Optimal Beamforming for MIMO Shared Relaying in Downlink Cellular Networks with ARQ

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***Abstract*—In this paper, we study the performance of the downlink of a cellular network with automatic repeat-request (ARQ) and a half duplex decode-and-forward shared relay. In this system, two multiple-input-multiple-output (MIMO) base stations serve two single antenna users. A MIMO shared relay retransmits the lost packets to the target users. First, we study the system with direct retransmission from the base station and derive a closed form expression for the outage probability of the system. We show that the direct retransmission can overcome the fading, however, it cannot overcome the interference. After that, we invoke the shared relay and design the relay beamforming matrices such that the signal-to-interference-and-noise ratio (SINR) is improved at the users subject to power constraints on the relay. In the case when the transmission of only one user fails, we derive a closed form solution for the relay beamformers. On the other hand when both transmissions fail, we pose the beamforming problem as a sequence of non-convex feasibility problems. We use semidefinite relaxation (SDR) to convert each feasibility problem into a convex optimization problem. We ensure a rank one solution, and hence, there is no loss of optimality in SDR. Simulation results are presented showing the superior performance of the proposed relay beamforming strategy compared to direct ARQ system in terms of the outage probability.**

I. INTRODUCTION

Fading and interference are fundamental phenomena of wireless communications. They limit the performance of many wireless systems. Automatic-repeat-request (ARQ) protocols have been developed as a mechanism to combat fading and ensure reliable data transmission for wireless communication systems. The idea of ARQ protocols is that a user requests retransmission when a message is not correctly decoded. A review of ARQ protocols is presented in [1] and the performance of different ARQ protocols is discussed in [2]. In ARQ systems, there is a natural tradeoff between the data transmission rate and ARQ re-transmission rate. In [3], it was shown that the performance of ARQ systems can be improved by selecting the data transmission rate based on the fading statistics. However, direct ARQ cannot significantly improve communication reliability in multiuser communication as interference is the main causes of transmission failure.

This work proposes to manage the inter-cell interference of cellular networks with ARQ using multiple-input-multiple-output (MIMO) shared relay. Many techniques have been proposed to manage inter-cell interference in cellular networks. The coordination of base stations (BSs) can significantly improve the system performance. However, it requires sharing large amounts of data between different BSs under tight coordination[4]. Also, interference alignment and cooperative systems require the availability of complete channel state information at all the transmitting nodes. In contrast, cellular relay networks present a promising practical solution not only for improving the network coverage but also for managing interference[5]. In [6], a hybrid-ARQ (HARQ) technique was proposed for wireless relay networks where the relay helps the source to communicate with a single user terminal by retransmitting failed packets. In [7], a comparison is presented between the performance of decode-and-forward (DF) ARQ relay systems relative to direct ARQ systems (where the retransmission is performed by the BS) in terms of the outage probability showing that the system performance can be significantly improved with relaying. In [8], the performance of distributed HARQ protocol was shown to be much better than direct transmission in terms of outage probability as the outage event occurs when the relay and the destination fail to decode simultaneously. However, in all this prior work, only single user ARQ systems were investigated and the effect of multiuser interference was not considered.

In this paper, we consider the downlink of a multiple-input-single-output (MISO) cellular ARQ system where the BSs are equipped with multiple antennas. We derive closed-form expressions for the outage probability in the presence and absence of inter-cell interference showing that direct ARQ cannot improve the outage probability of the system. Next, we consider a cellular relay network where a MIMO half-duplex DF shared relay retransmits the lost packets to the target users. We design the relay beamforming matrices such that the signal-to-interference-and-noise ratio (SINR) is improved at the users subject to power constraints on the relay. We consider two cases for the relay retransmission. The first case is when only one user fails where we derive a closed-form solution for the relay beamformers. On the other hand when both transmissions fail, we pose the beamforming problem as a sequence of non-convex feasibility problems. We use semidefinite relaxation (SDR) to convert each feasibility problem to a convex optimization problem. We ensure a rank one solution, and hence, there is no loss of optimality in SDR. Numerical simulations are presented showing that the proposed beamforming algorithms can significantly improve the outage probability compared to direct ARQ systems.

