Composite IG/FTR Channel Performance in Wireless Communication Systems

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Abstract—We present a composite wireless fading model encompassing multipath fading and shadowing based on fluctuating two-ray (FTR) fading and inverse gamma (IG) shadowing. We first determine an alternative framework for the statistical characterization and performance evaluation of the FTR fading model, which is based on the fact that the FTR fading distribution can be described as an underlying Rician Shadowed (RS) distribution with continuously varying parameter K_r (ratio of specular to diffuse components). We demonstrate that this new formulation permits to obtain a closed-form expression of the generalized moment generating function (GMGF) of the FTR model, from which the PDF and CDF of the composite IG/FTR model can be obtained in closed-form. The exact and asymptotic outage probability of the IG/FTR model are analyzed and verified by Monte Carlo simulations.

Index Terms—Fluctuating Two-Ray, Rician Shadowed, Generalized MGF, Composite fading, Inverse Gamma distribution.

I. INTRODUCTION

The random fluctuations affecting the radio signals have been classically divided into fast fading, resulting from multipath propagation, and (slow) shadowing, caused by the presence of large objects like trees or buildings. These two effects are often characterized independently, however, fast fading and shadowing happen simultaneously at different time scales. Thus, for a comprehensive analysis of wireless communications systems, a joint analysis is a more convenient approach by means of composite (i.e. integrated shadowing/fading) wireless channel models.

Regarding multipath fading, the Fluctuating Two-Ray (FTR) fading model was introduced in [1] as a general model to capture the rapid fluctuations of the radio channel in a wide variety of environments, through only three physically-motivated fading model parameters: K, Δ and m. The FTR model provides a better match to field measurements than most existing stochastic fading models in different environments, in particular at 28 GHz [2]. Consequently, the FTR fading

model has been widely used for the performance analysis of millimeter wave (mmWave) communications [3, 4].

Two different expressions for the probability density function (PDF) of the signal-to-noise ratio (SNR) in FTR fading are available in the literature. The original formulation for the PDF of the SNR of this model was introduced in [1], and an analytical expression based on the hypergeometric function Φ_2 [5] was given for the case of integer *m*. Later, an alternative expression for the PDF of the SNR valid for arbitrary *m* was proposed in [6, 7] based on the use of infinite series, and therefore all the performance results based on this formulation entail evaluating an infinite series, which must be truncated to be numerically computed, yielding error bounds, which in general will depend on the channel parameters values.

In this work we introduce an alternative framework for the FTR model that avoids the use of infinite series and is valid for arbitrary real *m*. The new formulation is based on the fact that the distribution of the FTR fading can be readily obtained by computing a finite-range integral over the distribution of the simpler Rician Shadowed (RS) fading model. We demonstrate that this new framework permits to obtain a closed-form expression of the generalized moment generating function (GMGF) of the FTR fading (which is a novel contribution of this work) that allows to readily obtain many different performance metrics including channel capacity, symbol error rate, outage probability, energy detection probability, secrecy outage probability, etc.

Moreover, we use the derived GMGF expression to represent a composite channel model based on FTR fading and inverse gamma (IG) shadowing. We use the general approach recently introduced in [8], where it was shown, through an extensive empirical validation based on data measurements from a wide variety of scenarios, that the use of the IG distribution to model shadowing is well-justified in cases of mild and moderate shadowing conditions, even providing a better match to empirical measurements than the more common lognormal distribution in some cases. We will provide closedform expressions for the PDF and CDF of the composite IG/FTR channel model, and calculate the outage probability, both in exact and asymptotic forms.

The rest of this paper is organized as follows: In Section II, the preliminary definitions are introduced. Section III presents the connection between the FTR and RS fading models. Based

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on this connection, a mathematical framework to characterize the outage probability analysis on IG/FTR composite model is shown in Sectionv IV. Finally, Section V provides numerical and simulation results followed by the conclusions in Section VI.

