On The Impact of Time-Varying Interference-Channel on the Spatial Approach of Spectrum Sharing between S-band Radar and Communication System

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Abstract—Spectrum sharing is a new approach to solve the congestion problem in RF spectrum. A spatial approach for spectrum sharing between radar and communication system was proposed, which mitigates the radar interference to communication by projecting the radar waveform onto null space of interference channel, between radar and communication system [1]. In this work, we extend this approach to maritime MIMO radar which experiences time varying interference channel due to the oscillatory motion of ship, because of the breaking of sea/ocean waves. We model this variation by using the matrix perturbation theory and the statistical distribution of the breaking waves. This model is then used to study the impact of perturbed interference channel on the spatial approach of spectrum sharing. We use the maximum likelihood (ML) estimate of target's angle of arrival to study the radar's performance when its waveform is projected onto the null space of the perturbed interference channel. Through our analytical and simulation results, we study the loss in the radar's performance due to the null space projection (NSP) of its waveform on the perturbed interference channel.

Index Terms—MIMO radar, null space, spectrum sharing, perturbed interference channel, coexistence.

I. INTRODUCTION

Spectrum sharing between radars and communication system is a new way forward to solve the growing problem of spectrum congestion [2]. The 3550-3650 MHz band is identified for spectrum sharing between military radars and broadband wireless access (BWA) communication systems, in the National Telecommunications and Information Administration's (NTIA) 2010 Fast Track Report [3].

Electromagnetic interference (EMI) to radars and BWA systems is expected from spectrum sharing [4]. In this paper, we focus on mitigating radar's interference to communication systems. The emission pattern of radar, especially high transmit power and high-peak sidelobes, saturates communication system receiver, which traditionally operate at very small power levels. However, radar's interference can be mitigated at communication systems by exploiting the advancements in transmitter and receiver designs, and through a combination of spatial and temporal signal processing algorithms.

Spatial algorithms for interference mitigation is a well explored topic in the cognitive radio research community [5], however, a spatial algorithm to mitigate radar's interference to communication system was first proposed in [1]. The waveform of radar is projected onto null space of interference channel between the radar and the communication system. This NSP of the radar waveform mitigates radar interference but in turn causes slight degradation in the radar performance, which is studied in [1].

For a maritime radar, i.e., a radar mounted on a ship, interference channel between the radar and communication system is subject to change due to many factors, motion of the ship due to waves is one of them. In this paper, we focus on the accuracy of available interference-channel-state-information (ICSI) for the NSP spectrum-sharing algorithm. We

model the imperfection or uncertainty in the ICSI by using the matrix perturbation theory. The NSP algorithm is sensitive to perturbations in the ICSI as it can alter the null space of the interference channel. This can cause interference leakage to a communication system, if it is not accounted for, and it can also degrade the performance of a radar system, which is addressed in this paper. Our goal is to study the impact of perturbed ICSI on the NSP algorithm and the radar performance.

Relation to prior work: Prior work on the NSP technique to share RF spectrum between radar and communication system focuses on the assumption that perfect ICSI is available at the radar [1]. However, this assumption is impractical since ICSI can have variations due to errors in the estimation process, quantization of feedback, or time variations in interference-channel, due to motion. In our case, we consider variation in interference-channel due to the up-and-down motion of the ship, because of waves, and not due to the communication system, which we assume is fixed. Thus, this brings in imperfect ICSI at the radar which is not considered in [1]. We study the impact of an inaccurate ICSI on the performance of radar.

Notations: Bold capital letters, e.g., $\mathbf{A}$, correspond to matrices and bold letters, e.g., $\mathbf{a}$, correspond to column vectors. The operators $(\cdot)^H$ and $(\cdot)^*$ corresponds to the Hermitian transpose and complex-conjugate, respectively.

The remainder of this paper is organized as follows. Section II discusses spectrum sharing architecture. Section III briefly discusses the theory of colocated MIMO radar. Section IV models perturbed ICSI. In Section V, we study perturbed channel's impact on the NSP algorithm and the ML estimate. Section VI discusses the simulation setup and provides quantitative results along with the discussion. Section VII concludes the paper.

II. SPECTRUM SHARING ARCHITECTURE

In this paper, we consider a MIMO communication system equipped with N_T transmit antennas and N_R receive antennas sharing the 3550-3600 MHz RF band with a MIMO radar with M_T transmit antennas and M_R receive antennas.

