

HDR - A Hysteresis-Driven Routing Algorithm for Energy Harvesting Tag Networks*

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Abstract

The work contains a first attempt to treat the problem of routing in networks with energy harvesting units. We propose HDR - a Hysteresis Based Routing Algorithm and analyse it in a simple diamond network. We also consider a network with three forwarding nodes. The results are used to give insight into its application in general topology networks and to general harvesting patterns.

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1 Introduction

Recent advances in the design of ultra-low power transceivers and solar cells has made it possible to develop and implement networks with self sustainable energy harvesting devices (*EnHANTs*) that communicate with neighboring devices over wireless links (see e.g. [GWZ], [GMSplus] and references therein). In such networks, node energy increases via harvesting, in addition to it being spent by data transmission and reception. As a result, the algorithms in *EnHANTs* networks differ considerably from the ones in legacy sensor and ad-hoc networks. Moreover, with the devices we are considering, the available energy for control and processing is extremely low and thus the employed algorithms must be simple and the amount of transmitted control data must be minimized.

The present work is the first attempt to design routing protocols for *EnHANTs* networks. We shall analyse the performance of a routing protocol for a very simple network. Then we indicate how this simple analysis can be employed to provide insight into the design of routing algorithms in larger networks.

2 The Model

Consider a diamond network with 4 nodes as in Fig. 1. Nodes s, d are respectively the source and destination nodes. Node s generates data to be transferred via the energy-harvesting nodes 1 and 2 to the destination d . Data is included in packets of fixed size. Time is divided in slots, where all nodes can change activity only at the end of a slot. Therefore, throughout this paper, we shall interchangeably use the terms "time" and "end of slot". Each wireless transmission by the source is overheard by both forwarding nodes, but in any given slot only one of them, referred to as *the active node* and denoted by v , is forwarding the packets to the destination. Only the active node spends energy in receiving the packet and forwarding it. The other node is said to be *inactive* and is denoted by x . If node 1,2 is *active*, we also say that we use *route 1,2* respectively.

At the end of each slot, the intermediate nodes inform the destination node of their current energy levels. The active node can do this by piggy-backing the information to the last data packet in the given slot, whereas the inactive node needs to use a control message, referred to as a *status message*. Based on this information, the destination node decides what route to use and informs the forwarding nodes accordingly. The decision is sent by the destination node just after having received the status messages, in a control message referred to as a *switch-command message*. We assume that it takes negligible time, but not negligible energy, to send the status messages, to receive the switch command and to apply it by the forwarding nodes.

For the first part of this work we assume that the energy harvesting rates at the two forwarding nodes is constant over a sufficiently long time, at the rate of e_1, e_2 $mJ/slot$ respectively. We also assume for most of this work that the source generates and transmits data at a fixed rate of g $packets/slot$. The source and the destination are assumed to be energy unlimited. If we momentarily disregard the energy spent by control messages, the maximal data rate that can be transmitted from source to destination in this network is

$$g_{max} = (e_1 + e_2)/c ; packets/slot$$

where c is the total combined energy required to receive and send a data packet. This is consistent with the results in many works that treat conditions for maximum flow, for example [MSZ], as applied to this simple network. In order to achieve the maximal flow, the traffic must be split between the two routes at average rates e_1/c and e_2/c respectively. Obviously, measurement of the harvesting rates and splitting the traffic accordingly are not easy tasks. The purpose of this work is to design a simple routing protocol that hopefully achieves this split without the need to measure the harvesting rates. We may mention in addition that if this goal is indeed achieved, the split will automatically adapt to (slow) changes in the harvesting rates.

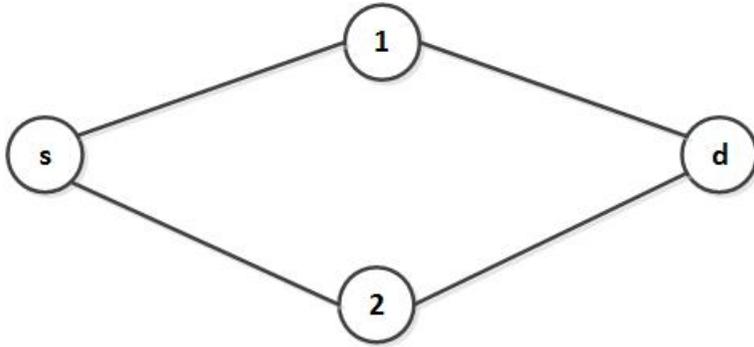


Figure 1: The Model

The proposed algorithm, referred to as the *Hysteresis-Driven Routing(HDR) Algorithm* is as follows. The destination assigns as *active* the node with the *higher energy level*, except that, in order to avoid fast oscillations and high control overhead, it switches routes only when the energy level at the inactive node exceeds the level at the active node by a certain *threshold*. The threshold for switching in one direction may be different from the one in the opposite direction. Since activity can change only at the end of a slot, if the threshold is reached during a slot, the *actual switch time is at the end of that slot*.

2.1 Notations and assumptions

For simplicity, we shall take the transmission energy for each status message to be the same, whether the information is piggybacked on a data message or sent separately. We need to distinguish between the time *just before* the transfer of the control messages and the time *just after*. We shall refer to the former as *the time just before the end of the slot* and to the latter as *the time at the end of the slot*.

We shall use the following notations. The period between two consecutive switches in the same direction is referred to as a *cycle*.

e_u - energy harvested by node u in each slot (mJ/slot)

g - number of data packets generated by the source node in each slot (packets/slot)

c = combined energy to receive and transmit a data packet (mJ/packet)

c_t = energy to transmit a status control packet or status information piggybacked on a data packet (mJ)

c_r = energy to receive a switch command packet (mJ)

$B_u^-(i)$ - battery level at node u *just before* the end of slot i in units of energy (mJ)

$B_u(i)$ - battery level at node u *at* the end of slot i , in units of energy (mJ)

B_{max} - maximum battery level (mJ)

$B_{min} = c_t + c_r$ - minimum battery level that allows transmission of control messages (mJ)

h_1 - threshold to switch from route 1 to route 2 (mJ)

h_2 - threshold to switch from route 2 to route 1 (mJ)

$h = h_1 + h_2$ (mJ)

γ_u^c = *node cycle throughput* = number of data packets transferred via node u in each cycle (packets/cycle)

γ^c = *total cycle throughput* = $\gamma_1^c + \gamma_2^c$

$1_z = 1$ if z is true and 0 otherwise

3 Energy equations

In the sequel, we develop formulas for the battery level at each node just before the end of each slot and at the end of each slot. We also determine the maximal steady state flow g_s of data. Since the harvesting rates are not known and they may, hopefully slowly, change with time, we shall also examine the behaviour of the system for arbitrary input rates g , both larger and smaller than g_s . In order to avoid the treatment of too many cases however, we shall restrict the input rate to be larger than $\max(e_1, e_2)/c$. In fact, if for example, $cg < e_1 - c_t$, then the energy of node 1 always increases, even when node 1 is active; not a very interesting scenario. We shall also assume that $\min(e_1, e_2) > c_t + c_r$.

With the above assumptions, when the battery levels are away from the boundaries B_{min} and B_{max} , the battery level at the inactive node x increases from *the end* of a slot until *just before* the end of the next slot by $e_x mJ$. The battery level at the active node v decreases from *the end* of some slot until *just before the end* of the next slot by $(cg - e_v) mJ$. If the battery at the inactive node reaches B_{max} , its harvesting stops. For input rate g , when the battery level at the active node v is away from B_{min} , it transfers g packets/slot.

It remains to consider the activity of the active node when it is close to the boundary B_{min} . If the active node battery reaches B_{min} *during* a slot, say a fraction α of the slot period into the slot time, then it will transfer αg packets before it reaches B_{min} and $(1 - \alpha) e_v/c$ packets afterwards¹. Next consider the situation at the end of some slot i . Recall the order of transmissions starting just before the end of a slot time: first the status messages and then, if necessary, the switch command. If just before the end of the slot, node v is at $B_v^-(i) < B_{min}$, it may run into a problem. If we allow it to send the status message, thereby spending $c_t mJ$ for control, it will not have sufficient energy to receive the switch command, if any. Thus in order to be safe, in this case we instruct the node *to refrain from sending the status message* at the end of slot i . The destination will assume that the node is at B_{min} when it does not hear the status message and will act accordingly². Finally, consider data transmission. If after sending the status message, the energy is less than B_{min} , we do not allow transfer of data messages in the next slot, with the hope that the harvested energy will sufficiently increase the battery level to allow it to send the status message at the end of the next slot. Otherwise, the normal algorithm applies.

With the above considerations, we can write down the dynamics of the system.

¹For simplicity of notation and analysis, we disregard the fact that we only send whole packets. The simulation does take this fact into account.

²Here, as well as throughout this work, we assume no lost messages.

```

A1      switch = 0
A2      if  $B_x^-(i) - B_v^-(i) \geq h_v$ 
A3          switch = 1
A4      end
A5       $B_x(i) = B_x^-(i) - c_t$ 
A6      if  $(B_v^-(i) \geq B_{min})$ 
A7           $B_v(i) = B_v^-(i) - c_t$ 
A8      else
A9           $B_v(i) = B_v^-(i)$ 
A10     end
A11     if (switch == 1)
A12          $B_v(i) = B_v(i) - c_r ; B_x(i) = B_x(i) - c_r$ 
A13         v and x switch
A14          $B_x^-(i+1) = B_x(i) + e_x$ 
A15          $B_v^-(i+1) = B_v(i) + e_v - c g$ 
A16          $\gamma_v^c(i+1) = \gamma_v^c(i) + g$ 
A17     else
A18          $B_x^-(i+1) = \min(B_x(i) + e_x, B_{max})$ 
A19         if  $(B_v(i) < B_{min})$ 
A20              $B_v^-(i+1) = B_v(i) + e_v$ 
A21              $\gamma_v^c(i+1) = \gamma_v^c(i)$ 
A22         else
A23              $B_v^-(i+1) = \max(B_{min}, B_v(i) + e_v - c g)$ 
A24              $\alpha = \max(0, \min(1, (B_v(i) - B_{min}) / (c g - e_v)))$ 
A25              $\gamma_v^c(i+1) = \gamma_v^c(i) + \alpha g + (1 - \alpha) e_v / c$ 
A26         end
A27     end

```

It is easy to see that the energy just after the transmission of the status control message, if any, is no smaller than c_r , so there is always energy to receive the switch message (see lines A12 and A3 in the Code above). To avoid confusion, we point out that the code above is not performed by any node. It merely describes the dynamics of the system.

