Capacity Regions of a MAC With a Wireless-Powered Relay-to-Destination Link Under Different Relay Strategies

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***Abstract*—We consider a two-user multiple access channel (MAC) with a wireless-powered relay-to-destination (R-D) link, where the relay harvests energy from a radio frequency (RF) signal sent by a dedicated Power Beacon (PB). Each frame is divided into three phases. In the first phase, the relay harvests energy from an RF signal sent by a dedicated PB. The relay then receives information from user nodes in the second phase and forwards it to the destination in the third phase using its harvested energy. We investigate the sum rate maximization problem and characterize the capacity region of such a channel with the relay's maximum transmit power constraint, under both the amplify-and-forward (AF) and decode-and-forward (DF) relay strategies. Optimal solutions are obtained for both cases. It is interesting to find that the shape of the capacity region is still pentagonal with the wireless-powered relay. And the relay's maximum transmission power constraint greatly affects the system's performance. Finally, simulation results demonstrate the correctness of our analysis.**

I. Introduction

Long battery lifetime and low-cost communication capability are two attractive attributes for future portable devices. The recent advance of microwave wireless power transfer (WPT) enables wireless-powered communication networks (WPCNs) to be built, which offer the aforementioned advantages [1].

There are two main differences between conventional networks and WPCNs. First, there is an extra energy transfer phase before the information transmission phases in WPCNs. Second, by replacing some conventional user nodes with wireless-powered nodes, the system's objective or power constraints can be dramatically changed. This would require re-design of communication protocols and optimal resource allocation for many conventional networks.

In the literature, many problems have been investigated. The authors in [2] considered the optimal time allocations to maximize the sum-throughput of a system in which one hybrid access point (HAP) with constant power supply coordinates the wireless energy/information transmissions to/from a set of distributed users that do not have other energy sources. Optimal resource allocations has also been studied in [3]. It was pointed out in [3] that when the information receiver and the energy transmitter are co-located as a HAP, there exists the "doubly-near-far" problem, which causes issues of user fairness. One way to cope with the fairness issue is to deploy multiple antennas at the HAP such that beamforming techniques can be applied. In [4], a joint design of downlink (DL)-uplink (UL) time allocation, DL energy beamforming, and UL transmit power allocation, as well as receive beamforming was investigated to optimize the users' throughput and yet guarantee their rate fairness. Except for the case that the energy transmitter and information receiver are co-located, the scenario that the power station and information sink are located separately has also been considered in [5]. Optimal time allocation and beamforming were derived to maximize the system throughput. Another way to tackle user fairness is through user cooperation. This comes naturally from the consideration that user nodes with a larger amount of energy may help those nodes with less energy, to achieve a better system performance [6].

Apart from the problems like beamforming or user cooperation mentioned above, some researchers have been interested in more fundamental properties in wireless powered communication systems. The authors in [7] obtained the capacity region for a multiple access channel (MAC) with transmitters equipped with energy harvesters, while the sum-capacity in a three-user MAC and a two-way channel with bi-directional energy cooperation were investigated in [8]. However, few works have considered a MAC with a relay-to-destination (R-D) link.

In this paper, we consider a two-user MAC with a wireless-powered relay, where the relay harvests energy from a dedicated power beacon and helps forward users' information to the destination. We investigate the maximum sum rate and capacity regions for both the amplify-and-forward (AF) and decode-and-forward (DF) relay strategies. Optimal solutions are obtained for both cases and simulation results demonstrate the correctness of our theoretical analysis.

II. System Model

As shown in Fig. 1, we consider a two-user MAC with a wireless-powered relay, where the relay is denoted as R, the destination D and the two user nodes U_1 and U_2, respectively. It is assumed that the relay harvests energy from a dedicated

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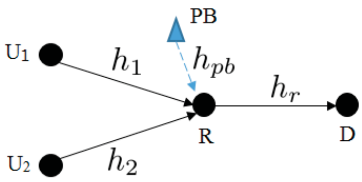

Fig. 1: System model.

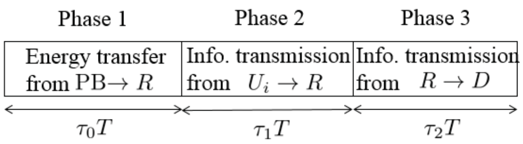

Fig. 2: Frame structure.

