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***Abstract*—The linear closed-loop MIMO precoding technique employs the channel state information (CSI) at both sides of the link to optimize various criteria such as the capacity, the mean square error, the signal to noise ratio (SNR), etc. Besides classical criteria such as capacity or bit error rate, the diversity-multiplexing trade-off (DMT) is now widely used to evaluate the performance of designed precoders. Indeed, it is known that a fundamental trade-off between the spatial multiplexing and the diversity order exists. The first definition was given for asymptotic SNR, then was extended to finite values. The DMT was studied for open-loop scheme (Alamouti or V-BLAST) and we propose in this paper a method to obtain DMT of multi-form MIMO precoders. Although several multi-form solutions were found, to obtain their theoretical performance is still difficult. In order to tackle this challenge, we propose to investigate the minimal distance approach: starting from the probability density function of a square minimum distance, we obtain the outage probability and diversity-multiplexing trade-off (DMT) at operational SNR. We arbitrarily choose the $\max\text{-}d_{\min}$ precoder based on the maximization of a minimal distance using the CSI at the transmitter (closed-loop). This expression is validated by simulations and comparisons between different MIMO precoding schemes are performed. The method can be applied to others precoders and fading channels.**

I. Introduction

The multiple-input multiple-output (MIMO) technology will play an important role in the future generation of wireless systems. Particularly, when the channel state information (CSI) is available at transmitter, it can be exploited by precoders. Those closed-loop solutions that are designed in order to optimize various criteria, demonstrated a great improvement of performance [1], [2], [3], [4], [5], [6]. Zheng and Tse firstly compared in [7] the performance between diversity-based and multiplexing-based schemes thanks to the diversity-multiplexing trade-off (DMT) curves. The DMT gives the theoretical analysis for MIMO system and presents the trade-off between diversity gain and multiplexing gain, i.e. between the data rate and the error probability in a channel. By studying asymptotic DMT at high SNR, we have the overview of the potential provided by MIMO systems, and it is known as upper bounded for any $n_T \times n_R$ MIMO system. The trade-off curves are used to evaluate the performance of existing schemes according to different design goals, e.g. to maximize the diversity gain as Alamouti [8] or to maximize the multiplexing gain as V-BLAST scheme [9]. Besides, following the work of Zheng and Tse, the authors in [10] investigated the fundamental DMT for diagonal precoders [1], [3] that exploit CSI at transmitter. Most practical MIMO systems usually operate at low or medium SNR and, following the seminal work of Zheng and Tse, the finite-SNR DMT framework for practical schemes is proposed in [11], [12]. The DMT as a function of SNR for orthogonal space-time block codes (OSTBC) [8], [13] is derived. In [14], the authors derived an analytical expression for the probability density function (pdf) of mutual information between the transmitter and receiver using the joint pdf of two largest eigenvalues of Wishart matrix [15]. Then, the finite-SNR DMT is calculated based on the outage probability versus SNR for any uncorrelated Rayleigh fading $n_T \times 2$ and $2 \times n_R$ MIMO systems. On the other hand, the authors in [16] worked on zero forcing and matched filter precodings by using eigenvalues distribution. In this paper, we deal with the closed-loop precoders such as Lattice [6], X and Y codes [17] and $\max\text{-}d_{\min}$ [4], [5] precoders whose solution is multi-form. The theoretical study is thus very complex because *i)* dealing with all the matrix forms, and *ii)* starting of the distribution of a subset of eigenvalues. However, to tackle this challenge, an approach for the mutual information based on the probability of the minimum distance was proposed in [18]. The advantage of this method is its ability to be applied to other criteria like DMT. The contributions are *i)* validate the outage probability of the mutual information as a function of the $d_{\min}$, and *ii)* obtain the DMT for a multi-form precoder for any $n_T \times n_R$ MIMO system with two M-QAM symbols.

The remainder of this paper is organized as follows: the system model is introduced in section II. Section III discusses the outage probability and the finite-SNR DMT when CSI is available at the transmitter. The obtained results are presented in section IV. Finally, section V offers the conclusion.

