

CD-PHY: Physical Layer Security in Wireless Networks through Constellation Diversity

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Abstract—A common approach for introducing security at the physical layer is to rely on the channel variations of the wireless environment. This type of approach is not always suitable for wireless networks where the channel remains static for most of the network lifetime. For these scenarios, a channel independent physical layer security measure is more appropriate which will rely on a secret known to the sender and the receiver but not to the eavesdropper. In this paper, we propose CD-PHY, a physical layer security technique that exploits the constellation diversity of wireless networks which is independent of the channel variations. The sender and the receiver use a custom bit sequence to constellation symbol mapping to secure the physical layer communication which is not known a priori to the eavesdropper. Through theoretical modeling and experimental simulation, we show that this information theoretic construct can achieve Shannon secrecy and any brute force attack from the eavesdropper incurs high overhead and minuscule probability of success. Our results also show that the high bit error rate also makes decoding practically infeasible for the eavesdropper, thus securing the communication between the sender and receiver.

I. INTRODUCTION

In wireless networks, physical (PHY) layer security enables nodes to communicate securely without using resource intensive encryption mechanisms at the application layer. PHY layer security measures are resource friendly due to their information theoretic construct based on *perfect secrecy* [1] in contrast with the computational hardness approaches [2]. By introducing security at the PHY layer, communication in wireless networks can avoid the stepping stone of most attacks: *eavesdropping*. In general, the broadcast nature of the the communication makes wireless networks more vulnerable to eavesdropping attacks than the wired counterpart. PHY layer security measures are able thwart such attacks to a considerable extent [3], [4].

Most of the existing PHY layer security schemes are based on the variation of channel characteristics [5], [6], [7]. However, without highly mobile or dynamic environment which can introduce significant variation in channel characteristics, these schemes do not perform as expected [8]. Experimental results show that in static scenarios, these scheme mostly provide keys with very low entropy which is not desired in many cases [6]. In this paper, we propose a PHY layer security technique, CD-PHY, based on *constellation diversity*, which is not dependent on channel characteristics and the performance does not vary depending on static or mobile scenario.

The underlying technique for CD-PHY is simple. At the physical layer, the sender and the intended receiver uses a custom constellation mapping [9] which acts as a secret key to secure the communication from an eavesdropper. In other

words, a sequence of bits from the sender is converted into symbols on the constellation space based on a mapping known only to the sender and the intended receiver. Using the correct mapping, the intended receiver will be able to decode the signal and reconstruct the original message. However, the eavesdropper will not even be able to decode the signal correctly without the knowledge of constellation mapping, let alone reconstruction of the message.

The guarantee of security provided by CD-PHY is much stronger than just keeping the modulation type (BPSK, QPSK, and QAM¹, for example) a secret between the sender and the receiver. Because, if the sender and receiver uses the standard constellation mapping for these modulations, an eavesdropper can use advanced machine learning techniques [11], [12] to identify the modulation type and then use the standard mapping to decode the signal. In case of CD-PHY, the custom constellation mapping is known only to the sender and the receiver which is the basis of security for this information theoretic construct.

Our theoretical modelling, security analysis and experimental simulation reveals the following about CD-PHY:

- For the eavesdropper, the probability of successfully decode the symbols range from 10^{-3} at $10dB$ SNR² to 0.015 at $0dB$ SNR, which is very low (Section IV),
- CD-PHY achieves perfect secrecy as a cipher and has a very high unicity distance which ensures that the eavesdropper will not be able to find the correct decoding regardless of the amount of ciphertexts it collects (Section V-A),
- A brute-force key search attack on CD-PHY has complexity $\#P$ (Sharp P)³ which is believed to be much harder than polynomial time algorithms (Section V-B), and
- Performance wise, in the presence of CD-PHY, regardless of the location, the bit error rate at the eavesdropper is always as high as 50% which is equivalent to random guessing for the decoding purposes (Section VI).

II. BACKGROUND AND OBSERVATIONS

At the physical layer, a modulation technique prepares the digital bit sequences for transmission over the analog wireless medium. A crucial part of this operation is to map the bit

¹BPSK and QPSK refers to Binary and Quadrature Phase Shift Keying, respectively. QAM refers to Quadrature Amplitude Modulation. An overview of modulation schemes by Zeimer can be found at [10].

²Signal-to-noise ratio.

³The set of the counting problems associated with the decision problems in the set NP.

sequences to symbols which can be represented as points on a two dimensional complex plane called the *constellation diagram*. Figure 1 shows an example constellation diagram from 16-ary Quadrature Amplitude Modulation (16QAM circular). An alternate constellation diagram is shown in Figure 2 which is known as 16QAM rectangular. If the transmitter wishes to send a bit sequence, it sets the real (x-axis) and imaginary (y-axis) part according to the constellation diagram. Mathematically, a signal can be expressed by the following equation:

$$s(t) = I(t).cos(2\pi f_o t) + Q(t).sin(2\pi f_o t)$$

where $I(t)$ and $Q(t)$ are real and imaginary parts of the symbols from the constellation diagram and f_o is the modulating frequency. The receiver recovers the real and imaginary values after demodulation, and plots each symbol on the constellation plane. To correctly decode the original message, the receiver needs to know both the type of modulation as well as symbol to bit sequence mapping⁴.

When only the modulation type is the secret, the eavesdropper can use machine learning based techniques [11], [12] to identify the modulation type and use standard bit sequence to symbol mapping to decode the data. However, if the sender and receiver use a custom constellation mapping which is not known to the eavesdropper, the complexity of correct decoding becomes very high. For an M -ary QAM, the eavesdropper has to try all $M!$ mappings to find out the correct decoding, which is very impractical for scenarios when the value of $M \geq 8$.

Figure 3 shows the decoding failure when the eavesdropper tries to decode an original 16QAM circular modulated signal using different modulation types: BPSK, QPSK and 16QAM rectangular. The input data stream contained 8 bits, 01100101. In 16QAM, each symbol consists of 4 bits. So, two symbols will be received by the eavesdropper. The QPSK receiver decodes two symbols as 4 bits and the BPSK receiver decodes it to 2 bits. Since the modulation classification was wrong, obviously the mapping will also be wrong resulting to a decoding failure. In the case of 16QAM rectangular, the receiver will correctly expand the symbols to 8 bits. However, since the constellation mapping was different⁵, the final decoded data will be different from the input: 11110111. Another decoding failure, where the original symbols belonged to 16QAM rectangular, is shown in Figure 4.