II. PRELIMINARY DEFINITIONS

In the FTR fading model, the wireless channel consists of two fluctuating dominant waves, referred to as specular components, to which other diffusely propagating waves are added. The complex baseband received signal can be expressed as

$$V = \sqrt{\zeta} V_1 \exp(j\phi_1) + \sqrt{\zeta} V_2 \exp(j\phi_2) + X + jY, \quad (1)$$

where V_n and ϕ_n , for n = 1, 2, represent, respectively, the average amplitude and the uniformly distributed random phase of the n-th specular component, such that $\phi_n \sim \mathcal{U}[0, 2\pi)$. The term X + jY is a complex Gaussian random variable, with $X, Y \sim \mathcal{N}(0, \sigma^2)$, representing the diffuse received signal component due to the combined reception of numerous weak scattered waves. On the other hand, ζ is a unit-mean Gamma distributed random variable modulating the specular components, whose PDF is given by

$$f_{\zeta}(u) = \frac{m^m u^{m-1}}{\Gamma(m)} e^{-mu}.$$
 (2)

This model is conveniently expressed in terms of the parameters K and Δ , defined as

$$\Delta = \frac{2V_1V_2}{V_1^2 + V_2^2}, \quad K = \frac{V_1^2 + V_2^2}{2\sigma^2}, \tag{3}$$

where K denotes the power ratio of the specular components to the diffuse components, and Δ provides a measure of the similarity of the two specular components, ranging from 0 (one specular component is absent) to 1 (the specular components have the same amplitude). Additionally, we define the ancillary random variable $\theta \triangleq \phi_1 - \phi_2$. Note that, as the phase difference is modulo 2π , θ will be uniformly distributed, and therefore we can write $\theta \sim \mathcal{U}[0, 2\pi)$.

Throughout this paper, we characterize the distribution of the instantaneous envelope power $\gamma = |V|^2$ at the receiver or, equivalently, the SNR with normalized noise power.

Definition 1: A random variable γ following a FTR distribution with parameters m, K, Δ and mean $\overline{\gamma}$ will be denoted by $\gamma \sim \mathcal{FTR}(\overline{\gamma}, m, K, \Delta)$, and its PDF will be denoted by $f_{\gamma}^{\text{FTR}}(x; \overline{\gamma}, m, K, \Delta)$, where the parameters may be dropped from the notation when there is no confusion.

Definition 2: A random variable γ following a squared RS distribution with parameters m, K_r and $\overline{\gamma}$ will be denoted by $\gamma \sim \mathcal{RS}(\overline{\gamma}, m, K_r)$, and its PDF can be written as

$$f_{\gamma}^{\rm RS}(x;\overline{\gamma},m,K_r) = \left(\frac{m}{m+K_r}\right)^m \frac{1+K_r}{\overline{\gamma}} \\ \times \exp\left(-\frac{1+K_r}{\overline{\gamma}}x\right) {}_1F_1\left(m;1;\frac{K_r\left(1+K_r\right)}{\overline{\gamma}\left(m+K_r\right)}x\right), \quad (4)$$

where ${}_{1}F_{1}(\cdot)$ is the confluent hypergeometric function of the first kind [9, eq. (9.210.1)], $\overline{\gamma} = 2\sigma^{2}(1+K_{r})$, and $K_{r} = \frac{\Omega}{2\sigma^{2}}$, where Ω and $2\sigma^{2}$ are the powers of the specular and diffuse components, respectively.

III. FTR FORMULATION AS A CONTINUOUS MIXTURE OF RS VARIATES

In this section, we explore the connection between the FTR and the RS fading models. We demonstrate that the statistics of the RS fading can readily be extended to the FTR case.

A. Connection between the FTR and the RS fading models

The PDF of the power of a wireless channel consisting of N specular components and a diffuse complex Gaussian component was obtained in [10] in terms of the powers of the individual components. Thus, for the case N = 2, the PDF of a random variable $\gamma \sim \mathcal{FTR}(\overline{\gamma}; m, K, \Delta)$, can be written as continuous mixture of squared RS variates as

$$f_{\gamma}^{\text{FTR}}(x;\overline{\gamma},m,K,\Delta) = \frac{1}{\pi} \int_{0}^{\pi} f_{\gamma|\theta}^{\text{RS}}(x;\overline{\gamma},m,K\left(1+\Delta\cos(\theta)\right)d\theta, \quad (5)$$

with $\gamma | \theta \sim \mathcal{RS}(\overline{\gamma}, m, K(1 + \Delta \cos(\theta))).$