II. SYSTEM MODEL AND DEFINITIONS

A. MIMO system and precoders

Let us consider a MIMO transmission with n_T transmit and n_R receive antennas. The precoder model [1], [2], [3], [4], [5] is given by

$$\mathbf{y} = \mathbf{G}(\mathbf{HFs} + \mathbf{n}) = \mathbf{G}_d\mathbf{H}_v\mathbf{F}_d\mathbf{s} + \mathbf{G}_d\mathbf{n}_v \tag{1}$$

where the data vector [1] $\mathbf{s} \in \mathbb{C}^b$ consists of $b \leq \min\{n_T, n_R\}$ data symbols to be transmitted, $\mathbf{y} \in \mathbb{C}^b$ is the received vector, $\mathbf{n} \in \mathbb{C}^{n_R}$ is the additive white Gaussian noise, $\mathbf{H} \in \mathbb{C}^{n_R \times n_T}$ is the channel matrix, $\mathbf{F} \in \mathbb{C}^{n_T \times b}$ is the linear precoder matrix, $\mathbf{G} \in \mathbb{C}^{b \times n_R}$ is the decoder matrix. Using the SVD, the channel matrix is diagonalized where the virtual eigen-channel matrix $\mathbf{H}_v = \mathbf{G}_v\mathbf{H}\mathbf{F}_v = \operatorname{diag}(\sigma_1, ..., \sigma_b)$ with σ_i sorted by decreasing order; $\mathbf{F} = \mathbf{F}_v\mathbf{F}_d$ and $\mathbf{G} = \mathbf{G}_d\mathbf{G}_v$. When the Maximum Likelihood decision is used at the receiver, the decoder matrix $\mathbf{G}_d$ has no longer significant role, therefore, is assigned to be an identity matrix. The virtual precoder matrix $\mathbf{F}_v$ is an orthogonal matrix (e.g. $\mathbf{F}_v^*\mathbf{F}_v = \mathbf{I}$), thus the problem becomes determinating the precoder matrix $\mathbf{F}_d$ to optimize a particular criterion subject to the power constraint $\operatorname{trace}\{\mathbf{FF}^*\} = \operatorname{trace}\{\mathbf{F}_d\mathbf{F}_d^*\} = E_s$.

We consider the dual-streams case $b = 2$. By changing of variables from Cartesian to polar coordinates, the virtual channel is given by

$$\sigma_1 = \sqrt{\lambda_1} = \rho\cos\gamma \quad \text{and} \quad \sigma_2 = \sqrt{\lambda_2} = \rho\sin\gamma \tag{2}$$

where ρ and γ are the channel gain and channel angle, respectively. Note that $\sigma_1 \geq \sigma_2 > 0$, so we have $0 < \gamma \leq \pi/4$. The max-SNR precoder designed to maximize the SNR at the receiver [2] only uses the strongest virtual sub-channel, thus the structure is simple $f_1 = \sqrt{E_s}$. The max-$d_{\min}$ precoder, aiming at maximizing the minimum distance between dual-symbol vectors at the receiver [4], can be given by

$$\mathbf{F}_d = \arg\max_{\mathbf{F}_d} d_{\min}^2 = \arg\max_{\mathbf{F}_d} \min_{\mathbf{s}_k \neq \mathbf{s}_l} \|\mathbf{H}_v\mathbf{F}_d(\mathbf{s}_k - \mathbf{s}_l)\|^2. \tag{3}$$

The authors in [5] introduced the suboptimal solution for 4^k-QAM ($b = 2$) that requires two forms and depends on the modulation order (SNR-independent). If $0 \leq \gamma \leq \gamma_0$,

$$\mathbf{F}_d = \mathbf{F}_1 = \sqrt{E_s}\begin{pmatrix} \cos\theta & \sin\theta e^{i\varphi} \\ 0 & 0 \end{pmatrix} \tag{4}$$

[1] Notations: Boldface capital letters denote the matrices, boldface lower-case letters denote the vectors. The superscript $(.)^*$ stands for conjugate transpose matrices. λ_i denotes the eigenvalues and σ_i denotes the singular values of a Hermitian matrix; $\operatorname{trace}(.)$ denotes the sum of the elements on the diagonal of a matrix; $(.)^+$ stands for $\max(., 0)$; the operator $\det(.)$ denotes the determinant; $\operatorname{diag}(.)$ denotes the diagonal matrix and $||\mathbf{X}||_{\mathrm{F}}^2 \triangleq \sum_{ij} ||\mathbf{X}_{ij}||^2 = \operatorname{trace}\{\mathbf{XX}^*\}$ denotes the square Frobenius norm of a matrix.

