HEAVY TRAFFIC PERFORMANCE OF A CLASS OF CHANNEL ASSIGNMENT ALGORITHMS*

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ABSTRACT

In this paper we present a study of the performance of a general class of channel assignment algorithms. These algorithms, which we call Ω-algorithms, are completely characterized by the set of carried-traffic "states" which they allow. We shall see that for any such algorithm, there is a closed-form expression for the carried traffic function, which lends itself to several kinds of asymptotic analysis. As an application, we shall study a particular Ω-algorithm, which has been previously studied under the name "maximum packing algorithm," and which is a "greedy" dynamic channel assignment algorithm, and show that its performance is in many cases inferior to that of simple fixed channel assignment algorithms. We shall see that the cause of this unexpected phenomenon, which was first observed by Kelly [3], it the tendency of dynamic algorithms to get trapped in states that are locally, but not globally, maximal.

INTRODUCTION

In this paper we consider "channelized" cellular telephone systems, i.e., those systems in which the available frequency spectrum is divided into channels in time, or frequency, or a combination of the two. In the case of FDMA systems, a "channel" is a frequency slot, and in the case of TDMA systems, it is a time slot. Our results do not apply to spread spectrum systems — direct sequence or frequency-hopped.

In our models, we assume that there are N cells, and that the offered traffic is uniform and independent from cell to cell. Thus if X_i denotes the offered traffic in cell i, the X_is are i.i.d random variables. Each X_i is a Poisson birth-death process with rate of arrival λ and rate of departure μ per call (see Feller [2], Chapter 17, Sections 5 and 6.). Thus the intensity of the offered traffic in each cell is $\rho = \lambda/\mu$ Erlangs. We do not allow handoffs, i.e., a call may not leave a cell when it is in progress.

We assume that there are a finite number C of channels available. When a call request arrives in a particular cell, it is either assigned to one of the C channels, or blocked, by a *channel assignment algorithm.* The channels assigned to calls cannot be arbitrary; they must satisfy certain *channel reuse constraints.* In this paper, we will consider only reuse constraints which can be represented by undirected graphs without loops or multiple edges. The vertices of the graph represent the cells of the system and pairs of cells that are forbidden from using the same channel simultaneously are joined by an edge. An example of a 3-cell system is shown in Figure 1. (In [5], we have studied more general cellular systems in which the reuse constraints are represented by hypergraphs, and the traffic may be nonuniform. The results in this paper generalize relatively easily to hypergraphs and nonuniform traffic, but for simplicity of exposition we will not present these generalizations here.)

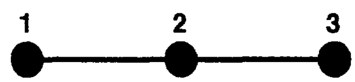

Figure 1. A 3-cell system.

For a cellular system as described above, a list $\tilde{n} = (n_1, n_2, \ldots, n_N)$ of N nonnegative integers is said to be a *permissible state* if it is possible to assign n_i channels to cell i, for $i = 1, 2, \ldots N$, without violating the reuse constraints. The permissible states thus represent the possible configurations of the cellular system, which might occur as the result of the action of some channel assignment algorithm. For example, in the cellular system of Figure 1, if

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there are two channels, the state $(1,1,1)$ is permissible, but $(2,1,1)$ is not. If $\tilde{n}$ and $\tilde{n}'$ are states such that $n_i \le n'_i$ for all i, we write $\tilde{n} \le \tilde{n}'$ and say that $\tilde{n}$ is a *predecessor* of $\tilde{n}'$. Thus for example $(1,1,1) \le (2,1,1)$.

We can now define the class of channel assignment algorithms we shall consider. If Ω is a set of states with the property that all predecessors of every state in Ω are also in Ω, we say that Ω is a *closed* set of states. If Ω is a closed set of permissible states and ϵ_i denotes the vector whose ith component is one and the rest are zeroes, we define the Ω channel assignment algorithm as follows:

If the system is in state $\tilde{n}$ and a call departs from cell i, the channel used by that call is freed, and the system moves to state $\tilde{n} - \epsilon_i$. If the system is in state $\tilde{n}$ and a call arrives in cell i, if state $\tilde{n} + \epsilon_i$ is not in Ω, the call is blocked. On the other hand, if $\tilde{n} + \epsilon_i$ is in Ω, the algorithm must accept the call and move to state $\tilde{n} + \epsilon_i$, regardless of how many calls must be rearranged to do so.