The intuitive design of CD-PHY is based on the above mentioned observations that without knowing the correct constellation mapping, it is not practically feasible for an eavesdropper to correctly decode the original message even though it might have the knowledge of modulation type.

III. ADVERSARIAL MODEL

We assume that the adversary (eavesdropper) is able to detect and will try to decode the communication between the sender and receiver. It can be either mobile or static.

⁴Constellation mapping.

⁵Refers to Figure 1 and 2.

An adversary can also measure the channel parameters. It can exploit some machine learning techniques to identify the modulation type of the wireless communication, but it does not have prior knowledge of the constellation mapping between the sender and intended receiver.

We also assume the eavesdropper's computation and communication capability as powerful as the sender and receiver. The adversary can try to handle the original signal as noise or try interference cancellation and joint decoding. Finally, we assume that the adversary is passive and has no intention to launch active attacks such as a man-in-the-middle attack. This is a common assumption among most of the practical wireless security schemes [8].

IV. THEORETICAL MODELLING

In this section, we derive the probability of an eavesdropper to correctly decode the message in the presence of gaussian noise when it knows the modulation type but does not know the constellation mapping. A very intuitive example of this case is the interaction between 16QAM circular and rectangular modulations discussed in Section II. We use this example to derive the probability measure of correct decoding when the sender modulation is 16QAM circular and eavesdropper modulation is 16QAM rectangular.

As discussed in Section II, each QAM symbol has a real and imaginary value associated with it in the constellation space. Mathematically, for an M -ary QAM, these real and imaginary values can range $\pm a, \dots, \pm(2m - 1)a$, where $m = \log_2 M$, $a^2 = 1.5E_s/(M - 1)$ with E_s being the symbol energy [13]. Table I shows the bit sequence to constellation symbol mapping in 16QAM circular and 16QAM rectangular Scheme These values are further factored by $a = \sqrt{E_s/10}$ to normalize the average symbol energy to 1.

The decision variable for demodulation in the presence of *additive white gaussian noise* can be obtained as

$$Y \approx X + n \quad (1)$$

where the noise term $n(t)$ is assumed with power spectral density $\frac{N_o}{2}$, zero mean and variance of $\sigma^2 = N_o$. Thus, the decision variable Y is a complex gaussian with a complex mean X and variance $\sigma^2 = N_o$. In other words, Y has a two dimensional gaussian distribution in complex plane. So, the real and imaginary parts of Y can be separated as independent gaussian variables as Y_R and Y_I with means at $Re(X)$ and $Im(X)$.

$$Y_R = Re(Y) = Re(X) + n_R = X_R + n_R$$

$$Y_I = Im(Y) = Im(X) + n_I = X_I + n_I$$

where n_R and n_I are the components of noise along real and imaginary axes with a mean zero and variance $\sigma_R^2 = \sigma_I^2 = \frac{N_o}{2}$. Now, the probability density function of Y_R can be expressed as:

$$\begin{aligned} f(Y_R) &= \frac{1}{\sqrt{2\pi\sigma_R^2}} \exp\left\{-\frac{(Y_R - X_R)^2}{2\sigma_R^2}\right\} \\ &= \frac{1}{\sqrt{\pi N_o}} \exp\left\{-\frac{(Y_R - X_R)^2}{N_o}\right\} \end{aligned} \quad (2)$$

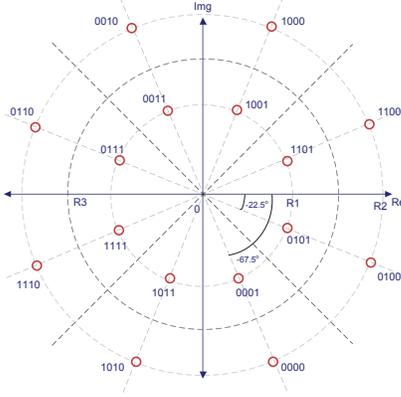


Fig. 1. 16QAM Circular Constellation

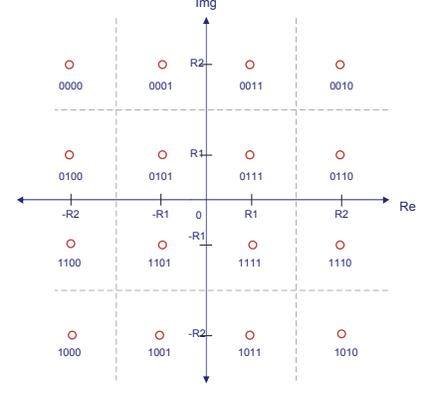


Fig. 2. 16QAM Rectangular Constellation

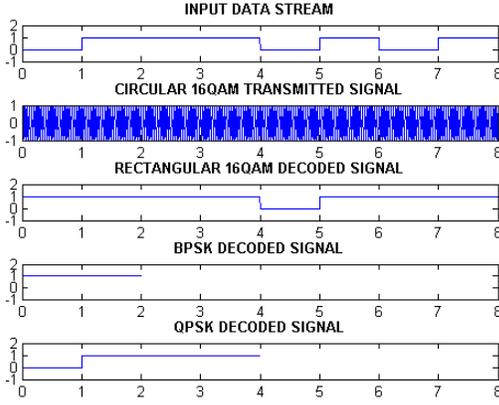


Fig. 3. Decoding failure when the original modulation is 16QAM circular.

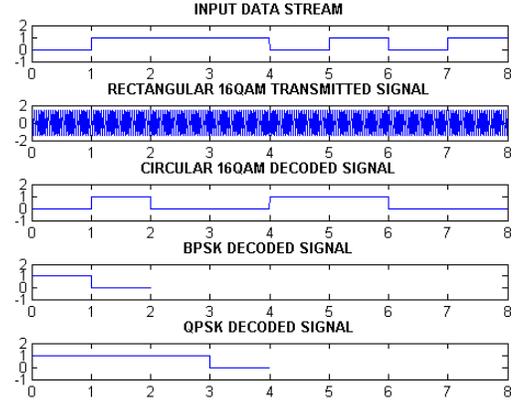


Fig. 4. Decoding failure when the original modulation is 16QAM rectangular.