and if $\gamma_0 \leq \gamma \leq \pi/4$,

$$\mathbf{F}_d = \mathbf{F}_2 = \sqrt{\frac{E_s}{2}}\begin{pmatrix} \cos\psi & 0 \\ 0 & \sin\psi \end{pmatrix}\begin{pmatrix} 1 & e^{i\frac{\pi}{4}} \\ -1 & e^{i\frac{\pi}{4}} \end{pmatrix} \tag{5}$$

where $N = 2^k - 1$ and different parameters used are given as:

$$\begin{aligned} &\varphi = \arctan\tfrac{1}{2N+\sqrt{3}}; \quad \theta = \arctan(2\sin\varphi); \\ &\psi = \arctan\tfrac{\sqrt{2}-1}{\tan\gamma}; \quad \tan^2\gamma_0 = \tfrac{\sqrt{2}-1}{\sqrt{2}N^2+\sqrt{6}N+\sqrt{2}-1}. \end{aligned} \tag{6}$$

This is the solution considered in this paper.

B. Asymptotic and finite-SNR DMT

In [7], the asymptotic multiplexing gain and diversity gain are defined by

$$r_{\text{asy}} = \lim_{\text{SNR}\to\infty} R(\text{SNR})/\log_2 \text{SNR}; \tag{7}$$

$$d_{\text{asy}} = -\lim_{\text{SNR}\to\infty} \log_2 P_e(\text{SNR})/\log_2 \text{SNR}. \tag{8}$$

where $R(\text{SNR})$ is the data rate and $P_e(\text{SNR})$ is the error probability. The asymptotic DMT is upper bounded for any $n_T \times n_R$ MIMO systems without CSI when SNR tends to infinity by:

$$d^*(u) = (n_T - u)(n_R - u) \tag{9}$$

where $u = 0, 1, \dots, \min\{n_T, n_R\}$. Following this work, some further researches [11], [12], [14] widened DMT at finite-SNR for Rayleigh MIMO model. At operational SNR, when capacity achieving codes are used over a block, the error probability is approximated by the outage probability. It is defined as the probability that a system cannot support a given data rate R (bits/symbol), written as

$$P_{\text{out}}(R) = \Pr[I \leq R] = \int_0^R f_I(I, \text{SNR})dI \tag{10}$$

where $f_I(I, \text{SNR})$ is the pdf of the mutual information at each value of SNR. The authors in [11], [12] proposed a framework to characterize the DMT at finite-SNR as follows. The multiplexing gain r is defined as the ratio of the system data rate R to the capacity of an AWGN channel with array gain G_a [12]:

$$r = \frac{R}{\log_2(1 + G_a \times \text{SNR})}. \tag{11}$$

With a fixed value of multiplexing gain r at a particular operating SNR, the diversity gain $d(r, \text{SNR})$ is defined by the negative slope of the log-log curve of the outage probability versus SNR [11]:

$$d(r, \text{SNR}) = -\frac{\partial \log_2 P_{\text{out}}(r, \text{SNR})}{\partial \log_2 \text{SNR}} \tag{12}$$

where $P_{\text{out}}(r, \text{SNR})$ is the outage probability of the MIMO link. The array gain G_a is chosen based on the mutual information as the SNR approaches 0, $I \approx \log_2(1 + \frac{\text{SNR}}{n_T}||\mathbf{H}||_F^2)$, thus, $G_a = E[||\mathbf{H}||_F^2/n_T] = n_R$.

$$\frac{d}{d\theta}\left(\int_{a(\theta)}^{b(\theta)} f(x,\theta)dx\right) = \int_{a(\theta)}^{b(\theta)} f_\theta(x,\theta)dx + f\left(b(\theta),\theta\right)b'(\theta) - f(a(\theta),\theta)a'(\theta) = A_1 + A_2 + A_3 \tag{23}$$

$$f_I(I,\mathrm{SNR}) = f_I(I,\mathrm{SNR}|\mathbf{F}_1).\Pr(\mathbf{F}_1) + f_I(I,\mathrm{SNR}|\mathbf{F}_2).\Pr(\mathbf{F}_2) = f_{I,\mathbf{F}_1}(I,\mathrm{SNR}) + f_{I,\mathbf{F}_2}(I,\mathrm{SNR}) \tag{24}$$