Assume that we have just switched at a slot, referred to as slot 0, from route 2 to route 1. This happens because the threshold h_2 is reached during slot 0 or just before its end and was not reached during the previous slot, namely

$$h_2 + e_1 - e_2 + c g > B_1^-(0) - B_2^-(0) \geq h_2 \quad (1)$$

The condition that we switch again I_1 slots afterwards is

$$h_1 + e_2 - e_1 + c g > B_2^-(I_1) - B_1^-(I_1) \geq h_1 \quad (2)$$

The differences in battery levels are calculated as sums of the expressions in lines A7, A9, A12, A23 in the Code above. We have a similar condition for the next switch from route 2 to route 1.

4 Operation away from the boundaries

Denote by

I_1^a = number of slots when node 1 is active in a given cycle a , when the system is away from the boundaries

I_2^a = number of slots when node 2 is active in a given cycle a , when the system is away from the boundaries

$$I^a = I_1^a + I_2^a$$

$\gamma = \gamma^c(a)/I^a$ = average throughput per slot in cycle a (packets/slot)

When the system operates away from the boundaries, all *max* and *min* operands in the Code above do not apply. Thus we have

$$h_1 + e_2 - e_1 + c g > B_2^-(I_1^a) - B_1^-(I_1^a) = I_1^a e_2 + I_1^a (c g - e_1) + B_2^-(0) - B_1^-(0) \geq h_1 \quad (3)$$

or

$$I_1^a = \lceil \frac{(h_1 + B_1^-(0) - B_2^-(0))}{(c g - e_1 + e_2)} \rceil \quad (4)$$

where $\lceil Z \rceil$ denotes *the smallest integer larger than or equal to Z*. Similarly

$$I_2^a = \lceil \frac{(h_2 + B_2^-(I_1^a) - B_1^-(I_1^a))}{(c g - e_2 + e_1)} \rceil \quad (5)$$

We shall be interested in the drift in total energy of the system $\Delta_{total} = B_1(I^a) - B_1(0) + B_2(I^a) - B_2(0)$ during a cycle $I^a = I_1^a + I_2^a$. The energy harvested during a cycle is $(e_1 + e_2) I^a$. Since g packets are sent in each slot, the energy spent is $c g I^a$. At the end of each slot, each node spends c_t for control. There are 2 switch times in a cycle and at every switch time each node spends c_r . The total drift in energy of both nodes per cycle is given by the difference between the harvested energy and the spent one:

$$\Delta_{total} = -4 c_r + (e_1 + e_2 - 2 c_t - c g) I^a \text{ mJ/cycle} \quad (6)$$

This is how far we can get analytically with arbitrary parameters. A useful approximation is to select thresholds as multiples of the net slot energy change. This implies that the thresholds are reached exactly at slot time, a fact that considerably simplifies the analysis .

5 Thresholds divide slot energies - Operation away from the boundaries

Suppose a switch from route 2 to route 1 occurs at the end of slot 0 and at that time, the energy level difference exactly matches the threshold value. This means $B_2^-(0) - B_1^-(0) = h_2$. Then, with $h = h_1 + h_2$, we have

$$I_1^a = \lceil \frac{h}{c g + (e_2 - e_1)} \rceil \quad (7)$$

and a similar expression for I_2^a . If h divides evenly the slot energies $cg + (e_2 - e_1)$ and $cg + (e_1 - e_2)$, then

$$I_1^a = \frac{h}{cg + (e_2 - e_1)} ; I_2^a = \frac{h}{cg + (e_1 - e_2)} \quad (8)$$

$$I^a = \frac{2cg h}{(cg)^2 - (e_2 - e_1)^2} \quad (9)$$

Moreover, the battery level difference returns to its initial value at the end of a cycle, namely $B_2^-(I^a) - B_1^-(I^a) = B_2^-(0) - B_1^-(0) = h_2$.

In each slot g packets are transferred, thus the average throughput is

$$\gamma = g \text{ packets/slot} \quad (10)$$

independently of the thresholds. The split ratio between the two routes is

$$\frac{\gamma_1^c}{\gamma_2^c} = \frac{g I_1^a}{g I_2^a} = \frac{cg + e_1 - e_2}{cg + e_2 - e_1} \quad (11)$$

In the sequel, we shall also need the following quantities. Let $\delta B_u(I)$ denote the change in the battery level at node u during an interval of I slots. Then

$$\delta B_1(I_1^a) = -\frac{h(cg - e_1)}{cg + (e_2 - e_1)} ; \delta B_2(I_1^a) = \frac{h e_2}{cg + (e_2 - e_1)} \quad (12)$$

and

$$\delta B_1(I_2^a) = \frac{h e_1}{cg + (e_1 - e_2)} ; \delta B_2(I_2^a) = -\frac{h(cg - e_2)}{cg + (e_1 - e_2)} \quad (13)$$

The drift $\Delta = B_1(I^a) - B_1(0) = B_2(I^a) - B_2(0)$ is half the total drift (6), namely

$$\Delta = -2c_r + \frac{(e_1 + e_2 - 2c_t - cg) I^a}{2} \text{ mJ/cycle} \quad (14)$$

6 Negligible Control - Thresholds divide slot energies

If the energy spent for sending and receiving control messages is negligible, we have $c_t = c_r = B_{min} = 0$.

6.1 Operation away from the boundaries

For negligible control, the drift is $\Delta = (e_1 + e_2 - cg) I^a / 2$. Let g_s denote the input that induces steady state, namely $\Delta = 0$. We get $g_s = (e_1 + e_2) / c$. If $g = g_s$, then the condition for the switches to occur at exactly battery level differences h_1 and h_2 becomes that $h / (2e_1)$

and $h/(2e_2)$ are integers. For example for $e_1 = 0.8, e_2 = 0.6$, any two threshold values h_1, h_2 that add up to 4.8 or its multiples will do the job.

We conclude that if $g = g_s$, then the *HDR Algorithm* indeed transfers in dynamic steady state the entire input rate g_s . For other values of the input rate g , the behavior is transient: the battery levels will drift up if $g < g_s$ until they reach steady state close to B_{max} and down if $g > g_s$ until they reach steady state close to B_{min} . The steady state behavior in these cases is treated in the following sections.

6.2 Drifts and Operation close to boundaries

As before, the analysis here is for negligible control and thresholds evenly dividing the slot energies. Since in practice, the harvesting rates vary, it is important to investigate the behavior of the system when $e_1 + e_2$ is not necessarily equal to cg .

If $e_1 + e_2 > cg$, then the battery levels drift up, until at least one of them reaches B_{max} . Similarly, if $e_1 + e_2 < cg$, the drift is down, until at least one of the batteries reaches the lower bound $B_{min} = 0$. Before reaching the boundary, the switch times are as before Eq. (8). The drift per slot is Δ/I^a , with Δ and I^a given in Eq. (14) and (9).

When a node is *inactive*, its battery charges. Recall that we have assumed $cg > e_1, cg > e_2$, so the battery level at the *active* node goes down. Thus, only the inactive node can reach the boundary B_{max} and only the active node can reach the lower boundary $B_{min} = 0$. We assume that B_{max} is much larger than the thresholds, so that if one of the nodes reaches some boundary, the other does not reach the opposite boundary in the same cycle.

In addition to the assumption of switching with differences in battery levels equal to the thresholds, in the analysis below we shall also assume that boundaries are reached by nodes exactly at slot times. As before, we point out that the simulation is performed without these assumptions.

Take 0 to indicate the time of a switch when node v becomes *active*. Recall that we denote by x the other node, namely the one that will become active *next*. The condition for the next switch (from route v to route x) to occur after slot I_v is given by Eq. (2). Note that the switch time I_v when boundaries are reached may not be the same as I_v^a , the switch time when no boundary is met. If neither node reaches a boundary (0 or B_{max}), then the switch time is as before $I_v = I_v^a$.

Let

I'_v = slot at the end of which node v reaches 0 before a switch, where $0 < I'_v \leq I_v^a$

I''_v = slot at the end of which node x reaches B_{max} before a switch, where $0 < I''_v \leq I_v^a$

If the corresponding boundary is not reached before a switch, then I'_v, I''_v respectively is defined to be equal to I_v . Note that if either node reaches some boundary before a switch, it

stays at that boundary until the next switch, and the other node cannot reach the opposite boundary. Also note that until a boundary is reached, both the *min* and the *max* operands in the Code above are inactive. The switch condition (2) becomes:

$$e_x \cdot I_v'' + (c g - e_v) I_v' = h \quad (15)$$

The cycle dynamics is given below.

```

Node v is actiVe, node x is neXt                                /* x = v mod 2 + 1
B1      I_v^a = h / (c g + e_x - e_v)
B2      case (B_max - B_x(0)) / e_x < I_v^a {                    /*node x reaches boundary B_max
B3          I_v'' = (B_max - B_x(0)) / e_x
B4          I_v = (h - (B_max - B_x(0))) / (c g - e_v)
B5          B_v(I_v) = B_v(0) - I_v (c g - e_v) ; B_x(I_v) = B_max
B6          \gamma_v^c = I_v g
      }
B7      case B_v(0) / (c g - e_v) < I_v^a {                      /*node v reaches boundary 0
B8          I_v' = B_v(0) / (c g - e_v)
B9          I_v = (h - (B_v(0))) / e_x
B10         B_v(I_v) = 0 ; B_x(I_v) = B_x(0) + I_v e_x
B11         \gamma_v^c = I_v' g + (I_v - I_v') (e_v) / c
      }
B12     else {                                                  neither node reaches boundary
B13         I_v = I_v^a
B14         B_v(I_v) = B_v(0) - I_v (c g - e_v) ; B_x(I_v) = B_v(0) + I_v c g
B15         \gamma_v^c = I_v g
      }

```

6.3 System Throughput and Operation close to the boundaries

In this Section we examine the operation of the system in terms of throughput and split ratio, at low and high battery level.