Similarly, the probability density function of Y_I can also be expressed as:

$$f(Y_I) = \frac{1}{\sqrt{\pi N_o}} \exp - \left\{ \frac{(Y_I - X_I)^2}{N_o} \right\} \quad (3)$$

Now, to calculate the probability of the successful decoding at the eavesdropper with 16QAM rectangular scheme when the original symbols were transmitted in 16QAM circular scheme, we first need to consider the probabilities at individual symbol level. These probabilities are then aggregated using the symmetry and mutual independence of the symbols. In the following derivations, S_i^r denotes a symbol S_i in 16QAM rectangular scheme, S_i^c represents a symbol S_i in 16QAM circular scheme and four symbols are chosen from the constellation diagram in such a way that symmetrically they represent all sixteen points of a QAM scheme.

A. Decoding of symbol 0000

First, we consider $S_0^c = 0000$ being transmitted. From Table I, the real and imaginary parts of 0000 are

$$X_R = 1.53\sqrt{\frac{E_s}{10}} \quad \& \quad X_I = -3.69\sqrt{\frac{E_s}{10}}$$

The received symbol Y has a complex gaussian distribution as discussed earlier with the mean at $X_R + jX_I$. Now, the probability that the symbol Y can be correctly decoded by the eavesdropper using 16QAM rectangular decoder can be found based on the decision space for $S_0^r = 0000$ in 16QAM rectangular scheme. Formally, the probability that decoded symbol is S_0^r given S_0^c was transmitted is:

$$\begin{aligned} P(Y = S_0^r | S_0^c) &= \\ &P\left(-\infty < Y_R < -2\sqrt{\frac{E_s}{10}}\right) P\left(2\sqrt{\frac{E_s}{10}} < Y_I < \infty\right) \\ P(Y = S_0^r | S_0^c) &= \int_{-\infty}^{-2\sqrt{\frac{E_s}{10}}} f(Y_R) dY_R \times \int_{2\sqrt{\frac{E_s}{10}}}^{\infty} f(Y_I) dY_I \end{aligned}$$

TABLE I
BIT SEQUENCE TO CONSTELLATION SYMBOL MAPPING IN 16QAM CIRCULAR AND 16QAM RECTANGULAR SCHEME

Bit Sequence	16QAM Circular	16QAM Rectangular	Bit Sequence	16QAM Circular	16QAM Rectangular
0000	1.53 - 3.69j	-3 + 3j	1000	1.53 + 3.69j	-3 - 3j
0001	.76 - 1.84j	-1 + 3j	1001	.76 + 1.84j	-1 - 3j
0010	-1.53 + 3.69j	3 + 3j	1010	-1.53 - 3.69j	3 - 3j
0011	-.76 + 1.84j	1 + 3j	1011	-.76 - 1.84j	1 - 3j
0100	3.69 - 1.53j	-3 + j	1100	3.69 + 1.53j	-3 - j
0101	1.84 - .76j	-1 + j	1101	1.84 + .76j	-1 - j
0110	-3.69 + 1.53j	3 + j	1110	-3.69 - 1.53j	3 - j
0111	-1.84 + .76j	1 + j	1111	-1.84 - .76j	1 - j

$$P(Y = S_0^r | S_0^c) = \frac{1}{\sqrt{\pi N_o}} \int_{-\infty}^{-2\sqrt{\frac{E_s}{10}}} \exp\left\{-\frac{(Y_R - 1.53\sqrt{\frac{E_s}{10}})^2}{N_o}\right\} dY_R \times \frac{1}{\sqrt{\pi N_o}} \int_{2\sqrt{\frac{E_s}{10}}}^{\infty} \exp\left\{-\frac{(Y_I - (-1.53\sqrt{\frac{E_s}{10}}))^2}{N_o}\right\} dY_I$$

Using the simplification of above integrals,

$$P(Y = S_0^r | S_0^c) = \frac{1}{\sqrt{\pi}} \int_{3.53\sqrt{\frac{E_s}{10N_o}}}^{\infty} \exp\{-t^2\} dt \times \frac{1}{\sqrt{\pi}} \int_{5.69\sqrt{\frac{E_s}{10N_o}}}^{\infty} \exp\{-z^2\} dz$$

$$\begin{aligned} P(Y = S_0^r | S_0^c) &= \frac{1}{2} \operatorname{erfc}\left(3.53\sqrt{\frac{E_s}{10N_o}}\right) \frac{1}{2} \operatorname{erfc}\left(5.69\sqrt{\frac{E_s}{10N_o}}\right) \\ &= \frac{1}{4} \operatorname{erfc}\left(3.53\sqrt{\frac{E_s}{10N_o}}\right) \operatorname{erfc}\left(5.69\sqrt{\frac{E_s}{10N_o}}\right) \end{aligned} \quad (4)$$

Here, $\operatorname{erfc}()$ is the complementary error function.

B. Decoding of symbol 0100

Now, we consider the symbol $S_1^c = 0100$ being transmitted. Similar to the previous example,

$$X_R = 3.69\sqrt{\frac{E_s}{10}} \quad \& \quad X_I = -1.53\sqrt{\frac{E_s}{10}}$$

So, the probability that the eavesdropper correctly decodes the symbol 0100 is:

$$P(Y = S_1^r | S_1^c) = P\left(-\infty < Y_R < -2\sqrt{\frac{E_s}{10}}\right) P\left(0 < Y_I < 2\sqrt{\frac{E_s}{10}}\right) \quad (5)$$

Now, the left part of the right hand side of Equation 5 gives us the following:

$$\begin{aligned} P\left(-\infty < Y_R < -2\sqrt{\frac{E_s}{10}}\right) &= \frac{1}{\sqrt{\pi N_o}} \int_{-\infty}^{-2\sqrt{\frac{E_s}{10}}} \exp\left\{-\frac{(Y_R - 3.69\sqrt{\frac{E_s}{10}})^2}{N_o}\right\} dY_R \\ &= \frac{1}{2} \operatorname{erfc}\left(5.69\sqrt{\frac{E_s}{10N_o}}\right) \end{aligned} \quad (6)$$