III. Outage probability and diversity gain

A. Basic idea: using the pdf of the minimum distance

Assuming that $\mathbf{s}$ is a zero-mean white complex Gaussian random vector, when CSI is exploited at both transmitter and receiver, the mutual information is given by

$$I = \log_2 \det\left(\mathbf{I} + \frac{\mathrm{SNR}}{n_T}(\mathbf{GHF})^*(\mathbf{GHF})\right). \tag{13}$$

For max-$d_{\min}$ precoder,

$$I = \begin{cases} \log_2\left(1 + \dfrac{\mathrm{SNR}}{n_T}z\right) & ;\mathbf{F}_1 \text{ case} \\ \log_2\left(1 + \dfrac{\mathrm{SNR}}{n_T}z + \left(\dfrac{\mathrm{SNR}}{n_T}\dfrac{z}{2\sqrt{2}}\right)^2\right) & ;\mathbf{F}_2 \text{ case} \end{cases} \tag{14}$$

where $z = \rho^2\cos^2\gamma$ for $\mathbf{F}_1$ case and $z = \rho^2\frac{(2-\sqrt{2})\cos^2\gamma\sin^2\gamma}{1+(2-2\sqrt{2})\cos^2\gamma}$ for $\mathbf{F}_2$ case.

The authors in [19] proposed the changing variables in order to obtain the pdf of minimum distance between two data vectors at receiver $d^2_{\min} = \alpha_k z$, where α_k is minimum distance between two data vectors at transmitter and the gain z depends on the channel characteristics. From the pdf of z, the authors derived the pdf of $d^2_{\min}$. In this paper, we take advantage of this result. By using the changing variable,

$$\begin{cases} \Gamma = \lambda_1 + \lambda_2 = \rho^2 \\ \beta = \dfrac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} = \cos 2\gamma \end{cases} \Rightarrow \quad z = \Gamma\delta(\beta) \tag{15}$$

where $\delta(\beta) = \dfrac{1+\beta}{2}$ for $\mathbf{F}_1$ and $\delta(\beta) = \dfrac{2(1-\beta^2)}{2-\sqrt{2}\beta}$ for $\mathbf{F}_2$. Let us consider the statistical channel. The joint law for two largest eigenvalues introduced in [15] that can be expressed under polynomial and exponential forms

$$\begin{aligned} f_{\lambda_1,\lambda_2}(\lambda_1,\lambda_2) = \quad & \kappa(\lambda_1\lambda_2)^{n_s}e^{-(\lambda_1+\lambda_2)}(\lambda_1-\lambda_2)^2 \\ & \times\sum_{n=0}^{m-2} e^{-n\lambda_2}\sum_{i,j} p_{n,i,j}\lambda_1^i\lambda_2^j \end{aligned} \tag{16}$$

where $\kappa^{-1} = \prod_{i=1}^{m}(n_T-i)!(n_R-i)!$, $m = \min\{n_T, n_R\}$, $n_s = |n_T - n_R|$ and $p_{n,i,j}$ are coefficients which can be determinated with the aid of Matlab or Mathematica. First of all, we find the joint law of δ and Γ, $f_{\Gamma,\beta}(\Gamma,\beta)$, then calculate the marginal law of $z = \Gamma\delta(\beta)$, denoted $f_{\Delta,\mathbf{F}_i}(z)$.

For the max-$d_{\min}$ case, we should note that this precoder is designed for finite alphabet input, 4^k-QAM.

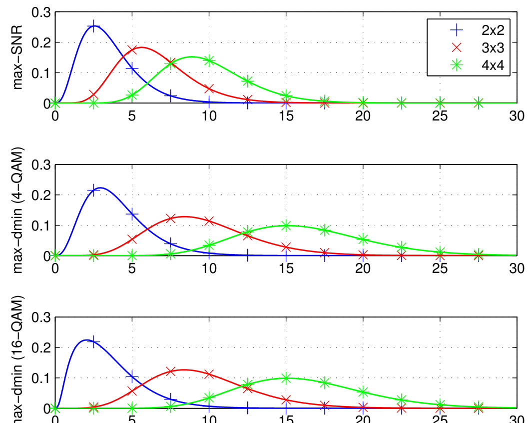


Fig. 1: Analytical pdf of variable z for max-SNR, 4-QAM and 16-QAM max-$d_{\min}$ precoders.

