SELF-NOISE IN MIMO SPACE-TIME CODED SYSTEMS WITH IMPERFECT SYMBOL TIMING

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Abstract - This paper analyzes an influence of an imperfect symbol timing estimate on the space-time coded modulation error performance. We use a concept of the self-noise to describe this influence. We derive an expression for the white self-noise approximation in slowly and fast Rayleigh flat fading MIMO channel. The self-noise is shown to cause a substantial performance degradation which is emphasized by dimensionality of the MIMO channel.

I. Introduction

RECENTLY we could see an enormous research interest in the MIMO (Multiple-input Multiple-output) type space-time communication systems. These systems provide very promising potential in increasing the information capacity of the communication channel. The research effort attacks the problem from various angles. The background is in the analysis of the information capacity ([7], [1]). Search for space-time modulations exploiting the potential of the MIMO channel mostly concentrates on linear space-time trellis codes ([6], [5]). Also nonlinear schemes are investigated ([2], [4]).

However, so far most of the effort in the space-time modulation design and performance evaluation has been focused to simplifying assumption of perfect channel state information (CSI) knowledge on the receiver. Some of the early attempts was made to include an assumption of imperfect CSI on the modulator design and performance ([5]). Unfortunately, a very important issue of the imperfect symbol timing estimate remains almost untouched.

The precision of the symbol timing estimate is very important from the following points. Currently, most communication systems are packet oriented with relatively short packets. All auxiliary information for the synchronization purposes (preamble, training sequence) must be part of the packet. In order to keep system efficient, this auxiliary information should be only a small portion of the packet. The quality of the symbol timing estimate (based on Cramer-Rao lower bound) is proportional to the length of the measurement (preamble length) and effective bandwidth of the signal. Unfortunately both these quantities are the bottlenecks of the communication system. The MIMO system seems to be even more vulnerable to the symbol timing error. This is because of the crosstalk from the multiple transmit antenna when the *propagation delays* and estimation *errors* in these particular branches are mutually *unequal*. As a consequence a potential original Nyquist orthogonality of the modulation impulses is damaged and intersymbol interference degrading bit error rate appears.

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The problem can be generally approached from the two viewpoints. First, we might try to design a space-time modulation systematically taking into account possibly unequal propagation delays or estimation errors. Second, given a modulation designed for equal propagation delays and with perfect symbol timing estimation in mind, we can analyze the effect of the estimation error on the bit error rate performance. This paper follows the second approach by analyzing the influence of the symbol timing error on the detector performance. We will use a concept of self-noise to describe this influence.

II. System Model

The whole treatment is performed in the space of signal complex envelopes. We consider a space-time linear coded modulation in the system with N_t transmit antennas. A modulated signal on i-th transmit antenna is

$$s_i(t) = \sum_n q_{i,n} h_i(t - nT_S) \tag{1}$$

where T_S is a symbol period, $h_i(t)$ is a modulation impulse (signal on i-th antenna is generally allowed to have a specific impulse $h_i(t)$), and $q_{i,n}$ are coded channel symbols. All modulation impulses have a double-sided bandwidth B_h. In the ubiquitous case of the Root Raised Cosine (RRC) impulse with roll-off α this is $B_h = (1+\alpha)/T_S$. Channel symbols depend on the data $d_n \in \{d^{(i)}\}_{i=0}^{M_d-1}$ and the state of the modulator $\sigma_n \in \{\sigma^{(i)}\}_{i=0}^{M_\sigma-1}$

$$q_{i,n} = q_i(d_n, \sigma_n) \tag{2}$$

and the states itself obey the modulator state equation

$$\sigma_{n+1} = \sigma(d_n, \sigma_n). \tag{3}$$

We denote $\mathbf{d} = [\ldots, d_n, \ldots]^T$, $\mathbf{q}_i = [\ldots, q_{i,n}, \ldots]^T$ and $\vec{\mathbf{q}} = [\mathbf{q}_1, \ldots, \mathbf{q}_{N_t}]$.