The class of Ω channel assignment algorithms is quite broad, and includes as special cases all *fixed* and many *dynamic* channel assignment algorithms, as we shall now see.

A *fixed channel assignment algorithm* (FCAA) allocates a fixed number of channels permanently to each cell under the constraint that two cells joined by an edge are not assigned the same channel. A new call in a cell is assigned one of the channels allocated to that cell, if such a channel is available. Otherwise the call is blocked. A cell is not allowed to borrow a channel from another cell even if the other cell has not assigned that channel to any call. Thus an FCAA is an Ω algorithm in which Ω consists of all the predecessors of one fixed permissible state, say $(c_1, c_2, \ldots, c_N)$, and we shall call such an algorithm a $(c_1, c_2, \ldots, c_N)$-FCAA. Clearly, the maximum number of calls that can be progress in such an algorithm, which we denote by M, is $\sum_i c_i$.

For example, consider the cellular system of Figure 1 and let the total number of channels available in this system be $C = 2$. Two possible FCAAs for this system are the $(1,1,1)$-FCAA which assigns one channel to each of the cells ($M = 3$) and the (2, 0, 2)-FCAA which assigns two channels to cells 1 and 3 and none to cell 2 ($M = 4$).

In contrast, a *dynamic channel assignment algorithm* (DCAA) is one in which, when a call arrives in a given cell, the algorithm may be allowed to rearrange some or all of the calls already in progress in order to accommodate the new call. Strictly speaking, any FCAA is also a DCAA, but when we refer to a DCAA we shall normally be considering an algorithm which actually rearranges calls. In this paper, we shall consider only one such algorithm, the so-called *maximum packing algorithm* introduced by Everitt and MacFadyen [1] and studied by Kelly [3], which we shall call the *greedy dynamic channel assignment algorithm* (GDCAA). In our terminology, the GDCAA is simply the Ω-algorithm in which Ω is the set of all permissible states. Thus under the operation of the GDCAA, when a call request arrives, it is assigned a channel whenever it is possible to do so without violating the channel reuse constraints, even if this can only be done by rearranging the channels already assigned to the calls in progress. If no channel can be found in spite of this, the call is blocked.

We shall measure the performance of any channel assignment algorithm, either fixed or dymanic, by its *carried traffic function*. This function, denoted by $T(\rho)$, is defined to be the expected number of calls in progess under the operation of the algorithm, as a function of the offered traffic ρ. The carried traffic is related to the blocking probability $B(\rho)$ by

$$T(\rho) = N\rho(1 - B(\rho)). \tag{1}$$

It is natural to expect that the GDCAA would be *better*, i.e., have a larger value of $T(\rho)$, than any FCAA for the same value of ρ. And indeed, it can be shown that, except in the trivial case of no reuse constraints, for any cellular system, for sufficiently small ρ, the GDCAA has a higher carried traffic than any FCAA [3]. The same is true for all ρ if no channel may be reused in the cellular system, i.e., if the cellular system is represented by a complete graph. (This phenomenon, which is basically a law of large numbers, is usually termed "trunking efficiency" in telephony.) Surprisingly, however, this is not always true. Indeed, it was shown in [3] that, for the infinite cellular system shown in Figure 2, when $C = 2$, the $(\ldots,1,1,1,\ldots)$-FCAA has a higher carried traffic than the GDCAA, for large ρ. This was the first example of a cellular system where an FCAA has a higher carried traffic than the GDCAA.

Figure 2. An infinite cellular system.