The effect of modulation order k is expressed through the switching point $\beta_0(k)$ between the $\mathbf{F}_1$ and $\mathbf{F}_2$. Consequently, the modulation size has an impact on the random variable z. Note that max-SNR precoder is similar to $\mathbf{F}_1$ case of max-$d_{\min}$ precoder and the result z does not depend on k. Figure 1 depicts the pdf of z for max-SNR and max-$d_{\min}$ solutions for 4-QAM and 16-QAM. We can observe that the impact of the modulation size is visible for the 2×2 system but limited for 3×3 and 4×4 systems.

From the pdf of variable z, $f_\Delta(z)$, we can derive the pdf of the mutual information, the outage probability and the diversity gain. The max-$d_{\min}$ solution requires dealing with two forms $\mathbf{F}_1$ and $\mathbf{F}_2$. Let us consider each case.

B. $\mathbf{F}_1$ case

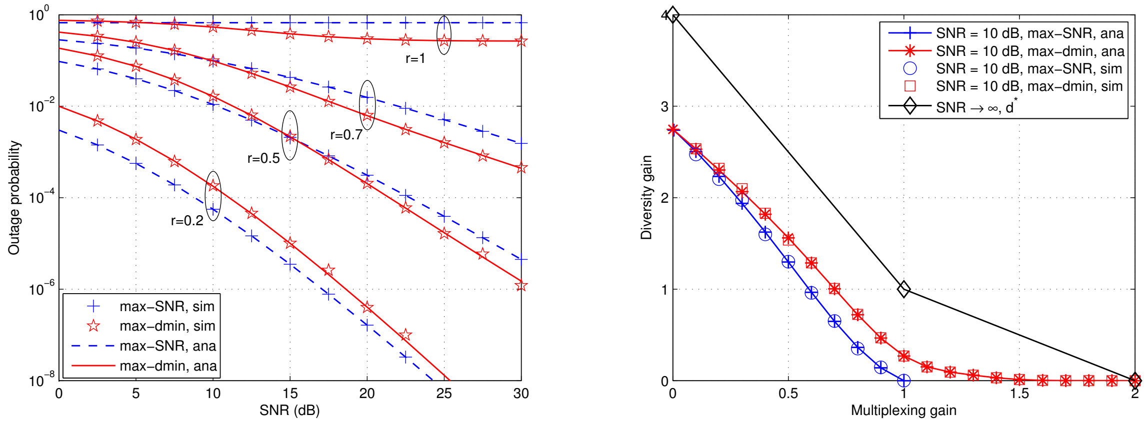
The pdf of the mutual information and outage probability can be derived from pdf of z at each SNR as

$$f_{I,\mathbf{F}_1}(I,\mathrm{SNR}) = f_{\Delta,\mathbf{F}_1}\left(\frac{n_T}{\mathrm{SNR}}(2^I-1)\right)\frac{n_T}{\mathrm{SNR}}2^I.\ln 2 \tag{17}$$

$$P_{\mathrm{out},\mathbf{F}_1}(r,\mathrm{SNR}) = \int_0^{g(r,\mathrm{SNR})} f_{\Delta,\mathbf{F}_1}(u)du \tag{18}$$

where $g(r,\mathrm{SNR}) = \frac{n_T}{\mathrm{SNR}}(2^R-1)$ and $R = r\log_2(1+G_a\times\mathrm{SNR})$. Using Leibniz integral rule [20] as Eq.(23) where $f_\theta(x,\theta)$ is partial derivative of $f(x,\theta)$, the derivative of P_{out} is given by ($A_1 = 0, A_3 = 0$):

$$\frac{\partial P_{\mathrm{out},\mathbf{F}_1}(r,\mathrm{SNR})}{\partial\mathrm{SNR}} = \frac{\partial g(r,\mathrm{SNR})}{\partial\mathrm{SNR}}f_{\Delta,\mathbf{F}_1}(g(r,\mathrm{SNR})). \tag{19}$$



(a) Outage probability

(b) Finite-SNR DMT

Fig. 2: Simulated and analytical outage probability and finite-SNR DMT for a 2×2 MIMO system with max-SNR and max-$d_{\min}$ precoders.

Results can be found for max-SNR and are similar to the $\mathbf{F}_1$ case.