Transmitted signal propagates through the Rayleigh flat fading channel. We assume N_r receive antennas on the receiver side. The received signal on m-th receive antenna is

$$x_m(t) = u_m(t) + w_m(t) \tag{4}$$

where $w_m(t)$ is AWGN IID[1] over the receiver branches with power spectral density $S_w(f) = 2N_0$. The useful signal on the m-th receiver branch is

$$u_m(t) = \sum_{i=1}^{N_t} a_{mi} s_i(t - \tau_{mi}) \tag{5}$$

where a_{mi} are IID complex Gaussian channel transfer coefficients with zero mean and unity[2] variance $\sigma_a^2 = 1$. Each propagation path introduces a delay τ_{mi}. These delays are assumed to be random IID with probability density $p_\tau(\tau)$. We denote $\mathbf{a}_m = [a_{m1}, \ldots, a_{mN_t}]^T$ and $\boldsymbol{\tau}_m = [\tau_{m1}, \ldots, \tau_{mN_t}]^T$.

The receiver is assumed to have *perfect* estimates of the coefficients $\hat{a}_{mi} = a_{mi}$ and *imperfect* estimates of delays $\hat{\tau}_{mi} = \tau_{mi} + \epsilon_{mi}$ suffering by ϵ_{mi} IID errors with probability density $p_\epsilon(\epsilon)$. We also denote $\hat{\boldsymbol{\tau}}_m = [\hat{\tau}_{m1}, \ldots, \hat{\tau}_{mN_t}]^T$ and $\boldsymbol{\epsilon}_m = [\epsilon_{m1}, \ldots, \epsilon_{mN_t}]^T$.

We consider a *packet* oriented communication system with the data message frame of the length L channel symbols. This roughly corresponds to the time duration LT_S (neglecting the end-tails of the outer modulation impulses). We will treat two cases of the channel dynamics. The first case is the *slowly* fading channel. Here we assume that the channel coefficients a_{mi}, delays τ_{mi} and estimation errors[3] ϵ_{mi} are constant within the packet of the length L. Each frame transfer realization is considered to be a function of an independent realization of these variables. The second case is the *fast*[4] fading (relatively to the frame length) channel. We will understand this as a situation where the channel parameters a_{mi}, τ_{mi} and consequently also the estimation error ϵ_{mi} develop all their randomness within the channel observation period corresponding to one frame. That means that they can be considered as realizations of the finite observation period ergodic process.

III. Self-Noise

First we develop a general approach to the self-noise evaluation and then subsequently we apply the procedure on our particular case of imperfect symbol timing synchronization.

A. General case

We will use a vector signal space representation for the brevity of the explanation. Assume the received signal

$$\mathbf{x} = \mathbf{u}(\mathbf{d}, \boldsymbol{\theta}, \boldsymbol{\xi}) + \mathbf{w} \tag{6}$$

where $\mathbf{u}$ is the useful signal, $\mathbf{d}$ is the data vector, $\boldsymbol{\theta}$, $\boldsymbol{\xi}$ and $\mathbf{w}$ are channel nuisance parameters. Vector $\mathbf{w}$ represents AWGN with complex envelope power spectrum density $2N_0$. The channel is used (observed) over a finite period T. Parameters $\boldsymbol{\xi}$ and $\mathbf{w}$ are finite observation T ergodic[5] while $\boldsymbol{\theta}$ parameters are nonergodic for a finite T observation.

Assume that the detector decision metric ρ is derived with the perfect synchronization (perfect CSI[6] knowledge) assumption[7] for the $\boldsymbol{\theta}, \boldsymbol{\xi}$ parameters. In the case of AWGN channel with the $\mathbf{w}$ elimination, the particular form of the metric is

$$\rho(\mathbf{x}, \check{\mathbf{d}}, \boldsymbol{\theta}, \boldsymbol{\xi}) = \|\mathbf{x} - \mathbf{u}(\check{\mathbf{d}}, \boldsymbol{\theta}, \boldsymbol{\xi})\|^2. \tag{7}$$

The detector performs a search over the trial data $\check{\mathbf{d}}$ in order to obtain the data estimates

$$\hat{\mathbf{d}} = \arg\min_{\check{\mathbf{d}}} \rho(\mathbf{x}, \check{\mathbf{d}}, \boldsymbol{\theta}, \boldsymbol{\xi}). \tag{8}$$

