Traceable measurements of electrical impedance

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Introduction

The need for measurement of electrical impedance and related quantities is shared by scientists and engineers from different backgrounds. Equivalent lumped resistance, capacitance or inductance are common parameters in the characterization, specification and design of electrical and electronic components. Electric and magnetic material properties such as resistivity, permittivity, and permeability are derived from impedance and from geometrical measurements. Many sensors of physical quantities have impedance as their output quantity. Electrochemical impedance spectroscopy (EIS), and electrical impedance tomography (EIT) have impedance measurement as their basis.

As in all measurements, the basis of accurate impedance measurements are:

a) a proper *definition* of the measurand;

b) a *traceability* to the impedance units of the International System (SI), achieved through the use of calibrated impedance meters and impedance standards.

The definition of electrical impedance

In textbooks, impedance is defined by analogy to the dc resistance: given a two-terminal linear passive circuit element if, at a given frequency f, a sinusoidal current I flows *through* the element, then a sinusoidal voltage V develops across its terminals, and

$$Z = V / I\,, \qquad (1)$$

where it is intended that the input quantities of Eq. (1) are the phasor representations of V and I, complex-valued quantities that include magnitude and phase information of the corresponding sinusoidal waveform.

The above definition looks simple and easy, until one focuses on the problem of the measurement of V and I to determine Z. According to the electromagnetic theory, a voltage V is defined as the line integral of the electric field E over a given integration path. In the variable regime -- at variance with the static (dc) case -- V is dependent on the choice of the integration path: in practice, on the geometry of the electrical mesh that includes Z, the meter, and the connections. Moreover, in the variable regime, since displacement currents can flow from the impedance to other parts of the circuit and to the environment, I is not constant through the

measurement mesh. A change in the circuit geometry, or in the environment, can alter the *V* and *I* readings and, therefore, the outcome of the measurement.
In summary, the *definition* of the electrical impedance as a measurand requires more than Eq. (1): it is necessary to state a set of electromagnetic boundary conditions such that *V* and *I*, and therefore *Z*, are well-defined quantities, independent of the connection and environment geometry.

Several definitions of electrical impedances exist, trading off between the complexity of implementation and the minimization of the *definitional uncertainty*, the contribution to the measurement uncertainty "[...] resulting from the finite amount of detail in the definition of a measurand" (VIM, 2.27).

The *four terminal-pair* definition (Fig. 2), introduced by Cutkosky (1964), allows to achieve the minimum definitional uncertainty. Despite being conceptually complex and requiring specific active circuitry, it is nowadays commonly employed in commercial impedance meters. When measuring devices defined in different ways (e.g., two-terminal electronic components) a carefully designed (Kibble, 1999) fixture or adapter becomes necessary.

Impedance units and their realization

The metrological traceability is the "property of a measurement result whereby the result can be related to a reference through a documented unbroken chain of calibrations, [...] a 'reference' can be a definition of a measurement unit through its practical realization" (VIM, 2.41). In short, a traceable impedance measurement ensures that the quantity value is measured in units of the International System of units (Systéme International, SI).

The SI units related to impedance are the ohm (Ω) for resistance and impedance, the siemens (S) for conductance and admittance, the henry (H) for inductance, the farad (F) for capacitance. The following relation holds among these units:

$$1\ \Omega = 1\ \mathrm{S}^{-1} = 1\ \mathrm{H\ Hz} = 1\ \mathrm{F}^{-1}\ \mathrm{s}\ . \qquad (1)$$

Eq. (1) involves the units of time (s), or frequency (Hz): since these quantities are easily measured with very high accuracy, it is possible to trace the measurement of one impedance quantity to another. For example, in resonance methods (Callegaro 2012, Sec. 4.9), the measurement of an inductor is traceable to a capacitance standard through the measurement of the resonance frequency of the resulting *LC* circuit.

The origin of a traceability chain is the *mise en pratique* -- the practical realization -- of the impedance units: the establishment of the value of a quantity of the same kind as the unit that is consistent with the definition of the unit itself.

In the present SI, the practical realization of impedance is the calculable capacitor experiment (SI Brochure, Appendix 2), where in a specific geometry the capacitance *C* is related to a mechanical length *l* through the equation

$C = (\varepsilon_0 \log 2 / \pi)\, l$, (2)

involving only the electric constant $\varepsilon_0 = 8.854\ 187\ 817...$ pF m^{-1} , an exact value.

Metrologists are working nowadays on a redefinition of the SI, whose adoption is predicted for 2018 (CGPM, 2014). Each base unit will be redefined by direct reference to a corresponding fundamental constant, whose value will be fixed and exact. In particular, the base unit of electricity, the ampere (A) will be defined by fixing the value of the elementary charge e; the base unit of mass, the kilogram (kg), will be defined by fixing the value of the Planck constant h.

