Downlink Power Control in User-Centric and Cell-Free Massive MIMO Wireless Networks

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Abstract—Recently, the so-called cell-free Massive MIMO architecture has been introduced, wherein a very large number of distributed access points (APs) simultaneously and jointly serve a much smaller number of mobile stations (MSs). A variant of the cell-free technique is the user-centric approach, wherein each AP just decodes the MSs that it receives with the largest power. This paper considers both the cell-free and user-centric approaches, and, using an interplay of sequential optimization and alternating optimization, derives downlink power-control algorithms aimed at maximizing either the minimum users' SINR (to ensure fairness), or the system sum-rate. Numerical results show the effectiveness of the proposed algorithms, as well as that the user-centric approach generally outperforms the CF one.

I. INTRODUCTION

Cell-free (CF) massive MIMO is a recent system concept [1], [2] addressing the cell-edge problem: a very large number of distributed APs (connected via a backhaul network to a central CPU) simultaneously and jointly serve a much smaller number of MSs; each AP uses local channel estimates obtained from received uplink pilots and applies channel inversion beamforming to transmit data to the users. Papers [1], [2] show that the CF approach provides better performance than a smallcell system in terms of 95%-likely per-user throughput. Additionally, the paper [3] has analyzed the performance improvements granted by the use of a zero-forcing precoder in the downlink. In [4] the CF approach is extended to the case in which multiple antennas are employed and propose the use of a channel-independent beamformer at the MSs, so that channel estimation is performed only in the uplink at the APs, and channel reciprocity induced by time division duplex is exploited at the APs to implement coherent signal reception at the MSs. The paper [4] also introduces a user-centric (UC) approach wherein each AP chooses to serve only a predetermined number of MSs. The UC approach is shown to provide larger achievable rates (with respect to the CF approach) to the vast majority of the users, and, also works with reduced backhaul overhead; a comparison

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between the CF and the UC approaches in a singleantenna setting is provided in [5]. While the papers [4], [5] consider a simple uniform power control allocation on the downlink, in this paper we provide power control algorithms aimed at maximizing either the minimum rate across the users – to ensure fairness – or the system sum rate. The considered optimization problems are nonconvex and involve a very large number of variables (equal to the product of the number of APs times the number of MSs); to address these challenges, the frameworks of sequential optimization and alternating optimization are combined in a suitable way. Our results show that the proposed power control algorithms are effective, as well as that the UC approach generally outperforms the CF one.

II. SYSTEM MODEL

We consider an area with K MSs and M APs. MSs and APs are randomly located. The M APs are connected by means of a backhaul network to a central processing unit (CPU) wherein data-decoding is performed. The TDD protocol is used and the channel coherence interval is divided into three phases: (a) uplink channel estimation, (b) downlink data transmission, and (c) uplink data transmission. In phase (a) the MSs send pilot data in order to enable channel estimation at the APs. In phase (b) the APs use channel estimates to perform channel-matched beamforming and send data symbols on the downlink; while in the CF architecture the APs send data to all the MSs in the system, in the UC approach each AP sends data only to a subset of the MSs in the system. Phase (c) is not described since the focus of this paper is only on the downlink.

A. Channel model

We denote by the $(N_{AP} \times N_{MS})$ -dimensional matrix $\mathbf{G}_{k,m}$ the channel between the k-th MS and the m-th AP. We have

$$\mathbf{G}_{k,m} = \beta_{k,m}^{1/2} \mathbf{H}_{k,m} , \qquad (1)$$

with $\beta_{k,m}$ a scalar coefficient modeling the channel shadowing effects and $\mathbf{H}_{k,m}$ an $(N_{\text{AP}} \times N_{\text{MS}})$ -dimensional matrix whose entries are i.i.d $\mathcal{CN}(0,1)$ RVs. Path loss and shadow fading are modeled by the large scale coefficient $\beta_{k,m}$ in (1), according to the formula [2]

$$\beta_{k,m} = 10^{\frac{\mathrm{PL}_{k,m}}{10}} 10^{\frac{\sigma_{\mathrm{sh}} z_{k,m}}{10}}, \qquad (2)$$