At the time of receiver real operation, estimates $\hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\xi}}$ are substituted *instead* of the actual CSI values $\boldsymbol{\theta}, \boldsymbol{\xi}$. The detector then operates with the metric

$$\hat{\rho}(\mathbf{x}, \check{\mathbf{d}}, \boldsymbol{\theta}, \boldsymbol{\xi}) = \|\mathbf{x} - \mathbf{u}(\check{\mathbf{d}}, \hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\xi}})\|^2 \tag{9}$$

instead the correct one $\rho(\mathbf{x}, \check{\mathbf{d}}, \boldsymbol{\theta}, \boldsymbol{\xi})$. The metric $\hat{\rho}(\mathbf{x}, \check{\mathbf{d}}, \boldsymbol{\theta}, \boldsymbol{\xi})$ can be easily expressed as

$$\hat{\rho}(\mathbf{x}, \check{\mathbf{d}}, \boldsymbol{\theta}, \boldsymbol{\xi}) = \|\mathbf{x}' - \mathbf{u}(\check{\mathbf{d}}, \boldsymbol{\theta}, \boldsymbol{\xi})\|^2 \tag{10}$$

where

$$\mathbf{x}' = \mathbf{x} + \mathbf{u}(\check{\mathbf{d}}, \boldsymbol{\theta}, \boldsymbol{\xi}) - \mathbf{u}(\check{\mathbf{d}}, \hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\xi}}). \tag{11}$$

Expression

$$\boldsymbol{\zeta}(\check{\mathbf{d}}, \boldsymbol{\theta}, \hat{\boldsymbol{\theta}}, \boldsymbol{\xi}, \hat{\boldsymbol{\xi}}) = \mathbf{u}(\check{\mathbf{d}}, \boldsymbol{\theta}, \boldsymbol{\xi}) - \mathbf{u}(\check{\mathbf{d}}, \hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\xi}}) \tag{12}$$

is additive to the received signal and it is also random because of the random nature of channel nuisance parameters and data. Therefore it is called *self-noise*.

Self-noise can be conveniently used as a tool for approximate evaluation of the channel nuisance parameter errors on the detector performance. This approximation is based on the idea of replacing the actual self-noise by an equivalent (first and second moments) white Gaussian noise. This replacement is then equivalent to the perfectly synchronized system operating under new *effective* level of AWGN with spectral density $S'_w(f) = S_w(f) + Z_0$ where Z_0 is the white power spectrum density approximation of the self-noise. The evaluation of the self-noise stochastic properties (first and second moments) is performed over all channel parameters which are ergodic[8] with respect to the channel observation period T, i.e. over $\check{\mathbf{d}}, \boldsymbol{\xi}, \hat{\boldsymbol{\xi}}$.

[1] Independent and Identically Distributed.

[2] This can be done without the loss of generality since the required signal-noise ratio can be easily set by choosing proper N_0.

[3] This means that the parameter estimate is performed once per frame.

[4] It does not necessarily mean that the two successive channel symbols would undergo independent states of the channel.

[5] With sufficiently small error with probability close to one.

[6] Channel State Information.

[7] We can also systematically derive the metric correctly considering the stochastic properties of the channel nuisance parameters. Details can be found in [3]. However this approach is not followed here.

[8] That means that they develop their full randomness within the observation period and their influence can be judged through their average properties.

Strictly speaking, the above stated approximation violates three principles.

1) The self-noise is *not* strictly Gaussian. However self-noise becomes close to the Gaussian one according to the central limit theorem when the received signal is a function of large number of independent ergodic parameters.
2) The self-noise is *not* strictly white. Level of violation of this principle very much depends on particular form of the useful signal $\mathbf{u}$. In most practical cases, it can be considered as acceptable quite comfortably.
3) The self-noise is *not* generally independent with the useful signal model with respect to the ergodic[9] parameters. The mutual dependence can of course affect the performance evaluation especially when the self-noise is dominant.

In most practical situations, the above stated problems have only a mild influence on the results obtained from the approximation.

B. Using self-noise to assess the performance of the detector

The self-noise demonstrates itself (in the sense of the above approximation) on the detector error performance through the effective increase of the AWGN power spectrum density. The increase is equivalent to the white spectrum approximation of the self-noise power spectrum density $Z_0(\boldsymbol{\theta}, \hat{\boldsymbol{\theta}})$. It is a function of all finite observation T nonergodic parameters $\boldsymbol{\theta}, \hat{\boldsymbol{\theta}}$.