II. MISO CELLULAR SYSTEM WITH DIRECT ARQ

We consider the downlink of a cellular system containing two BSs each serving a user terminal. For the sake of simplicity, we consider a two-cell case where two BSs serve two users. However, the proposed algorithms can be directly extended to multiple cells. We assume that the transmitting BSs do not have any any information about the channel coefficients of the links to the users. Each BS is equipped with N antennas while each user has one antenna only. The ith BS transmits the data vector $\boldsymbol{x}_i$ to the ith user where $\boldsymbol{x}_i$ has zero-mean with a covariance matrix $\boldsymbol{I}_N$ and $\boldsymbol{I}_N$ denotes the $N \times N$ identity matrix. The received signal at the ith user is given by

$$y_i = \sqrt{\frac{P}{N}} \sum_{j=1}^{2} \boldsymbol{h}_{i,j}^H \boldsymbol{x}_j + n_i \tag{1}$$

where $\boldsymbol{h}_{i,j}$ is the $N \times 1$ vector containing the complex conjugate of the coefficients of the channel from the jth BS to the ith user i. All channel coefficients are independent and identically distributed circularly symmetric complex Gaussian random variable with zero-mean. We assume that the variance of the direct channels, i.e., from the ith BS to the ith user, is given by σ_1^2 and the variance of the interference channels is given by σ_2^2. In (1), n_i is the zero-mean circular Gaussian complex noise received at the ith user with variance σ^2 and P is the transmitted power from each BS.

Let us consider the case where there is no interference, i.e., $\boldsymbol{h}_{1,2} = \boldsymbol{h}_{2,1} = \boldsymbol{0}$. In this case, the mutual information between the ith BS and its corresponding user is given by

$$I_i^{(\mathrm{SU})} = \log_2 \left(1 + \frac{P}{N\sigma^2} \boldsymbol{h}_{i,i}^H \boldsymbol{h}_{i,i}\right) \tag{2}$$

The corresponding outage probability is given by

$$\begin{aligned} P_{\mathrm{out},i}^{(\mathrm{SU})} &= \Pr\left\{ I_i^{(\mathrm{SU})} < R \right\} \\ &= \Pr\left\{ \tilde{\boldsymbol{h}}_{i,i}^H \tilde{\boldsymbol{h}}_{i,i} < \frac{N\sigma^2}{P\sigma_1^2}\left(2^R - 1\right) \right\}. \end{aligned} \tag{3}$$

where $\tilde{\boldsymbol{h}}_{i,i} = \boldsymbol{h}_{i,i}/\sigma_1$ and R is the attempted transmission rate by the BS. Since $\tilde{\boldsymbol{h}}_{i,i} \sim \mathcal{CN}(0, \boldsymbol{I}_N)$, then $\tilde{\boldsymbol{h}}_{i,i}^H \tilde{\boldsymbol{h}}_{i,i} \sim \mathcal{X}_{(2N)}^2$, i.e., Chi-square distribution with $2N$ degrees of freedom. The outage probability (3) is given by

$$P_{\mathrm{out},i}^{(\mathrm{SU})} = Q_{\mathcal{X}_{(2N)}^2}\left(\frac{2N\sigma^2}{P\sigma_1^2}\left(2^R - 1\right)\right) \tag{4}$$

where $Q_{\mathcal{X}_{(2N)}^2}(x)$ is the cumulative density function of a Chi-square distribution with $2N$ degrees of freedom.

Next, we consider the case with inter-cell interference. The mutual information between the first user and its BS in this case is given by

$$I_1 = \log_2\left(1 + \frac{\frac{P}{N}\boldsymbol{h}_{1,1}^H \boldsymbol{h}_{1,1}}{\frac{P}{N}\boldsymbol{h}_{1,2}^H \boldsymbol{h}_{1,2} + \sigma^2}\right) \tag{5}$$