where $PL_{k,m}$ represents the path loss (expressed in dB) from the *k*-th MS to the *m*-th AP, and $10^{\frac{\sigma_{\rm sh}z_{k,m}}{10}}$ represents the shadow fading with standard deviation $\sigma_{\rm sh}$, while $z_{k,m}$ will be specified later. For the path loss we use the following three slope path loss model [6]:

$$PL_{k,m} = \begin{cases} -L - 35 \log_{10} (d_{k,m}), & \text{if } d_{k,m} > d_1 \\ -L - 10 \log_{10} \left(d_1^{1.5} d_{k,m}^2 \right), & \text{if } d_0 < d_{k,m} \le d_1 \\ -L - 10 \log_{10} \left(d_1^{1.5} d_0^2 \right), & \text{if } d_{k,m} < d_0 \end{cases}$$
(3)

where $d_{k,m}$ denotes the distance between the *m*-th AP and the *k*-th user, *L* is

$$L = 46.3 + 33.9 \log_{10} (f) - 13.82 \log_{10} (h_{\rm AP}) - [1.11 \log_{10} (f) - 0.7] h_{\rm MS} + 1.56 \log_{10} (f) - 0.8,$$
(4)

f is the carrier frequency in MHz, $h_{\rm AP}$ and $h_{\rm MS}$ denote the AP and MS antenna heights, respectively. In realworld scenarios, transmitters and receivers that are in close vicinity of each other may be surrounded by common obstacles, and hence, the shadow fading RVs are correlated; for the shadow fading coefficient we thus use a model with two components [7]

$$z_{k,m} = \sqrt{\delta}a_m + \sqrt{1-\delta}b_k, \quad m = 1, \dots, M, \quad k = 1, \dots, K$$
(5)

where $a_m \sim \mathcal{N}(0, 1)$ and $b_k \sim \mathcal{N}(0, 1)$ are independent RVs, and δ , $0 \leq \delta \leq 1$ is a parameter. The covariance functions of a_m and b_k are given by:

$$E\left[a_{m}a_{m'}\right] = 2^{-\frac{d_{\mathrm{AP}(m,m')}}{d_{\mathrm{decorr}}}} \quad E\left[b_{k}b_{k'}\right] = 2^{-\frac{d_{\mathrm{MS}(k,k')}}{d_{\mathrm{decorr}}}},$$
(6)

where $d_{AP(m,m')}$ is the geographical distance between the *m*-th and *m'*-th APs and $d_{MS(k,k')}$ is the geographical distance between the *k*-th and the *k'*-th MSs. The parameter d_{decorr} is a decorrelation distance which depends on the environment, typically this value is in the range 20-200 m.

B. Uplink training

During this phase the MSs send uplink training pilots in order to permit channel estimation at the APs. This phase is the same for both the UC and CF approaches. We denote by τ_c the length (in samples) of the channel coherence time, and by τ_p the length (in samples) of the uplink training phase. Of course we must ensure that $\tau_p < \tau_c$. Denote by $\Phi_k \in C^{N_{MS} \times \tau_p}$ the pilot sequence sent by the k-th MS, and assume that $\|\Phi_k\|_F^2 = 1$; we also assume that the pilot sequences assigned to each user are mutually orthogonal, so that $\Phi_k \Phi_k^H = \mathbf{I}_{N_{MS}}$, while, instead, pilot sequences from different users are non-orthogonal. The signal received at the m-th AP in the *n*-th signaling time is represented by the following $N_{\rm AP}$ -dimensional vector:

$$\mathbf{y}_m(n) = \sum_{k=1}^K \sqrt{p}_k \mathbf{G}_{k,m} \mathbf{\Phi}_k(:,n) + \mathbf{w}_m(n) , \quad (7)$$

with \sqrt{p}_k the k-th user's transmit power during the training phase. Collecting all the observable vectors $\mathbf{y}_m(n)$, for $n = 1, \ldots, \tau_p$ into the $(N_{\text{AP}} \times \tau_p)$ -dimensional matrix \mathbf{Y}_m , and assuming simple pilot-matched (PM) single-user channel estimation, the estimate, $\widehat{\mathbf{G}}_{k,m}$ say, dof the channel matrix $\mathbf{G}_{k,m}$ is obtained at the m-th AP according to $\widehat{\mathbf{G}}_{k,m} = \frac{1}{\sqrt{p_k}} \mathbf{Y}_m \mathbf{\Phi}_k^H$.