C. $\mathbf{F}_2$ case

By changing the variable,

$$a = \frac{\text{SNR}}{2\sqrt{2}n_T} z \Rightarrow I = \log_2(1 + 2\sqrt{2}a + a^2).$$

The pdf of mutual information is given by

$$f_{I,\mathbf{F}_2}(I, \text{SNR}) = f_a(\sqrt{1+2^I} - \sqrt{2}, \text{SNR}) \qquad (20)$$

where $f_a(a, \text{SNR}) = f_{\Delta,\mathbf{F}_2}\left(\frac{2\sqrt{2}n_T}{\text{SNR}} a\right) \frac{2\sqrt{2}n_T}{\text{SNR}}$. The outage probability and its derivative versus SNR are given by

$$P_{\text{out},\mathbf{F}_2}(r, \text{SNR}) = \int_0^{h(r,\text{SNR})} f_{\Delta,\mathbf{F}_2}(u) du \qquad (21)$$

$$\frac{\partial P_{\text{out},\mathbf{F}_2}(r, \text{SNR})}{\partial \text{SNR}} = \frac{\partial h(r, \text{SNR})}{\partial \text{SNR}} f_{\Delta,\mathbf{F}_2}(h(r, \text{SNR})). \qquad (22)$$

where $h(r, \text{SNR}) = 2\sqrt{2}\frac{n_T}{\text{SNR}}\sqrt{1+2^R} - \sqrt{2}$. Thanks to a numerical method, Eq.(18) and (21) are calculated.

D. Whole max-$d_{\min}$ precoder:

The pdf of the mutual information is expressed as Eq.(24) and the outage probability is given by

$$P_{\text{out}}(r, \text{SNR}) = P_{\text{out},\mathbf{F}_1}(r, \text{SNR}) + P_{\text{out},\mathbf{F}_2}(r, \text{SNR}). \qquad (26)$$

By using Eq.(19) and Eq.(22) in Eq.(26), the diversity gain defined in Eq.(12) can be derived.

IV. Analysis of the Results

Figure 2a shows the outage probability obtained by Monte Carlo simulations and analytical expression of Eq.(18) and Eq.(26) for max-SNR and max-$d_{\min}$ precoders. Firstly, as we can see, the analytical results are identical to ones obtained by Monte Carlo simulation.

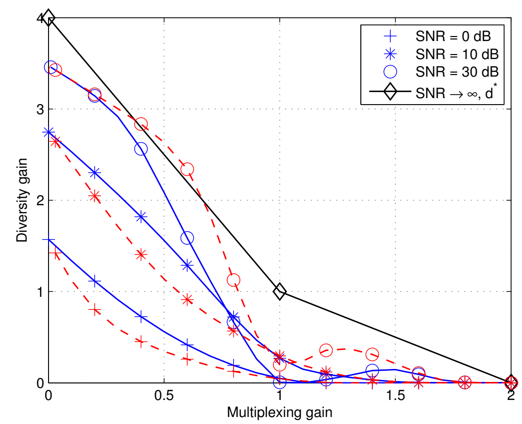
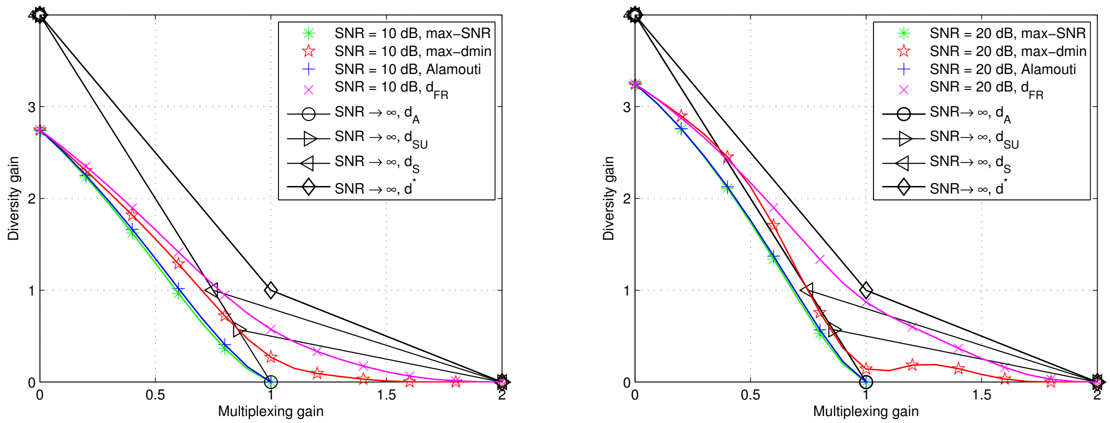