The probability of outage of the first user is given by

$$\begin{aligned} P_{\mathrm{out},1} &= \Pr\left\{ \frac{\boldsymbol{h}_{1,1}^H \boldsymbol{h}_{1,1}}{\boldsymbol{h}_{1,2}^H \boldsymbol{h}_{1,2} + \frac{N\sigma^2}{P}} < \gamma \right\} && (6) \\ &= \Pr\left\{ \sum_{k=1}^{N} y_k < \frac{\gamma N \sigma^2}{P} \right\} && (7) \end{aligned}$$

where $\gamma = 2^R - 1$, $y_k = |h_{1,1}(k)|^2 - \gamma |h_{1,2}(k)|^2$, and $h_{i,j}(k)$ is the kth element of the vector $\boldsymbol{h}_{i,j}$. Since $\boldsymbol{h}_{1,1} \sim \mathcal{CN}\left(\boldsymbol{0}, \sigma_1^2 \boldsymbol{I}\right)$ and $\boldsymbol{h}_{1,2} \sim \mathcal{CN}\left(\boldsymbol{0}, \sigma_2^2 \boldsymbol{I}\right)$, then y_k are independent identically distributed random variables with probability density function

$$f_{y_k}(y_k) = \begin{cases} \frac{\lambda}{\lambda+\mu}\mu e^{-\mu y_k} & \text{if } y_k > 0. \\ \frac{\mu}{\lambda+\mu}\lambda e^{\lambda y_k} & \text{if } y_k < 0. \end{cases} \tag{8}$$

where $\lambda = \frac{1}{\sigma_1^2}$ and $\mu = \frac{1}{\gamma\sigma_2^2}$. The characteristic function $\phi_{Y_k}(t)$ of the random variable Y_k is given by

$$\begin{aligned} \phi_{Y_k}(t) &= \mathrm{E}\left\{e^{j y_k t}\right\} && (9) \\ &= \left(\frac{\lambda\mu}{\lambda+\mu}\right)\left(\frac{1}{\lambda + jt} + \frac{1}{\mu - jt}\right) && (10) \end{aligned}$$

where $j = \sqrt{-1}$. Since the random variables Y_k are independent, the characteristic function of the random variable $Z = \sum_{k=1}^{N} Y_k$ is given by

$$\begin{aligned} \phi_Z(t) &= \prod_{k=1}^{N} \phi_{Y_k}(t) && (11) \\ &= \left(\frac{\lambda\mu}{\lambda+\mu}\right)^N \left(\frac{1}{\lambda + jt} + \frac{1}{\mu - jt}\right)^N && (12) \end{aligned}$$

The probability density function of Z is the inverse Fourier transform of $\phi_Z(t)$

$$f_Z(z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-jtz}\phi_Z(t)\, dt \tag{13}$$

The integration (13) can be solved by using contour integration. The result of the integration equals $2\pi \sum_i r_i(z)$ where $r_i(z)$ is the residue of $e^{-jtz}\phi_Z(t)$ at the ith singular point. We have two singular points $t_1 = j\lambda$ and $t_2 = -j\mu$ repeated N times. If the singular point is repeated N times, its residue is given by

$$r_i(z) = \lim_{t \to t_i} \frac{1}{(N-1)!} \frac{d^{N-1}}{dt^{N-1}} (t - t_i)^N \phi_Z(t) e^{-itz} \tag{14}$$

When $z > 0$, the residue $r_1(z)$ is due to t_1. The residue $r_2(z)$ is due to t_2 when $z < 0$.

As an example, we consider the case when the number of the transmit antennas is three, $N = 3$. Evaluating the residues $r_1(z)$ and $r_2(z)$ using (36), we get

$$\begin{aligned} r_1(z) &= \frac{(\lambda\mu)^3}{2}\left(\frac{z^2 e^{-\mu z}}{(\lambda+\mu)^3} + \frac{6z e^{-\mu z}}{(\lambda+\mu)^4} + \frac{12 e^{-\mu z}}{(\lambda+\mu)^5}\right) && (15) \\ r_2(z) &= \frac{(\lambda\mu)^3}{2}\left(\frac{z^2 e^{\lambda z}}{(\lambda+\mu)^3} - \frac{6z e^{\lambda z}}{(\lambda+\mu)^4} + \frac{12 e^{\lambda z}}{(\lambda+\mu)^5}\right) && (16) \end{aligned}$$