Remark 1. It must be noted that the factor $\frac{\overline{\gamma}(\theta)}{1+K_r(\theta)}$ remains invariant in the transformation defined in (5), and in all subsequent expressions derived from it. This is due to the fact that parameters K and $\overline{\gamma}$ of the FTR model are related by the expression $\overline{\gamma} = (V_1^2 + V_2^2 + 2\sigma^2) = 2\sigma^2(1 + K)$. On the other hand, parameter K_r of the RS model in (5) is a function of the particular realization of the RS fading, as it verifies $K_r(\theta) = K(1 + \Delta \cos(\theta))$. Therefore, the average power for the RS fading will depend also on θ according to $\overline{\gamma}(\theta) = 2\sigma^2(1 + K_r(\theta))$, yielding,

$$\frac{\overline{\gamma}(\theta)}{1+K_r(\theta)} = 2\sigma^2 = \frac{\overline{\gamma}}{1+K}.$$
(6)

Proposition 1. Let $\gamma \sim \mathcal{FTR}(\overline{\gamma}, m, K, \Delta)$, then, its PDF can be computed as

$$f_{\gamma}^{\text{FTR}}(x;\overline{\gamma},m,K,\Delta) = \frac{1}{\pi} \int_{0}^{\pi} \left(\frac{m}{m+K(1+\Delta\cos(\theta))}\right)^{m} \exp\left(-\frac{1+K}{\overline{\gamma}}x\right) \times \frac{1+K}{\overline{\gamma}} {}_{1}F_{1}\left(m;1;\frac{(1+K)K(1+\Delta\cos(\theta))}{\overline{\gamma}m+\overline{\gamma}K(1+\Delta\cos(\theta))}x\right) d\theta.$$
(7)

Proof. This expression is obtained by plugging (4) into (5) and considering (6). \Box

It was demonstrated in [1] that the FTR fading model collapses to the Hoyt fading model for m = 1. In this regard, it can be easily shown that the integral connection between the Hoyt and Rayleigh models presented in [11] is actually a particular case of (7) when m = 1.

Remark 2. Proposition 1 implies that, conditioning on θ , the FTR distribution is actually a RS distribution, and we can write the conditional PDF as

$$f_{\gamma|\theta}^{\rm FTR}(x;m,K,\Delta) = f_{\gamma|\theta}^{\rm RS}(x;m,K(1+\Delta\cos(\theta))) = \left(\frac{m}{m+K(1+\Delta\cos(\theta))}\right)^m \frac{1+K}{\overline{\gamma}} \exp\left(-\frac{1+K}{\overline{\gamma}}x\right) \times {}_1F_1\left(m;1;\frac{K(1+K)\left(1+\Delta\cos(\theta)\right)}{\overline{\gamma}m+K\overline{\gamma}\left(1+\Delta\cos(\theta)\right)}x\right),$$
(8)

which will be used in subsequent derivations.

Proposition 2. Let $\gamma \sim \mathcal{FTR}(\overline{\gamma}, m, K, \Delta)$, then, its cumulative distribution function (CDF) can be calculated as

$$F_{\gamma}^{\text{FTR}}(x;m,K,\Delta) = \frac{1}{\pi} \int_{0}^{\pi} \frac{1+K}{\overline{\gamma}} x \left(\frac{m}{m+K\left(1+\Delta\cos(\theta)\right)}\right)^{m} \times \Phi_{2}\left(1-m,m;2;-\frac{1+K}{\overline{\gamma}}x;\frac{-(1+K)mx}{\overline{\gamma}(m+K\left(1+\Delta\cos(\theta)\right))}\right) d\theta,$$
(9)

where Φ_2 is the bivariate confluent hypergeometric function defined in [5, p. 34, eq. (8)]. An efficient algorithm to compute this function can be found in [12].

Proof. This expression is readily obtained from [10] for the case N = 2 and considering (6).

B. Generalized MGF

The generalized MGF is an important statistical function relevant to wireless communication theory, as it naturally appears when analyzing different scenarios such as energy detection probability, outage probability with co-channel interference in interference limited scenarios, physical layer security analysis [13], or in the context of composite fading channel modeling [8]. The GMGF of a random variable X is defined:

$$M_X^{(n)}(s) = \int_0^\infty x^n \exp(xs) f_X(x) dx.$$
 (10)

Note that in the case of $n \in \mathbb{N}$, the generalized MGF coincides with the n^{th} order derivative of the MGF.

