we directly evaluate the precision of the prediction, which is made possible by the availability of the ground truth.

IV. Evaluation

In our evaluation, we put the emphasis on an important endpoint of the system, namely the accuracy of the prediction regarding precisely when and where each particle passes the separation mechanism. Performing the separation immediately after the belt is advantageous for designing compact belt sorters. Therefore, we generated predictions for the position and time at which each particle passes the edge of the belt. The belt has a total length of $40\,\mathrm{cm}$. We designated the last $10\,\mathrm{cm}$ of the belt to be the prediction phase, which is a reasonable distance between the last observation and the separation for optical belt sorters. The belt was simulated to run at $1.5\,\mathrm{m/s}$. In our evaluation, we regarded wooden spheres that were also used in the base case of the evaluation performed in [6]. Predictions were generated for over 1000 particles.

The entire evaluation was performed offline. First, the computationally very expensive DEM-simulation was performed. The centroids of the particles and additional information such as the actual velocities of the particles were saved. Furthermore, synthetic images of the scenario were generated as shown in Fig. 5. To determine the true position and time of the intersection, we interpolated the ground truth positions of each particle in all phases. For the predictions evaluated, only information available during the tracking phase was used to predict the intersection of the path with the edge of the belt.

We have subdivided the evaluation into two parts that are explained in the first two subsections of this section. In the first part, we validate the models and the tracking under optimal conditions by using noise-free information obtained by the simulation. The second part is an evaluation based on the synthetic image data that allows us to evaluate the entire approach including the image processing component. In the third subsection, we present the results of our evaluation.

A. Evaluation Based on the Ground Truth Data

To test our model under optimal conditions, we first used all noise-free ground truth information obtained during the tracking phase for the prediction. We chose the last point of the ground truth track before the start of the prediction phase (meaning the last known position of the particle before it enters the prediction phase ranging from a belt position of $30\,\mathrm{cm}$ to $40\,\mathrm{cm}$) and then created a prediction based on the velocity of the particle at that precise moment, as given by the DEM-simulation. To account for the fact that even with perfect localization, the actual speed would not be known, we also regarded predictions generated by using the tracking on the noise-free position measurements. In this case, the filter steps become trivial due to the absence of measurement uncertainty.

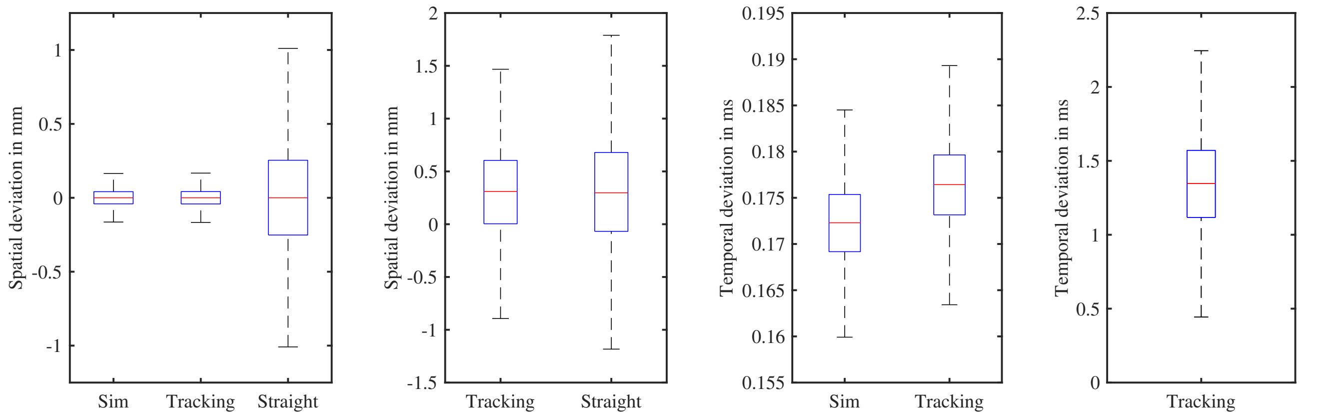
For comparison, we evaluated a third approach based on the ground truth data in which we performed a prediction resembling the old, implicit model. For this, we also used the position of the particle at the last time step before it enters the prediction phase as the starting point. Based on this, we determined the intersection with the edge of the belt when assuming a movement straight in the transport direction.