Therefore, the PDF of z is given by

$$f_Z(z) = \begin{cases} \frac{(\lambda\mu)^3 e^{-\mu z}}{2(\lambda+\mu)^3}\left(z^2 + \frac{6z}{(\lambda+\mu)} + \frac{12}{(\lambda+\mu)^2}\right) & z > 0 \\ \frac{(\lambda\mu)^3 e^{\lambda z}}{2(\lambda+\mu)^3}\left(z^2 - \frac{6z}{(\lambda+\mu)} + \frac{12 e^{\lambda z}}{(\lambda+\mu)^2}\right) & z < 0 \end{cases} \tag{17}$$

A closed-form expression of the outage probability of the ith user can be obtained then as

$$P_{\text{out},i} = \int_{-\infty}^{\frac{N\sigma^2\gamma}{P}} f_z(z)\, dz \tag{18}$$

In direct ARQ systems, if an outage event occurs at any user, the user transmits a NACK signal to the BS. The BS retransmits the data vector again to the user. Assuming that the channel coefficients are independent from one transmission to another, the outage probability after L retransmissions is:

$$P_{\text{out},i}^{(\text{ARQ,L})} = (P_{\text{out},i})^L \tag{19}$$

As an illustrative example, let us consider a 2-cell MISO system with $N = 3$ antennas at the BSs. We use $\sigma^2 = 10^{-3}$, $\sigma_1^2 = 2$, $\sigma_2^2 = 1$, and $R = 2$ b/s/Hz. We define the transmit signal-to-noise ratio (SNR) as $SNR = \frac{P}{\sigma^2}$. Fig. 3 shows the outage probability of the system in the absence and presence of interference for different number of retransmission attempts. Even with ten retransmissions, the outage probability of the system in presence of inter-cell interference is inferior to that of a single user system. This can be attributed to the fact that direct retransmission can overcome the fading, however, it cannot overcome the interference.

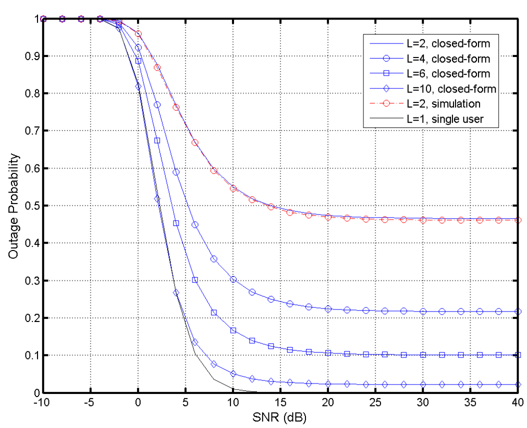

Fig. 1. Outage probability in the presence and absence of interference for a direct ARQ system.

III. Shared Relaying for MISO ARQ Systems

In order to manage the inter-cell interference, we assume that a MIMO half-duplex DF shared relay with M antennas is deployed to retransmit the lost packets to the target users. We assume that the relay can successfully decode the messages transmitted from the two BSs and the NACK signals transmitted by the users. This assumption is well justified, since the relay can be placed such that it has line of sight communication with the BSs. We also assume that the relay can acquire the transmit channel state information (CSI) to the users at the cell edge. This can be achieved with much smaller training overhead than that required to transmit the CSI to the BSs as the relay is placed in close proximity to the cell edge users. Nevertheless, we assume that the relay does not have any information about the channels from the BSs to the users.

The relay operates in one of two modes. The first mode is the single-user retransmission where only one user has failed to receive its message. In this case, the relay retransmits the signal for this user while the BS associated with the other user *simultaneously* transmits a new message to its designated user. In contrast, in the second mode, both users have failed to receive their messages, and hence, the relay forwards these messages to the users while the BSs are silent. In each case, we present a relay beamforming algorithm that aims to improve the outage probability of the system by managing the inter-cell interference associated with downlink transmission.