Power Beacon (PB), which has a maximum transmission power P_{pb}. U_1 and U_2 have a steady power supply and their transmission powers, p_1 and p_2, are upper bounded by P_1 and P_2, respectively. All nodes are equipped with a single antenna.

Each frame is divided into three phases as shown in Fig. 2, starting from the first phase in which the relay harvests energy from the PB, to the second phase in which the relay receives information from the users, followed by the last phase in which the relay decodes information and forwards it to the destination. It is assumed the relay operates in half-duplex mode. Without loss of generality, each frame length is taken as $T = 1$. We denote by h_1 and h_2 the channel gains from the source nodes to the relay, by h_r the channel gain from the relay to the destination and by h_{pb} the channel gain from the PB to the relay. The noise variance of the relay and the destination are denoted as σ^2 and σ_d^2, respectively. We let $\gamma_1 = h_1/\sigma^2, \gamma_2 = h_2/\sigma^2$, and $\gamma_r = h_r/\sigma_d^2$ in the rest of this paper.

A. Amplify-and-Forward

For the AF, the durations of the information phases are equal, i.e., $\tau_1 = \tau_2$, and the time causality constraint requires that $\tau_0 + 2\tau_1 \le 1$.

In phase 1, the PB sends $\sqrt{P_{pb}}x_0$, with $E\left[|x_0|^2\right] = 1$. The received signal at the relay is

$$y_{r1} = \sqrt{P_{pb}h_{pb}}x_0 + n_1. \tag{1}$$

It is assumed that P_{pb} is large and the energy harvested from the noise can be neglected. Therefore, the amount of harvested energy at the relay in phase 1 equals

$$E_h = \eta h_{pb}P_{pb}(1 - 2\tau_1), \tag{2}$$

where η represents the energy harvesting efficiency at the relay and it is assumed to be a constant for convenience.

In phase 2, the user nodes send signals $u_i = \sqrt{p_i}x_i$, with $E\left[|x_i|^2\right] = 1$, $i = 1, 2$. The relay receives

$$y_{r2} = \sqrt{h_1p_1}x_1 + \sqrt{h_2p_2}x_2 + n_2, \tag{3}$$

where $n_2 \sim \mathcal{CN}(0, \sigma^2)$, a circularly symmetric complex Gaussian (CSCG) random variable with zero mean and variance σ^2.

The relay then amplifies the signal and sends

$$t_r = \sqrt{\alpha h_1p_1}x_1 + \sqrt{\alpha h_2p_2}x_2 + \sqrt{\alpha}n_2. \tag{4}$$

In phase 3, the destination receives

$$y_d = \sqrt{h_r\alpha h_1p_1}x_1 + \sqrt{h_r\alpha h_2p_2}x_2 + \sqrt{h_r\alpha}n_2 + n_d. \tag{5}$$

Using succesive interference cancellation (SIC) at the destination and assuming user 2's information is decoded first, the end-to-end rates are given as follows:

$$r_1 = \tau_1 \log_2\left(1 + \frac{\gamma_r\alpha h_1p_1}{\gamma_r\alpha\sigma^2 + 1}\right) \tag{6}$$

$$r_2 = \tau_1 \log_2\left(1 + \frac{\gamma_r\alpha h_2p_2}{\gamma_r\alpha h_1p_1 + \gamma_r\alpha\sigma^2 + 1}\right), \tag{7}$$

where α satisfies the relay's maximum transmission power constraint $|t_r|^2 \le \min\{\beta(1 - 2\tau_1)/\tau_1,\ P_R\}$.