We start with low level. If $e_1 + e_2 < c g$, the battery levels will drift down while seesawing. An example appears in Fig. 2.

As we shall see in the sequel, after the time when one of the nodes reaches the boundary 0, the system operates in a periodic dynamic steady state. There are two cases: i) only one node is ever empty for a non-zero period of time ; ii) both nodes alternately reach 0. The condition for these to happen is given below.

Lemma 1 *If $h_1/h_2 \geq (c g - e_2)/e_1$, then B_2 is never empty for a non-zero period of time and the throughput during a cycle, the cycle length, and the average throughput are as in Eq. (18) below.*

Proof: The condition above is equivalent to $h_1 e_1 + h_2 e_2 - h_2 c g \geq 0$. The switch from route 1 to route 2 occurs after slot I when $B_2(I) = B_1(I) + h_1 \geq h_1$. Afterwards, B_2 goes down at a rate $(c g - e_2) \mu J / \text{slot}$. Node B_2 will reach 0 only if during the next I_2^g slots

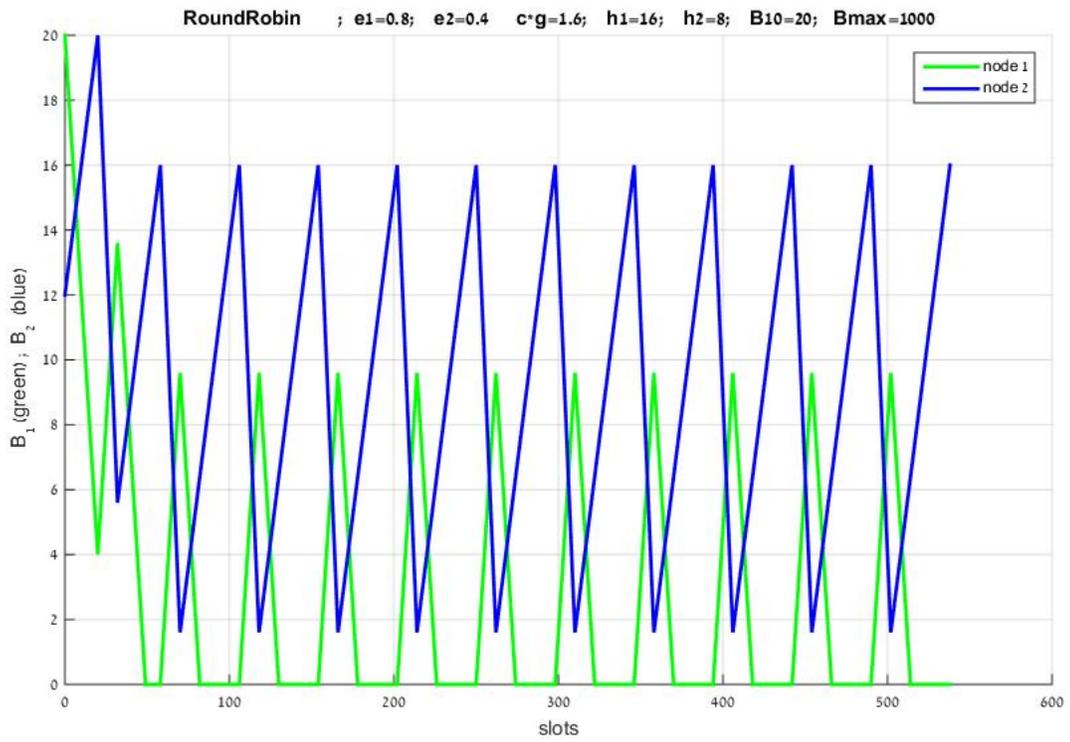


Figure 2: Only node 1 is at boundary 0 for a non-zero period of time

(namely before the next switch) it will do so. Thus if $h_1 - I_2^a \cdot (cg - e_2) \geq 0$, the battery B_2 will never be at B_{min} for a non-zero period. Substituting for I_2^a from Eq. (9), we obtain the condition above.

Since the system drifts down, at least one battery must hit 0 and if the condition in the Lemma holds, only B_1 can do so. After the first time it hits 0, it waits for B_2 to reach h_1 , at which time a switch from route 1 to route 2 occurs. Rename the slot at the end of which this occurs as $I = 0$. From this slot on, the system is in dynamic steady state.

The next switch, from 2 to 1, occurs at time I_2^a , and the battery levels are

$$B_1(I_2^a) = I_2^a \cdot e_1 = \frac{e_1 h}{cg + (e_1 - e_2)} \quad (16)$$

and

$$B_2(I_2^a) = B_2(0) + \delta B_2(I_2^a) = h_1 - \frac{(h_1 + h_2)(cg - e_2)}{cg + (e_1 - e_2)} = \frac{h_1 e_1 - h_2 (cg - e_2)}{cg + (e_1 - e_2)} \quad (17)$$

If we again rename the slot after which the switch from route 2 to route 1 occurs as $I = 0$, the battery B_1 drains out at time

$$I_1' = \frac{e_1 h}{(cg + (e_1 - e_2))(cg - e_1)}$$

and then node 1 waits for B_2 to charge to level h_1 at time

$$\left(h - \frac{e_1 h}{cg + (e_1 - e_2)}\right)/e_2 = \frac{h(cg - e_2)}{e_2 \cdot (cg + (e_1 - e_2))},$$

where the switch from 1 to 2 occurs at battery levels (16), (17). The scenario repeats from now on.

While active and in dynamic steady state, node 2 transfers a total of $\gamma_2^c = (gh)/(e_1 - e_2 + cg)$ packets/cycle, and node 1 transfers $\gamma_1^c = (e_1 gh)/(e_2(e_1 - e_2 + cg))$ packets/cycle. The split ratio is

$$\gamma_1/\gamma_2 = e_1/e_2$$

The total throughput in a cycle γ^c , the number of slots in a cycle I and the average throughput $\gamma = \gamma^c/I$ are:

$$\gamma^c = \frac{g(e_1 + e_2)h}{e_2(e_1 - e_2 + cg)}; I = \frac{cg h}{e_2 \cdot (cg + e_1 - e_2)}; \gamma = \frac{e_1 + e_2}{c} \text{ packets/slot} \quad (18)$$

Note that, since $cg > e_1 + e_2$, the average throughput above is the maximal possible. \otimes

Lemma 2 *If $h_1/h_2 \leq e_2/(cg - e_1)$, then B_1 never empties out and the throughput during a cycle, the cycle length and the average throughput are as in Eq. (18) with indexes 1 and 2 interchanged.*

Lemma 3 *If $e_2/(cg - e_1) < h_1/h_2 < (cg - e_2)/e_1$, then both batteries alternately reach level 0 and stay there for non-zero numbers of slots, and the throughput during a cycle, the cycle length and the average throughput are as in Eq. (20).*

Proof: An example appears in Fig. 3. Since the system drifts down, at least one battery must eventually drain out. Suppose node 1 is the **first** that hits battery level 0 at a time *that is not its lower tip*. Then it waits for B_2 to reach h_1 , at which time a switch from route 1 to route 2 occurs. Let $I = 0$ indicate the slot after which this happens. From this time on, the system is in dynamic steady state.

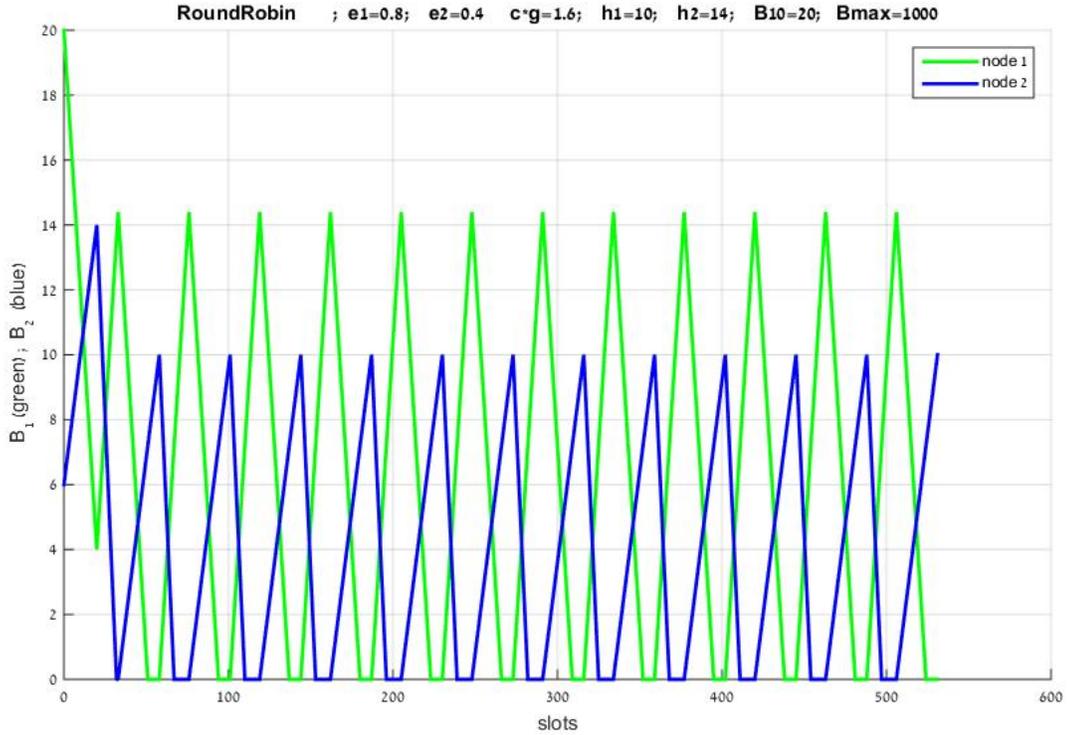


Figure 3: Both nodes empty out alternately

At time $I'_2 = h_1/(cg - e_2) < h_2/e_1$, the battery B_2 reaches 0. It waits for B_1 to reach h_2 at time $I_2 = h_2/e_1$, at which time the switch from node 2 to node 1 occurs (see Fig. 3). The battery levels at this time are

$$B_1(I_2) = h_2 ; B_2(I_2) = 0 \tag{19}$$

Counting slots after that switch, B_1 drains out at time $I'_1 = h_2/(cg - e_1) < h_1/e_2$, waits for B_2 to charge to level h_1 at time h_1/e_2 , where the switch from 1 to 2 occurs at battery levels corresponding to (19) with exchanged indices.