In the new SI, the resistance quantum, the von Klitzing constant R_K, will have the exact value $R_K = h / e^2 = 25812.8074555\ \Omega$.[1] The resistance $R_H = R_K/i$ displayed by the quantum Hall effect is a simple integer i submultiple of the resistance quantum. Hence, in the new SI the quantum Hall effect is a *practical realization* of the ohm, that is, the establishment of the value of a resistance consistent with the definition of the ohm. Given Eq. (1), the quantum Hall effect can also be the basis of the practical realizations of the henry and the farad.

The quantum Hall effect is displayed by certain semiconductor devices, under conditions of low temperature T and high magnetic induction B. Devices routinely employed in resistance and impedance metrology are GaAs heterostructures, for which B ranging from 6 to 10 T and T must be lower than 1.6 K; these conditions require large cryomagnets with pumped liquid helium, devices requiring dedicated laboratories and trained personnel, expensive to acquire and to maintain. Recently, quantum Hall effect in graphene devices has been demonstrated in less stringent conditions (B = 5 T and T = 5.1 K) (Ribeiro, 2015) which can be achieved by smaller liquid helium or cryogen-free cryomagnets (Janssen, 2015). The availability of a practical realization of the ohm in an industrial environment can thus be foreseen.

Traceability in impedance measurements

As has been said, traceability involves an unbroken chain of comparisons. The instrument that allows the comparison of two impedance standards is called the impedance *bridge,* a measuring method invented by Christie (1833) and popularized by Wheatstone (1843). In a voltage ratio bridge, see Fig. 1, the two impedance standards Z_1 and Z_2 under comparison are connected in series, and crossed by the same current. Two voltages V_1 and V_2 develop across Z_1 and Z_2. The ratio V_1/V_2 is measured by comparison with a voltage ratio standard: the impedance ratio is related to the voltage ratio by the equation

$$Z_1/Z_2 = V_1/V_2 .$$

The current ratio bridge is the dual circuit, where a current ratio is measured.

In *transformer* bridges (Kibble, 1984; Awan, 2011), the voltage (or current) ratio standard is an inductive divider, whose ratio is determined by the turns ratio of its electrical windings on a

1 The value of R_K and of the other fundamental constants will be fixed at the time of the adoption of the new SI, on the basis of the Committee on Data for Science and Technology (CODATA) recommended values of the fundamental physical constants.

ferromagnetic core. Transformer bridges allow extremely accurate ratio measurements, but are large and complex electrical networks, can measure only impedance ratios very close to a limited set of nominal ratios fixed by construction, and have limitations in the frequency bandwidth available.

The realization of impedance units typically involve *voltage* ratio bridges, where the ratio standard is an inductive voltage divider; relative uncertainties of parts in 10^9 have been demonstrated (Schurr, 2009). Current ratio bridges, often in a configuration called the *current comparator*, are of typical interest in power and energy applications, where measurements involve high voltages or currents.

Commercially available electronic impedance meters (often with alternative names like *LCR* or *RCL meters* or *bridges*, *impedance analyzers*) are typically voltage ratio bridges, the ratio standard given by an analog or digital electronic circuit. The impedance under measurement is compared with an internal reference impedance (typically automatically selected from a set, to cover different impedance ranges). The base accuracy is limited to a few parts in 10^4.

The calibration procedure of impedance meters requires the use of a set of impedance standards of different kind (resistors, capacitors and inductors) and different nominal values. Each standard has to be manually connected to the meter, and the difference between the measured value and the reference value (coming from a calibration certificate) is the meter reading error for that particular nominal impedance value and measurement frequency. Electronic impedance bridges often allow the so-called *artifact calibration*, in fact an adjustment procedure: for each impedance standard measured, the corresponding reference numerical value is entered in the bridge firmware, which recalculates a set of adjustment numerical coefficients and store them in a permanent memory. The coefficients are then employed during normal measurements, to convert raw data into readings.

New trends in calibration

Digital bridges

Digital bridges are impedance bridges based on mixed-signal electronic devices (analog-to-digital converters, ADC, and/or digital-to-analog converters DAC) and a digital representation of the sinusoidal voltage and current waveforms in the bridge mesh. Research on digital bridges dates back to decades ago: only recently, however, the performances of ADCs and DACs have improved sufficiently to achieve significant results in impedance metrology. With respect to traditional impedance bridges, digital bridges have simpler electrical networks, are less expensive and can be automatically operated.

We can classify digital bridges into two broad categories.