The end-to-end sum rate can be expressed as

$$R_{sum} = \tau_1 \log_2\left(1 + \frac{\alpha\gamma_r(h_1p_1 + h_2p_2)}{\gamma_r\alpha\sigma^2 + 1}\right). \tag{8}$$

B. Decode-and-Forward

For the DF, the first phase is the same as that of the AF. In phase 2, we use SIC at the relay and without loss of generality assume $h_1 > h_2$. Thus the relay decodes user 2's information first and cancels it from the signal before decoding user 1's information. The case that $h_1 \le h_2$ can be solved similarly. The individual rates from the users to the relay under this decoding order are given as

$$r_{11} = \tau_1 \log_2(1 + \gamma_1p_1) \tag{9}$$

$$r_{21} = \tau_1 \log_2\left(1 + \frac{\gamma_2p_2}{1 + \gamma_1p_1}\right). \tag{10}$$

In phase 3, the relay re-encodes and transmits x_r to the destination. At the end of phase 3, the destination decodes the users' information and the end-to-end user rates are expressed as $R_1 = \min\{r_{11}, r_{12}\}$ and $R_2 = \min\{r_{21}, r_{22}\}$ with the rate constraint of the relay-to-destination link, which is given by $r_{12}+r_{22} \le \tau_2 \log_2(1 + \gamma_rp_r)$ and the relay's power constraint $p_r \le \min\{\beta(1 - \tau_1 - \tau_2)/\tau_2,\ P_R\}$.

The end-to-end sum rate can be expressed as

$$R_{sum} = \min\left\{\tau_1 \log_2(1 + \gamma_1p_1 + \gamma_2p_2),\ \tau_2 \log_2(1 + \gamma_rp_r)\right\}. \tag{11}$$

For notation simplicity, in the rest of the paper, we define $\mathbb{S}_\tau = \{(\tau_1, \tau_2) : \tau_1 + \tau_2 < 1, \tau_1, \tau_2 > 0\}$, $\mathbb{S}_{\mathbf{p}} = \{\mathbf{p} : p_1 \le P_1, p_2 \le P_2,\ p_1, p_2, p_r \ge 0\}$, $\mathbb{S}_{\mathbf{p}_s} = \{\mathbf{p}_s : p_1 \le P_1, p_2 \le P_2,\ p_1, p_2 \ge 0\}$, where $\mathbf{p} = [p_1, p_2, p_r],\ \mathbf{p}_s = [p_1, p_2]$. We further let $\beta = \eta h_pP_{pb}$, $c_1 = \log_2(1 + \gamma_1P_1 + \gamma_2P_2)$, $c_2 = \log_2(1+\gamma_rP_R)$, $c_3 = \log_2(1+\gamma_1P_1)$, $c_4 = \log_2(1+\frac{\gamma_2P_2}{1+\gamma_1P_1})$, $m_1 = \beta\gamma_r$, $m_2 = \beta\gamma_r/(\gamma_1P_1 + \gamma_2P_2 + 1)$, $a = \gamma_r\gamma_2P_2\beta$, $b = \gamma_1P_1 + \gamma_2P_2 + 1$, and $c = (\gamma_1P_1 + 1)\gamma_r\beta$.

III. Sum Rate Maximization and Capacity Region With a Wireless-Powered Relay

In this section, we investigate the problems with a wireless-powered relay. The case with a conventional relay is much easier and thus omitted due to space limitation.

A. Amplify-and-Forward

1) Sum Rate Maximization: For the AF, the problem can be formulated as follows:

$$\max_{\tau_1,\alpha,\mathbf{p}_s\in\mathbb{S}_{\mathbf{p}_s}} \quad \tau_1\log_2\left(1+\frac{\alpha\gamma_r(h_1p_1+h_2p_2)}{\gamma_r\alpha\sigma^2+1}\right) \tag{12}$$

$$\text{s.t.} \quad \alpha h_1p_1+\alpha h_2p_2+\alpha\sigma^2 \le \min\left\{\frac{\beta(1-2\tau_1)}{\tau_1},P_R\right\} \tag{13}$$

$$0<\tau_1<1/2.$$

Theorem 3.1: The maximum sum rate $R^*_{sum} = \arg\max g_1(\tau_1)$ with $\tau_1 \in [\frac{\beta}{2\beta+P_R},\frac{1}{2})$, where $g_1(\tau_1) = \tau_1\log_2(\frac{m_1+(1-2m_1)\tau_1}{m_2+(1-2m_2)\tau_1})$. The optimal τ_1^* is determined correspondingly.