In this paper, we study this interesting phenomenon further. We begin by deriving an explicit, though unwieldy, expression for the carried traffic function $T(\rho)$ for an arbitrary Ω channel assignment algorithm, (equation (6), below). We then make two different asymptotic analyses of this expression. First, we study $T(\rho)$ in the limit as $\rho \to \infty$, for a fixed value of C. Second, we study the limiting behavior of $T(\rho)$ as ρ and C go to infinity at the same rate. In both cases, we shall see that the GDCAA is, in general, far from optimal when the traffic is heavy.

A FORMULA FOR $T(\rho)$

Recall that N is the number of cells, C the number of available channels, ρ the offered traffic per cell (measured in Erlangs) and, for any channel assignment algorithm, M is the maximum number of calls that can be in progress simultaneously in the system. If n_i is the number of calls in progress in cell i under some channel assignment algorithm, we define $\tilde{n} = (n_1, \ldots, n_N)$ as the state of the cellular system. As we have seen, all FCAAs and the GDCAA can be described in terms of the set of allowed states, Ω, of the system under their operation.

Our results are all based on the following generalization of the well-known Erlang B formula.

Theorem 1. *For any Ω-algorithm, in particular for all FCAAs and for the GDCAA, the (steady-state) probability that the system is in state $\tilde{n}'$ is*

$$\pi(\tilde{n}') = \frac{\rho^{n'_1+\cdots+n'_N}/n'_1!\ldots n'_N!}{\sum_{\tilde{n}\in\Omega}\rho^{n_1+\cdots+n_N}/n_1!\ldots n_N!}. \qquad (2)$$

The proof of Theorem 1, which is based on the proof of the $N=1$ case, i.e., the Erlang B formula, given in [2], pp. 460–468, is lengthy and is omitted. For the special case of the GDCAA, i.e., when Ω is the set of all permissible states, it was stated by Kelly ([3], equation (3.1)).

Let Π_k be the probability that k calls are in progress in the system. Then

$$\Pi_k = \sum_{\tilde{n}\in\Omega: n_1+\ldots+n_N=k} \pi(\tilde{n}), \qquad (3)$$

where $\pi(\tilde{n})$ is as defined in Theorem 1. We define

$$p_k = \sum_{\tilde{n}\in\Omega: n_1+\ldots+n_N=k} 1/n_1!\ldots n_N!. \qquad (4)$$

Combining (2), (3), and (4), therefore, we have

$$\Pi_k = \frac{p_k\rho^k}{\sum_{j=0}^{M} p_j\rho^j}. \qquad (5)$$

The carried traffic $T_\Omega(\rho)$ for the Ω-algorithm is the expected number of calls in progress, under the operation of the Ω-algorithm, algorithm and is therefore given by

$$T_\Omega(\rho) = \sum_{k=0}^{M} k\Pi_k. \qquad (6)$$

In general the number of states in the system is large and the carried traffic function (6) is difficult to compute. However, in the following two sections, we will see that certain asymptotic limits of (6) shed considerable light on the relative performances of FCAAs and the GDCAA.

FIXED C AND LARGE ρ

For a given cellular system with a fixed number C of available channels, we denote by α the largest number of cells that can simultaneously use the same channel. (α is the independence or stability number of the corresponding graph [5].) Since no channel can be used more than α times, and since there are C channels, it follows that for any channel assignment algorithm, M, the maximum number of calls that can be in progress simultaneously in the system, must satisfy $M \le C\alpha$. On the other hand, by the definition of α, there must be a set of α cells, all of which may use the same channel. An FCAA that assigns all C available channels to each cell in this set is therefore one example of a channel assignment algorithm that achieves $M = C\alpha$. We shall call a FCAA for which $M = C\alpha$ a *maximal* FCAA. Since the GDCAA includes all states achieved by any FCAA, it follows that $M = C\alpha$ for the GDCAA as well. The following theorem gives an approximation to $T_\Omega(\rho)$ for large ρ in terms of the quantity M.