We assume ICSI of the communication system, or its distribution, is available at the radar system.

Fig. 1. A maritime MIMO radar sharing spectrum with a MIMO communication system. The interference channel $\mathbf{H}_I$ is perturbed by $\Delta\mathbf{H}_I$ due to the motion of the ship.

In practice, this assumption can be justified when both the radar and communication systems belong to military. In this case, let the received signal at the communication system's receiver be

$$\mathbf{y}(t) = \mathbf{H}_I^{N_R \times M_T}\mathbf{x}_{\text{Radar}}(t) + \mathbf{H}^{N_R \times N_T}\mathbf{x}(t) + \mathbf{n}(t)$$

where $\mathbf{H}_I$ is the $N_R \times M_T$ interference channel between the radar and the communication system [6], $\mathbf{H}$ is the $N_R \times N_T$ communication channel between the communication systems, and $\mathbf{n}(t)$ is the additive white Gaussian noise. In order to avoid the interference to the communication system, the spatial approach is to project radar signal $\mathbf{x}_{\text{Radar}}(t)$ onto the null-space of $\mathbf{H}_I$ such that $\mathbf{H}_I\mathbf{x}_{\text{Radar}}(t) = \mathbf{0}$.

However, due to oscillatory motion of the ship, because of sea/ocean waves even if the ship is docked at harbor, the interference channel between the radar and the communication system is perturbed. In this case, the received signal at the communication system's receiver is

$$\mathbf{y}(t) = [\mathbf{H}_I + \Delta\mathbf{H}_I]\mathbf{x}_{\text{Radar}}(t) + \mathbf{H}\mathbf{x}(t) + \mathbf{n}(t).$$

where $\Delta\mathbf{H}_I$ denotes perturbation in ICSI and is modeled using Rayleigh distribution, see Section IV for details, with covariance matrix

$$\text{Cov}_{\Delta\mathbf{H}_I} = \mathbb{E}\left\{vec(\Delta\mathbf{H}_I)vec(\Delta\mathbf{H}_I)^H\right\}$$

where $vec(\cdot)$ is the column stacking operator. This spectrum sharing scenario is shown in Figure 1.

As previously stated, spatial approach of interference mitigation is projection of radar signal onto

the null space of interference channel [7]. After perturbation, in order for the spatial approach to work, the radar signal is projected onto the null space of the perturbed interference channel such that $[\mathbf{H}_\text{I} + \Delta\mathbf{H}_\text{I}]\mathbf{x}_\text{Radar}(t) = \mathbf{0}$.

III. COLOCATED MIMO RADAR

In this section, we introduce preliminaries of colocated MIMO radar, having M_T transmit and M_R receive antennas, which transmits finite-alphabet constant-envelop BPSK waveforms designed in [8]. The radar waveform $\mathbf{x}_\text{Radar}(t)$ can be expressed as

$$\mathbf{x}_\text{Radar}(t) = \begin{bmatrix} x_1(t)e^{j\omega_c t} & x_2(t)e^{j\omega_c t} & \cdots & x_{M_T}e^{j\omega_c t}(t) \end{bmatrix}^T$$

where $x_k(t)$ is the baseband signal from the kth transmit element, ω_c is the carrier angular frequency, $t \in [0, T_o]$, with T_o being the observation time. Then the received signal, from a single point target at an angle θ, is

$$\mathbf{y}_\text{Radar}(t) = \alpha\,\mathbf{A}(\theta)\,\mathbf{x}_\text{Radar}(t - \tau(t)) + \mathbf{n}(t)$$

where $\mathbf{n}(t)$ is additive white Gaussian noise, $\tau(t) = \tau_{T_k}(t) + \tau_{R_l}(t)$, denoting the sum of propagation delays between the target and the kth transmit element and the lth receive element, respectively; and α represents the complex path loss including the propagation loss and the reflection coefficient; $\mathbf{a}_T(\theta)$ is the transmit steering matrix defined as

$$\mathbf{a}_T(\theta) \triangleq \begin{bmatrix} e^{-j\omega_c\tau_{T_1}(\theta)} & e^{-j\omega_c\tau_{T_2}(\theta)} & \cdots & e^{-j\omega_c\tau_{T_{M_T}}(\theta)} \end{bmatrix}^T$$