We also evaluated the temporal offset for all approaches except for the old, implicit model. For the implicit model, it is hard to generate a temporal offset—strictly assuming that all particles move at the velocity of the belt would not be in line with implementations in real systems. In these, experimental fine-tuning is used to generate predictions that are usually more accurate than those obtained by simply assuming that each particle moves at the velocity of the belt.

B. Evaluation Based on the Synthetic Image Data

As the part of our evaluation that is closest to the actual application, we performed an evaluation based on the synthetic image data generated using the simulation. The images generated for the tracking mainly capture the belt and have a resolution of $1064\,\mathrm{px} \times 708\,\mathrm{px}$ at about $0.44\,\mathrm{mm/px}$ with a format shown in Fig. 5. The entire belt is in the field of view of the camera. Such a wide field of view is desirable when using the tracking to improve the classification, e.g., for motion-based classification, but poses a challenge to the image processing and results in a higher measurement uncertainty. While the ground truth data was generated at $1000\,\mathrm{Hz}$, we only used 200 frames per second of the image data. This frame rate makes sense for multiple reasons. First, cameras with this frame rate are readily available. Second, due to the run time of the image processing and the tracking, higher frame rates cannot easily be used in real time systems.

We did not use a perfect calibration between the image and world coordinates to emulate the imperfect calibration in real applications. We based our calibration on detecting the corners of the belt and then deriving a transformation that maps the belt in pixel coordinates to the belt in the world coordinates of the simulation. Due to limitations of the software used for rendering, no further challenges common in image acquisition such as lens distortion were considered. However, due to lens distortion correction employed for properly calibrated



(a) Spatial deviation when the 1000 Hz ground truth data was used for the predictions.

(b) Spatial deviation when the image data with 200 frames per second was used for the predictions.

(c) Temporal deviation when the ground truth data was used for the predictions.

(d) Temporal deviation when the image data was used for the predictions.

Fig. 6: Spatial and temporal deviation for all evaluated approaches to generate predictions. Evaluations based on the ground truth data and the image data are depicted in separate plots. “Sim” indicates that the velocity obtained by the DEM-simulation was used, whereas “Tracking” indicates that the tracking was used to derive a velocity. The prediction straight in the transport direction as used in the old, implicit model was denoted as “Straight”. A positive temporal offset indicates that the particle has arrived earlier than predicted, whereas a negative temporal offset indicates that it arrived later.

cameras, we believe that this effect is not essential to our evaluation.

Performing tracking based on actual image data instead of merely adding an artificial stochastic noise is important for a multitude of reasons. First, errors introduced by image acquisition are more of a discretization type of error whose characteristics are not purely stochastic. Thus, generating the position measurements using image processing on synthetic images yields noise characteristics that are more similar to those in real applications. Second, artifacts affecting the multitarget tracking problem that are characteristic to using image processing have to be taken into account, e.g., that colliding particles may be detected as one, resulting in an incorrect number of measurements and an incorrect centroid.

First, the tracking approach as described in Sec. II-B was performed on the results of the image processing. Necessary parameters of the filter such as the observable region were set beforehand but no specific adjustments were made to optimize the performance for this scenario. Then, the tracks obtained as the result of the tracking were matched to ground truth tracks by finding the track that minimizes a distance function. As the distance function, we calculated the euclidean norm of the euclidean distances between all estimated positions of the tracking and all corresponding ground truth positions at the respective time steps. As only tracks that are deemed valid while entering the prediction phase trigger a separation event in the real machinery, we only regarded these tracks in our evaluation.

We also evaluated the old, implicit model on the measurements derived from the image data. In this case, we used the results of the tracking to determine the last instant a particle is observed at before entering the prediction phase. Based on the corresponding measurement, we predicted the particles to move straight in the transport direction and compared the results with the true intersection. As before, the temporal offset of this approach was omitted in the analysis.