Digitally-assisted bridges (Fig. 3) still rely on electromagnetic components (transformers and inductive voltage dividers) as ratio standards. The bridge is fed by one large-amplitude signal and by a number of small correction signals, necessary to achieve the main equilibrium (that gives the bridge reading) and the auxiliary equilibria necessary to set the impedance standards in the proper defining conditions. All signals are generated by direct digital synthesis with DACs.

In digitally-assisted bridges, the accuracy is guaranteed by the electromagnetic ratio standard, and thus the performances - and the limitations (fixed set of ratios and frequencies available) are similar to traditional transformer bridges. A set of digitally-assisted bridges for the realization of the farad from the quantum Hall effect with a relative combined accuracy of 64×10^{-9} has been reported (Callegaro, 2010).

Fully-digital bridges (Fig. 1) are voltage ratio bidges where the bridge ratio standard is based on the linearity properties of DACs (or ADCs). Since sampling and digital processing allow to generate (or measure) arbitrary voltage ratios and frequencies, fully-digital bridges overcome the intrinsic limitations of transformer bridges, either tradional or digitally-assisted. Presently, relative accuracies in the order of one part in 10^5 or better can be achieved, sufficient for the calibration of impedance standards for traceability.

Josephson bridges

The accuracy limitation of electronic DACs employed in a fully-digital bridge can be overcome by employing Josephson DACs. These are composed of integrated circuits including thousands of Josephson junctions in series, called Josephson arrays, driven by room-temperature electronics.

Two main types of Josephson DACs are available:

Binary Josephson arrays work as binary-weighed DACs: the array is divided into segments composed of 1, 2, 4, 8, ... junctions. Each segment can generate a positive, zero or negative quantized voltage, proportional to the number of junctions in it. The selection is performed by driving the segment with a proper bias current.
The output of a binary array is a quantized voltage $V = b\ (h\ /\ 2e)\ f$, where b is an integer dependent on the selected code and f is the frequency of a bias microwave. Perfectly quantized stepwise approximations of the desired waveform (e. g., a sinewave in an ac bridge) can be generated. The limitations of binary Josepshon array DACs are the relatively low resolution in bits (limited by the number of integrated Josephson junctions) and the settling time, which allows accurate waveform generation limited up to the kHz range. Josephson voltage ratio bridges with relative accuracies in the 10^{-8} range have been demonstrated (Lee, 2011).

Pulse-driven arrays are related to delta-sigma DACs. The array of N junctions in series is driven by a sequence of microwave pulses. The Josephson array transform each driving pulse in a corresponding voltage impulse of quantized amplitude $\int V\mathrm{d}t = Nh / 2e.$ Because of the intrinsic time constants of the circuit, the output voltage pulses are averaged; with a constant pulse repetition rate (typically in the GHz range), a quantized dc voltage is available at the output. To synthesize a voltage waveform, the microwave pulses are generated at high speed with a digital pattern generator.
Pulse-driven array DACs allow to generate high-resolution sinewaves with frequencies up to the MHz range, and waveform amplitudes up to 1 V have been recently achieved (Kieler, 2015). The present research focus on the implementation of these sources in Josephson impedance bridges.

Impedance simulators

The classical calibration / adjustment procedure of electronic impedance meters described previously has two main drawbacks. Each standard has to be manually connected to the meter, and a large set of standards is required. To give an example, the "performance test" specified by the manufacturer for the HP/Agilent/Keysight 4284A precision LCR meter requires 14 dedicated standards, each one calibrated at several frequencies.
Recently, Overney et al. (2014), improving over an older paper (Oldham, 1994), have revived the concept of a *digital impedance simulator* (Fig. 4); the voltage and current that are read by the meter under calibration are digitally synthesized. Impedances of arbitrary magnitude and phase can be synthesized starting from a very small set of physical impedance standards (resistors), and the bridge error can be mapped over the full complex plane.

Conclusions

Traceable electrical impedance measurements require a careful definition of the measurand and calibrated meter. The new SI links the impedance units to the fundamental constants of nature, the Planck constant and the electron charge. The practical realization of these units is possible with a solid-state quantum phenomenon, the quantum Hall effect. Impedance metrology research is now focusing on graphene devices, where the effect is observed at temperatures and magnetic fields reachable with a small tabletop experiment; and in digital impedance bridges and simulators, that allow the realization of compact and automated traceability chains for meter calibrations suitable for an implementation in the industrial environment, such as calibration centers.

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Figure Captions

Fig. 1 Principle schematic diagram of a voltage ratio bridge. Voltage generators V_1 and V_2, are connected to the series of the two impedances Z_1 and Z_2. At equilibrium ($V_D = I_D = 0$) the relation $Z_1/Z_2 = V_1/V_2$ holds. In the Wheatstone bridge, V_1 and V_2 are the voltage drops of other two impedances in series, that constitute the ratio arm. In a transformer bridge, V_1 and V_2 are generated in two secondary windings of a transformer, see also Fig. 3b. In a fully-digital bridge, V_1 and V_2 are generated by two DACs.