Proof: Constraint (13) achieves equality for the optimal solution, and α should be as small as possible. Thus $p_1^*=P_1$ and $p_2^*=P_2$. The objective function is only related to τ_1. For $\beta(1-2\tau_1)/\tau_1 \le P_R$, $R^*_{sum}=\arg\max g_1(\tau_1)$ for $\tau_1\in[\frac{\beta}{2\beta+P_R},\frac{1}{2})$. And for $\beta(1-2\tau_1)/\tau_1\ge P_R$, the optimal value of the objective function is achieved at $\beta(1-2\tau_1)/\tau_1=P_R$, which can also be included in the previous case. ■

2) Capacity Region: Assuming SIC is used at the destination and user 2's information is decoded first, the problem can be formulated as follows:

$$\max_{\tau_1,\alpha,\mathbf{p}_s\in\mathbb{S}_{\mathbf{p}_s}} \quad \tau_1\log_2\left(1+\frac{\gamma_r\alpha h_1p_1}{\gamma_r\alpha\sigma^2+1}\right) \tag{14}$$

$$\text{s.t.} \quad \tau_1\log_2\left(1+\frac{\gamma_r\alpha h_2p_2}{\gamma_r\alpha h_1p_1+\gamma_r\alpha\sigma^2+1}\right) \ge \bar{R} \tag{15}$$

$$\alpha h_1p_1+\alpha h_2p_2+\alpha\sigma^2 \le \min\left\{\frac{\beta(1-2\tau_1)}{\tau_1},P_R\right\} \tag{16}$$

$$0<\tau_1<1/2. \tag{17}$$

First we claim that constraint (16) achieves equality in the optimal solution. When $\bar{R}$ is small, constraint (15) can be satisfied with $p_1=P_1$. And R_2 is an increasing function of p_2. With the increasing of $\bar{R}$, p_2 will eventually achieve P_2, which corresponds to the case of the maximum sum rate. After that, p_1 should be decreased to guarantee user 2's rate constraint and this would make the rate pair lie strictly inside the capacity region. Thus we do not need to consider the case that $\bar{R}\ge\bar{R}^c$ when user 2's information is decoded first at the destination. Similarly, when user 1's information is first decoded at the destination, we can derive the other part of the capacity region and the segment between them is achieved by time sharing between these two decoding orders. The result is concluded in the following theorem.

Theorem 3.2: For $\bar{R}\in[0,\bar{R}^c]$, the optimal solution is given in two cases.

Case 1: If $g_2'(\tau_1^0)\le 0$, $\bar{R}^c=\bar{R}^{c_1}=g_2(\tau_1^0)$. For $\bar{R}\le\bar{R}^c$, the optimal rate pair is $(R_1^*,\bar{R})$, where $R_1^*=\arg\max g_3(\tau_1)$ with $\tau_1\in[\tau_1^0,\tau_1^1]$.

Case 2: If $g_2'(\tau_1^0)\ge 0$, $\bar{R}^c=\bar{R}^{c_2}=\arg\max g_2(\tau_1)$ with $\tau_1\in[\tau_1^0,1/2)$. The optimal rate pair is $(R_1^*,\bar{R})$, where

$$R_1^*=\begin{cases}\arg\max\limits_{\tau_1\in[\tau_1^0,\tau_1^1]} g_3(\tau_1), & \text{if } \bar{R}\in[0,\bar{R}^{c_1}],\\ \arg\max\limits_{\tau_1\in[\tau_1^1,\tau_1^2]} g_3(\tau_1), & \text{if } \bar{R}\in[\bar{R}^{c_1},\bar{R}^{c_2}].\end{cases} \tag{18}$$

We have $g_2(\tau_1)=\tau_1\log_2\left(1+\frac{a(1-2\tau_1)/\tau_1}{b+c(1-2\tau_1)/\tau_1}\right)$, $g_3(\tau_1)=\tau_1\log_2\frac{(1+\gamma_1P_1)[\gamma_r\beta(1/\tau_1-2)+1]}{\gamma_1P_12^{\bar{R}/\tau_1}+\gamma_r\beta(1/\tau_1-2)+1}$, $p_1^*=P_1$ and $\tau_1^0=\frac{\beta}{2\beta+P_R}$. τ_1^1 and τ_1^2 correspond to the two roots of $g_2(\tau_1)=\bar{R}$ when $\bar{R}\ge\bar{R}^{c_1}$. The optimal τ_1^* is determined correspondingly.