Lemma 1. Let $\gamma \sim \mathcal{FTR}(\overline{\gamma}; m, K, \Delta)$, then, the GMGF of γ can be obtained in closed-form as (11), where ${}_2F_1(\cdot)$ is the Gaussian hypergeometric function [9, eq. (9.100)] and where $(a)_n$ is the Pochhammer symbol.

Proof. The conditional generalized MGF can be written by plugging (8) into (10), i.e.:

$$M_{\gamma|\theta}^{(n)}(s) = \int_{0}^{\infty} x^{n} \exp(xs) \left(\frac{m}{m+K(1+\Delta\cos(\theta))}\right)^{m} \\ \times \frac{1+K}{\overline{\gamma}} \exp\left(-\frac{1+K}{\overline{\gamma}}x\right) \\ \times {}_{1}F_{1}\left(m; 1; \frac{K(1+K)(1+\Delta\cos(\theta))}{\overline{\gamma}m+K\overline{\gamma}(1+\Delta\cos(\theta))}x\right) dx.$$
(12)

The integral in (12) can be solved with the help of [9, eq. (7.621.4)] as

$$\begin{split} M_{\gamma|\theta}^{(n)}(s) &= \\ \left(\frac{m}{m+K+K\Delta\cos(\theta)}\right)^m \frac{(1+K)\Gamma(n+1)}{(1+K-\overline{\gamma}s)^{n+1}}\overline{\gamma}^n \times \\ {}_2F_1\left(m,n+1;1;\frac{(1+K)\left(K+K\Delta\cos(\theta)\right)}{(m+K+K\Delta\cos(\theta))\left(1+K-\overline{\gamma}s\right)}\right) \\ &= \left(\frac{m(1+K-\overline{\gamma}s)}{m(1+K)-(m+K+K\Delta\cos(\theta))\overline{\gamma}s}\right)^m \\ \times \frac{(1+K)\Gamma(n+1)}{(1+K-\overline{\gamma}s)^{n+1}}\overline{\gamma}^n \times \\ {}_2F_1\left(m,-n;1;-\frac{(1+K)\left(K+K\Delta\cos(\theta)\right)}{m(1+K)-(m+K+K\Delta\cos(\theta))\overline{\gamma}s}\right), \end{split}$$
(13)

where the last equality is obtained with the help of [9, eq. (9.131)]. Besides, based on [5, p.17 eq. (12)] for integer *n*, the hypergeometric function can be written as

$${}_{2}F_{1}(m, -n; c; z) = {}_{2}F_{1}(-n, m; c; z)$$
$$= \sum_{l=0}^{n} (-1)^{l} {n \choose l} \frac{(m)_{l}}{(c)_{l}} z^{l}, \qquad (14)$$

thus, the conditional GMGF is determined as

$$M_{\gamma|\theta}^{(n)}(s) = n! \frac{m^m (1+K-\overline{\gamma}s)^m}{(1+K-\overline{\gamma}s)^{n+1}} \overline{\gamma}^n \sum_{l=0}^n \binom{n}{l} \frac{(m)_l}{l!} \times \frac{(1+K)^{l+1} K^l (1+\Delta\cos(\theta))^l}{\left[m\left(1+K\right) - (m+K+K\Delta\cos(\theta))\overline{\gamma}s\right]^{l+m}}.$$
(15)

Therefore, the unconditional GMGF will be given by

$$M_{\gamma}^{(n)} = \frac{1}{\pi} \int_0^{\pi} M_{\gamma|\theta}^{(n)}(s) d\theta,$$

and with the help of integral I1 solved in the Appendix the proof is completed. $\hfill \Box$

It should be noted that (11) is valid for all possible values of the channel parameters when s is a non-positive real number (which is usually the case in communication theory applications), since for the Gaussian hypergeometric function ${}_{2}F_{1}(a, b; c; z)$ in the expression it is always true that $z \in \mathbb{R}$ with $z \leq 0$, which permits its computation using its integral representation [9, eq. (9.111)]. The moments of the distribution can be easily obtained from the GMGF by simply setting s = 0.

IV. A COMPOSITE IG/FTR FADING MODEL

We now use the obtained GMGF to represent the composite IG/FTR channel model using the general approach in [8].