A. Single-User Retransmission

Let us assume that the first user has decoded its message while the other user has failed to decode its message. In the next time slot, the first BS retransmits a new message to its target user while the relay simultaneously retransmits the lost message to the second user. The received signal by the ith user in this case is given by

$$y_i = \sqrt{\frac{P}{N}}\boldsymbol{h}_{i,1}^H \boldsymbol{x}_1 + \boldsymbol{h}_{i,r}^H \boldsymbol{B}\boldsymbol{x}_r + \boldsymbol{n}_i \tag{20}$$

where $\boldsymbol{x}_r$ is the retransmitted message from the relay and is equal to the decoded message of the second user in the previous time slot, the $M \times N$ matrix $\boldsymbol{B}$ is the relay beamforming matrix, $\boldsymbol{h}_{i,r}$ is the $M \times 1$ vector containing the complex conjugate of the channel coefficients between the relay and the ith user. These channel coefficients are independent identically distributed zero-mean Gaussian random variables with variance σ_3^2.

In order to decrease the outage probability for the first user, we design the relay beamforming matrix such that the relay transmission does not cause any interference. In order to improve the outage probability of the second user, we maximize the received signal power at the second user given the constraint on the relay transmission power. Hence, we can write the relay design problem as

$$\max_{\boldsymbol{B}} \left\|\boldsymbol{B}^H \boldsymbol{h}_{2,r}\right\|^2 \quad \text{s.t. } \boldsymbol{B}^H \boldsymbol{h}_{1,r} = \boldsymbol{0}_N,\ \text{tr}\left\{\boldsymbol{B}\boldsymbol{B}^H\right\} = P_r \tag{21}$$

where tr$\{\cdot\}$ denotes the trace of a matrix and P_r is the relay transmission power. For the sake of fairness of the comparison with direct ARQ case, we use $P_r = P$. Using the identity $\text{vec}\{\boldsymbol{ABC}\} = \left(\boldsymbol{C}^T \otimes \boldsymbol{A}\right)\text{vec}\{\boldsymbol{B}\}$ where vec$\{\cdot\}$ is the vectorization operator and $\otimes$ is the matrix Kronecker product. The optimization problem in (21) can be expressed as

$$\begin{aligned} \max_{\tilde{\boldsymbol{b}}} \quad & \tilde{\boldsymbol{b}}^H \left(\boldsymbol{I}_N \otimes \boldsymbol{h}_{2,r}\boldsymbol{h}_{2,r}^H\right)\tilde{\boldsymbol{b}} \\ \text{s.t.} \quad & \left(\boldsymbol{I}_N \otimes \boldsymbol{h}_{1,r}^H\right)\tilde{\boldsymbol{b}} = \boldsymbol{0}_{MN},\ \left\|\tilde{\boldsymbol{b}}\right\|^2 = P_r \end{aligned} \tag{22}$$

where $\tilde{\boldsymbol{b}} = \text{vec}\{\boldsymbol{B}\}$. Let us define the orthonormal $MN \times (M-1)N$ matrix $\boldsymbol{V}$ such that its columns span the null space of the matrix $\boldsymbol{I}_N \otimes \boldsymbol{h}_{1,r}^H$. Therefore, we can write the optimal solution of the optimization problem in (22) as

$$\tilde{\boldsymbol{b}} = \sqrt{P_r}\,\boldsymbol{V}\nu_{\max}\left\{\boldsymbol{V}^H\left(\boldsymbol{I}_N \otimes \boldsymbol{h}_{2,r}\boldsymbol{h}_{2,r}^H\right)\boldsymbol{V}\right\} \tag{23}$$

where $\nu_{\max}\{\boldsymbol{A}\}$ is the eigen vector of the matrix $\boldsymbol{A}$ associated with its maximum eigen value.