Next, the right part yields the following:

$$P\left(0 < Y_I < 2\sqrt{\frac{E_s}{10}}\right) = 1 - P\left(Y_I < 0, Y_I > 2\sqrt{\frac{E_s}{10}}\right)$$

$$P\left(0 < Y_I < 2\sqrt{\frac{E_s}{10}}\right) = 1 - \left[\frac{1}{\sqrt{\pi N_o}} \int_{-\infty}^0 \exp\left\{-\frac{(Y_I - (-1.53\sqrt{\frac{E_s}{10}}))^2}{N_o}\right\} dY_I + \frac{1}{\sqrt{\pi N_o}} \int_{2\sqrt{\frac{E_s}{10}}}^{\infty} \exp\left\{-\frac{(Y_I - (-1.53\sqrt{\frac{E_s}{10}}))^2}{N_o}\right\} dY_I \right]$$

$$P\left(0 < Y_I < 2\sqrt{\frac{E_s}{10}}\right) = 1 - \frac{1}{2} \operatorname{erfc}\left(1.53\sqrt{\frac{E_s}{10N_o}}\right) - \frac{1}{2} \operatorname{erfc}\left(3.53\sqrt{\frac{E_s}{10N_o}}\right) \quad (7)$$

Using Equation 6,7 on Equation 5, we have the following:

$$P(Y = S_1^r | S_1^c) = \frac{1}{2} \operatorname{erfc}\left(5.69\sqrt{\frac{E_s}{10N_o}}\right) \times \left[1 - \frac{1}{2} \operatorname{erfc}\left(1.53\sqrt{\frac{E_s}{10N_o}}\right) - \frac{1}{2} \operatorname{erfc}\left(3.53\sqrt{\frac{E_s}{10N_o}}\right) \right]$$

$$\begin{aligned} P(Y = S_1^r | S_1^c) &= \frac{1}{2} \operatorname{erfc}\left(5.69\sqrt{\frac{E_s}{10N_o}}\right) \\ &\quad - \frac{1}{4} \operatorname{erfc}\left(5.69\sqrt{\frac{E_s}{10N_o}}\right) \operatorname{erfc}\left(1.53\sqrt{\frac{E_s}{10N_o}}\right) \\ &\quad - \frac{1}{4} \operatorname{erfc}\left(5.69\sqrt{\frac{E_s}{10N_o}}\right) \operatorname{erfc}\left(3.53\sqrt{\frac{E_s}{10N_o}}\right) \end{aligned} \quad (8)$$

C. Decoding of symbol 0101

Now, we consider, $S_2^c = 0101$ is being transmitted. In this case:

$$X_R = 1.84\sqrt{\frac{E_s}{10}} \quad \& \quad X_I = -0.76\sqrt{\frac{E_s}{10}}$$

So, the probability that the eavesdropper correctly decodes the symbol is:

$$P(Y = S_2^r | S_2^c) = P\left(-2\sqrt{\frac{E_s}{10}} < Y_R < 0\right) P\left(0 < Y_I < 2\sqrt{\frac{E_s}{10}}\right) \quad (9)$$

We first consider the left part of the right hand side of Equation 9:

$$P\left(-2\sqrt{\frac{E_s}{10}} < Y_R < 0\right) = 1 - P\left(Y_R > 0, Y_R < -2\sqrt{\frac{E_s}{10}}\right)$$

$$\begin{aligned} P\left(-2\sqrt{\frac{E_s}{10}} < Y_R < 0\right) &= 1 - \left[\frac{1}{\sqrt{\pi N_o}} \int_0^{\infty} \exp\left\{-\frac{(Y_R - 1.84\sqrt{\frac{E_s}{10}})^2}{N_o}\right\} dY_R \times \frac{1}{\sqrt{\pi N_o}} \int_{-\infty}^{-2\sqrt{\frac{E_s}{10}}} \exp\left\{-\frac{(Y_R - 1.84\sqrt{\frac{E_s}{10}})^2}{N_o}\right\} dY_R \right] \end{aligned}$$

$$P\left(-2\sqrt{\frac{E_s}{10}} < Y_R < 0\right) = 1 - \frac{1}{2} \operatorname{erfc}\left(3.84\sqrt{\frac{E_s}{10N_o}}\right) - \frac{1}{2} \operatorname{erfc}\left(-1.84\sqrt{\frac{E_s}{10N_o}}\right) \quad (10)$$

Similarly, we consider the right part of the right hand side of Equation 9:

$$\begin{aligned} P\left(0 < Y_I < 2\sqrt{\frac{E_s}{10}}\right) &= 1 - P\left(Y_I < 0, Y_I > 2\sqrt{\frac{E_s}{10}}\right) \\ P\left(0 < Y_I < 2\sqrt{\frac{E_s}{10}}\right) &= 1 - \\ &\left[\frac{1}{\sqrt{\pi N_o}} \int_{-\infty}^0 \exp\left\{-\frac{(Y_I - (-0.76\sqrt{\frac{E_s}{10}}))^2}{N_o}\right\} dY_I\right. \\ &\left. + \frac{1}{\sqrt{\pi N_o}} \int_{2\sqrt{\frac{E_s}{10}}}^{\infty} \exp\left\{-\frac{(Y_I - (-0.76\sqrt{\frac{E_s}{10}}))^2}{N_o}\right\} dY_I\right] \\ P\left(0 < Y_I < 2\sqrt{\frac{E_s}{10}}\right) &= 1 - \frac{1}{2} \operatorname{erfc}\left(-0.76\sqrt{\frac{E_s}{10N_o}}\right) \\ &\quad - \frac{1}{2} \operatorname{erfc}\left(2.76\sqrt{\frac{E_s}{10N_o}}\right) \end{aligned} \quad (11)$$

Thus, combining Equation 10, 11, we have:

$$\begin{aligned} P(Y = S_2^r | S_2^c) &= \\ &\left[1 - \frac{1}{2} \operatorname{erfc}\left(3.84\sqrt{\frac{E_s}{10N_o}}\right) - \frac{1}{2} \operatorname{erfc}\left(-1.84\sqrt{\frac{E_s}{10N_o}}\right)\right] \\ &\times \left[1 - \frac{1}{2} \operatorname{erfc}\left(-0.76\sqrt{\frac{E_s}{10N_o}}\right) - \frac{1}{2} \operatorname{erfc}\left(2.76\sqrt{\frac{E_s}{10N_o}}\right)\right] \end{aligned} \quad (12)$$

D. Decoding of symbol 0001

Finally, we consider symbol $S_3^c = 0001$ being transmitted. In this case:

$$X_R = 0.76\sqrt{\frac{E_s}{10}} \quad \& \quad X_I = -1.84\sqrt{\frac{E_s}{10}}$$