Proof: The difficulty lies in the impact of τ_1 on both the objective function and the other user's rate constraint. We solve the problem by considering its two sub-problems; i.e., the right hand side of (16) equals $\frac{\beta(1-2\tau_1)}{\tau_1}$ or P_R. If $\frac{\beta(1-2\tau_1)}{\tau_1}\ge P_R$, the critical value of $\bar{R}$ with $p_2=P_2$ is given by $\bar{R}^{c_1}$ and $\tau_1^*=\tau_1^0$. However, if $\frac{\beta(1-2\tau_1)}{\tau_1}\le P_R$, there might be the case that for $p_2=P_2$, $\bar{R}$ can achieve a larger $\bar{R}^{c_2}$ and also $\arg\max g_3(\tau_1)\ge g_3(\tau_1^0)$. In fact, $\bar{R}^c$ can be found by checking $g_2'(\tau_1^0)=0$ since $g_2''(\tau_1)<0$. If $g_2'(\tau_1^0)>0$, it means there exists a larger τ_1 that satisfies $g_2(\tau_1)>g_2(\tau_1^0)$. And for $\bar{R}\in[\bar{R}^{c_1},\bar{R}^{c_2}]$, $p_r=P_R$ no longer holds because this would violate constraint (16) with $p_2=P_2$. Therefore, for this part the optimal rate is simply given by $\arg\max g_3(\tau_1)$ with $\tau_1\in[\tau_1^1,\tau_1^2]$. ■

B. Decode-and-Forward

1) Sum Rate Maximization: The problem can be formulated as follows:

$$\max_{\tau\in\mathbb{S}_\tau,\mathbf{p}\in\mathbb{S}_\mathbf{p}} \quad \min\left\{\tau_1\log_2(1+\gamma_1p_1+\gamma_2p_2),\ \tau_2\log_2(1+\gamma_rp_r)\right\} \tag{19}$$

$$\text{s.t.} \quad p_r\le\min\left\{\beta(1-\tau_1-\tau_2)/\tau_2,P_R\right\}. \tag{20}$$

Note that for the optimal solution, p_1 and p_2 have to achieve their maximum, since otherwise we could always increase p_1 or p_2 and decrease τ_1 to get a larger sum rate. Thus the problem can be reformulated as

$$\max_{r,E_r,\tau\in\mathbb{S}_\tau} \quad r \tag{21}$$

$$\text{s.t.} \quad r\le\tau_1\log_2(1+\gamma_1P_1+\gamma_2P_2) \tag{22}$$

$$r\le\tau_2\log_2\left(1+\frac{\gamma_rE_r}{\tau_2}\right) \tag{23}$$

$$E_r\le\beta(1-\tau_1-\tau_2) \tag{24}$$

$$E_r\le P_R\tau_2. \tag{25}$$

This problem is convex as the feasible set, objective function and constraints are all convex. The convexity of constraint (23) can be proved by checking its Hessian and the rest are all linear functions. The optimal solution is summarized in the following theorem.

Theorem 3.3: The maximum sum rate equals $\tau_1^* c_1$ and

$$\tau_1^* = \begin{cases} \max(\tau_1^1, \tau_1^2), & \text{if } p_r^* \le P_R, \\ \tau_1^1, & \text{otherwise}, \end{cases} \tag{26}$$

where $\tau_1^1 = \frac{\beta c_2}{\beta c_2 + c_1(\beta + P_R)}$, $\tau_1^2 = \frac{\beta \log_2(1+\gamma_r p_r^*)}{\beta \log_2(1+\gamma_r p_r^*) + c_1(\beta + p_r^*)}$, and p_r^* is the unique solution of equation $\log(1+\gamma_r p_r) - \frac{\gamma_r(\beta+p_r)}{1+\gamma_r p_r} = 0$.