Similarly, we can calculate the mutual information between the transmitting and receiving nodes as

$$I_1^{(\mathrm{I})} = \log_2\left(1 + \frac{\frac{P}{N}\boldsymbol{h}_{1,1}^H\boldsymbol{h}_{1,1}}{\boldsymbol{h}_{1,r}^H\boldsymbol{B}\boldsymbol{B}^H\boldsymbol{h}_{1,r} + \sigma^2}\right) \tag{24}$$

$$I_2^{(\mathrm{I})} = \log_2\left(1 + \frac{\boldsymbol{h}_{2,r}^H\boldsymbol{B}\boldsymbol{B}^H\boldsymbol{h}_{2,r}}{\frac{P}{N}\boldsymbol{h}_{2,1}^H\boldsymbol{h}_{2,1} + \sigma^2}\right) \tag{25}$$

t

B. Multiuser Retransmission

In the case when both users fail to decode their transmitted messages, the relay forwards them to the users in the next time slot while the BSs remain silent. The transmitted signal by the relay in this scenario is given by

$$\boldsymbol{x}_r = \boldsymbol{B}_1\boldsymbol{x}_1 + \boldsymbol{B}_2\boldsymbol{x}_2 \tag{26}$$

The received signal at the ith user is given by

$$y_i = \boldsymbol{h}_{i,r}^H\boldsymbol{B}_1\boldsymbol{x}_1 + \boldsymbol{h}_{i,r}^H\boldsymbol{B}_2\boldsymbol{x}_2 + \boldsymbol{n}_i \tag{27}$$

As a result, we can write the received SINR of the ith user as

$$\text{SINR}_i = \frac{\boldsymbol{h}_{i,r}^H\boldsymbol{B}_i\boldsymbol{B}_i^H\boldsymbol{h}_{i,r}}{\boldsymbol{h}_{i,r}^H\boldsymbol{B}_j\boldsymbol{B}_j^H\boldsymbol{h}_{i,r} + \sigma^2} \tag{28}$$

where $i \in \{1,2\}$ and $i \neq j$.

In order to decrease the outage probability, we design the relay beamforming matrices such that the minimum SINR of the two users is maximized under the constraint on the relay transmission power. Using the auxiliary variable t, we can write the relay design problem as

$$\begin{aligned} \max_{\boldsymbol{B}_1,\boldsymbol{B}_2,t} \quad & t \\ \text{s.t.} \quad & \text{SINR}_i \geq t \qquad i = 1,2 \\ & \text{tr}\left(\boldsymbol{B}_1\boldsymbol{B}_1^H + \boldsymbol{B}_2\boldsymbol{B}_2^H\right) \leq P_r \end{aligned} \tag{29}$$

where P_r is chosen in this case as $P_r = 2P$ for the sake of fairness when comparing with the direct ARQ system. Problem (29) can be solved by using the Bisection method as a sequence of feasibility problems as follows [9]. We initialize the lower bound b_l and the upper bound b_u on the objective function respectively as, $b_l = 0$, and

$$b_u = \min_{i=1,2} \frac{P_r\boldsymbol{h}_{i,r}^H\boldsymbol{h}_{i,r}}{\sigma^2} \tag{30}$$

At each iteration, we solve a feasibility problem in the variables $\boldsymbol{B}_1$ and $\boldsymbol{B}_2$ at $t = (b_l + b_u)/2$ for the constraints in (29). If the problem is feasible we set $b_l = t$, else, we set $b_u = t$. The procedure is repeated until $b_u - b_l \leq \epsilon$ where ϵ is the required accuracy for the solution. The resulting feasibility problem is not convex due to the SINR constraints in (29). Let $\tilde{\boldsymbol{b}}_i = \text{vec}\{\boldsymbol{B}_i\}$, then we can write the feasibility problem as

$$\begin{aligned} \text{Find} \quad & \tilde{\boldsymbol{b}}_1, \tilde{\boldsymbol{b}}_2 \\ \text{s.t.} \quad & \text{tr}\left\{\boldsymbol{C}_i\tilde{\boldsymbol{b}}_i\tilde{\boldsymbol{b}}_i^H\right\} \geq t\text{tr}\left\{\boldsymbol{C}_i\tilde{\boldsymbol{b}}_j\tilde{\boldsymbol{b}}_j^H\right\} + t\sigma^2 \quad i,j = 1,2, i \neq j \\ & \text{tr}\left\{\tilde{\boldsymbol{b}}_1\tilde{\boldsymbol{b}}_1^H\right\} + \text{tr}\left\{\tilde{\boldsymbol{b}}_2\tilde{\boldsymbol{b}}_2^H\right\} \leq P_r \end{aligned} \tag{31}$$