So, the probability that eavesdropper correctly decodes symbol 0001 is:

$$P(Y = S_3^r | S_3^c) = P\left(-2\sqrt{\frac{E_s}{10}} < Y_R < 0\right) P\left(2\sqrt{\frac{E_s}{10}} < Y_I < \infty\right) \quad (13)$$

Considering the left part of the right hand side of Equation 13:

$$\begin{aligned} P\left(-2\sqrt{\frac{E_s}{10}} < Y_R < 0\right) &= 1 - P\left(Y_R < -2\sqrt{\frac{E_s}{10}}, Y_R > 0\right) \\ P\left(-2\sqrt{\frac{E_s}{10}} < Y_R < 0\right) &= 1 - \\ &\left[\frac{1}{\sqrt{\pi N_o}} \int_{-\infty}^{-2\sqrt{\frac{E_s}{10}}} \exp\left\{-\frac{(Y_R - 0.76\sqrt{\frac{E_s}{10}})^2}{N_o}\right\} dY_R\right. \\ &\left. + \frac{1}{\sqrt{\pi N_o}} \int_0^{\infty} \exp\left\{-\frac{(Y_R - 0.76\sqrt{\frac{E_s}{10}})^2}{N_o}\right\} dY_R\right] \\ P\left(-2\sqrt{\frac{E_s}{10}} < Y_R < 0\right) &= 1 - \frac{1}{2} \operatorname{erfc}\left(2.76\sqrt{\frac{E_s}{10N_o}}\right) \\ &\quad - \frac{1}{2} \operatorname{erfc}\left(-0.76\sqrt{\frac{E_s}{10N_o}}\right) \end{aligned} \quad (14)$$

Similarly, the right part yields:

$$\begin{aligned} P\left(2\sqrt{\frac{E_s}{10}} < Y_I < \infty\right) &= \\ &\frac{1}{\sqrt{\pi N_o}} \int_{2\sqrt{\frac{E_s}{10}}}^{\infty} \exp\left\{-\frac{(Y_I - (-1.84\sqrt{\frac{E_s}{10}}))^2}{N_o}\right\} dY_I \end{aligned}$$

$$P\left(2\sqrt{\frac{E_s}{10}} < Y_I < \infty\right) = \frac{1}{2} \operatorname{erfc}\left(3.84\sqrt{\frac{E_s}{10N_o}}\right) \quad (15)$$

By combining the outcomes of Equation 14, 15, we get the following:

$$\begin{aligned} P(Y = S_3^r | S_3^c) &= \frac{1}{2} \operatorname{erfc}\left(3.84\sqrt{\frac{E_s}{10N_o}}\right) \\ &\quad - \frac{1}{4} \operatorname{erfc}\left(3.84\sqrt{\frac{E_s}{10N_o}}\right) \operatorname{erfc}\left(2.76\sqrt{\frac{E_s}{10N_o}}\right) \\ &\quad - \frac{1}{4} \operatorname{erfc}\left(3.84\sqrt{\frac{E_s}{10N_o}}\right) \operatorname{erfc}\left(-0.76\sqrt{\frac{E_s}{10N_o}}\right) \end{aligned} \quad (16)$$

As mentioned earlier, based on the symmetry of QAM constellation diagrams, other symbols will also have probabilities equal to one of the following symbols: S_0, S_1, S_2 or S_3 . Assuming all symbols have equal probability of being generated and transmitted i.e. $P(S_k) = 1/16$ where $(k = 0 \dots 15)$, the total probability $P(C)$ that the data transmitted by 16QAM circular transmitter and correctly decoded by 16QAM rectangular eavesdropper is:

$$\begin{aligned} P(C) &= P(S_k) \times 4 \times [P(Y = S_0^r | S_0^c) + P(Y = S_1^r | S_1^c) \\ &\quad + P(Y = S_2^r | S_2^c) + P(Y = S_3^r | S_3^c)] \end{aligned}$$

$$\begin{aligned} P(C) &= \frac{1}{4} \left[-\frac{1}{4} \operatorname{erfc}\left(5.69\sqrt{\frac{E_s}{10N_o}}\right) \operatorname{erfc}\left(1.53\sqrt{\frac{E_s}{10N_o}}\right) \right. \\ &\quad + \frac{1}{4} \operatorname{erfc}\left(-1.84\sqrt{\frac{E_s}{10N_o}}\right) \operatorname{erfc}\left(2.76\sqrt{\frac{E_s}{10N_o}}\right) \\ &\quad + \frac{1}{4} \operatorname{erfc}\left(-1.84\sqrt{\frac{E_s}{10N_o}}\right) \operatorname{erfc}\left(-0.76\sqrt{\frac{E_s}{10N_o}}\right) \\ &\quad - \frac{1}{2} \operatorname{erfc}\left(-0.76\sqrt{\frac{E_s}{10N_o}}\right) - \frac{1}{2} \operatorname{erfc}\left(2.76\sqrt{\frac{E_s}{10N_o}}\right) \\ &\quad \left. + \frac{1}{2} \operatorname{erfc}\left(5.69\sqrt{\frac{E_s}{10N_o}}\right) - \frac{1}{2} \operatorname{erfc}\left(-1.84\sqrt{\frac{E_s}{10N_o}}\right) + 1 \right] \end{aligned} \quad (17)$$

Here, N_o is the power spectral density of the noise and E_s is the symbol energy of the signal. So, the term E_s/N_o is a representative of the SNR at the receiver. Since Equation 17 contains $\operatorname{erfc}()$ function, as we increase the value of SNR in the $\operatorname{erfc}()$ function, the probability will decrease. So, the probability of correct decoding is adversely affected by the SNR of the wireless medium at receivers. This theoretical fact is illustrated further in Figure 5. The line with circles refers to the probability of correct decoding and the line with crosses refers to the probability of error. At $0dB$ SNR, the probability of error for the eavesdropper is 0.002. At SNR values above $20dB$, the probability of error is nearly 1 which makes the decoding almost infeasible in practice. In comparison, for an intended receiver with 16QAM circular scheme, the probability of error at $0dB$ SNR is around 1, and 0 for a SNR of $20dB$ [13].