Theorem 2. *For any Ω channel assignment algorithm,*

$$T_\Omega(\rho) = M - \frac{p_{M-1}}{p_M}\frac{1}{\rho} + \frac{p_{M-1}^2 - 2p_{M-2}p_M}{p_M^2}\frac{1}{\rho^2} + O(\frac{1}{\rho^3}), \qquad \textit{for large } \rho. \qquad (7)$$

Proof. From (5) we can derive the following asymptotic expressions for large ρ, using a Taylor's expansion in ρ^{-1}.

$$\Pi_M = 1 - \frac{p_{M-1}}{p_M}\frac{1}{\rho} + \frac{p_{M-1}^2 - p_{M-2}p_M}{p_M^2}\frac{1}{\rho^2} + O(\frac{1}{\rho^3}),$$

$$\Pi_{M-1} = \frac{p_{M-1}}{p_M}\frac{1}{\rho} - \frac{p_{M-1}^2}{p_M^2}\frac{1}{\rho^2} + O(\frac{1}{\rho^3}),$$

$$\Pi_{M-2} = \frac{p_{M-2}}{p_M}\frac{1}{\rho^2} + O(\frac{1}{\rho^3}),$$

and

$$\Pi_k = O(\frac{1}{\rho^3}), \qquad k \le M-3.$$

The theorem follows by substituting these expressions for the Π_k in the expression (6) for $T_\Omega(\rho)$. ∎

Example 1. To illustrate Theorem 2 we consider the GDCAA for the 3-cell system of Figure 1. Let $C = 2$. For this system $\alpha = 2$ since cells 1 and 3 may simultaneously use the same channel but all three cells may not. Therefore, $M = C\alpha = 4$. The only state of the system that corresponds to $M = 4$ calls in progress is (2,0,2). Hence, $p_M = p_4 = 1/2!0!2! = 1/4$. The states of the system that correspond to $M = 3$ calls in progress are (2,0,1), (1,0,2) and (1,1,1). Hence, $p_{M-1} = p_3 = 1/2!0!1! + 1/1!0!2! + 1/1!1!1! = 2$. Therefore, from Theorem 2, the carried traffic $T(\rho) = 4 - 4/\rho + O(1/\rho^2)$ for large ρ.

We can similarly apply Theorem 2 to compute the heavy traffic performance of the (1,1,1)-FCAA and the (2,0,2)-FCAA in this example but the following corollary to Theorem 2 will save us the trouble.

Corollary 1. A $(c_1, c_2, \ldots, c_N)$-FCAA satisfies

$$T(\rho) = M - \frac{M}{\rho} + O(\frac{1}{\rho^2}), \qquad \text{for large } \rho,$$

where $M = \sum_{i=1}^{N} c_i$.

Proof. The only state of the system that corresponds to M calls in progress is $(c_1, \ldots, c_N)$. The only states of the system that correspond to $M - 1$ calls in progress are those obtained by deleting one call from this state, i.e., $(c_1 - 1, c_2, \ldots, c_N)$, $(c_1, c_2 - 1, c_3, \ldots, c_N)$ and $(c_1, \ldots, c_{N-1}, c_N - 1)$, assuming $c_i > 0$ for all i. If any of the $c_i = 0$ the corresponding states are absent. In either case, a straightforward calculation shows that $p_{M-1}/p_M = \sum_i c_i = M$ and the result follows by applying Theorem 2. ∎

Example 2. Again, consider the 3-cell system of Figure 1 and let $C = 2$. Since $M = 3$ for the (1, 1, 1)-FCAA, using Corollary 1, we obtain $T(\rho) = 3 - 3/\rho + O(1/\rho^2)$ for large ρ. But, for large ρ, the (2, 0, 2)-FCAA for which $M = 4$ and $T(\rho) = 4 - 4/\rho + O(1/\rho^2)$ has a higher carried traffic. In fact, its carried traffic is even higher than that of the GDCAA (using the same number of channels viz. $C = 2$) which, from Example 1, satisfies $T(\rho) = 4 - 8/\rho + O(1/\rho^2)$.