While active, node 2 transfers a total of $\gamma_2^c = (e_1 h_1 + e_2 h_2)/(c e_1)$ packets, and node 1 transfers $\gamma_1^c = (e_1 h_1 + e_2 h_2)/(c e_2)$. The split ratio is $\gamma_1^c/\gamma_2^c = e_1/e_2$. The total throughput in a cycle γ^c , the total number of slots in a cycle I and the average throughput $\gamma = \gamma^c/I$ are

$$\gamma^c = \frac{(e_1 + e_2)(e_1 h_1 + e_2 h_2)}{c e_1 e_2} ; I = \frac{h_1 e_1 + h_2 e_2}{e_1 e_2} ; \gamma = \frac{e_1 + e_2}{c} \text{ packets/slot} \quad (20)$$

⊗

Next we consider the case when the input rate is less than the total harvesting rates, namely $e_1 + e_2 > c g$. In this case the battery levels will drift up while seesawing. The operation here is similar to the one at low-level batteries. An example when only node 2 reaches B_{max} appears in Fig. 4. The properties for all cases are stated in the summary section below without proofs. The proofs are similar to the ones above.

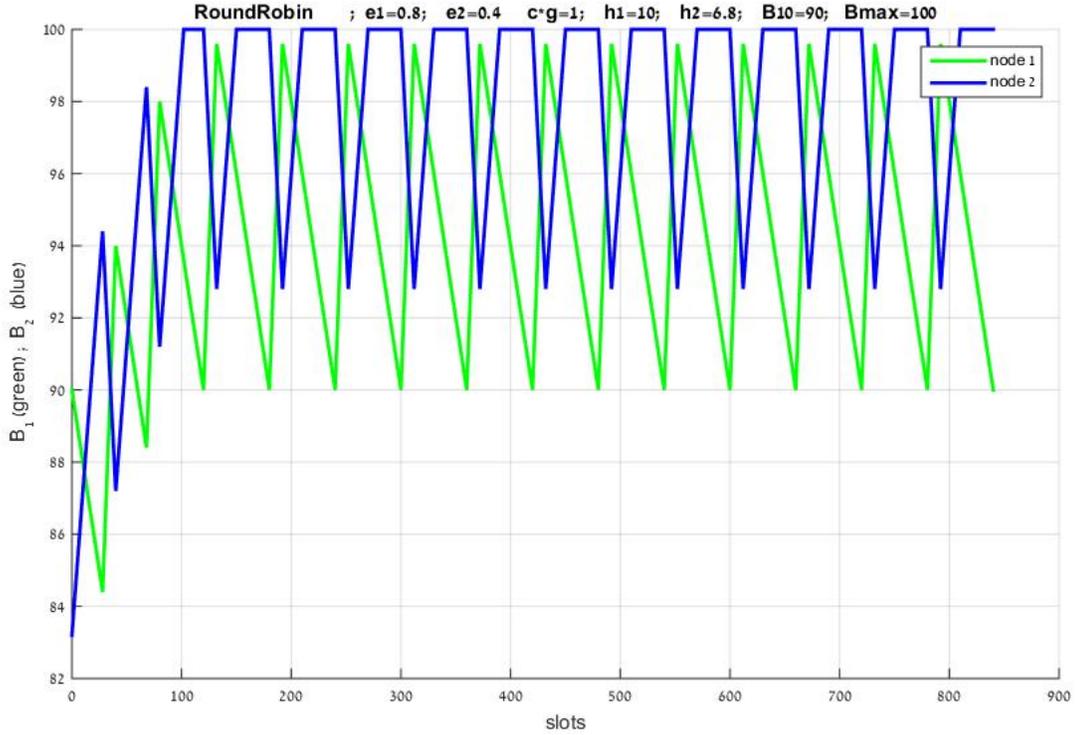


Figure 4: Node 2 reaches B_{max}

6.4 Summary of Analytic Results

Theorem 1

Under the assumptions that:

1. the energies c_t, c_r spent for control are negligible
2. all switches occur at times when the difference in battery levels is exactly equal to the thresholds
3. boundaries are reached at slot time

the system behaves as follows

1. While away from the boundaries

$$I_1 = I_1^a = \frac{h}{c g + e_2 - e_1} \text{ slots} ; I_2 = I_2^a = \frac{h}{c g + e_1 - e_2} \text{ slots}$$

$$\gamma_1^c = \frac{g h}{c g + e_2 - e_1} \text{ packets} ; \gamma_2^c = \frac{g h}{c g + e_1 - e_2} \text{ packets}$$

$$\frac{\gamma_1^c}{\gamma_2^c} = \frac{c g + e_1 - e_2}{c g + e_2 - e_1} ; \gamma^c = \frac{2 c g^2 h}{(c g)^2 - (e_1 - e_2)^2} \text{ packets} ;$$

$$I = \frac{2 c g h}{(c g)^2 - (e_1 - e_2)^2} \text{ slots} ; \gamma = g \text{ packets/slot} ; \text{Drift per cycle} = \frac{e_1 + e_2 - c g}{2} \mu J$$

2. If $e_1 + e_2 = c g$ and the battery levels are initially far from the boundaries, then they are in dynamic steady state with the following:

$$I_1 = I_1^a = \frac{h}{2 e_2} \text{ slots} ; I_2 = I_2^a = \frac{h}{2 e_1} \text{ slots}$$

$$\gamma_1^c = \frac{g h}{2 e_2} \text{ packets} ; \gamma_2^c = \frac{g h}{2 e_1} \text{ packets}$$

$$\frac{\gamma_1^c}{\gamma_2^c} = \frac{e_1}{e_2} ; \gamma^c = \frac{c g^2 h}{2 e_1 e_2} \text{ packets} ;$$

$$I = \frac{c g h}{2 e_1 e_2} \text{ slots} ; \gamma = g \text{ packets/slot} ; \text{Drift per cycle} = 0$$

3. If $e_1 + e_2 < c g$, then the battery levels will drift down while seesawing and

- (a) If $h_1/h_2 \leq e_2/(c g - e_1)$, then B_1 is never at level zero level for a non-zero amount of time and in steady state:

$$I_1 = \frac{h}{c g + (e_2 - e_1)} \text{ slots} ; I_2 = \frac{h(c g - e_1)}{e_1 \cdot (c g + (e_2 - e_1))} \text{ slots} ;$$

$$\gamma_1^c = \frac{g h}{e_2 - e_1 + c g} \text{ packets} ; \gamma_2^c = \frac{e_2 g h}{e_1 (e_2 - e_1 + c g)} \text{ packets} ;$$

$$\frac{\gamma_1^c}{\gamma_2^c} = \frac{e_1}{e_2} ; \gamma^c = \frac{g (e_2 + e_1) h}{e_1 (e_2 - e_1 + c g)} ; I = \frac{c g h}{e_1 \cdot (c g + e_2 - e_1)} ; \gamma = \frac{e_1 + e_2}{c} \text{ packets/slot}$$

(b) If $h_1/h_2 \geq (cg - e_2)/e_1$, then B_2 is never at level zero for a non-zero amount of time and in steady state (see Eq. (18)):

$$I_1 = \frac{h(cg - e_2)}{e_2 \cdot (cg + (e_1 - e_2))} ; I_2 = \frac{h}{cg + (e_1 - e_2)} ; \gamma_1^c = \frac{e_1 gh}{e_2(e_1 - e_2 + cg)} ; \gamma_2^c = \frac{gh}{e_1 - e_2 + cg}$$

$$\frac{\gamma_1^c}{\gamma_2^c} = \frac{e_1}{e_2} ; \gamma^c = \frac{g(e_1 + e_2)h}{e_2(e_1 - e_2 + cg)} ; I = \frac{cgh}{e_2 \cdot (cg + e_1 - e_2)} ; \gamma = \frac{e_1 + e_2}{c} \text{ packets/slot}$$

(c) If $e_2/(cg - e_1) < h_1/h_2 < (cg - e_2)/e_1$, then in steady state both batteries alternately reach level 0 for non-zero amounts of time and (see Eq. (20)):

$$I_1 = \frac{h(cg - e_2)}{e_2(cg + e_1 - e_2)} ; I_2 = \frac{h}{cg + e_1 - e_2} ; \gamma_2^c = \frac{e_1 h_1 + e_2 h_2}{c e_1} ; \gamma_1^c = \frac{e_1 h_1 + e_2 h_2}{c e_2}$$

$$\frac{\gamma_1^c}{\gamma_2^c} = \frac{e_1}{e_2} ; \gamma^c = \frac{(e_1 + e_2)(e_1 h_1 + e_2 h_2)}{c e_1 e_2} ; I = \frac{h_1 e_1 + h_2 e_2}{e_1 e_2} ; \gamma = \frac{e_1 + e_2}{c} \text{ packets/slot}$$

4. If $e_1 + e_2 > cg$, then the battery levels will drift up while seesawing³

(a) If $h_1/h_2 \geq e_1/(cg - e_2)$, then B_1 is never at B_{max} for a non-zero amount of time and in steady state:

$$I_1 = \frac{e_1 h}{(cg - e_1)(cg + e_1 - e_2)} ; I_2 = \frac{h}{cg + e_1 - e_2} ; \gamma_1^c = \frac{e_1 h g}{(cg - e_1)(cg + e_1 - e_2)} ;$$

$$\gamma_2^c = \frac{h g}{cg + e_1 - e_2} ; \gamma^c = \frac{c g^2 h}{(cg - e_1)(cg + e_1 - e_2)} ; \frac{\gamma_1^c}{\gamma_2^c} = \frac{e_1}{cg - e_1}$$

$$I = \frac{cgh}{(cg - e_1)(cg + e_1 - e_2)} ; \gamma = g$$

(b) If $h_1/h_2 \leq (cg - e_1)/e_2$, then B_2 is never at B_{max} for a non-zero amount of time and in steady state:

$$I_1 = \frac{h}{cg + e_2 - e_1} ; I_2 = \frac{e_2 h}{(cg - e_2)(cg + e_2 - e_1)} ; \gamma_1^c = \frac{h g}{cg + e_2 - e_1}$$

$$\gamma_2^c = \frac{e_2 h g}{(cg - e_2)(cg + e_2 - e_1)} ; \gamma^c = \frac{c g^2 h}{(cg - e_2)(cg + e_2 - e_1)} ; \frac{\gamma_1^c}{\gamma_2^c} = \frac{cg - e_2}{e_2}$$

$$I = \frac{cgh}{(cg - e_2)(cg + e_2 - e_1)} ; \gamma = g$$

³In a previous version of this report, the conditions below were wrongly stated. Now they are correct - thanks to Alex Lavzin for perceiving the error.