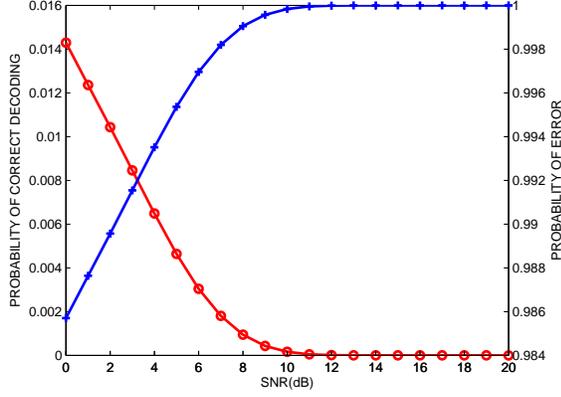


Fig. 5. Probability that the eavesdropper decodes correctly and incorrectly at different signal-to-noise ratio.

V. SECURITY ANALYSIS

In this section, we analyze CD-PHY in terms of information theoretic security, security by complexity and resistance to potential modulation classification schemes such as Automatic Modulation Classification (AMC) [12] and Digital Modulation Classification (DMC) [11].

The basis of information theoretic security is the fact that the bit sequence to constellation symbol mapping is known only to the sender and receiver(s). The eavesdropper does not have any a priori knowledge of the mapping. In the subsequent section, by applying Shannon's secrecy model (Figure 6) to CD-PHY, we show that it can in deed achieve information theoretic security. In addition, any decoding attempt on the eavesdropper side incurs high complexity as it blindly tries to find the mapping. Finally, we show how CD-PHY thwarts the classification attempts by AMC and DMC.

A. Information theoretic security

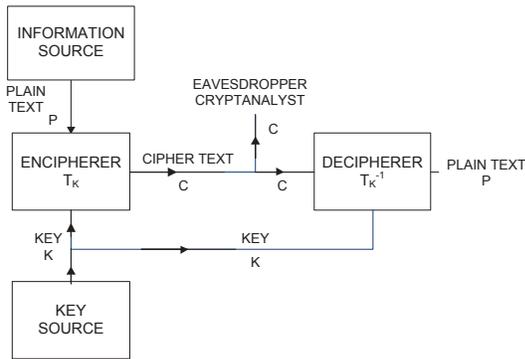


Fig. 6. Shannon's Secrecy Model

In CD-PHY, the act of finding the correct mapping from the constellation points to bit sequences is essentially a deciphering operation for the eavesdropper. Here, the transmitted bit sequences are plaintext P , signal received by the eavesdropper is the ciphertext C , the mapping is the key K . For an

M -ary QAM, the plaintext can have M symbols, each of which are $\log_2 M$ bits. The key, mapping of bit sequences to constellation points, has $M!$ variations. Now, we define *perfect secrecy* and *unicity distance* which is due to Shannon [1].

Definition 1. A cipher achieves perfect secrecy, if without knowing the secret key, the plaintext P is independent of the ciphertext C , formally:

$$\text{prob}(\mathbf{P} = P | \mathbf{C} = E_K(P)) = \text{prob}(\mathbf{P} = P) \quad (18)$$

Equivalently,

$$\text{prob}(\mathbf{C} = C | \mathbf{P} = E_K^{-1}(C)) = \text{prob}(\mathbf{C} = C) \quad (19)$$

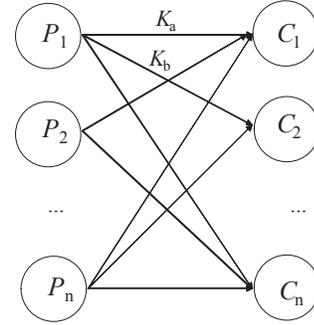


Fig. 7. An illustration of plaintext to ciphertext mapping.

Definition 2. Unicity distance of a cipher is the minimum amount of ciphertext needed for brute-force attack to succeed. Formally:

$$U = H(K)/D \quad (20)$$

where $H(K)$ is the entropy of the key and D is the redundancy of the message.

Definition 1 leads us to the following theorem:

Theorem 1. CD-PHY achieves perfect secrecy.

Proof: Perfect secrecy requires that without the knowledge of the key, each ciphertext is equally probably to map to any plaintext of that domain. Since the symbols are independent of each other and equally probable to map any of the constellation points, for an M -ary QAM scheme, we have the following:

$$\text{prob}(\mathbf{C} = C | \mathbf{P} = E_K^{-1}(C)) = 1/M = \text{prob}(\mathbf{C} = C) \quad (21)$$

which meets the requirements of perfect secrecy. In other words, since the key K is independent of plaintext P and follows uniform distribution, it leads us to:

$$\text{prob}(\mathbf{P} = P | \mathbf{C} = E_K(P)) = 1/M = \text{prob}(\mathbf{P} = P) \quad (22)$$

$$\begin{aligned}
& \text{More rigorously: } \text{prob}(\mathbf{P} = P | \mathbf{C} = C) \\
&= \frac{\text{prob}(\mathbf{P} = P, \mathbf{C} = C)}{\text{prob}(\mathbf{C} = C)} \\
&= \frac{\text{prob}(\mathbf{C} = C | \mathbf{P} = P) \text{prob}(\mathbf{P} = P)}{\sum_{P' \in \mathbf{P}} \text{prob}(\mathbf{C} = C | \mathbf{P} = P') \text{prob}(\mathbf{P} = P')} \\
&= \frac{\text{prob}(K = C \rightarrow P) \text{prob}(\mathbf{P} = P)}{\sum_{P' \in \mathbf{P}} \text{prob}(K = C \rightarrow P') \text{prob}(\mathbf{P} = P')} \\
&= \frac{\frac{1}{M} \text{prob}(\mathbf{P} = P)}{\sum_{P' \in \mathbf{P}} \frac{1}{M} \text{prob}(\mathbf{P} = P')} \\
&= \text{prob}(\mathbf{P} = P) \tag{23}
\end{aligned}$$

where $K=C \rightarrow P$ refers that key K is a mapping between plaintext P and ciphertext C . ■

In addition, according to *perfect cipher keyspace theorem* [1] ⁶, if a cipher is perfect, there must be at least as many keys (l) as there are possible messages (n). This leads us to the following corollary:

Corollary 1. *Messages in CD-PHY with M-ary QAM scheme should contain less than n symbols such that $M! \geq M^n$ to maintain perfect secrecy.*

Definition 2 leads us to the following theorem:

Theorem 2. *The unicity distance of CD-PHY tends to infinity.*

Proof: For a CD-PHY with M-ary QAM, entropy of the key $H(K) \approx \log M!$. Since, the symbols are independent of each other, the redundancy $D = 0$ for the message. So, the unicity distance is $U \approx (\log M! / 0) = \infty$. ■

Unicity distance is a theoretical measure of how many ciphertexts are required to determine a unique plaintext. If one has less than unicity distance ciphertext, it is not possible to identify if the deciphering is correct. In fact, when the redundancy approaches to zero, it is hard to attack even simple cipher. For CD-PHY, a unicity distance of infinity means that the eavesdropper won't be able to determine whether the deciphering is correct regardless of the number of the ciphertexts it has in its possession. This is, in fact, a very strong information theoretic guarantee of CD-PHY security.