Fig. 3: Finite-SNR DMT curves for the max-$d_{\min}$ precoder and 2×2 MIMO system. The optimal solution for 4-QAM is in blue solid line, and the optimal solution for 16-QAM is in red dash line

Secondly, max-SNR and max-$d_{\min}$ outage probabilities can be compared: max-SNR is the best for r close to zero but max-$d_{\min}$ outperforms max-SNR when $r > 0.5$. From numerical Monte Carlo results for the outage probability, the simulated finite-SNR DMT is represented in Figure 2b. It is observed that the analytical results again are very close to simulation results and validate our method. It also permits us to compare finite-SNR DMT: max-$d_{\min}$ achieves better DMT than max-SNR and both converge to the same point at $r = 0$.

With the assumption of a Gaussian input, Figure 3 compares the finite-SNR DMT for max-$d_{\min}$ optimal solutions for 4-QAM and for 16-QAM at various SNR values (remember that this precoder needs to know the modulation size). At low SNR, the 4-QAM modulation offers a better DMT than the 16-QAM, while the contrary occurs at hight SNR. A first remark is about the non-monotonic behavior of the max-$d_{\min}$ DMT.



(a) 10 dB (b) 20 dB

Fig. 4: Asymptotic and finite-SNR DMT curves for a 2×2 MIMO system for different schemes.

Indeed, the precoder chooses between two forms: *i)* $\mathbf{F}_1$ is equivalent to max-SNR and favors the diversity, *ii)* $\mathbf{F}_1$ gives greater importance to multiplexing. The inflection point corresponds to a multiplexing gain equal to 1 and coincide with the frontier between $\mathbf{F}_1$ and $\mathbf{F}_2$. Thus, the behavior is explained by the liberty of choice of max-$d_{\min}$, based on the $d_{\min}$ and not the DMT. It is also worth noting that at some value of r, the achievable diversity gain is greater than the asymptotic diversity gain d^* defined in Eq.(9).

Figure 4 shows the analytical finite-SNR DMT curves for 2×2 MIMO systems at 10, 20 dB for max-SNR and max-$d_{\min}$ (solution for 4-QAM) precoders in comparison with the following published results

1) Asymptotic DMT d^* described as Eq.(9) for $n_T \times n_R$ MIMO systems and asymptotic DMT for Alamouti [7]

$$d_{\mathrm{A}}(b) = n_T \times n_R (1-b)^+, 0 \leq b \leq \min\{n_T, n_R\}, \tag{27}$$

2) Finite-SNR DMT for Alamouti [11],
3) Finite-SNR DMT for uncorrelated Rayleigh MIMO channels d_{FR} [14],
4) Asymptotic DMT for some diagonal precoders [1], [3], given by the piecewise-linear function connecting the points $(0, n_T \times n_R), (r(b), d^*(b))$ and $(\min\{n_T, n_R\}, 0)$. $r(b)$ denotes the values of multiplexing gain r when the number of active substreams increases from b to $b+1$ and $d_b = (n_T - b + 1)(n_R - b + 1)$; if the data rate is poured over the optimal number of active substreams $b^*(r)$ with uniform rate allocation [10, Theo.2]:

$$\begin{aligned} r_j &= r/b^*(r), \quad j = 1, ..., b^*(r) \\ r(b) &= b - \frac{b d_{b+1}}{(b+1)d_b - b d_{b+1}}; \quad (28) \\ d_{SU} &= \frac{d_b d_{b+1}}{(b+1)d_b - b d_{b+1}} \quad (29) \end{aligned}$$

for $b = 0, \ldots, \min\{n_T, n_R\} - 1$; with optimal rate allocation [10, Theo.3]: $(j = 1, \ldots, b^*(r))$

$$\begin{aligned} r_j &= 1 - \frac{1/d_j}{\sum_{i=1}^{b^*(r)} 1/d_i}(b^*(r) - r), \\ r(b) &= b - d_{b+1}\left(\sum_{i=1}^{b} 1/d_i\right); \quad (30) \\ d_S &= (n_T - b)(n_R - b) \quad (31) \end{aligned}$$

for $b = 1, \ldots, \min\{n_T, n_R\} - 1$.