(c) If $(cg - e_1)/e_2 < h_1/h_2 < e_1/(cg - e_2)$, then both B_1 and B_2 alternately reach B_{max} and stay there for non-zero amounts of time and in steady state:

$$I_1 = \frac{h_1}{cg - e_1} ; I_2 = \frac{h_2}{cg - e_2} ; \gamma_1^c = \frac{gh_1}{cg - e_1} ; \gamma_2^c = \frac{gh_2}{cg - e_2}$$

$$\gamma^c = \frac{g(cgh - e_1h_2 - e_2h_1)}{(cg - e_2)(cg - e_1)} ; \gamma_1^c = \frac{h_1(cg - e_2)}{h_2(cg - e_1)} ; I = \frac{cgh - e_1h_2 - e_2h_1}{(cg - e_2)(cg - e_1)} ; \gamma = g$$

In words, we can summarize the dynamic steady state activity as follows:

1. If $e_1 + e_2 = cg$, steady state occurs at all battery levels away from the boundaries. The throughput is g , with split ratio e_1/e_2 , independent of the thresholds. The thresholds affect only the frequency of switching.
2. If $e_1 + e_2 < cg$, steady state occurs close to empty battery levels. The throughput is $(e_1 + e_2)/c$, with split ratio e_1/e_2 , independent of the thresholds. The thresholds affect only the frequency of switching.
3. If $e_1 + e_2 > cg$, steady state occurs close to full battery levels. The throughput is g with split ratio $\neq e_1/e_2$. The thresholds affect the split ratio, as well as the frequency of switching.

7 Case when differences in battery levels at switch time do not exactly match the thresholds

The switch from route 1 to route 2 (and viceversa) occurs at the end of the slot during which h_1 (respectively h_2) is reached. If h_1 or h_2 is reached during a slot (as opposed to just before the end of the slot), the difference in battery levels at switch time is larger than the corresponding threshold.

Consider the case $e_1 + e_2 = cg$, namely steady state when away from the boundaries. If the system starts with $B_2 = B_1 + h_2$ and if $h = h_1 + h_2$ is a multiple of both $2e_1$ and $2e_2$, then all switches will occur with battery level differences equal to the thresholds. Moreover, the battery levels at the end of each cycle are the same as the ones at the beginning of the cycle. Namely the dynamic steady state has a *period of one cycle*.

If h is not a multiple of $2e_1$ and $2e_2$, then thresholds will be reached not necessarily at the end of slots. As a result, the difference in battery levels at switch time will be larger than the threshold values. On the other hand, since $e_1 + e_2 = cg$, the battery levels do not drift and thus the system must be in dynamic steady state. The behavior is now that the battery levels do not return to their initial values after each cycle, but rather after more than one cycle. As a topic for future research, it will be interesting to find conditions for periods

containing 1,2,3,.. cycles. We present in Table 1 a few numerical examples for $e_1 = 0.6$, $e_2 = 0.8$, $c = 0.08$, $g = 17.5$. The third column is the number of cycles in each period. The first row in each entry are the switch times, namely the slots at the end of which the switch occurs. The second row represents the battery level differences at switch time.

h_1	h_2	period	switch at slot ; $B(1) - B(2)$						
4	0.8	1	0	3	7				
			0.8	-4	0.8				
2	1	1	0	3	7				
			1.4	-3.4	1.4				
6.2	5	2	0	7	17	25	35		
			5	-6.2	5.8	-7	5		
5	5	3	0	7	17	24	33	40	49
			5	-6.2	5.8	-5.4	5.4	-5.8	5

Table 1: Switching Patterns

8 Non-negligible Power Consumption for Control Messages

Here we consider the situation with non-zero c_t and c_r and look only at the operation away from the boundaries. If h evenly divides the slot energies, the drift Δ and the cycle length are as in 14, (9)

Substituting I^a into Δ , we get

$$\Delta = -2c_r + \frac{cgh(\tilde{e} - cg)}{(cg)^2 - (e_2 - e_1)^2} \text{ mJ/cycle} \quad (21)$$

where $\tilde{e} = e_1 + e_2 - 2c_t$. The condition for dynamic steady state is $\Delta = 0$. We obtain a quadratic equation for g , whose solution, denoted by g_s , is given by

$$g_s = \frac{h\tilde{e} + \sqrt{h^2\tilde{e}^2 + 8c_r(2c_r + h)(e_2 - e_1)^2}}{2c(2c_r + h)} \quad (22)$$

The throughput g_s as a function of the thresholds is shown in Fig. 5. As expected, the higher the thresholds the less energy is spent on control, since the inter-switch period goes up, and thus more energy is left for data messages. However, high thresholds can lead to the batteries reaching the boundary. With the values of e_1 , e_2 , c as in the graph, the value of g_s for negligible control is $(e_1 + e_2)/c = 20$.

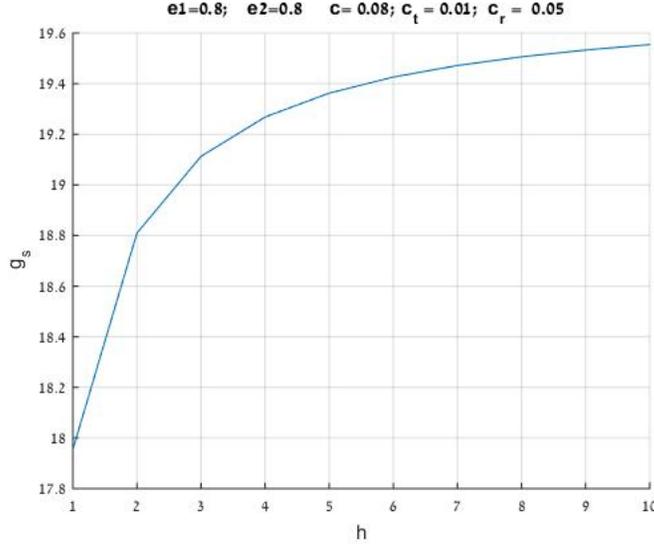


Figure 5: g_a as a function of h

At this point, recall that the analysis is correct if I_1^a and I_2^a are integers. It is difficult to get explicit conditions for this to hold for arbitrary control values c_t and c_r and we have to resort to numerical simulations. Consider for example the situation in Fig. 6, where $e_1 = 0.8$; $e_2 = 0.6$; $c = 0.08$; $h_1 = 6.2$; $h_2 = 5$; $c_r = 0.05$; $c_t = 0.01$. We get $g_s = 17.10$. The plot shows a drift of about $0.5 mJ$ in 1000 slots. This is due to the fact that an average cycle length turns out to be 18.72 slots, as opposed to the theoretical 16.73 slots given by Eq. (9). As an exercise, we have performed the same simulation, but at each step we estimate the current average cycle length and adjust the input rate g so that Δ given by Eq. (14) is zero. Not surprisingly, with this feedback, there is no drift (see Fig.7).

9 Summary so far

The analysis above is performed for a simple system with several helpful assumptions. We have extended it in several directions, like non-negligible power spent on control messages, but we can look at its implications to more general practical systems. In practice, the harvesting rates are time varying, according to various parameters, like time of day and illumination conditions. Our discussion here assumes that those variations are relatively slow. If the pattern is known, the source can try to adapt to the current parameters. For example, it should increase the input rate g when the harvesting rates increase. Also, in this simple network, the wireless transmissions of the intermediate nodes are overheard at the source. The latter can use this information in order to adapt the input rate.