C. Downlink data transmission

After that each AP has obtained estimates of the channel matrix from all the MSs in the system, the downlink data transmission phase begins. The APs treat the channel estimates as the true channels, and channel inversion beamforming is performed to transmit data to the MSs. Denoting by P_k the multiplexing order (i.e., the number of simultaneous data-streams) for user k, and by $\mathbf{x}_k^{\mathrm{DL}}(n)$ the P_k -dimensional unit-norm vector containing the k-th user data symbols to be sent in the n-th sample time, and letting $\mathbf{L}_k = \mathbf{I}_{P_k} \otimes \mathbf{1}_{\mathrm{MMS}/P_k}$, the downlink precoder at the m-th AP for the k-th MS is expressed as

$$\mathbf{Q}_{k,m}^{\mathrm{DL}} = \frac{\widehat{\mathbf{G}}_{k,m} \left(\widehat{\mathbf{G}}_{k,m}^{H} \widehat{\mathbf{G}}_{k,m}\right)^{-1} \mathbf{L}_{k}}{\sqrt{\mathrm{tr} \left(\mathbf{L}_{k}^{H} \left(\widehat{\mathbf{G}}_{k,m}^{H} \widehat{\mathbf{G}}_{k,m}\right)^{-1} \mathbf{L}_{k}\right)}} .$$
(8)

1) UC massive MIMO architecture: In the usercentric approach, we assume that each AP communicates only with a fixed number N of MSs. The generic m-th AP, based on the knowledge of the channel estimates $\widehat{\mathbf{G}}_{k,m}$, for $k = 1, \ldots, K$, sorts them in descending order according to the Frobenius norm, and selects the first N. We denote by $\mathcal{K}(m)$ the set of MSs served by the m-th AP. Given the sets $\mathcal{K}(m)$, for all $m = 1, \ldots, M$, we can define the set $\mathcal{M}(k)$ of the APs that communicate with the k-th user:

$$\mathcal{M}(k) = \{m : k \in \mathcal{K}(m)\}\tag{9}$$

So, in this case, the signal transmitted by the *m*-th AP in the *n*-th interval is the following $N_{\rm AP}$ -dimensional vector

$$\mathbf{s}_{m}^{\mathrm{uc}}(n) = \sum_{k \in \mathcal{K}(m)} \sqrt{\eta_{k,m}^{\mathrm{DL},\mathrm{uc}}} \mathbf{Q}_{k,m}^{\mathrm{DL}} \mathbf{x}_{k}^{\mathrm{DL}}(n) , \qquad (10)$$

with $\eta_{k,m}^{\text{DL,uc}}$ a scalar coefficient representing the power transmitted by the *m*-th AP to the *k*-th MS. The generic

k-th MS receives signal contributions from all the APs and so the observable vector is expressed as

$$\mathbf{r}_{k}^{\mathrm{uc}}(n) = \sum_{m=1}^{M} \mathbf{G}_{k,m}^{H} \mathbf{s}_{m}^{\mathrm{uc}}(n) + \mathbf{z}_{k}(n)$$
$$= \sum_{m \in \mathcal{M}(k)} \sqrt{\eta_{k,m}^{\mathrm{DL},\mathrm{uc}}} \mathbf{G}_{k,m}^{H} \mathbf{Q}_{k,m}^{\mathrm{DL}} \mathbf{x}_{k}^{\mathrm{DL}}(n) +$$
$$\sum_{j=1, j \neq k}^{K} \sum_{m \in \mathcal{M}(j)} \sqrt{\eta_{j,m}^{\mathrm{DL},\mathrm{uc}}} \mathbf{G}_{k,m}^{H} \mathbf{Q}_{j,m}^{\mathrm{DL}} \mathbf{x}_{j}^{\mathrm{DL}}(n) +$$
(11)

In (11), the $N_{\rm MS}$ -dimensional vector $\mathbf{z}_k(n)$ represents the thermal noise and out-of-cluster interference at the *k*-th MS, and its entries are modeled as i.i.d. $\mathcal{CN}(0, \sigma_z^2)$ RVs. Based on the observation of the vector $\mathbf{r}_k^{\rm uc}(n)$, a soft estimate of the data symbols $\mathbf{x}_k^{\rm DL}(n)$ is obtained at the *k*-th MS as

$$\widehat{\mathbf{x}}_{k}^{\mathrm{DL},\mathrm{uc}}(n) = \mathbf{L}_{k}^{H} \mathbf{r}_{k}^{\mathrm{uc}}(n) .$$
 (12)

2) *CF* massive MIMO architecture: In the CF architecture all the APs communicate with all the MSs in the systems, so the CF case can be obtained as a special case of the UC approach letting N = K, which leads to $\mathcal{K}(m) = \{1, \ldots, K\}$, for all $m = 1, \ldots, M$ and $\mathcal{M}(k) = \{1, \ldots, M\}$, for all $k = 1, \ldots, K$.