It is obvious that the practical schemes such as Alamouti, max-SNR and max-$d_{\min}$ precoders can accomplish a same maximum achievable diversity gain when the multiplexing gain r approaches zero. In other words, these schemes profit from all the available diversity provided by the system at operational SNRs. The finite-SNR DMT curves for Alamouti and max-SNR precoder for the case 2×2 MIMO system are analogous. At low multiplexing gain, the DMT curve at 20 dB (cf. Figure 4b) for max-$d_{\min}$ precoder overcomes the finite-SNR DMT for Rayleigh MIMO channels [14]. At operational SNR, the channel knowledge can increase the diversity versus multiplexing trade-off. Moreover, the finite-SNR diversity gain for max-$d_{\min}$ precoder is higher than the asymptotic diversity gain for some diagonal precoders [10] at low multiplexing gain regime.

Figure 5 shows the finite-SNR DMT in the case 4×2 MIMO channel in dashed lines and 2×4 MIMO channel in solid lines. It is obvious that receive antennas provide a higher diversity gain than transmit antennas at low to moderate SNRs. Besides, applying the proposed method, we can derive the achievable diversity gain for larger MIMO systems. Figure 6 depicts the analytical finite-SNR DMT in the case 3×3 MIMO channel at 10 and 20 dB. It highlights a characteristic behavior of max-$d_{\min}$ DMT with a significant gain at low (0.5) and high (1.5) multiplexing gains with a hole at $r = 1$.

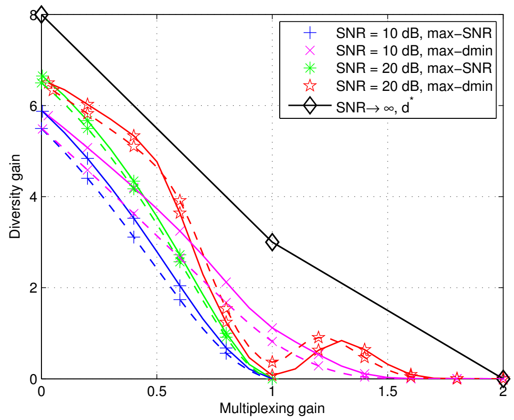


Fig. 5: Analytical finite-SNR DMT for a 4×2 MIMO channel in dashed lines and a 2×4 MIMO channel in solid lines at 10, 20 dB.

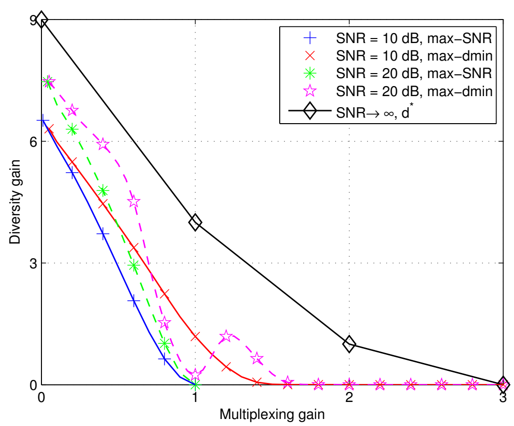


Fig. 6: Analytical finite-SNR DMT for a 3×3 MIMO channel at 10, 20 dB.

V. Conclusion

In this paper, we have focused on the evaluation of the performance of MIMO precoding technique such as max-SNR and max-$d_{\min}$ precoders in which the channel state information is exploited at the transmitter side. The outage probability is firstly analytically evaluated in this work and, afterwards, the finite-SNR DMT is obtained. This theoretical tool that analyzes the performance of schemes at operational SNR, is extended for max-$d_{\min}$ precoders for any $n_T \times n_R$ MIMO system with two 4^k-QAM symbols, dealing with the difficulty of a two-forms solution. The analytical results pointed out that when CSI is exploited at the transmitter, the finite-SNR DMT can overcome the asymptotic upper bound of the DMT for values of SNR. The next step of those results is to analytically study the convergence of the obtained finite-SNR DMT when SNR tends toward infinity. Moreover, this method is easy to apply to other precoders with multi-form, such as Lattice in [6].

Acknowledgment

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