Comparing Examples 1 and 2, we see that the 3-cell system of Figure 1 with $C = 2$ is another example (the first being Kelly's infinite system of Figure 2) where the GDCAA has a lower carried traffic than an FCAA for heavy traffic — but the FCAA to be considered is not the (1, 1, 1)-FCAA but the (2, 0, 2)-FCAA. Of course, one cannot expect the (1, 1, 1)-FCAA to be better than the GDCAA for heavy traffic since it can only carry a maximum of 3 calls whereas the GDCAA can carry 4. In general, any FCAA with $M < C\alpha$ must be inferior to the GDCAA for heavy traffic.

Example 3. Consider the 7-cell system shown in Figure 3 and let $C = 2$. Using Corollary 1, the (1, 1, 1, 1, 1, 1, 0)-FCAA, the (2, 0, 2, 0, 2, 0, 0)-FCAA and the (0, 2, 0, 2, 0, 2, 0)-FCAA satisfy $T(\rho) = 6 - 6/\rho + O(1/\rho^2)$ whereas the GDCAA, using Theorem 1, satisfies $T(\rho) = 6 - 42/5\rho + O(1/\rho^2)$ for large ρ. (We omit the details of the calculations.) This is another example of a cellular system where the GDCAA is worse than an FCAA when the traffic is heavy.

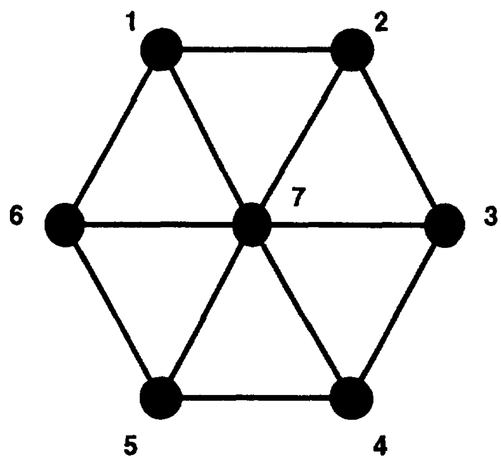

Figure 3. A 7-cell system.

Examples 2 and 3 raise the question: Does there always exist an FCAA that is better than the GDCAA when ρ is sufficiently large for every cellular system? The following example shows that the answer to this question is *no*.

Example 4. Consider the 4-cell system shown in Figure 4 and let $C = 2$. The (1, 1, 1, 1)-FCAA satisfies $T(\rho) = 4 - 4/\rho + 4/\rho^2 + O(1/\rho^3)$ and the (2, 0, 2, 0)-FCAA satisfies $T(\rho) = 4 - 4/\rho + O(1/\rho^3)$ whereas the GDCAA satisfies $T(\rho) = 4 - 4/\rho + 16/3\rho^2 + O(1/\rho^3)$ for large ρ. Therefore, the GDCAA is better than both the (1, 1, 1, 1)-FCAA and the (2, 0, 2, 0)-FCAA, which are the only FCAAs (up to symmetry) with $M = 4$, for heavy traffic.

A more complete calculation yields that the carried traffic for the (1,1,1,1)-FCAA is

$$T(\rho) = \frac{4\rho + 12\rho^2 + 12\rho^3 + 4\rho^4}{1 + 4\rho + 6\rho^2 + 4\rho^3 + \rho^4},$$

for the (2,0,2,0)-FCAA is

$$T(\rho) = \frac{2\rho + 4\rho^2 + 3\rho^3 + \rho^4}{1 + 2\rho + 2\rho^2 + \rho^3 + \frac{1}{4}\rho^4},$$

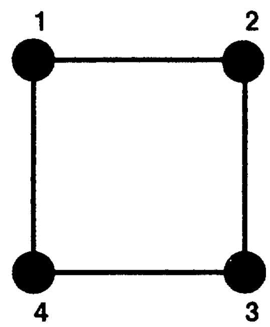

Figure 4. A 4-cell system.

and for the GDCAA is

$$T(\rho) = \frac{4\rho + 16\rho^2 + 18\rho^3 + 6\rho^4}{1 + 4\rho + 8\rho^2 + 6\rho^3 + \frac{3}{2}\rho^4}.$$

Using these expressions, we can show that, in this example, the GDCAA is better than all FCAAs for all values of ρ.