Proof: We can prove this by KKT conditions and the details are omitted due to limited space. ■

2) Capacity Region: The problem can be formulated as

$$\max_{R_1, \tau \in \mathbb{S}_\tau, \mathbf{p} \in \mathbb{S}_\mathbf{p}} \quad R_1 \tag{27}$$

$$\text{s.t.} \quad R_1 \le \tau_1 \log_2(1+\gamma_1 p_1) \tag{28}$$

$$\bar{R} \le \tau_1 \log_2\left(1 + \frac{\gamma_2 p_2}{\gamma_1 p_1 + 1}\right) \tag{29}$$

$$R_1 + \bar{R} \le \tau_2 \log_2(1+\gamma_r p_r) \tag{30}$$

$$p_r \le \min\left\{\beta(1-\tau_1-\tau_2)/\tau_2, P_R\right\}. \tag{31}$$

The key observation is that constraint (29) can be dropped after the discussion of $\bar{R}$ and it is again a convex optimization problem. The result is concluded in the following theorem.

Theorem 3.4: For given $\bar{R} \in [0, \max(\bar{R}^{c_1}, \bar{R}^{c_2})]$, the rate pair (R_1, R_2) on the capacity region corresponds to $(\tau_1^* c_3, \bar{R})$, where

$$\tau_1^* = \begin{cases} f(p_r^*), & \text{if } p_r^* \le P_R, \text{ and } f(p_r^*) \ge f(P_R), \\ & \text{or } \bar{R} \in [\bar{R}^{c_1}, \bar{R}^{c_2}], \\ f(P_R), & \text{otherwise}, \end{cases} \tag{32}$$

$f(x) = \frac{\beta \log_2(1+\gamma_r x) - \bar{R}(\beta + x)}{c_3(\beta+x) + \beta \log_2(1+\gamma_r x)}$. $\bar{R}^{c_1} = g(P_R), \bar{R}^{c_2} = g(p_r^*)$, $g(x) = \frac{\beta c_4 \log_2(1+\gamma_r x)}{(\beta+x)(c_3+c_4) + \beta \log_2(1+\gamma_r x)}$, and p_r^* is the same as that in theorem 3.3.

Proof: When $\bar{R}$ is small, constraint (29) will not be a problem. However, we have to find out how large $\bar{R}$ can be before we need to consider this constraint. For the same reason as in the AF case, we only need to consider the critical situation that $p_2 = P_2$ when deriving the capacity region. Luckily, by first looking into the problem without considering constraint (29), we find that the right hand side of constraint (29) is always a decreasing function of $\bar{R}$. Thus it is straightforward to say there indeed exists a critical $\bar{R}^c$ such that for $\bar{R} \le \bar{R}^c$, constraint (29) always holds for the optimal case. Therefore, we solve the original problem by the discussion of $\bar{R}$ and by checking the KKT conditions. Specifically, we solve the two sub-problems individually and the optimal solution corresponds to the case with a larger R_1.

Case 1: $\beta(1-\tau_1-\tau_2)/\tau_2 \le P_R$. By using variable substitutions $E_1 = P_1\tau_1$ and $E_r = P_R\tau_2$, the Lagrangian of the reduced problem is

$$\begin{aligned} \mathcal{L}(R_1, E_1, E_r, \tau) &= R_1 - \lambda_1\left[R_1 - \tau_1 \log_2(1+\gamma_1 E_1/\tau_1)\right] \\ &-\lambda_2\left[R_1 + \bar{R} - \tau_2 \log_2(1+\gamma_r E_r/\tau_2)\right] \\ &-\lambda_3(E_1 - P_1\tau_1) - \lambda_4(E_r - P_R\tau_2) \\ &-\mu\left[E_r - \beta(1-\tau_1-\tau_2)\right]. \end{aligned}$$

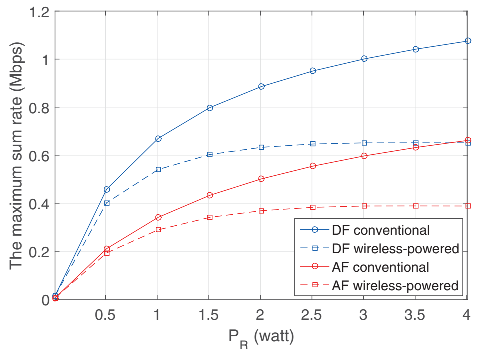

Fig. 3: The maximum sum rate vs P_R with $\gamma_1 = 2, \gamma_2 = 1, \gamma_r = 1$, and $\beta = 2.8$.