$\mathbf{a}_R(\theta)$ is the receive steering matrix defined as

$$\mathbf{a}_R(\theta) \triangleq \begin{bmatrix} e^{-j\omega_c\tau_{R_1}(\theta)} & e^{-j\omega_c\tau_{R_2}(\theta)} & \cdots & e^{-j\omega_c\tau_{R_{M_R}}(\theta)} \end{bmatrix}^T$$

and $\mathbf{A}(\theta)$ is the transmit-receive steering matrix defined as

$$\mathbf{A}(\theta) \triangleq \mathbf{a}_R(\theta)\mathbf{a}_T^T(\theta).$$

Our performance metric for the MIMO radar's performance is the maximum likelihood (ML) estimate of the target's angle of arrival, which is expressed as, as in [9],

$$(\hat{\theta}, \hat{\tau}_r, \hat{\omega}_D)_\text{ML} = \arg\max_{\theta, \tau_r, \omega_D} \frac{\left|\mathbf{a}_R^H(\theta)\mathbf{E}(\tau_r, \omega_D)\mathbf{a}_T^*(\theta)\right|^2}{M_R\mathbf{a}_T^H(\theta)\mathbf{R}_{\mathbf{x}_\text{Radar}}^T\mathbf{a}_T(\theta)} \tag{1}$$

where

$$\mathbf{R}_{\mathbf{x}_\text{Radar}} = \int_{T_0} \mathbf{x}_\text{Radar}(t)\,\mathbf{x}_\text{Radar}^H(t)\,dt$$

$$\mathbf{E}(\tau_r, \omega_D) = \int_{T_0} \mathbf{y}_\text{Radar}(t)\,\mathbf{x}_\text{Radar}^H(t - \tau_r)\,e^{j\omega_D t}\,dt$$

where τ_r is the propagation delay between the target and the reference point, and ω_D is the Doppler frequency shift as defined in Table 2 [1].

IV. PHYSICAL MODELING OF THE PERTURBED INTERFERENCE CHANNEL

In this paper, the cause of perturbation in ICSI is due to the motion of the maritime radar, mounted on ship. This motion is induced by sea/ocean waves. So, in order to statistically describe the perturbation we look at the statistics of the wave height. The probability density function (pdf) of the wave height h is given by the Rayleigh distribution as, as in [10],

$$p(h) = \frac{2h}{h_\text{rms}^2}\exp\left[-\left(\frac{h}{h_\text{rms}}\right)^2\right]$$

where h_rms is the root-mean-square (rms) wave height defined as

$$h_\text{rms} = \left[\frac{1}{N}\sum_{n=1}^{N} h_n^2\right]^{1/2}$$

where N corresponds to the number of observed waves and the nth wave has height h_n. Thus, h_rms can be computed by observing N wave heights. We choose Rayleigh pdf for the wave height distribution as it gives a very good estimate of the wave properties for various nearshore conditions.

V. IMPACT OF THE PERTURBED INTERFERENCE CHANNEL ON THE NSP ALGORITHM

The projection of the radar signals onto the null space of the interference channel via a projection algorithm was proposed in [1] for a stationary interference channel. However, due to the motion of the ship, the interference channel gets perturbed and can be written as

$$\mathbf{H}_{\Delta\text{I}} = \mathbf{H}_\text{I} + \Delta\mathbf{H}_\text{I}.$$

For the NSP algorithm, the null space of the perturbed interference channel can be calculated from the singular value decomposition (SVD) theorem as

$$\mathbf{H}_{\Delta\text{I}} = \mathbf{U}_{\Delta\text{I}}\mathbf{\Sigma}_{\Delta\text{I}}\mathbf{V}_{\Delta\text{I}}^H$$

where $\mathbf{U}_{\Delta I}$ is the perturbed complex unitary matrix, $\boldsymbol{\Sigma}_{\Delta I}$ is the diagonal matrix of the perturbed singular values, and $\mathbf{V}_{\Delta I}^H$ is the perturbed complex unitary matrix and its columns corresponding to the vanishing singular values span the null space of $\mathbf{H}_{\Delta I}$. This is denoted by $\breve{\mathbf{V}}_{\Delta I}$. We project the radar signal onto the null space of the perturbed interference-channel using the projection algorithm, as in [1],

$$\mathbf{P}_{\breve{\mathbf{V}}_{\Delta I}} = \breve{\mathbf{V}}_{\Delta I}\breve{\mathbf{V}}_{\Delta I}^H.$$

The radar waveform projected onto the null space of $\mathbf{H}_{\Delta I}$ can be written as

$$\breve{\mathbf{x}}_{\text{Radar}} = \mathbf{P}_{\breve{\mathbf{V}}_{\Delta I}}\mathbf{x}_{\text{Radar}}. \tag{2}$$