where $\boldsymbol{C}_i = \boldsymbol{I}_N \otimes \boldsymbol{h}_{i,r}\boldsymbol{h}_{i,r}^H$ and $\text{tr}\{\cdot\}$ denotes the trace of a matrix. Let us define the $MN \times MN$ matrices $\boldsymbol{X}_i = \tilde{\boldsymbol{b}}_i\tilde{\boldsymbol{b}}_i^H$. We use SDR to convert the problem in (31) into a convex optimization problem. The relaxed problem can be expressed as the following semi-definite program

$$\begin{aligned} \text{Find} \quad & \boldsymbol{X}_1, \boldsymbol{X}_2 \\ \text{s.t.} \quad & \text{tr}\{\boldsymbol{C}_i\boldsymbol{X}_i\} \geq t\text{tr}\{\boldsymbol{C}_i\boldsymbol{X}_j\} + t\sigma^2 \quad i,j = 1,2, i \neq j \\ & \text{tr}\{\boldsymbol{X}_1\} + \text{tr}\{\boldsymbol{X}_2\} \leq P_r \\ & \boldsymbol{X}_i \succeq 0 \qquad i = 1,2 \end{aligned} \tag{32}$$

where we have relaxed the problem by dropping the rank one constraints on the matrices $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$.

The optimization problem (32) is similar to problem (15) in [10] and has an arbitrary rank profile. Nevertheless, Theorem 3.2 in [10] states that, we can generate another optimal solution $(\boldsymbol{Z}_1^\star, \boldsymbol{Z}_2^\star)$ from the optimal solution of (32), $(\boldsymbol{X}_1^\star, \boldsymbol{X}_2^\star)$, with a constrained rank profile that satisfies

$$2 \leq \text{rank}^2\{\boldsymbol{Z}_1^\star\} + \text{rank}^2\{\boldsymbol{Z}_2^\star\} \leq 3 \tag{33}$$

Since the summation at (33) includes 2 matrices and the rank of each matrix is greater than zero, this implies that the rank of $(\boldsymbol{Z}_n^\star) = 1$, for all $n = 1,2$.

In the following, we present Algorithm1 [10] to generate a rank one optimal solution, $\{\boldsymbol{Z}_i^\star\}_{i=1}^2$, from the arbitrary rank solution of (32), $\{\boldsymbol{X}_i^\star\}_{i=1}^2$, without any loss of optimality. Let $r_n = \text{rank}(\boldsymbol{X}_n^\star)$ and $W = r_1^2 + r_2^2$.

while $W > 3$ do

1) Decompose $\boldsymbol{X}_n^\star = \boldsymbol{V}_n\boldsymbol{V}_n^H$, $n = 1,2$.
2) Find $(\boldsymbol{\Delta}_1, \boldsymbol{\Delta}_2)$ a nonzero solution of the following system of linear equations:

$$\text{tr}\left\{\boldsymbol{V}_1^H\,\boldsymbol{C}_1\boldsymbol{V}_1\boldsymbol{\Delta}_1\right\} - t\,\text{tr}\left\{\boldsymbol{V}_2^H\,(\boldsymbol{C}_1)\,\boldsymbol{V}_2\boldsymbol{\Delta}_2\right\} = 0 \tag{34}$$

$$\text{tr}\left\{\boldsymbol{V}_2^H\boldsymbol{C}_2\boldsymbol{V}_2\boldsymbol{\Delta}_2\right\} - t\,\text{tr}\left\{\boldsymbol{V}_1^H\,(\boldsymbol{C}_2)\,\boldsymbol{V}_1\boldsymbol{\Delta}_1\right\} = 0 \tag{35}$$

$$\text{tr}\left\{\boldsymbol{V}_1^H\boldsymbol{V}_1\boldsymbol{\Delta}_1\right\} + \text{tr}\left\{\boldsymbol{V}_2^H\,\boldsymbol{V}_2\boldsymbol{\Delta}_2\right\} = 0 \tag{36}$$