We assume that we know the detection error[10] performance for the *perfectly* synchronized case $P_e(N_0, \boldsymbol{\theta})$. This characteristic is unavoidably conditioned by nonergodic parameters $\boldsymbol{\theta}$. In order to obtain the performance in the imperfect synchronization case we replace N_0 by[11] $N_0' = N_0 + Z_0(\boldsymbol{\theta}, \hat{\boldsymbol{\theta}})/2$ to obtain

$$P_e'(N_0, \boldsymbol{\theta}, \hat{\boldsymbol{\theta}}) = P_e\left(N_0 + \frac{1}{2} Z_0(\boldsymbol{\theta}, \hat{\boldsymbol{\theta}}), \boldsymbol{\theta}\right) \tag{13}$$

where

$$Z_0(\boldsymbol{\theta}, \hat{\boldsymbol{\theta}}) = \mathrm{E}\left[Z_0(\check{\mathbf{d}}, \boldsymbol{\xi}, \hat{\boldsymbol{\xi}}, \boldsymbol{\theta}, \hat{\boldsymbol{\theta}}) | \boldsymbol{\theta}, \hat{\boldsymbol{\theta}}\right]. \tag{14}$$

Very often, the required measure of the performance is the *average* error probability over large number of successive message transfers (frames). Then the imperfect synchronization error probability approximation using the self-noise is

$$\begin{aligned} \bar{P}_e'(N_0) &= \mathrm{E}_{\boldsymbol{\theta}, \hat{\boldsymbol{\theta}}}\left[P_e'(N_0, \boldsymbol{\theta}, \hat{\boldsymbol{\theta}})\right] \\ &= \mathrm{E}_{\boldsymbol{\theta}, \hat{\boldsymbol{\theta}}}\left[P_e\left(N_0 + \frac{1}{2} Z_0(\boldsymbol{\theta}, \hat{\boldsymbol{\theta}}), \boldsymbol{\theta}\right)\right]. \end{aligned} \tag{15}$$

It is very important to notice that the averaging over the finite observation nonergodic parameters must be accomplished on the overall error probability P_e. The averaging over the finite observation ergodic parameters can be accomplished inside the function of the error probability.

[9]Whose influence is being considered only through the average.
[10]Or any other performance characteristic we might be interested in.
[11]Recollect that $S_w(f) = 2N_0$.

C. Application on the imperfect symbol timing synchronization case

1) Expression for the self-noise: Now we turn our attention to the particular system described in Section II. We derive the expression for the self-noise white power spectrum approximation. The self-noise on the m-th receiver input is (trial channel symbols corresponding to the trial data $\check{d}_n$ are denoted by $\check{q}_{i,n}$)

$$\begin{aligned} \zeta_m &= u_m(t, \check{\mathbf{d}}, \mathbf{a}_m, \boldsymbol{\tau}_m) - u_m(t, \check{\mathbf{d}}, \mathbf{a}_m, \hat{\boldsymbol{\tau}}_m) \\ &= \sum_{i=1}^{N_t} a_{mi} \sum_n \check{q}_{i,n} \Delta h_i(\epsilon_{mi}, t - \tau_{mi} - nT_S) \end{aligned} \tag{16}$$

where

$$\begin{aligned} &\Delta h_i(\epsilon_{mi}, t - \tau_{mi} - nT_S) = \\ &= h_i(t - \tau_{mi} - nT_S) - h_i(t - \tau_{mi} - \epsilon_{mi} - nT_S) \end{aligned} \tag{17}$$

can be interpreted as modulation impulse error.