Fig. 2 Four terminal-pair impedance definition. The device under measurement is provided with four coaxial ports, here labelled High Current (HC), High Potential (HP), Low Potential (LC), Low Current (LC). The device is energized by connecting a generator to port HC. The measurement circuitry must achieve coaxiality (equal and opposite currents in the inner and outer conductor of each port) and the definition conditions $I_{HP} = I_{LP} = 0$, $V_{LP} = 0$. Under these conditions the four terminal-pair impedance of the standard is defined as $Z_{4TP} = V_{HP}/I_{LC}$.

Fig. 3 A digitally-assisted bridge. Generator V energizes the series of the two impedances Z_1 and Z_2 and the inductive voltage divider, which winding has a tap at ratio k. The generator V_0 is adjusted until detector D is nulled. Calling $e = V_0 / V$ is the adjustment term, if $e \ll 1$ the measurement model (correct to first order) $Z_1 / Z_2 = (1-k)/k + e/k$ holds.

Fig. 4 Principle of the impedance simulator devloped at METAS, Switzerland. The LCR meter (UUT) under calibration measures a voltage V and a current I generated by digital sources (S_1 and S_2, generating voltages V_V and V_I) and a reference resistor R in a connecting box. A four terminal-pair impedance is simulated. The sampling system acts as an automated loop that controls the complex ratio V_V / V_I match the desired programmed value to simulate an impedance $Z = (V_V / V_I)R$. Reprinted from Overney (2014); Copyright IEEE, 2014.

Biography

Luca Callegaro is an Electronic Engineer with a PhD in Physics. He joined the Istituto Nazionale di Ricerca Metrologica, INRIM, in 1996, where he is responsible of the activities on impedance metrology. He is the Chairman of EURAMET Technical Committee on Electricity and Magnetism. He is author of about 80 papers published on international reviews, and of the book *Electrical impedance: principles, measurement, applications*.

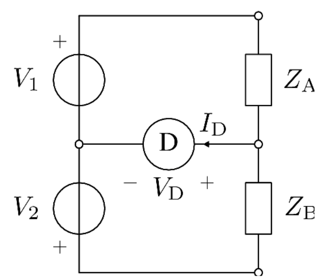


Fig.1.

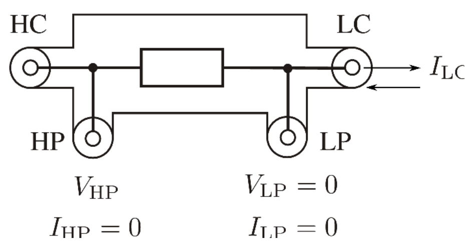


Fig. 2.

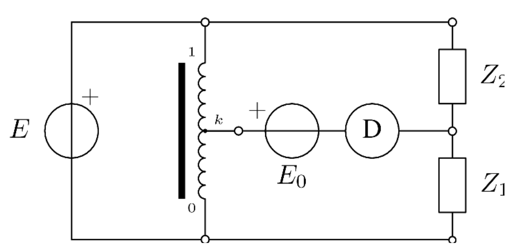
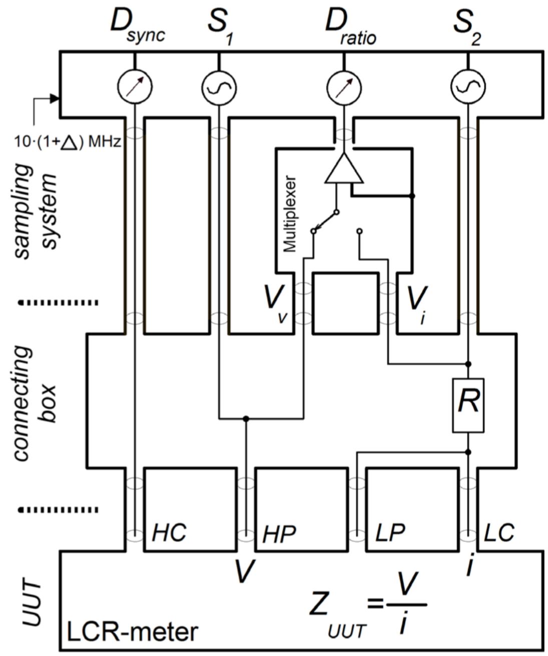


Fig. 3



D_{sync}
S_1
D_{ratio}
S_2
10·(1+△) MHz
sampling system
Multiplexer
V_v
V_i
connecting box
R
HC
HP
LP
LC
V
i
UUT
LCR-meter
$Z_{UUT} = \frac{V}{i}$

Fig. 4.