III. DOWNLINK POWER CONTROL

From (11), we have that the achievable rate for the user k in the user-centric case is written as

$$\mathcal{R}_{k} = W \log_{2} \det \left[\mathbf{I} + \mathbf{R}_{k}^{-1} \mathbf{A}_{k,k} \mathbf{A}_{k,k}^{H} \right] , \qquad (13)$$

where $\mathbf{R}_k = \sigma_z^2 \mathbf{L}_k^H \mathbf{L}_k + \sum_{j=1, j \neq k}^K \mathbf{A}_{k,j} \mathbf{A}_{k,j}^H$, and

$$\mathbf{A}_{k,j} = \mathbf{L}_{k}^{H} \sum_{m \in \mathcal{M}(j)} \sqrt{\eta_{j,m}^{\mathrm{DL},\mathrm{uc}}} \mathbf{G}_{k,m}^{H} \widehat{\mathbf{G}}_{j,m} \left(\widehat{\mathbf{G}}_{j,m}^{H} \widehat{\mathbf{G}}_{j,m} \right)^{-1} \mathbf{L}_{j}$$
(14)

In order to obtain the achievable rate expressions for the classical cell-free MIMO case, it suffices to consider in the above expressions $\mathcal{M}(k) = \{1, 2, \dots, K\}$, for all $k = 1, \dots, K$.

The rest of this work will be concerned with the optimization of the downlink transmit powers for the maximization of the system sum-rate and minimum users' rate, subject to maximum power constraints. Mathematically, the sum-rate maximization problem is formulated as the optimization program:

$$\max_{\boldsymbol{\eta}} \sum_{k=1}^{K} \mathcal{R}_k(\boldsymbol{\eta})$$
(15a)

s.t.
$$\sum_{k \in \mathcal{K}_m} \eta_{k,m} \le P_{max,m} , \forall \ m = 1, \dots, M$$
(15b)

$$\eta_{k,m} \ge 0, \forall m = 1, \dots, M, \ k = 1, \dots, K$$
 (15c)

whereas the minimum rate maximization problem is

$$\max_{\boldsymbol{\eta}} \min_{1 \le k \le K} \mathcal{R}_k(\boldsymbol{\eta}) \tag{16a}$$

s.t.
$$\sum_{k \in \mathcal{K}_m} \eta_{k,m} \le P_{max,m} , \forall \ m = 1, \dots, M$$
(16b)

$$\eta_{k,m} \ge 0 , \forall \ m = 1, \dots, M, \ k = 1, \dots, K$$
 (16c)

with η the KM×1 vector collecting the transmit powers
z_k (f_h)all access points. Both problems have non-concave objective functions, which makes their solution challenging. Moreover, even if the problems were concave, the large number of optimization variables, KM, would still pose a significant complexity challenge¹. In order to face
these issues, we will resort to the framework of successaries lower-bound maximization, recently introduced in²
t. [9], and briefly reviewed next.

A. Successive lower-bound maximization

The main idea of the method is to merge the tools of alternating optimization [10, Section 2.7] and sequential convex programming [11]. To elaborate, consider the generic optimization problem

$$\max_{\boldsymbol{x}\in\mathcal{X}}f(\boldsymbol{x}),\qquad(17)$$

with $f : \mathbb{R}^n \to \mathbb{R}$ a differentiable function, and \mathcal{X} a compact set. As in the alternating optimization method, the successive lower-bound maximization partitions the variable space into M blocks, $\boldsymbol{x} = (\boldsymbol{x}_1, \ldots, \boldsymbol{x}_M)$, which are cyclically optimized one at a time, while keeping the other variable blocks fixed. This effectively decomposes (17) into M subproblems, with the generic subproblem stated as

$$\max_{\boldsymbol{x}} f(\boldsymbol{x}_m, \boldsymbol{x}_{-m}) , \qquad (18)$$

with \boldsymbol{x}_{-m} collecting all variable blocks except the *m* th. It is proved in [10, Proposition 2.7.1] that iteratively solving (18) monotonically improves the value of the objective of (17), and converges to a first-order optimal point if the solution of (18) is unique for any *m*, and if $\mathcal{X} = \mathcal{X}_1 \times \ldots \times \mathcal{X}_M$, with $\boldsymbol{x}_m \in \mathcal{X}_m$ for all *m*.