2) Self-noise power spectrum density for slowly fading channel: We start with the evaluation of the total mean power of the self-noise. In the case of slowly fading channel, the only finite observation ergodic signal parameter is the data vector $\check{\mathbf{d}}$ and corresponding channel symbols $\check{\mathbf{q}}$. Therefore the only averaging we can perform on the self-noise power is the one over $\check{\mathbf{q}}$. We assume stationary channel symbols. The average power of the self-noise in the m-th receiver branch is[12]

$$\begin{aligned} \bar{\mathcal{P}}_{\zeta_m} &= \mathrm{AvE}\left[|\zeta_m(t)|^2\right] \\ &= \frac{1}{T_S} \sum_{i=1}^{N_t} \sum_{i'=1}^{N_t} a_{mi} a_{mi'}^* \sum_\ell R_{\check{q}_{ii'}}[\ell] \eta_\ell(\Delta\tau_{m,ii'}, \epsilon_{mi}, \epsilon_{mi'}) \end{aligned} \tag{18}$$

where $\Delta\tau_{m,ii'} = \tau_{mi} - \tau_{mi'}$, $R_{\check{q}_{ii'}}[\ell] = \mathrm{E}[\check{q}_{i,\ell} \check{q}_{i',0}^*]$ and

$$\begin{aligned} &\eta_\ell(\Delta\tau_{m,ii'}, \epsilon_{mi}, \epsilon_{mi'}) = \\ &= \mathcal{R}_{h_{ii'}}^{\mathcal{E}}(-\Delta\tau_{m,ii'} - \ell T_S) \\ &- \mathcal{R}_{h_{ii'}}^{\mathcal{E}}(-\Delta\tau_{m,ii'} + \epsilon_{mi'} - \ell T_S) \\ &- \mathcal{R}_{h_{ii'}}^{\mathcal{E}}(-\Delta\tau_{m,ii'} - \epsilon_{mi} - \ell T_S) \\ &+ \mathcal{R}_{h_{ii'}}^{\mathcal{E}}(-\Delta\tau_{m,ii'} - \epsilon_{mi} + \epsilon_{mi'} - \ell T_S). \end{aligned} \tag{19}$$

Function $\mathcal{R}_{h_{ii'}}^{\mathcal{E}}$ is energy correlation function of the modulation impulses

$$\mathcal{R}_{h_{ii'}}^{\mathcal{E}}(\tau) = \int_{-\infty}^{\infty} h_i(t + \tau) h_{i'}^*(t)\, dt. \tag{20}$$

The bandwidth of the self-noise (16) will be given by the modulation impulse spectrum properties. Following our assumptions in the system model, the double-sided bandwidth

[12]$\mathrm{Av}[(\cdot)] = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} (\cdot)\, dt$

of ζ_m is B_h. Then the effective value power spectrum density of the *white* self-noise approximation is

$$Z_0(\mathbf{a}_m, \boldsymbol{\tau}_m, \boldsymbol{\epsilon}_m) = \frac{\bar{\mathcal{P}}_{\zeta_m}}{B_h}. \tag{21}$$

We can clearly see that the self-noise is heavily influenced by the crosstalk of the signals from different transmitter antennas. A potential Nyquist orthogonality of the modulation impulses is dissolved by the random mutual time shifts.

3) Self-noise power spectrum density for fast fading channel: In the case of relatively fast fading channel, we can make averaging over all channel parameters. Utilizing their properties defined in the system model (zero mean IID a_{mi} and $\Delta\tau_{m,ii} = 0$) we obtain

$$\bar{\mathcal{P}}_{\zeta_{\mathrm{F},m}} = \frac{\sigma_a^2}{T_S} \sum_{i=1}^{N_t} \sum_{\ell} R_{\check{q}_{ii}}[\ell] \bar{\eta}_{\mathrm{F},\ell} \tag{22}$$

where

$$\begin{aligned} \bar{\eta}_{\mathrm{F},\ell} &= \mathrm{E}\left[\eta_\ell(\Delta\tau_{m,ii}, \epsilon_{mi}, \epsilon_{mi})\right] \\ &= 2\mathcal{R}^{\varepsilon}_{h_{ii}}(-\ell T_S) \\ &\quad - \mathrm{E}\left[\mathcal{R}^{\varepsilon}_{h_{ii}}(\epsilon_{mi} - \ell T_S) - \mathcal{R}^{\varepsilon}_{h_{ii}}(-\epsilon_{mi} - \ell T_S)\right]. \end{aligned} \tag{23}$$