By inserting the projected signal, equation (2) in equation (1), we get the ML estimate of the target's angle of arrival for the NSP projected radar waveform as

$$(\hat{\theta}, \hat{\tau}_r, \hat{\omega}_D)_{\text{ML}_{\text{NSP}}} = \arg\max_{\theta, \tau_r, \omega_D} \frac{\left|\mathbf{a}_R^H(\theta)\mathbf{E}(\tau_r, \omega_D)\mathbf{a}_T^*(\theta)\right|^2}{M_R\mathbf{a}_T^H(\theta)\mathbf{R}_{\breve{\mathbf{x}}_{\text{Radar}}}^T\mathbf{a}_T(\theta)}. \tag{3}$$

VI. SIMULATION RESULTS

In this section, we simulate the impact of time varying interference channel on MIMO radar-communication system spectrum-sharing scenario. The interference channel is modeled to have a Rayleigh distribution. The elements of the error channel $\Delta\mathbf{H}_I$ are modeled as Rayleigh distribution, as it describes the pdf of the height of the waves, with h_{rms} taking values 1, 2, 3, and 4, indicating perturbation in the channel coefficients. The ML results are based on the average of 10,000 independent trials. The simulation parameters used are listed in Table 2 [1].

The ML estimate of the target's angle of arrival is calculated by equations (1) and (3) for the original radar waveform and the NSP radar waveform on the perturbed interference-channel, respectively. The impact of the NSP of the radar waveform, on the perturbed interference-channel, can be studied by the ML estimation error of the angle of arrival. First, in Figure 2, we compare the original and the estimated angles using the ML estimation for the original radar waveform and the NSP waveform onto the un-perturbed interference channel. It is important to note that the NSP waveform doesn't degrade the

Fig. 2. The ML estimate of target's angle of arrival. The performance of the original waveform is compared with the null space projected waveform.

ML estimates or radar performance. This endorses the claims of [1] that the NSP for spectrum-sharing is a viable approach for MIMO radars. Second, we analyze the effect of the perturbation on the NSP algorithm. In Figure 3, we compare the original and the estimated angles for various magnitudes of the perturbation in the channel coefficients. It is important to note that due to the perturbation of the interference-channel the degradation in the performance of the NSP waveform is significant. The degradation also depends on the magnitude of the perturbation. It is evident that as the perturbation in the channel coefficients increases the performance of the ML, for the NSP waveform, degrades significantly. Thus, perturbation in the interference channel can have a detrimental effect on the radar performance, in a spectrum-sharing setting, when NSP approaches are used. This can be avoided by having a frequent exchange of the ICSI between radars and communication systems.

VII. CONCLUSION

In future, federal radar spectrum will be shared with commercial operators. In this paper, we explore the spectrum-sharing scenario when the interference channel between a radar and a communication system is subject to variations due to the oscillatory motion of ship. We study the effect of perturbation, in the interference channel, when the NSP approach of spectrum sharing is used. The NSP

Fig. 3. The ML estimate of target's angle of arrival. The performance of the original waveform is compared with the null space projected waveform onto the perturbed and un-perturbed interference channel. The values of the rms wave height, i.e., $h_{\mathrm{rms}} = 1$, $h_{\mathrm{rms}} = 2$, $h_{\mathrm{rms}} = 3$, and $h_{\mathrm{rms}} = 4$ indicate a perturbation in the channel coefficients.

algorithm does not degrade the radar performance when complete ICSI is available, however, when the interference channel is perturbed, by the waves, the radar's performance is degraded. The magnitude of the degradation depends upon the magnitude of the perturbation in the channel coefficients, which is due to the height of the waves. The radar performance, in such a scenario, can be improved by having frequent updates of the ICSI.

VIII. FUTURE WORK

In this paper, we considered only a single communication system sharing spectrum with a radar. However, we are looking into extending this work to multi-cell communication system [7] and studying target detection performance of spectrum sharing MIMO radars [11]. Moreover, instead of using orthogonal waveforms, MIMO radar waveforms with spectrum sharing constraints can be designed which can mitigate radar interference to communication systems [12], [13]. We are also working on various algorithms that exploit the NSP, at radar, and resource allocation along the lines of [14]–[20], at communication system, to mitigate interference in a radar-communication system spectrum-sharing scenario.

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