B. Security by complexity

Now, we model the problem of brute-force key search attack⁷ on CD-PHY as a *complete bipartite graph perfect matching* problem and analyze the algorithmic complexity of it.

Definition 3. *A complete bipartite graph is a bipartite graph where every vertex of the one set is connected to each vertex of the other set. Formally, a complete bipartite graph, $G = (V_1 \cup V_2, E)$, is a bipartite graph such that for any two vertices, $v_1 \in V_1$ and $v_2 \in V_2$, $v_1 v_2$ is an edge in G .*

⁶Also known as *Shannon bound*.

⁷Finding the bit sequence to constellation point mapping.

From the definition of a complete bipartite graph [14], it is straightforward to see the following theorem.

Theorem 3. *The bit sequence to constellation point mapping in CD-PHY is a complete bipartite graph.*

Proof: A complete bipartite graph partitions the vertices into two sets $|V_1| = p$ and $|V_2| = q$. Now, we can see from Figure 7 that each plaintext (bit sequence) on the left side of the graph can be considered a vertex of V_1 and each ciphertext (constellation points) on the right can be considered a vertex of V_2 . Based on the key, it is possible to map every member of V_1 to any member of V_2 . Thus, it constitutes a complete bipartite graph where $|V_1| = |V_2| = \log_2 M$ for an M-ary QAM scheme. ■

Now, to explain perfect matching [15] of the complete bipartite graph, we need the following definition.

Definition 4. *A matching in a graph is a set of edges without common vertices. In a perfect matching, every vertex of the graph is connected to only one edge of the matching.*

The counting version of complete bipartite graph perfect matching problem returns the total number of perfect matching where each edge in the matching connects two unique vertices from V_1 and V_2 . Theorem 3 and Definition 4 leads us to the following theorem:

Theorem 4. *The brute-force key search attack on CD-PHY is:*

- 1) *equivalent to counting version of complete bipartite graph perfect matching problem, and*
- 2) *in complexity class #P (Sharp P) complete.*

Proof: Based on Theorem 3 and Definition 4, proof of part 1 is trivial. The problem of counting the number of perfect matching of a complete bipartite graph can be solved by computing the permanent of the bi-adjacency matrix [16] of the graph. The permanent of a matrix $A = n \times n$ is defined as:

$$\text{perm}(A) = \sum_{\sigma} \prod_{i=1}^n a_{i, \sigma(i)} \tag{24}$$

where σ is a permutation over $\{1, 2, \dots, n\}$. The complexity of computing permanent of a matrix is in complexity class #P complete, as proved by the seminal work [17] of Valiant in 1979. ■

In general, computing the permanent of a matrix is believed to be harder than its determinant. While one can compute the determinant in polynomial time by Gaussian elimination, the same cannot be used to compute the permanent. Thus, the computational complexity of the brute-force key search attack on CD-PHY also adds to the security of the scheme.

C. Defense against modulation classification schemes

The section explains where does CD-PHY stand when the eavesdropper tries to apply some modulation classification techniques such as AMC [12] and DMC [11].

AMC is based on cyclic feature detection technique considering the *cyclostationary* property of the modulated signals.

It considers the fact that modulated signals in practice have parameters that vary periodically with time. These hidden periodicities are used to classify the modulation techniques. Although, AMC is able to differentiate modulations such as BPSK, QPSK, and QAM based on large amount of training data and supervised learning, it can not identify the shape of the constellation and constellation mapping of symbols to constellation points. Also, for higher order QAM, the complexity of AMC makes it practically infeasible even to classify the modulation.

DMC uses constellation shape as the basis of modulation classification. In this algorithm, the receiver constructs a scatter diagram of the received noisy symbols in a complex plane and uses fuzzy c-means clustering to recover robust constellation. The modulation type is identified using maximum likelihood (ML) classification with predefined constellation templates. Similar to AMC, digital modulation classification also requires a large amount of training data and supervised learning to identify templates. Thus, although it can identify pre-defined constellation shapes, it is not able to identify constellation mapping from symbols to constellation points.

In summary, CD-PHY can withstand existing modulation classification techniques and secure against the attacks exploiting such techniques in practice.

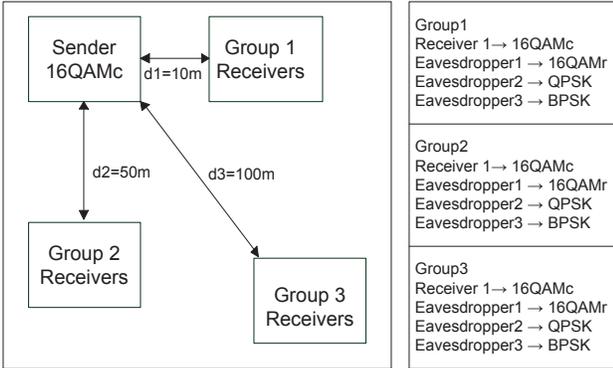


Fig. 14. Simulated wireless network scenario. The sender uses 16QAM circular scheme. At different distances, each group has an intended receiver with 16QAM circular scheme and three eavesdroppers each with 16QAM rectangular, QPSK and BPSK scheme.

VI. PERFORMANCE EVALUATION AND SIMULATION RESULTS

In this section, we show the impact of CD-PHY on the network performance of the eavesdropper. A very intuitive measure of such performance evaluation is to show how many bits are received in error at different signal and noise power. Typically, when the signal power increases, the receiver is able to decode the bits more accurately leading to a lower bit error rate (BER). In the following experiment, we show that the BER of CD-PHY receiver conforms to this pattern whereas the BER of the eavesdroppers does not decrease even for higher signal power.