Clearly, alternating optimization proves useful when (18) can be solved with minor complexity. If this is not the case, the successive lower-bound maximization method proposes to tackle (18) by means of sequential convex programming. This does not guarantee to globally solve (18), but can lead to a computationally feasible algorithm. Moreover, it is guaranteed to preserve the properties of the alternating optimization method [9]. The idea of sequential optimization is to tackle a difficult

¹Although polynomial, the best known upper-bound for the complexity of generic convex problems scales with the fourth power of the number of variables, while many classes of convex problems admit a cubic complexity [8].

²In [9] the method is labeled successive upper-bound minimization, since minimization problems are considered.

maximization problem by solving a sequence of easier maximization problems. To elaborate, let us denote by $g_i(\boldsymbol{x}_m)$ the *i*-th constraint of (18), for $i = 1, \ldots, C$. Then, consider a sequence of approximate problems $\{\mathcal{P}_\ell\}_\ell$ with objectives $\{f_\ell\}_\ell$ and constraint functions $\{g_{i,\ell}\}_{i=1}^C$, such that the following three properties are fulfilled, for all ℓ :

(P1)
$$f_{\ell}(\boldsymbol{x}_m) \leq f(\boldsymbol{x}_m), \ g_{i,\ell}(\boldsymbol{x}_m) \leq g_{i,\ell}(\boldsymbol{x}_m), \ \text{for all } i \text{ and } \boldsymbol{x}_m;$$

(P2)
$$f_{\ell}(\boldsymbol{x}_{m}^{(\ell-1)}) = f(\boldsymbol{x}_{m}^{(\ell-1)}), \quad g_{i,\ell}(\boldsymbol{x}_{m}^{(\ell-1)}) = g_{i,\ell}(\boldsymbol{x}_{m}^{(\ell-1)})$$

(P3)
$$\nabla f_{\ell}(\boldsymbol{x}_{m}^{(\ell-1)}) = \nabla f(\boldsymbol{x}_{m}^{(\ell-1)}), \ \nabla g_{i,\ell}(\boldsymbol{x}_{m}^{(\ell-1)}) = \nabla g_{i}(\boldsymbol{x}_{m}^{(\ell-1)}).$$

In [11] (see also [9], [12]) it is shown that, subject to constraint qualifications, the sequence $\{f(x_m^{(\ell)})\}_{\ell}$ of the solutions of the ℓ -th Problem \mathcal{P}_{ℓ} , is monotonically increasing and converges. Moreover, every convergent sequence $\{x_m^{(\ell)}\}_{\ell}$ attains a first-order optimal point of the original Problem (18). Thus, the sequential approach enjoys strong optimality properties, fulfilling at the same time the monotonic improvement property for the objective function, and the Karush Kuhn Tucker (KKT) first-order optimality conditions for the original problem. Nevertheless, its applicability hinges on determining suitable lower bounds for the original objective to maximize, which fulfill all three properties **P1**, **P2**, **P3**, while at the same time leading to manageable optimization problems.

In conclusion, the successive lower-bound maximization method can be formulated as variation of the alternating optimization method, in which each subproblem (18) is not globally solved, but instead is tackled by sequential optimization theory. It is proved in [9] that successive lower-bound maximization has the same optimality properties as the true alternating optimization method, under similar assumptions, even though each subproblem might not be globally solved³.