Assuming the Nyquist[13] modulation impulse and the case of symmetrical probability density $p_\epsilon(\epsilon) = p_\epsilon(-\epsilon)$ we can further manipulate the expression into

$$\bar{\eta}_{\mathrm{F},\ell} = 2\left(\delta[\ell] - \mathrm{E}\left[\mathcal{R}^{\varepsilon}_{h_{ii}}(\epsilon_{mi} - \ell T_S)\right]\right). \tag{24}$$

The white noise approximation is again

$$Z_{\mathrm{F},0} = \frac{\bar{\mathcal{P}}_{\zeta_{\mathrm{F},m}}}{B_h} \tag{25}$$

The fast fading channel, unlike the slow one, does not suffer from the crosstalk (in a statistical sense) between different transmitter branches. However the potential Nyquist orthogonality of the modulation impulse is again dissolved by the averaging over ϵ_{mi}.

IV. Pairwise Error Probability

Now we apply the previously gained results for the self-noise properties to evaluation of its influence on the Pairwise Error Probability (PEP) of the space-time trellis coded modulation. Space-time trellis modulation design in the spirit of [6] does not count with the possibility of nonzero unequal delays τ_{mi} or estimation errors. These delays generally cause loss of modulation impulse Nyquist orthogonality and substantially damage space-time code performance especially the diversity gain. The issue of the space-time code design resistant to this phenomenon is a separate problem. We will consider a case of *all zero delays* $\tau_{mi} = 0$ for the rest of the paper in the evaluation of the error performance. This corresponds to the assumption of all paths having almost equal delay or a presence of the equalizer.

Error performance characteristics will be parametrized by the ratio of the mean received signal energy per symbol to noise spectral density per one receiver branch. It is defined as

$$\begin{aligned} \gamma &= \frac{T_S \,\mathrm{AvE}\left[|u_m(t)|^2\right]}{N_0} \\ &= \frac{T_S \sigma_a^2 N_t \bar{\mathcal{P}}_{s_i}}{N_0} \end{aligned} \tag{26}$$

where $\bar{\mathcal{P}}_{s_i} = \mathrm{AvE}[|s_i(t)|^2]$ is mean power in one transmitted signal branch which is assumed to invariant of i. We used zero mean and IID property of a_{mi}.

A. Perfect CSI

In the case of perfect CSI information knowledge on the receiver side, the *mean* PEP[14] averaged over the channel states in the stream of frames is given by[15] (see [6])

$$\bar{P}_{2e} = \frac{1}{2\sqrt{\pi}} \prod_{m=1}^{N_r} \int e^{-\frac{\mathbf{a}_m^H \mathbf{Q}_{\min} \mathbf{a}_m}{8N_0}} p(\mathbf{a}_m)\, d\mathbf{a}_m \tag{27}$$

where ($\Delta\vec{\mathbf{q}} = \vec{\mathbf{q}}^{(1)} - \vec{\mathbf{q}}^{(2)}$ for two given data messages)

$$\mathbf{Q}_{\min} = \Delta\vec{\mathbf{q}}_{\min}^H \Delta\vec{\mathbf{q}}_{\min}, \tag{28}$$

and[16]

$$\Delta\vec{\mathbf{q}}_{\min} = \arg \min_{\Delta\check{\vec{\mathbf{q}}} \neq \mathbf{0}} \det\left(\Delta\check{\vec{\mathbf{q}}}^H \Delta\check{\vec{\mathbf{q}}}\right). \tag{29}$$

That is, $\Delta\vec{\mathbf{q}}_{\min}$ is a codeword difference between the two codewords most vulnerable to the pairwise error.

B. Imperfect symbol timing estimate

Now we apply previously derived procedure using the self-noise approximation for the case of imperfect symbol timing estimate. We assume $\boldsymbol{\tau}_m = \mathbf{0}$. For a *slowly* fading channel, we get for mean (over all successive frames) PEP

$$\begin{aligned} \bar{P}_{2e}^{(\mathrm{imp})} &= \frac{1}{2\sqrt{\pi}} \prod_{m=1}^{N_r} \int\int e^{-\frac{\mathbf{a}_m^H \mathbf{Q}_{\min} \mathbf{a}_m}{8\left(N_0 + \frac{Z_0(\mathbf{a}_m, \mathbf{0}, \boldsymbol{\epsilon}_m)}{2}\right)}} \\ &\qquad \times p(\mathbf{a}_m) p_\epsilon(\boldsymbol{\epsilon}_m)\, d\mathbf{a}_m d\boldsymbol{\epsilon}_m. \end{aligned} \tag{30}$$

[13] $\int_{-\infty}^{\infty} h(t + \ell T_S) h^*(t)\, dt = \delta[\ell]$

[14] Actually this is an exponential approximation of the PEP.