C. Evaluation Results

The evaluation results are visualized as box plots in Fig. 6. Outliers were omitted as over a thousand data points result in too many outliers to properly visualize. We first regard the spatial offset orthogonal to the transport direction and then the temporal deviation.

As shown in Fig. 6a, using the noise-free information obtained at least $10\,\mathrm{cm}$ before the end of the belt to predict the positions of the particles at the edge of the belt (and thus at the separation mechanism) yields results with very high precision. When using the velocity obtained by the simulation, the vast majority of all predictions are accurate up to less than $0.2\,\mathrm{mm}$ of precision. Very similar results were obtained when using only noise-free position measurements to predict each particle’s motions via tracking. This was an expected result as the particles of the bulk material do not change their velocity very fast and thus, the velocity can be approximated well when all positions of the particle are perfectly known. However, as seen on the right of Fig. 6a, the assumption that the particles move straight in the transport direction is not valid in general as the old, implicit model performs far worse on the ground truth data.

The results of the evaluation based the synthetic image data are given in Fig. 6b and show that the tracking approach achieves a performance comparable with the performance that the old, implicit model achieves using noise-free position measurements. Further analysis showed that the bias observed is a result of the inaccurate calibration used. With the vast majority of particles deviating less than $1.5\,\mathrm{mm}$ from the ground truth, a reliable separation is ensured for commonly used separation mechanisms. The predictions based on the old, implicit model are worse, but not far inferior to those of the predictive tracking approach using the image data. For perfectly round particles, the error of the image processing can dominate, lessening the importance of the improved motion model. In line with observations in [4], we expect the models to be more important for irregularly shaped particles of biological origin such as peppercorns. However, accurate simulations for these kinds of bulk materials are significantly harder and are subject to future work.

The temporal errors of the approaches based on the noise-free information are shown in Fig. 6c. A conspicuous difference to the spatial deviations are the biases in the temporal deviations. The particles are still on the belt and accelerate further in the prediction phase in this scenario, which is not accounted for by the constant velocity model. This induces a need for more sophisticated models that take such effects into account. In Fig. 6d, it can be seen that the temporal deviation when using the tracking approach based on the image data is higher, including a higher bias. The reasons are twofold. First, the spatial calibration error along the transport direction induces an error in the predicted time of arrival at the separation mechanism. Second, the higher uncertainties in the tracking based on the image data make this approach slower to adapt to the changes in velocity.

V. Conclusion

Our evaluation of the predictive tracking approach based on a DEM-simulation shows that using tracking to generate predictions for accurately targeting particles in optical belt sorters is a valid and useful concept. If the particles can be localized very accurately, the derived predictions are highly precise, a result that is in line with observations in [6]. While the tracking performs well using noise-free information, the obtained results based on the output of the image processing are limited by the accuracy of the determined centroids passed on to the tracking. Nonetheless, we showed in our evaluation that even when using a perspective that was not chosen to optimize tracking results, useful position measurements can be derived using suitable image processing algorithms.

However, evidently, the accuracy of the measurements obtained using the image processing has a large impact on the performance of the tracking system and thus, optimization of the tracking performance for industrial applications should always include tailoring the parameters of the camera and the image processing to the problem at hand. E.g., while using a large field of view that covers the entire belt can be useful for classification purposes and for combining the knowledge of multiple sensors along the belt, the lower precision of the derived measurements may induce worse prediction results.

One very useful insight is that while a constant velocity model works very well to reduce spatial inaccuracies, improving the model may lead to lower temporal errors. One possibility to improve the tracking performance would be to also observe the particles during the prediction phase to be able to tell whether the particles tend to further accelerate or decelerate during the prediction phase. This information could be used to adjust the predictions of particles that have yet to enter the prediction phase. The change in the velocities could also be determined for each class of particle individually to improve the predictions even further using the classification decision.