B. Sum-rate maximization

Consider Problem (15) and define the variable blocks η_m , m = 1, ..., M, collecting the transmit powers of access point m. Then, the sum-rate maximization with respect to the variable block η_m is cast as

$$\max_{\boldsymbol{\eta}_m} \sum_{k=1}^{K} \mathcal{R}_k(\boldsymbol{\eta}_m, \boldsymbol{\eta}_{-m})$$
(19a)

s.t.
$$\sum_{k \in \mathcal{K}_m} \eta_{k,m} \le P_{max,m} \tag{19b}$$

$$\eta_{k,m} \ge 0 , \forall \ k \in \mathcal{K}_m \tag{19c}$$

The complexity of (19) is significantly lower than that of (15), since only the M transmit powers of access point m

³Of course, this holds provided the additional assumption of the sequential method are fulfilled in each iteration

are being optimized. Nevertheless, Problem (19) is still non-convex, which makes its solution difficult. Indeed, defining

$$\mathbf{A}_{k,j,m} = \mathbf{L}_{k}^{H} \mathbf{G}_{k,m}^{H} \widehat{\mathbf{G}}_{j,m} \left(\widehat{\mathbf{G}}_{j,m}^{H} \widehat{\mathbf{G}}_{j,m} \right)^{-1} \mathbf{L}_{j} , \quad (20)$$

the k-th user's achievable rate can be expressed as

$$\mathcal{R}_{k}(\boldsymbol{\eta}) = \underbrace{W \log_{2} \left| \sigma_{z}^{2} \mathbf{L}_{k}^{H} \mathbf{L}_{k} + \sum_{j=1}^{K} \sum_{m,\ell} \sqrt{\eta_{j,m} \eta_{j,\ell}} \mathbf{A}_{k,j,m} \mathbf{A}_{k,j,\ell}^{H} \right|}_{g_{1}(\boldsymbol{\eta})}$$
(21)

$$-\underbrace{W \log_2 \left| \sigma_z^2 \mathbf{L}_k^H \mathbf{L}_k + \sum_{j=1, j \neq k}^K \sum_{m, \ell} \sqrt{\eta_{j,m} \eta_{j,\ell}} \mathbf{A}_{k,j,m} \mathbf{A}_{k,j,\ell}^H \right|}_{g_2(\boldsymbol{\eta})} \right|$$

which can be seen to be non-concave, also with respect to only the variable block η_m . Thus, following the successive lower-bound maximization, (19) will be tackled by sequential optimization. To this end, it is necessary to derive a lower-bound of the objective of (19), which fulfills Properties **P1**, **P2**, and **P3**, while at the same time leading to a simple optimization problem. To this end, the following lemma proves useful.

Lemma 1. The function $f : (x, y) \in \mathbb{R}^2 \to \sqrt{xy}$ is jointly concave in x and y, for x, y > 0.

Proof: The proof follows upon computing the Hessian of \sqrt{xy} and showing that it is negative semi-definite. Details are omitted due to space constraints.

Lemma 1, coupled with the facts that the function $\log_2 |(\cdot)|$ is matrix-increasing, and that summation preserves concavity, implies that the rate function in (21) is the difference of two concave functions. This observation is instrumental for the derivation of the desired lower-bound. Indeed, recalling that any concave function is upper-bounded by its Taylor expansion around any given point $\eta_{m,0}$, a concave lower-bound of \mathcal{R}_k is obtained as

$$\mathcal{R}_{k}(\boldsymbol{\eta}) = g_{1}(\boldsymbol{\eta}_{m}) - g_{2}(\boldsymbol{\eta}_{m})$$

$$\geq g_{1}(\boldsymbol{\eta}_{m}) - g_{2}(\boldsymbol{\eta}_{m,0}) - \nabla_{\boldsymbol{\eta}_{m}}^{T} g_{2}|_{\boldsymbol{\eta}_{m,0}}(\boldsymbol{\eta} - \boldsymbol{\eta}_{m,0})$$

$$= \widetilde{\mathcal{R}}_{k}(\boldsymbol{\eta}_{m}, \boldsymbol{\eta}_{m,0}) .$$

$$(22)$$

Moreover, it is easy to check that \mathcal{R}_k fulfills by construction also properties **P2** and **P3** with respect to \mathcal{R}_k . Thus, Problem (19) can be tackled by the sequential optimization framework, by defining the ℓ -th problem of the sequence, \mathcal{P}_ℓ , as the convex optimization program:

$$\max_{\boldsymbol{\eta}_m} \sum_{k=1}^{K} \widetilde{\mathcal{R}}_k(\boldsymbol{\eta}_m, \boldsymbol{\eta}_{m,0}, \boldsymbol{\eta}_{-m})$$
(23a)