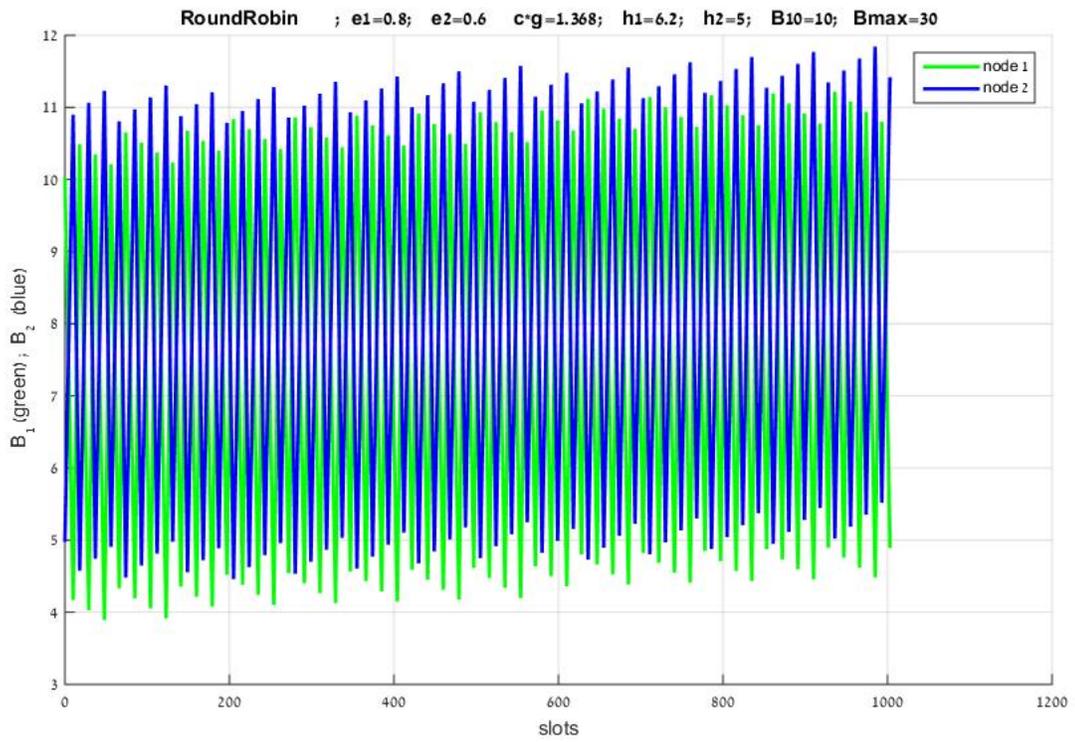


Figure 6: Behavior of system with $g = g_s$

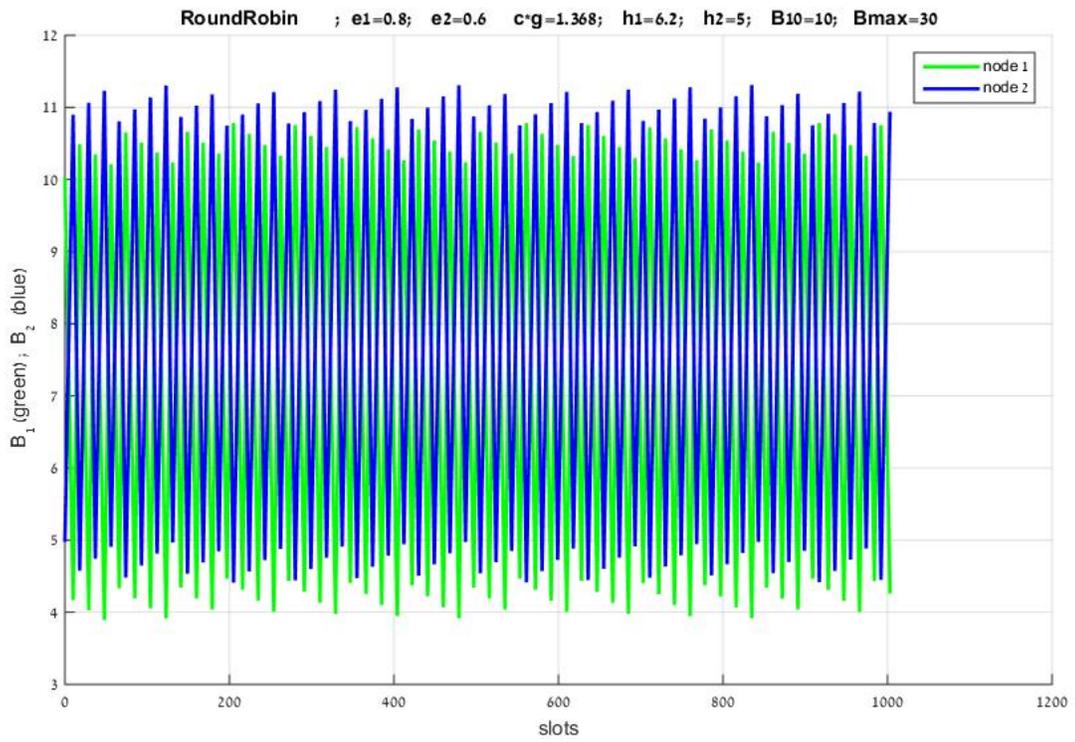


Figure 7: Behavior of system with g obtained by estimating I^a

Consider now extensions of the *HDR Algorithm* to larger networks, either with one data collection point (several sources, one destination) or with several source-destination pairs. Many routing procedures have been proposed for legacy ad-hoc and sensor networks with a variety of performance criteria, like maximum lifetime or maximum total throughput (see e.g. [AY], [ASSC], [KK], [LS] and references therein). In such networks the units spend energy and thus the battery level only goes down. Those algorithms do not seem to be applicable to networks with energy harvesting units, where energy also increases via harvesting. In networks with harvesting nodes, lifetime and total throughput have no meaning.

The analysis in this first work on the topic of routing in harvesting node networks can provide an insight for larger networks. Assuming that the topology of the network does not change often (e.g. tags on books in a library, static tags in a room or building), two or more paths can be established in advance for every source-destination pair. If all units have similar harvesting rates, it makes sense to select node-disjoint paths. If there are units with significantly larger harvesting rates than the others, they can participate in more than one path. One can think of several simple centralized or distributed algorithms to select the paths, but this topic is outside the scope of the present work. The energy levels of the nodes on the path can be periodically collected, either piggy-backed on data messages or via control messages. Using a threshold mechanism as given by the *HDR Algorithm* on the maximum battery level along the path, the destination can decide which path to use. Again, the exact procedures for collecting data and for informing nodes in the network upon the path selection are topics for future research. For instance, one can also consider situations when the collection node has a powerful transmitter that can be simultaneously heard by all nodes. In this case, it can directly communicate routes and switching decisions to all nodes.

10 Three Forwarding Nodes

Having extensively discussed the diamond configuration and having obtained detailed results for its behavior, we can now adventure into looking at larger networks. In this section we analyze the behaviour of the *HDR Algorithm* in a network with three parallel paths (see Fig. 8).

We first consider a simple switching policy, whereby the routes are switched in a round-robin fashion, with thresholds h_1, h_2, h_3 for the switches (1,2),(2,3),(3,1) respectively.

10.1 Operation away from the boundaries

Let v denote the current active node and x denote the next active node. Let I_v^a denote the number of slots when the destination employs route v while the batteries are away from

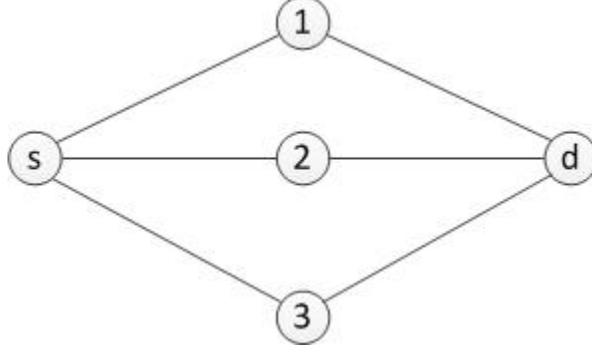


Figure 8: Three Forwarding Nodes

the boundaries. We also assume as before that switching occurs at battery level differences exactly matching the value of the thresholds.

If we denote $I^a = I_1^a + I_2^a + I_3^a$ and refer to I^a as a *cycle*, then the operation away from the boundaries is:

$$\begin{aligned}
B_1^-(0) &= B_{-3}(0) + h_3 ; B_u(0) = B_u^-(0) - c_t - c_r \\
B_2^-(I_1^a) &= B_1^-(I_1^a) + h_1 ; B_u(I_1^a) = B_u^-(I_1^a) - c_t - c_r \\
B_3^-(I_1^a + I_2^a) &= B_2^-(I_1^a + I_2^a) + h_2 ; B_u(I^a) = B_u^-(I^a) - c_t - c_r \\
B_1(I_1^a) &= B_1(0) - (c g + c_t - e_1) I_1^a - c_r \\
B_2(I_1^a) &= B_2(0) + (e_2 - c_t) I_1^a - c_r \\
B_3(I_1^a) &= B_3(0) + (e_3 - c_t) I_1^a - c_r \\
B_1(I_1^a + I_2^a) &= B_1(I_1^a) + (e_1 - c_t) I_2^a - c_r \\
B_2(I_1^a + I_2^a) &= B_2(I_1^a) - (c g + c_t - e_2) I_2^a - c_r \\
B_3(I_1^a + I_2^a) &= B_3(I_1^a) + (e_3 - c_t) I_2^a - c_r \\
B_1(I^a) &= B_1(I_1^a + I_2^a) + (e_1 - c_t) I_3^a - c_r \\
B_2(I^a) &= B_2(I_1^a + I_2^a) + (e_2 - c_t) I_3^a - c_r \\
B_3(I^a) &= B_3(I_1^a + I_2^a) - (c g + c_t - e_3) I_3^a - c_r
\end{aligned} \tag{23}$$

We also require that we are in dynamic steady state, namely that the change in battery level during a cycle is the same for all three batteries (denoted by Δ):

$$\Delta = B_1(I^a) - B_1(0) = B_2(I^a) - B_2(0) = B_3(I^a) - B_3(0)$$

Substituting above, and selecting the initial condition $B_1(0) = B_{10}$, the system of linear equations has a unique solution. We give here explicitly only the expressions for I_1^a, I_2^a, I_3^a

and for the drift.

$$\begin{aligned}
I_1^a &= \frac{h(cg + 2e_1 - e_2 - e_3)}{2(c^2g^2 - e_1^2 - e_2^2 - e_3^2 + e_1e_2 + e_2e_3 + e_3e_1)} \\
I_2^a &= \frac{h(cg + 2e_2 - e_1 - e_3)}{2(c^2g^2 - e_1^2 - e_2^2 - e_3^2 + e_1e_2 + e_2e_3 + e_3e_1)} \\
I_3^a &= \frac{h(cg + 2e_3 - e_1 - e_2)}{2(c^2g^2 - e_1^2 - e_2^2 - e_3^2 + e_1e_2 + e_2e_3 + e_3e_1)}
\end{aligned} \tag{24}$$

The throughput is g packets/slot and the split ratio is

$$\gamma_1 : \gamma_2 : \gamma_3 = (cg + 2e_1 - e_2 - e_3) : (cg + 2e_2 - e_1 - e_3) : (cg + 2e_3 - e_1 - e_2)$$

The cycle length and the drift are

$$\begin{aligned}
I^a &= \frac{3cg(h_1 + h_2 + h_3)}{2(c^2g^2 - e_1^2 - e_2^2 - e_3^2 + e_1e_2 + e_2e_3 + e_3e_1)} \\
\Delta &= -3c_r - \frac{cgh(cg - \tilde{e})}{2(c^2g^2 - E)}
\end{aligned} \tag{25}$$

where $\tilde{e} = e_1 + e_2 + e_3 - 3c_t$ and $E = e_1^2 + e_2^2 + e_3^2 - e_1e_2 - e_2e_3 - e_3e_1$.