If $\lambda_4 > 0$, then $\lambda_2 > 0$, if $\lambda_1, \lambda_3 = 0$, $\mu = 0$, then $p_r^* = P_R$ must hold. Else, $\lambda_1, \lambda_3 > 0$, $E_1 = P_1\tau_1, E_r = P_R\tau_2$. $\tau_1 \log_2(1+\gamma_1 P_1) = \tau_2 \log_2(1+\gamma_r P_R) - \bar{R}$, and $P_R\tau_2 = \beta(1-\tau_1-\tau_2)$. We can then get $\tau_1^* = f(P_R)$. Note that here in order to satisfy constraint (29), $\bar{R}$ cannot be too large. Specifically, there exists a critical value $\bar{R}^{c_1}$ and for $\bar{R} \ge \bar{R}^{c_1}$, $p_1 = P_1$ cannot hold. This is not hard to see since $f(P_R)$ is a decreasing function of $\bar{R}$. Thus $\bar{R}^{c_1}$ is found when $\bar{R} = f(P_R)c_4$, which gives $\bar{R}^{c_1} = g(P_R)$.

If $\lambda_4 = 0$, then $\lambda_1, \lambda_3 > 0$, if $\lambda_2 = 0$, then $p_1^* = P_1$ must hold. Else, $\lambda_2 > 0, \mu > 0$, $p_r^* \le P_R$ must hold, $E_1 = P_1\tau_1$. $\tau_1 \log_2(1+\gamma_1 P_1) = \tau_2 \log_2(1+\gamma_r p_r) - \bar{R}$, and $p_r^*\tau_2 = \beta(1-\tau_1-\tau_2)$. Similarly, we can solve τ_1 and τ_2, and $\bar{R}^{c_2}$ can be found as $g(p_r^*)$.

Case 2: $\beta(1-\tau_1-\tau_2)/\tau_2 \ge P_R$. In this case, we can directly derive the closed-form optimal solution since τ_1 and τ_2 are separable. Specifically, we have $\tau_1^* \log_2(1+\gamma_1 P_1) = \tau_2^* \log_2(1+\gamma_r P_R) - \bar{R}$ and $\tau_1^* = (1-\tau_2^*) - P_R\tau_2^*/\beta$, which requires $\bar{R} \le \bar{R}^{c_1}$. ■

IV. Simulation Results

In this section, we validate our analysis through simulations. We let the bandwidth be 1 MHz and the two users' maximum transmit powers are set to be $1W$.

First we investigate the impact of P_R on the maximum sum rate, for both the AF and DF relay. In Fig. 3, we set $\gamma_1 = 2, \gamma_2 = 1, \gamma_r = 1$, and $\beta = 2.8$ for illustration. $\beta = \eta h_p P_{pb}$ and it represents the condition of the wireless power transfer process. As shown in the figure, unlike the conventional relay case, the sum rate with a wireless-powered relay eventually saturates with the increasing of P_R. This is because for the given channel condition, P_R will no longer be the bottleneck after it exceeds a certain value.

In Fig. 4, we compare the capacity regions of the MAC with an AF R-D link for the conventional relay and wireless-powered relay. It is worth noting that like the case in sum rate maximization, the capacity region also approaches its upper bound with the increasing of P_R and is not affected after $P_R \ge p_r^* = \beta(1/\tau_1^* - 2)$, where τ_1^* achieves the maximum value of $g_3(\tau_1)$ with $\bar{R} = \bar{R}^{c_2}$. It can be seen that for the same P_R, the wireless-powered relay system suffers from a performance

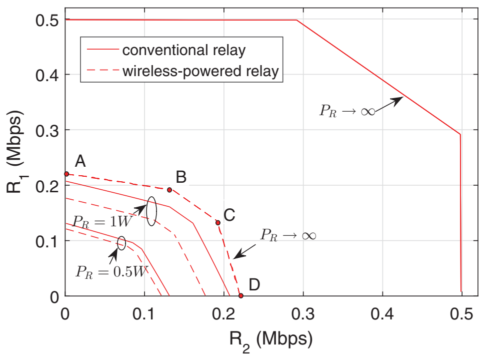

Fig. 4: The capacity regions vs P_R, for conventional and wireless-powered AF with $\gamma_1 = 1$, and $\gamma_2 = 1$.