s.t.
$$\sum_{k \in \mathcal{K}_m} \eta_{k,m} \le P_{max,m}$$
(23b)

$$\eta_{k,m} \ge 0 , \forall \ k \in \mathcal{K}_m \tag{23c}$$

Algorithm	1	Sum	rate	maximization
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1:	Set $i = 0$ and choose any feasible η_2, \ldots, η_M ;
2:	repeat
3:	for $m = 1 \rightarrow M$ do
4:	repeat
5:	Choose any feasible $\eta_{m,0}$;
6:	Let η_m^* be the solution of (23);
7:	$oldsymbol{\eta}_{m,0}=oldsymbol{\eta}_m^*;$
8:	until convergence
9:	$oldsymbol{\eta}_m=oldsymbol{\eta}_m^*;$
10:	end for
11:	until convergence

Algorithm 2 Minimum rate maximization 1: Set i = 0 and choose any feasible η_2, \ldots, η_M ; 2: repeat for $m = 1 \rightarrow M$ do 3: repeat 4: 5: Choose any feasible $\eta_{m,0}$; Let η_m^* be the solution of (26); 6: 7: $\boldsymbol{\eta}_{m,0} = \boldsymbol{\eta}_m^*;$ 8: until convergence 9: $\eta_m = \eta_m^*;$ end for 10: 11: until convergence

For any $\eta_{m,0}$, Problem (26) can be solved by means of standard convex optimization theory, since the objective is concave, and the constraints are affine. The resulting power control procedure can be stated as in Algorithm 1. Moreover, based on the general theory reviewed in Section III-A, the following result holds.

Proposition 1. After each iteration in Line 6 of Algorithm 1, the sum-rate value $\sum_{k=1}^{K} \mathcal{R}_k$ is not decreased, and the resulting sequence $\{\sum_{k=1}^{K} \mathcal{R}_k\}$ converges. Moreover, every limit point of the sequence $\{\eta_m\}_m$ fulfills the KKT first-order optimality conditions of Problem (19).

Two remarks are now in order. First of all an extreme case of Algorithm 1 is that in which only one variable block is used, namely optimizing all of the transmit powers simultaneously. In this scenario, Algorithm 1 reduces to a pure instance of sequential optimization, and no alternating optimization is required. Nevertheless, as already mentioned, the complexity of this approach seems prohibitive for large M and K. Then, another extreme case is that in which the KM transmit powers $\eta_{k,m}$ are optimized one at a time, thus leading to considering KM variable blocks. The advantage of this approach is that each subproblem (23) would have only one optimization variable, and thus could be solved in closed-form. This brings drastic computational complexity savings and proves to be useful especially in the CF scenario, since in this case each variable block η_m always has dimension K.

C. Minimum rate maximization

Consider Problem (16). Following similar steps as in Section III-B, Problem (16) with respect to variable block η_m becomes

$$\max_{\boldsymbol{\eta}_{m}} \min_{1 \le k \le K} \mathcal{R}_{k}(\boldsymbol{\eta}_{m}, \boldsymbol{\eta}_{-m})$$
(24a)
s.t.
$$\sum_{k \in \mathcal{K}_{m}} \eta_{k,m} \le P_{max,m}; \quad \eta_{k,m} \ge 0 , \forall k \in \mathcal{K}_{m}$$
(24b)

Besides the difficulties already encountered in the sumrate scenario, Problem (24) poses the additional challenge of having a non-differentiable objective due to the $\min(\cdot)$ operator. To circumvent this issue, (24) can be equivalently reformulated as the program:

$$\max_{\boldsymbol{\eta}_m, t} t \tag{25a}$$

s.t.
$$\sum_{k \in \mathcal{K}_m} \eta_{k,m} \le P_{max,m}$$
(25b)

$$\eta_{k,m} \ge 0 , \forall \ k \in \mathcal{K}_m \tag{25c}$$

$$\mathcal{R}_k(\boldsymbol{\eta}_m, \boldsymbol{\eta}_{-m}) \ge t \; , \forall \; k = 1, \dots, K \; .$$
 (25d)