WHEN C AND ρ ARE BOTH LARGE

In many cases of practical interest, the number of available channels is quite large (about 400 in current FDMA cellular systems and about 1200 in the next generation hybrid FDMA/TDMA systems). In such a situation, the analysis of the previous section will not be applicable, and another type of asymptotic expansion, valid for large values of C, may be needed. In this section we will present such an expansion.

Consider a fixed cellular system. Let Ω_1 be the set of permissible states when only one channel is available, i.e., when $C = 1$, and let $\overline{\Omega}_1$ be the convex hull of Ω_1. For a given value of ρ, let $\omega = \omega(\rho)$ be an element of $\overline{\Omega}_1$ that maximizes the function

$$\sum_{i=1}^{N} \omega_i \left(1 + \log \frac{\rho}{\omega_i}\right),$$

and let $|\omega(\rho)|$ denote the sum of the components of $\omega(\rho)$, i.e., $|\omega(\rho)| = \omega_1(\rho) + \cdots + \omega_N(\rho)$.

Theorem 3. *For any Ω channel assignment algorithm,*

$$\lim_{C\to\infty} \frac{1}{C} T_G(C\rho) = |\omega(\rho)|, \qquad \text{for all } \rho \geq 0.$$

The proof of Theorem 3 is similar to the proof of Corollary 3.3 in [4]. Although [4] deals with the asymptotic analysis of a routing problem in circuit-switched networks, the results can be extended to any application, such as ours, where the state probabilities (of the underlying multidimensional Markov process) are given by (2).

Example 5. To illustrate Theorem 3, we consider the 3-cell system of Figure 1 and the GDCAA. For this system,

$$\Omega_1 = \{(0,0,0),(1,0,0),(0,1,0),(0,0,1),(1,0,1)\},$$

$$\overline{\Omega}_1 = \{(x_1,x_2,x_3)|x_1,x_2,x_3 \geq 0, \text{real}, \\ x_1 + x_2 \leq 1, x_2 + x_3 \leq 1\}$$

and it is possible to show that

$$|\omega(\rho)| = \min\left\{3\rho, 1 + \frac{\rho}{2}\left(\sqrt{1 + \frac{4}{\rho}} - 1\right)\right\}.$$

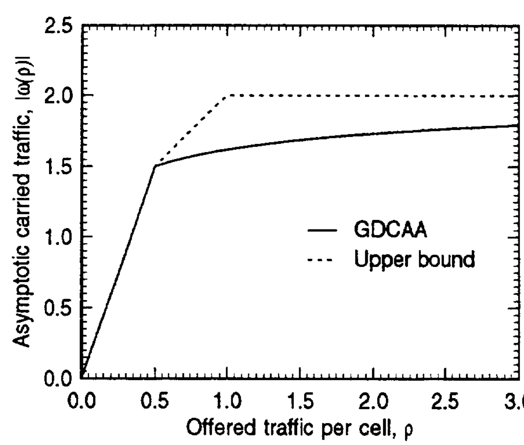

Figure 5. The asymptotic performance of the GDCAA for the 3-cell system.

Using this formula for $|\omega(\rho)|$, we have plotted the asymptotic carried traffic for the GDCAA for this example in Figure 5, along with an upper bound on the carried traffic for any channel assignment algorithm that we have proved in [5]. We also show in [5] that for each value of ρ, this upper bound is achievable by a FCAA. Therefore, we have the interesting result that, for every cellular system and for every fixed $\rho \geq 0$, there exists a FCAA whose asymptotic carried traffic is no worse than the asymptotic carried traffic of the GDCAA. Indeed, in our 3-cell example, there exists a FCAA that is strictly better (has a higher carried traffic) than the GDCAA for all $\rho > 1/2$. The following example shows that this is not always the case.