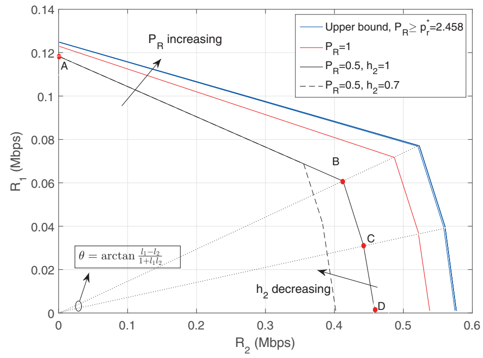

Fig. 5: The capacity regions vs P_R, for wireless-powered DF with $\gamma_1 = 0.1, \gamma_2 = 1, \gamma_r = 2$, and $\beta = 2.8$.

degradation, which is our expectation. However, a larger P_{pb} or a better PB to the relay channel would help decrease the gap, as we will show in Fig. 6 in the DF case.

Fig. 5 shows the capacity region under different P_R for the wireless-powered DF R-D link. It is shown that there exists a $\bar{P_R}$ such that after $P_R > \bar{P_R}$, increasing P_R no longer enlarges the capacity region. This is because a large P_R results in a small τ_1^* in case 2 and thus the optimal solution falls to case 1, which does not change with P_R. Moreover, it is found that $\bar{P_R} = p_r^*$, which depends on the channel condition of the R-D link and conditions of the wireless power transfer phase. The capacity region in the DF case is still a pentagon, as shown in theorem 3.4. The corner points of the capacity region for the wireless-powered relay lie on two lines, with the slopes given by $l_1 = \frac{\log_2(1+\gamma_1 P_1)}{\log_2(1+\gamma_2 P_2/(1+\gamma_1 P_1))}$ and $l_2 = \frac{\log_2(1+\gamma_1 P_1/(1+\gamma_2 P_2))}{\log_2(1+\gamma_2 P_2)}$, respectively. And we can characterize the angle $\theta = \arctan \frac{l_1 - l_2}{1 + l_1 l_2}$. Furthermore, the slope of part AB, i.e., the characteristic of user 1's rate with respect to user 2's rate, is not affected by user 2's channel condition h_2, as shown in this figure. Similarly, the slope of part CD would not be affected by h_1 either.

Finally, Fig. 6 illustrates the impact of P_{pb} on the capacity region with the wireless-powered DF R-D link under a given P_R. For a given P_R, the capacity region of the conventional

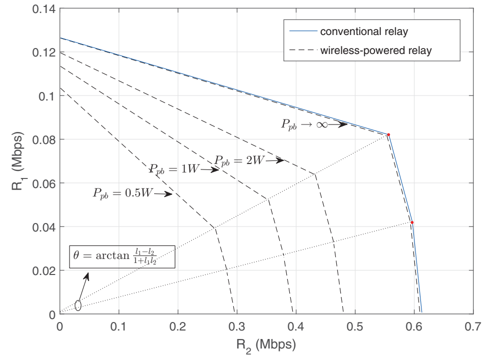

Fig. 6: The capacity regions vs P_{pb}, for conventional and wireless-powered DF with $\gamma_1 = 0.1, \gamma_2 = 1, \gamma_r = 2, h_{pb} = 1$, and $P_R = 1W$.

relay is shown as the outer bound, and we can see that with the increasing of P_{pb}, the capacity region of the wireless-powered relay eventually converges to the conventional relay case. The fact that the corner points lie on lines also holds for the impact of P_{pb}. Moreover, the slopes and thus the angle are the same as the different P_R case since they depend on l_1 and l_2, which are not affected by P_R or P_{pb}.

V. Conclusion

In this paper, we have investigated a MAC with a wireless-powered R-D link. We have considered sum rate maximization and derived the capacity regions of the systems, for both the AF and DF relay strategies. Simulation results show that DF outperforms AF and both are greatly affected by the relay's maximum transmit power constraint. Unlike conventional relaying, the capacity region in wireless-powered relaying is upper bounded even with P_R continuously increasing. In addition, a better PB to the relay channel helps decrease the gap in the capacity regions between the conventional relaying and wireless-powered relaying.

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