   where $\boldsymbol{\Delta}_n$ is $r_n \times r_n$ Hermitian matrix for all n;
3) Find the eigenvalues $\delta_{n,1}, \ldots, \delta_{n,r_n}$ of $\boldsymbol{\Delta}_n$ for $n = 1,2$;
4) Determine n_0 and k_0 such that $|\delta_{n_0,k_0}| = \max\{|\delta_{n,k}| : 1 \leqslant k \leqslant r_n, 1 \leqslant n \leqslant 2\}$.
5) Compute $\boldsymbol{Z}_n^\star = \boldsymbol{V}_n\left(\boldsymbol{I}_{r_n} - \frac{1}{\delta_{n_0,k_0}}\boldsymbol{\Delta}_n\right)\boldsymbol{V}_n^H$ for $n = 1,2$.
6) Evaluate $r_n = \text{rank}(\boldsymbol{Z}_n^\star)$ for $n = 1,2$ and $W = \sum_{n=1}^{2} r_n^2$;

end while

IV. SIMULATION AND ANALYTICAL RESULTS

In this section, we present simulation results that compare the performance of the proposed relaying scheme with direct ARQ systems. The outage probability is used as the performance metric. The comparison is performed under equal power allocation between the different systems. We assume only one retransmission round. In simulations, we use $\sigma^2 = 1$, $\sigma_1^2 = 2$, $\sigma_2^2 = 1$ and $\sigma_3^2 = 4$, and $N = 3$.

Fig. 2 shows the performance of the proposed ARQ system with a MIMO shared relay with $M = 3$ and the direct ARQ system at different transmission rates. The single user case serves as a reference for outage probability with no interference. The proposed ARQ relay system works efficiently at high transmission rates compared with the direct ARQ system even at very high transmission rates.

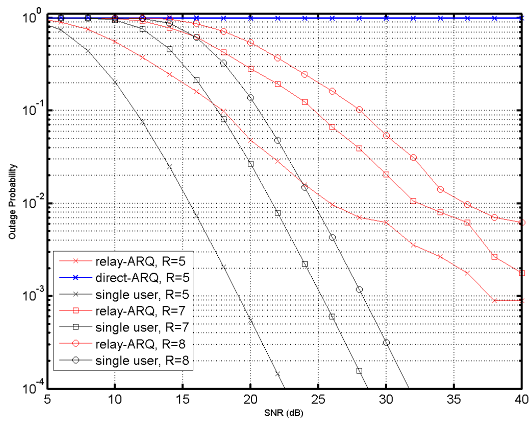

Fig. 2. Outage probability of the proposed ARQ relay system, direct ARQ and single user at different transmission rates .

Fig. 3 depicts the performance of the proposed ARQ system with the MIMO shared relay for different numbers of relay antennas at certain transmission rate $R = 6$. With sufficient number of antennas at the relay, the proposed system model attains high diversity gain, minimizes the interference, and achieves performance even better than the single user case.

V. CONCLUSION

In this paper, we have considered a MIMO shared relay operating in the downlink of an ARQ wireless cellular system. We have proposed relay beamforming techniques that improve the outage probability by maximizing the received SINR at the users under a constraint on the relay transmission power. The performance of the proposed algorithms was compared to that of direct ARQ systems via numerical simulations showing that significant performance improvement can be achieved by the proposed system.

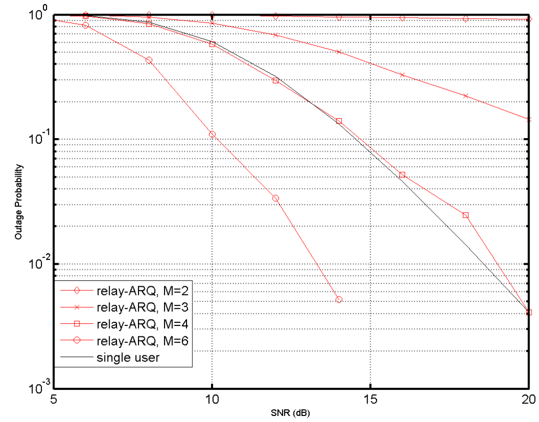

Fig. 3. Outage probability of the proposed ARQ relay system at different number of relay antennas and single user case under R=6.

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