Example 6. Consider the 4-cell system of Figure 4 and the GDCAA. Ω_1 consists of the maximal states $(1,0,1,0)$ and $(0,1,0,1)$, and their predecessors, so that

$$\overline{\Omega}_1 = \{(x_1,x_2,x_3,x_4)|x_1,x_2,x_3,x_4 \geq 0, \text{real},$$
$$x_1+x_2 \leq 1, x_2+x_3 \leq 1,$$
$$x_3+x_4 \leq 1, x_4+x_1 \leq 1\}$$

and it can be shown that

$$|\omega(\rho)| = \min\{4\rho, 2\}.$$

In this example, $|\omega(\rho)|$ coincides with the upper bound of [5] for all $\rho > 0$. Therefore, the GDCAA is asymptotically optimal for the 4-cell system for all values of $\rho > 0$.

CONCLUSIONS

The most important results in this paper concern the performance of the "greedy" dynamic channel assignment algorithm. This algorithm is worth studying because its implementation, like that of many dynamic channel assignment algorithms, is independent of the offered traffic ρ, whereas the asymptotically optimal FCAAs of [5] have the disadvantage that ρ must be known. Thus dynamic channel assignment algorithms (unlike FCAAs) have the capability of adapting to changes in the offered traffic.

However, we have seen that under some circumstances, the GDCAA can be inferior to a FCAA when the traffic is sufficiently heavy. Initially this seems paradoxical, but on closer study it becomes clear that it is the tendency of the GDCAA to become trapped in states that are locally but not globally maximal that causes the problem. We first consider the case where C is fixed and ρ is large. For example, consider the 3-cell system of Examples 1 and 2 ($C = 2$). If the system gets into the state (1,1,1), no new call can be accepted but the number of calls in progress is only 3 while $C\alpha = 4$. The (2,0,2)-FCAA avoids getting into this state since it never accepts a call in cell 2 (the central cell) whereas the GDCAA, being a greedy algorithm, accepts a call in the central cell if it is possible to do so and then pays for it in terms of a reduction in the carried traffic. The explanation is similar in the case of the 7-cell system of Example 2. The (1,1,1,1,1,1,0)-FCAA avoids states like (2,0,1,1,1,0,0) where no new calls can be accepted but the GDCAA does not. In the 4-cell system of Example 3 there are no such "bad" states and hence, the GDCAA outperforms all the FCAAs.

The explanation is similar for the case when both C and ρ are large. Consider the 3-cell system of Example 5. Using the results in [5], we can show that the $(C/2, C/2, C/2)$-FCAA is asymptotically optimal for $\rho \leq 1/2$, the $(\rho C, (1-\rho)C, \rho C)$-FCAA is optimal for $1/2 < \rho < 1$, and the $(C, 0, C)$-FCAA is asymptotically optimal for $\rho \geq 1$. What this suggests is that, for $\rho \leq 1/2$, there is enough "capacity" to accept all the offered calls and a greedy algorithm like the GDCAA is asymptotically optimal. But for $\rho > 1/2$ an asymptotically optimal algorithm must avoid getting into "bad" states where it has accepted too many calls in the central cell but the GDCAA does not do this. For $\rho > 1$, the asymptotically optimal algorithm must block all the calls in the central cell and hence, the $(C, 0, C)$-FCAA is optimal. Now consider the 4-cell system of Example 6. Again using the results in [5], the $(C/2, C/2, C/2, C/2)$-FCAA is asymptotically optimal for all $\rho > 0$ and, we have seen in Example 6, that the GDCAA has the same performance. This is again because there are no "bad" states in the 4-cell system.

What all of this suggests is that the performance of the GDCAA could be improved by modifying it so as to avoid certain bad states. Indeed, we can prove that a modified Ω-algorithm, where Ω consists of all globally maximal permissible states, and their descendants, is up to terms in ρ^{-1}, at least as good as any FCAA, for fixed C and large ρ. Possibly further study of "bad" states may lead to further improvements in the design of improved dynamic channel assignment algorithms.

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