The system is in steady state if $\Delta = 0$, which occurs at input rate $g = g_s$ given by the solution of the corresponding quadratic equation

$$g_s = \frac{h\tilde{e} + \sqrt{h^2\tilde{e}^2 + 24c_r(6c_r + h)E}}{2c(6c_r + h)} \tag{26}$$

For $c_r = c_t = 0$, the drift is zero if $e_1 + e_2 + e_3 = cg$, and then $\gamma_1 : \gamma_2 : \gamma_3 = e_1 : e_2 : e_3$, namely there is no drift and the throughput through each node is proportional to its harvesting rate.

10.2 Operation close to the boundaries - negligible control

Here we assume negligible energy spending for control messages. If $e_1 + e_2 + e_3 > cg$, the battery levels drift up, until at least one of them reaches B_{max} . Similarly, if $e_1 + e_2 + e_3 < cg$, then the drift is down, until at least one of the batteries empties out. Before reaching the boundary, the switch times are as before Eq. (24). The drift is Δ .

Obviously, when a node is inactive, its battery charges. As before, we assume $cg > \max(e_1, e_2, e_3)$, so the battery level at the active node goes down. Thus, only the inactive nodes can reach the boundary B_{max} and only the active node can reach the boundary 0.

We look first at the system just after a switch from route 3 to route 1. We shall temporarily refer to the time when this occurs as slot 0. The condition for the next switch (from route 1 to route 2) to occur at the end of slot I_1 is given by Eq. (2), which here translates to

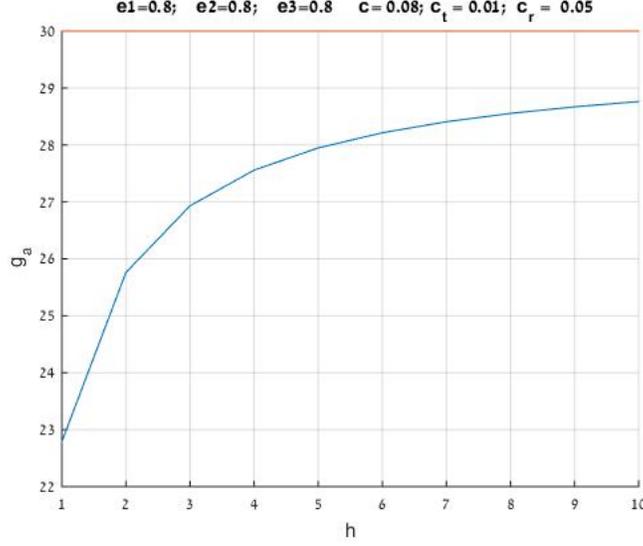


Figure 9: g_s as a function of h for a system with 3 forwarding nodes

$$\sum_{0 < j \leq I_1} (e_2 1_{(B_2(j-1) < B_{max})} - e_1 1_{(0 < B_1(j-1))}) + \sum_{0 < j \leq I_1} c g 1_{(0 < B_1(j-1))} = h_1 + B_1(0) - B_2(0) \quad (27)$$

Note that since the switch at time 0 is from node 3 and not from node 2, in general $B_1(0) - B_2(0) \neq h_2$. Note that the switch time I_1 when boundaries may be reached for a non-zero amount of time may not be the same as I_1^a , the switch time when no boundary is met.

Since nodes 2 and 3 are inactive during I_1 , their battery level increases, at rates e_2, e_3 $mJ/slot$ respectively. On the other hand, since $c g > e_1$, the battery level at the active node decreases, at a rate $(c g - e_1)$ $mJ/slot$. For now we assume that B_{max} is large and hence only one of the boundaries can be reached in a given cycle. If none of the three nodes reaches a boundary, then the switch time is at time

$$I_1^b = \max \left(0, \frac{h_1 + B_1(0) - B_2(0)}{c g + e_2 - e_1} \right)$$

Let

I_1' = slot when node 1 reaches 0 for a non-zero amount of time, where $0 < I_1' \leq I_1^b$

I_1'' = slot when node 2 reaches B_{max} for a non-zero amount of time, where $0 < I_1'' \leq I_1^b$

Each of the two quantities above are defined to equal I_1 if the corresponding boundary is not reached for a non-zero amount of time. Note that if any node reaches some boundary, it

stays at that boundary until the next switch. Therefore the switch condition (27) becomes:

$$e_2 \cdot I_1'' + (c g - e_1) I_1' = h_1 + B_1(0) - B_2(0) \quad (28)$$

if $h_1 + B_1(0) - B_2(0) > 0$ and $I_1 = I_1' = I_1'' = 0$ otherwise. The total number of packets transferred γ_1^c is $I_1' \cdot g + (I_1 - I_1') \cdot (e_1/c)$ if $I_1' < I_1^b$ and $I_1 \cdot g$ otherwise. The operation of the system is similar upon the switch from 2 to 3 and then from 3 to 1 (with a round robin substitution).

The evolution of the system is given below, where $B_1(0), B_2(0), B_3(0)$ are the battery levels at their respective time 0:

Node v is active, node x is next /* $x = v \bmod 3 + 1$

C1 $I^b = \max(0, (h_v + B_v(0) - B_x(0))/(c g + e_x - e_v))$

C2 **case** $((B_{max} - B_x(0))/e_x) < I^b$ { /*node x reaches limit B_{max}

C3 $I_v'' = (B_{max} - B_x(0))/e_x$

C4 $I_v = (h_v - (B_{max} - B_v(0))/(c g - e_v))$

C5 $B_v(I_v) = B_v(0) - I_v (c g - e_v)$; $B_x(I_v) = B_{max}$

C6 $B_u(I_v) = \min(B_u(0) + I_v e_u, B_{max})$ /* u is the third node

C7 $\gamma_v = I_v g$

}

C8 **case** $(B_v(0)/(c g - e_v) < I^b)$ { /*node v reaches limit 0

C9 $I_v' = B_v(0)/(c g - e_v)$

C10 $I_v = (h_v - B_x(0))/e_x$

C11 $B_v(I_v) = 0$; $B_x(I_v) = B_x(0) + I_v c g$

C12 $B_u(I_v) = B_u(0) + I_v e_u$ /* u is the third node

C13 $\gamma_v = I_v' g + (I_v - I_v') (e_v/c)$

}

C14 **else** { neither node reaches limit

C15 $I_v = I^b$

C16 $B_v(I_v) = B_v(0) - I_v (c g - e_v)$; $B_x(I_v) = B_v(0) + I_v c g$

C17 $B_u(I_v) = B_u(0) + I_v e_u$ /* u is the third node

C18 $\gamma_v = I_v g$

}

10.3 The Earliest Switch schedule

In the Round Robin (RR) switching policy, the active routes are pre-assigned. Suppose node 1 is active. A switch will occur only when the battery level at node 2 exceeds the battery level at node 1 by the corresponding threshold. On the other hand, the battery level at node 3 may exceed the corresponding threshold much earlier. If we allow switches from 1 to 3 as well, the performance of the algorithm might improve. To investigate this possibility, we consider here the *Earliest Switch (ES) schedule*. In *ES*, routes are switched to the node that first reaches the corresponding threshold. For simplicity, we look only at the case when the thresholds h_{12}, h_{13} for the switch from 1 to 2 and from 1 to 3 respectively, are the same,

namely $h_{12} = h_{13} = h_1$. Similarly $h_{23} = h_{21} = h_2$ and $h_{31} = h_{32} = h_3$. Also, we assume negligible c_t and c_r and take $c = 0.08mJ/\text{packet}$.

We have selected a series of sets of parameters as in Table 2. We have run all simulations for a duration of 2000 slots, with statistics gathered only beginning at slot 301, to allow for the system to reach dynamic steady state.

Config	cg	e_1	e_2	e_3	h_1	h_2	h_3	Bmax	Comment
A	1.6	0.1	0.7	0.8	5	10	10	100	$cg = e_1 + e_2 + e_3$
B	1.6	0.1	0.7	0.8	5	10	10	12	$cg = e_1 + e_2 + e_3$
C	2.4	0.1	0.7	0.8	5	10	10	100	$cg > e_1 + e_2 + e_3$
D	1.2	0.1	0.7	0.8	5	10	10	100	$cg < e_1 + e_2 + e_3$
E	0.2	0.1	0.01	0.01	5	10	10	60	$cg > e_1 + e_2 + e_3$
F	1.2	0.1	0.7	0.8	10	10	10	100	$cg < e_1 + e_2 + e_3$
G	1.6	0.1	0.7	0.8	5	10	10	60	$cg = e_1 + e_2 + e_3$

Table 2: Parameters

The results are given in the Table 3. We could not find a conclusive statement as to when does the ES schedule perform significantly better than RR. In general, when $Bmax$ is low, it seems that RR waists time in waiting for the corresponding threshold to be reached.

In the scenario of Fig. 10, node 2 becomes active starting at the end of slot 8 and its threshold is $h_2 = 10$. It reaches boundary 0 at end of slot 12 and waits for the other nodes to gather sufficient energy. In RR (not depicted), it would wait for the battery level at node 3, its pre-assigned successor, to reach value 10, which would happen during slot 21. However, the battery level at node 1 reaches value 10 earlier, during slot 17 and in ES, the destination will reroute to node 1 at the end of that slot. The number of packets forwarded from end of slot 17 until the end of slot 21 is the following. In RR, node 2 is active at level 0 and thus it forwards data at rate $e_2/c = 10 \text{ packets/slot}$, whereas in ES, node 1 is active away from boundaries and thus it transmits at rate $g = 31 \text{ packets/slot}$, for a total gain of $(31 - 10) \cdot 4 = 84 \text{ packets}$.