Future work may entail generating image data that is more realistic and closer to the image data obtained on real machinery to improve the emulation of the challenges in the actual image processing task. For example, this could be achieved by generating hybrid images by overlaying real image data with simulated particles in a realistic manner. In the long term, we plan to develop a simulation that integrates the tracking, the sorting decisions based on the tracking, and the actual separation. Using this simulation, the set-up of the tracking and the design of the entire belt sorter could be iteratively improved.

Acknowledgment

The IGF project 18798 N of the research association Forschungs-Gesellschaft Verfahrens-Technik e.V. (GVT) was supported via the AiF in a program to promote the Industrial Community Research and Development (IGF) by the Federal Ministry for Economic Affairs and Energy on the basis of a resolution of the German Bundestag.

References

[1] J. Duran, *Sands, Powders, and Grains: An Introduction to the Physics of Granular Materials*. Springer, New York, 2000.
[2] H. Wotruba, "Stand der Technik der sensorgestützten Sortierung (in German)," *BHM Berg- und Hüttenmännische Monatshefte*, vol. 153, pp. 221–224, June 2008.
[3] H. Kattentidt, T. De Jong, and W. Dalmijn, "Multi-Sensor Identification and Sorting of Bulk Solids," *Control engineering practice*, vol. 11, no. 1, 2003.
[4] F. Pfaff, M. Baum, B. Noack, U. D. Hanebeck, R. Gruna, T. Längle, and J. Beyerer, "TrackSort: Predictive Tracking for Sorting Uncooperative Bulk Materials," in *Proceedings of the 2015 IEEE International Conference on Multisensor Fusion and Integration for Intelligent Systems (MFI 2015)*, San Diego, California, USA, Sep. 2015.
[5] F. Pfaff, C. Pieper, G. Maier, B. Noack, H. Kruggel-Emden, R. Gruna, U. D. Hanebeck, S. Wirtz, V. Scherer, T. Längle, and J. Beyerer, "Improving Optical Sorting of Bulk Materials Using Sophisticated Motion Models," *tm - Technisches Messen, De Gruyter*, vol. 83, no. 2, pp. 77–84, Feb. 2016.
[6] C. Pieper, G. Maier, F. Pfaff, H. Kruggel-Emden, S. Wirtz, R. Gruna, B. Noack, V. Scherer, T. Längle, J. Beyerer, and U. D. Hanebeck, "Numerical Modeling of an Automated Optical Belt Sorter using the Discrete Element Method," *Powder Technology*, Jul. 2016.
[7] L. Shapiro and G. Stockman, *Computer Vision*. Prentice Hall, 2001.
[8] R. P. Mahler, *Statistical Multisource-Multitarget Information Fusion*. Artech House, Inc., 2007.
[9] D. Schuhmacher, B.-T. Vo, and B.-N. Vo, "A Consistent Metric for Performance Evaluation of Multi-Object Filters," *IEEE Transactions on Signal Processing*, vol. 56, no. 8, pp. 3447–3457, 2008.
[10] R. Jonker and A. Volgenant, "A Shortest Augmenting Path Algorithm for Dense and Sparse Linear Assignment Problems," *Computing*, vol. 38, no. 4, pp. 325–340, 1987.
[11] D. P. Bertsekas, *Encyclopedia of Optimization*, 2nd ed. Springer, 2009, ch. Auction Algorithms, pp. 128–132.
[12] C. N. Vasconcelos and B. Rosenhahn, "Bipartite Graph Matching Computation on GPU," 2009.
[13] M. Liggins II, D. Hall, and J. Llinas, *Handbook of Multisensor Data Fusion: Theory and Practice*, 2nd ed. CRC press, 2009.
[14] S. Blackman and R. Popoli, *Design and Analysis of Modern Tracking Systems*, 1999.
[15] D. Höhner, S. Wirtz, and V. Scherer, "Experimental and Numerical Investigation on the Influence of Particle Shape and Shape Approximation on Hopper Discharge Using the Discrete Element Method," *Powder Technology*, 2013.
[16] R. S. Fitzpatrick, H. J. Glass, and R. D. Pascoe, "CFD–DEM Modelling of Particle Ejection by a Sensor-Based Automated Sorter," *Minerals Engineering*, vol. 79, pp. 176 – 184, 2015.