The experimental scenario is shown in Figure 14. We designate a CD-PHY sender with 16QAM circular modulation scheme. The receivers are divided into three groups based on their distances from the sender. Group 1, group2 and group 3 are at 10m, 50m and 100m distance, respectively. Each group has an intended CD-PHY receiver with 16QAM circular scheme and three eavesdroppers with 16QAM rectangular, QPSK and BPSK scheme.

We measure the BER at different receivers for different SNRs. Experimental scenarios contain both free space and indoor environments. Figure 8, 9 and 10 show the measurements from free space environment. For the CD-PHY receiver, with the increment of SNR, the bit error rate decreases following the usual pattern of wireless communication. However, for eavesdroppers with different schemes, the bit error rate is more than 50% regardless of the increment of SNR. The error rate is the highest in BPSK which is consistent with our analysis in Section II. As the distance increases, BER of BPSK scheme can go as high as 60%, resulting in a near to impossible decoding process.

Figure 11, 12 and 13 show BER vs SNR for indoor environment. The bit error rates of the eavesdroppers are also as high as 50% throughout the measurements for different SNR values. Similar to the free space environment, the distance of the receivers also adversely affect the bit error rate.

Figure 15 aggregates the BER measurements for different locations of the eavesdropper. The median BER is around 50% and the range is 40% to 60%. It shows that in the presence of CD-PHY, the eavesdroppers experience such a high bit error rate that it is almost equivalent of randomly guessing the bits. This is true for both indoor and free space environment and ensures that the eavesdropper can not comprehend the signal when CD-PHY is in action.

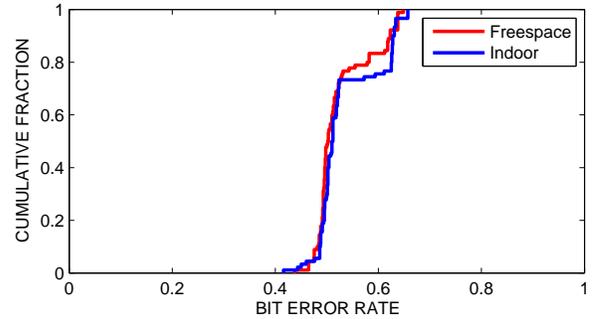
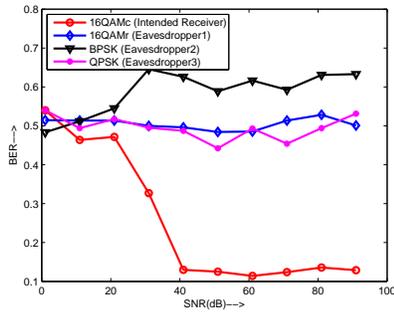
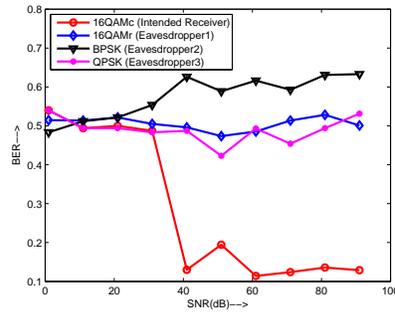
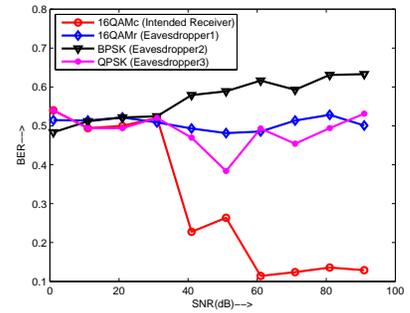
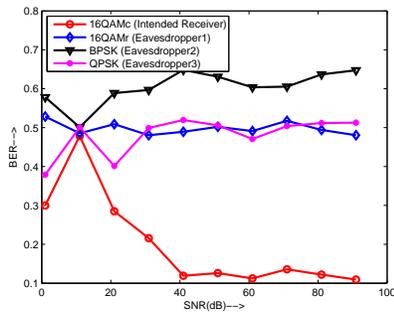
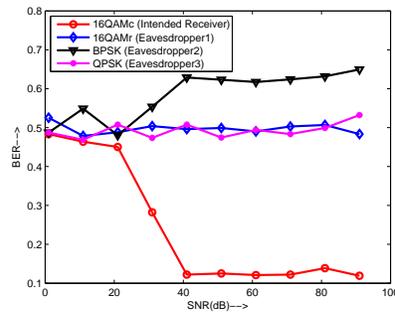
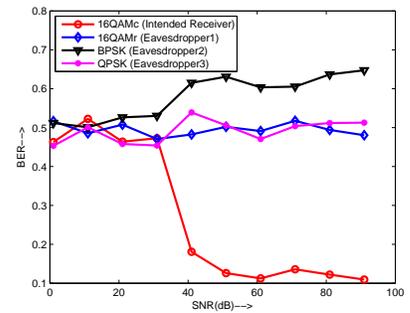


Fig. 15. Eavesdropper bit error rate from indoor and free space experiments.

VII. CONCLUSION

CD-PHY is a simple mechanism that introduces channel independent security at the physical layer of wireless communication. We have shown that in the presence of CD-PHY, the eavesdropper has a very low probability of successfully decoding the signal. The scheme achieves Shannon secrecy as a cipher and a brute-force key search attack on CD-PHY falls under complexity class $\#P$ which is believed to be harder than

Fig. 8. BER vs SNR for $\alpha = 2$ and $d = 10m$ Fig. 9. BER vs SNR for $\alpha = 2$ and $d = 50m$ Fig. 10. BER vs SNR for $\alpha = 2$ and $d = 100m$ Fig. 11. BER vs SNR for $\alpha = 1.4$ and $d = 10m$ Fig. 12. BER vs SNR for $\alpha = 1.4$ and $d = 50m$ Fig. 13. BER vs SNR for $\alpha = 1.4$ and $d = 100m$

polynomial time algorithms. Our experimental results confirm the theoretical derivations; the bit error rate at the eavesdropper is significantly high and it is practically infeasible to decode the signal which ensures the communication secrecy between the sender and the intended receiver.

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