11 Variable Harvesting Rates and Inputs

Practical systems cannot guarantee time invariant harvesting rates. When harvesting rates and inputs are time varying, the energy equations are the same as in the Algorithm in Sec. 3, except that the parameters $e_v(i)$, $e_x(i)$, $g(i)$ are time dependent. Fig.11 approximates the pattern of harvesting of two nodes over 8,000 slots that appears in [GMSplus]. We have investigated the behavior of the system with $c = 0.08$, $c_t = 0.01$, $c_r = 0.05$, $B_{max} = 200$ and

Config		RR	ES
A	Thruput	20	20
	ave cycle (slots)	32	34
	# of switches	163	156
B	Thruput	18.75	19.91
	ave cycle (slots)	13.3	28.03
	# of switches	163	157
C	Thruput	20	20
	ave cycle (slots)	46	18.52
	# of switches	109	152
D	Thruput	15	15
	ave cycle (slots)	31.3	28.8
	# of switches	100	91
E	Thruput	20	20
	ave cycle (slots)	32	34
	# of switches	163	156
F	Thruput	15	15
	ave cycle (slots)	36	33
	# of switches	82	85
G	Thruput	20	20
	ave cycle (slots)	30.8	33
	# of switches	163	153

Table 3: Comparison of Throughputs

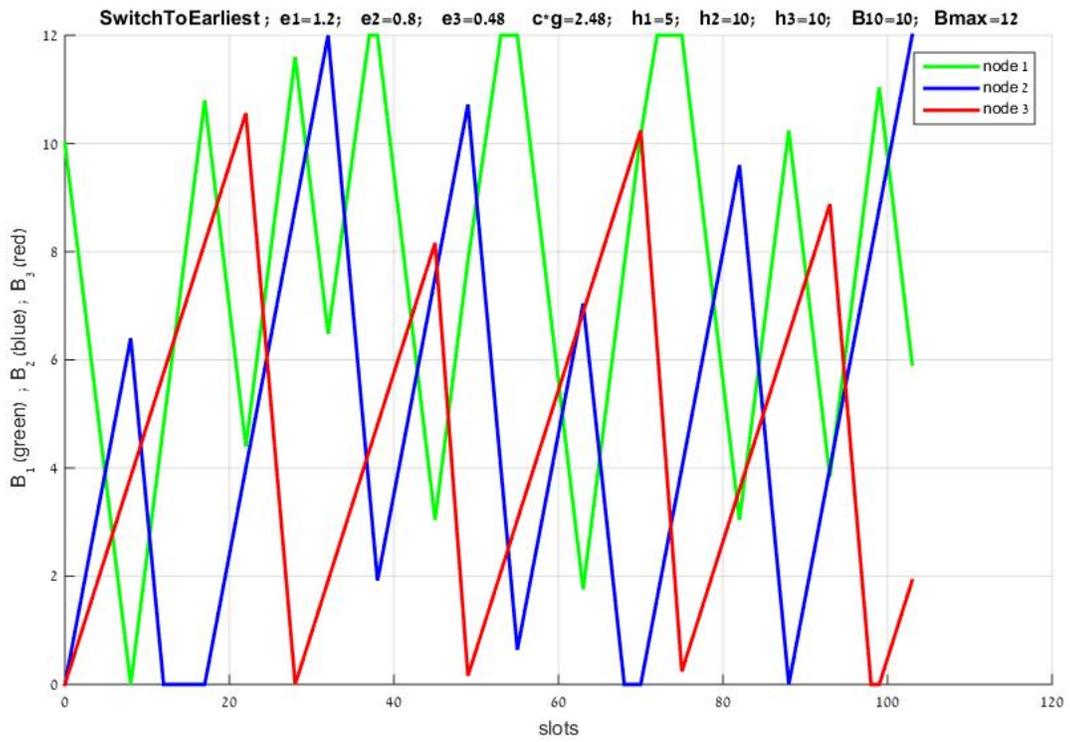


Figure 10: Node 1 becomes active after node 2

two types of inputs, each totaling 48,000 packets over 8,000 slots. With these parameters the total energy required to transfer the packets is $48,000 * 0.08 = 3,840$ mJ. During the period under consideration, the nodes harvest 2,553 and 1,595 mJ respectively, for a total of 4,148 mJ. With transmission status messages requiring 160 mJ and receipt of switch commands requiring about 10 mJ, if the harvest and inputs were uniformly distributed, the harvest would have been more than sufficient to transfer all packets.

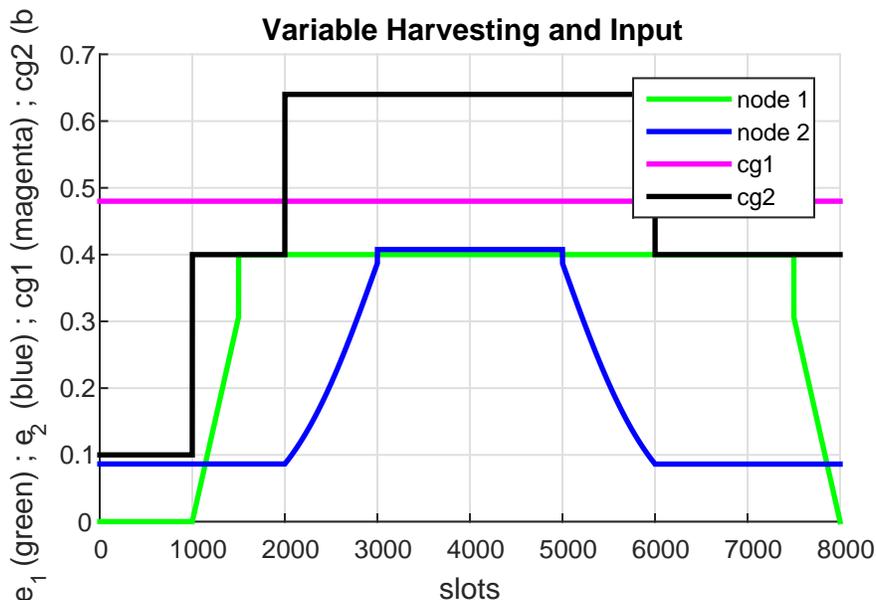


Figure 11: Variable harvesting and input

In our time-varying scenarios, the harvest changes with time. In the first scenario, the input is uniformly distributed at level of $0.48/c$. The contents of the batteries over the 8,000 slots is shown in Fig. 12 and the inputs vs. the throughput over periods of 1000 slots each appears in Table 4.

input	6000	6000	6000	6000	6000	6000	6000	6000
throughput	1072	4150	5983	5994	5994	5994	5994	5994

Table 4: Inputs vs. Throughput in Scenario 1

During the first 1000 slots, node 1 has no harvesting and, except for the first few slots where it uses the initial battery energy (10 mJ), it cannot transfer any packets. Node 2 harvests 0.086 mJ/slot and thus can transfer roughly $g = e_2/c \cong 1\text{packet/slot}$. The total transfer during the first 1000 slots is 1072 packets. During this period, the source tries to send $0.48/0.08 * 1000 = 6000$ packets. During the rest of the scenario, the system behaves,

for the periods when the harvesting parameters are constant, roughly as predicted in Sec. 6.3. For example, during the period from 3000 to 3500, the approximate cycle length is $I = (2 * 0.48 * 20) / ((0.48)^2 - (0.40754 - 0.4)^2) = 85.442$ slots with drift $(0.40754 + 0.4 - 0.48) / 2 = 0.16337$ $mJ/slot$. The simulation shows an 88 slot cycle with drift $13.1 / 88 = 0.149$ $mJ/slot$. We recall that the analytic results are just approximations calculated under the assumptions that $c_t = c_r = 0$ and that the thresholds are exact multiples of the slot energies.

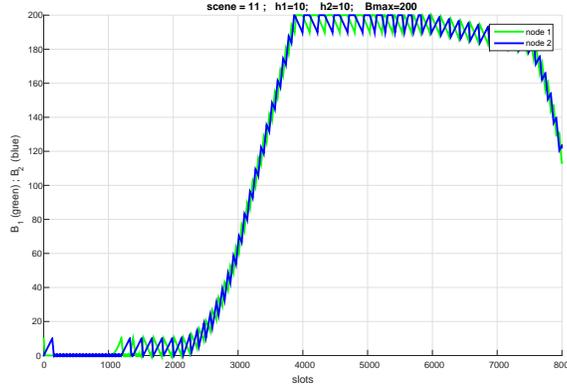


Figure 12: Behavior of the System with Fixed Input $cg1/c$

It is often the case that the node harvesting patterns are roughly known. In this case one can design an appropriate schedule for the input. Assuming that the destination has a transmitter that can be heard at the source, one way to implement this is for it to monitor the transfer and from time to time to transmit desirable input rates to the source. An example of a variable input rate is $cg2/c$ of Fig. 11, so that the total input over the 8000 slots is the same as in the first example. In the second scenario depicted in Fig. 13, the total transfer in the first 1000 slots is 977 packets, but the total input attempt is for 1000 packets only. The final result is that the total throughput in the second scenario is 46,353 packets, whereas in the first scenario is only 41,175 packets.

input	1000	5000	8000	8000	8000	8000	5000	5000
throughput	977	3830	7580	7992	7992	7992	4995	4995

Table 5: Inputs vs. Throughput in Scenario 2

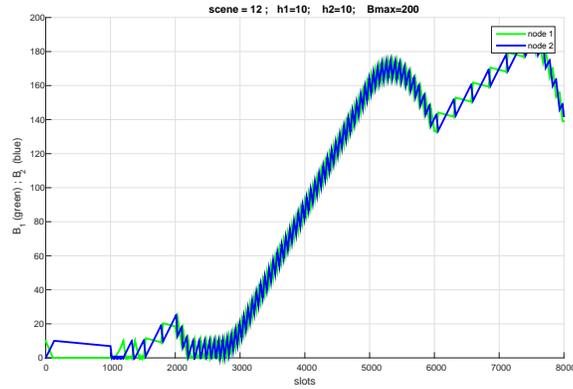


Figure 13: Behavior of the System with Variable Input $cg2/c$

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