At this point, it is possible to tackle (25) by the sequential method. Leveraging again the bound in (22) leads to considering the approximate problem

$$\max_{\boldsymbol{\eta}_m,t} t \tag{26a}$$

s.t.
$$\sum_{k \in \mathcal{K}_m} \eta_{k,m} \le P_{max,m}$$
(26b)

$$\eta_{k,m} \ge 0 , \forall \ k \in \mathcal{K}_m \tag{26c}$$

$$\mathcal{R}_k(\boldsymbol{\eta}_m, \boldsymbol{\eta}_{m,0}, \boldsymbol{\eta}_{-m}) \geq t, \forall k = 1, \dots, K.$$
 (26d)

For any $\eta_{m,0}$, Problem (26) can be solved by means of standard convex optimization theory, since the objective is linear, and the constraints are all convex. The resulting power control procedure can be stated as in Algorithm 2, which enjoys similar properties as Algorithm 1.

IV. NUMERICAL RESULTS

In our simulation setup, we consider a communication bandwidth of W = 20 MHz centered over the carrier frequency $f_0 = 1.9$ GHz. The antenna height at the AP is 15 m and at the MS is 1.65 m. The standard deviation of the shadow fading is $\sigma_{\rm sh} = 8$ dB, the parameters for the three slope path loss model in (3) are $d_1 = 50$ m and $d_0 = 10$ m, the parameter δ in (5) is 0.5 and the correlation distance in (6) is $d_{\rm decorr} = 100$ m. The additive thermal noise is assumed to have a power spectral density of -174 dBm/Hz, while the front-end receiver at the AP and at the MS is assumed to have a noise figure of 6 dB. The shown results come from

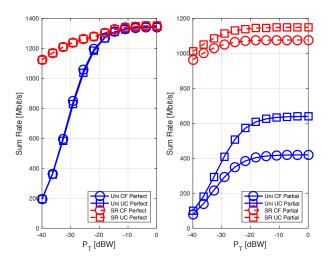


Figure 1. Downlink achievable sum-rate for the case of uniform power allocation and of sum-rate maximizing power allocation.

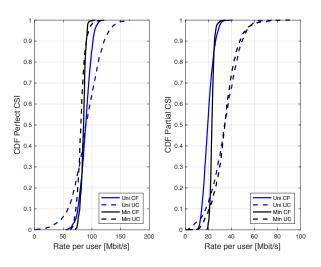


Figure 2. Downlink average achievable rate per user CDF for the case of uniform power allocation and of minimum-rate maximizing power allocation.

an average over 100 random scenario realizations with independent MSs and APs locations and channels. We quantitatively study and compare the performance of the CF and UC massive MIMO architectures. We consider M = 60 APs and K = 15 MSs spread over a square area of 800×800 sqm.; we assume $N_{AP} = 4$, $N_{MS} = 2$ and $P_k = 2, \ \forall k = 1, \dots, K$. The MSs transmit power is 0.1 W during uplink channel estimation, and the pilot length is $\tau_p = 32$. Fig. 1 reports the system sum-rate, versus the downlink maximum transmit power per AP P_T , for the CF and UC approaches, and for the case in which uniform power allocation or sum-rate maximizing power allocation is used. Both the cases of perfect channel state information (CSI) and of partial (i.e., estimated) CSI are considered. Results show that the sum-rate maximizing power allocation provides much better results than the uniform power allocation, especially for the case of

partial CSI. Also, while the CF and UC approaches are practically equivalent in the perfect CSI scenario, the UC approach outperforms the CF one in the relevant scenario of partial CSI. Indeed, for partial CSI, APs that are very far from the MS of interest perform very noisy channel estimates, and so their contribution to the communication process is detrimental; this effect is instead not observed in the case of perfect CSI. Fig. 2 shows the CDF of the rate-per-user for the case of uniform power allocation and minimum rate maximizing power allocation. It is seen that the latter strategy is effective in increasing the system fairness and in maximizing key performance measures such as the 95%-likely-per-user throughput. As an example, for the case of UC approach with partial CSI, when the proposed power control algorithm is used in place of uniform power allocation, this number increases from 12.6 Mbit/s to 16.5 Mbit/s (+30%).

V. CONCLUSION

Downlink power control algorithms aimed at maximizing either the sum-rate or the minimum rate for a CF massive MIMO system have been introduced, and sample numerical results have been provided. Current work is focused on the consideration of power allocation strategies maximizing the system energy efficiency.

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