[15] With a minor modification reflecting our definition of parameters.

[16] We assume space-time code with full diversity.

Utilizing the IID property of $\mathbf{a}_m$ and $\boldsymbol{\epsilon}_m$ we get

$$\bar{P}_{2e}^{(\mathrm{imp})} = \frac{1}{2\sqrt{\pi}} \left(\int \int e^{-\frac{\mathbf{a}_1^H \mathbf{Q}_{\min} \mathbf{a}_1}{8\left(N_0 + \frac{Z_0(\mathbf{a}_1, \mathbf{0}, \boldsymbol{\epsilon}_1)}{2}\right)}} \times p(\mathbf{a}_1) p_\epsilon(\boldsymbol{\epsilon}_1)\, d\mathbf{a}_1 d\boldsymbol{\epsilon}_1 \right)^{N_r} . \quad (31)$$

This integral can be numerically evaluated for given probability densities of $\mathbf{a}_m$ and $\boldsymbol{\epsilon}_m$.

V. Example application

As an example application, we evaluate the mean PEP performance of the Tarokh [6] 2-space 4-state 4PSK code ($M_d = 4$, $M_\sigma = 4$, $N_t = N_r = 2$) in the slowly fading channel. The state and output equations of this simple code are

$$q_{1,n} = \mathcal{Q}(\sigma_n), \quad (32)$$
$$q_{2,n} = \mathcal{Q}(d_n), \quad (33)$$
$$\sigma_{n+1} = d_n \quad (34)$$

where $\mathcal{Q}(k) = \exp(j2\pi k/M_d)$, $d_n \in \{0, 1, \ldots, M_d - 1\}$, $\sigma_n \in \{0, 1, \ldots, M_\sigma - 1\}$. In this particular case we easily find that

$$R_{q_{ii'}}[\ell] = \delta[\ell + i - i']. \quad (35)$$

The matrix $\mathbf{Q}_{\min}$ can be easily found to be

$$\mathbf{Q}_{\min} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}. \quad (36)$$

Symbol timing errors are assumed to be IID Gaussian zero mean random variables with variance σ_ϵ^2. Numerical results for this code are shown on Figure 1.

VI. Conclusions

We have evaluated influence of the imperfect symbol timing estimate on the space-time coded system. It was done with the utilization of the self-noise concept. First, the self-noise derivation was carried out in a general system framework. Then it was applied on two cases—slowly and fast Rayleigh flat fading channel. As an example application we applied the concept on a simple 2-space trellis coded space-time code.

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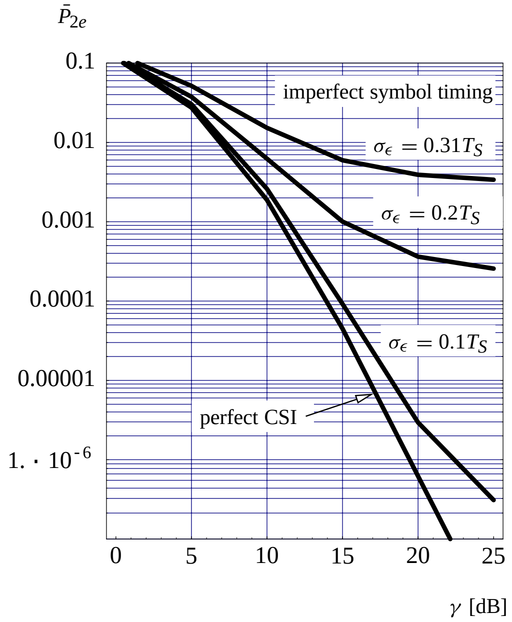

Fig. 1. Mean pairwise error probability of the Tarokh 2-space 4-state 4PSK code in the slowly fading Rayleigh channel.

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