The received power of the composite IG/FTR model can be expressed as

$$Z = \bar{Z}\mathcal{G}\mathcal{V},\tag{16}$$

$$M_{\gamma}^{(n)}(s) = n! m^{m} (1 + K - \overline{\gamma}s)^{m-n-1} \overline{\gamma}^{n} \sum_{l=0}^{n} \binom{n}{l} \frac{(m)_{l}}{l!} \frac{(1+K)^{l+1} K^{l}}{[m(1+K) - (m+K-K\Delta)\overline{\gamma}s]^{m+l}} \\ \times \sum_{q=0}^{l} \binom{l}{q} (1-\Delta)^{l-q} (2\Delta)^{q} \frac{\Gamma(\frac{1}{2}+q)}{\sqrt{\pi}\Gamma(q+1)} {}_{2}F_{1}\left(m+l, \frac{1}{2}+q; q+1; \frac{2K\Delta\overline{\gamma}s}{m(1+K) - (m+K-K\Delta)\overline{\gamma}s}\right).$$
(11)

where $\overline{Z} = \mathbb{E}\{Z\}$, and \mathcal{G} , \mathcal{V} are independent random variables representing the shadowing and the fast fading with normalized power, respectively. Thus, \mathcal{G} is an IG random variable with shape parameter λ and $\mathbb{E}\{\mathcal{G}\} = 1$, and \mathcal{V} is the FTR random variable with parameters m, K, Δ , and $\overline{\gamma} = 1$.

We will provide the PDF and CDF of the composite IG/FTR and obtain the outage probability. The PDF of Z can be obtained from the closed-form expression of the GMGF in (11) and [8, eq. 12] as

$$f_Z(z) = \frac{\bar{Z}^{\lambda}(\lambda-1)^{\lambda}}{z^{\lambda+1}\Gamma(\lambda)} M_{\mathcal{V}}^{(\lambda)}\left(\frac{(1-\lambda)\bar{Z}}{z}\right), \qquad (17)$$

and the CDF of Z for integer λ , by using [8, eq. 22] and (11), is expressed as

$$F_{Z}(z) = \sum_{n=0}^{\lambda-1} \frac{\bar{Z}^{n} (\lambda-1)^{n}}{z^{n} \Gamma(n+1)} M_{\mathcal{V}}^{(n)} \left(\frac{(1-\lambda)\bar{Z}}{z}\right).$$
(18)

The PDF and CDF of the received signal amplitude R can be obtained by means of the transformation $Z = R^2$, from which they can be calculated, respectively, as $f_R(r) = 2rf_Z(r^2)$ and $F_R(r) = F_Z(r^2)$.

Considering that the SNR at the receiver is defined as

$$\gamma_z = \frac{\bar{\gamma}_z Z}{\bar{Z}},\tag{19}$$

where $\bar{\gamma}_z = \mathbb{E}\{\gamma_z\}$, then the outage probability is given by [14]

$$P_{\text{out}}^{\text{IG/FTR}}(\gamma_{th}) \triangleq P(\gamma_z < \gamma_{th}) = F_{\gamma_z}(\gamma_{th}) = F_Z(\bar{Z}\gamma_{th}/\bar{\gamma}_z),$$
(20)

where γ_{th} is the SNR threshold for reliable communication, and the CDF $F_Z(z)$ is given in (18). Therefore, the asymptotic outage probability can be achieved by $\bar{\gamma}_z \rightarrow \infty$. Using an asymptotic approximation for the FTR CDF in [15, eq. 18] and the asymptotic outage probability for IG-based composite fading models for $\lambda > 1$ in [8, eq. 33], the composite IG/FTR asymptotic outage probability is given by

$$P_{\text{out}}^{\text{IG/FTR}}(\gamma_{th}|\bar{\gamma}_z \to \infty) \approx \frac{\Gamma(\lambda+1)}{\Gamma(\lambda)(\lambda-1)} \frac{(1+K)}{(1+\frac{K}{m})^m} \times_2 F_1(\frac{m}{2}, \frac{1+m}{2}; 1; \frac{\Delta^2}{(\frac{m}{K}+1)^2}) \frac{\gamma_{th}}{\bar{\gamma}_z}, \quad (21)$$

where $_2F_1(.)$ is the Gauss hypergeometric function.



Fig. 1. Analysis and Monte-Carlo simulation of the PDF of γ under FTR fading obtained using the FTR-RS connection for m = 1.5, 3.

V. NUMERICAL RESULTS

In this section, numerical results obtained from our analytical derivations are presented for the new expression of the PDF of the FTR model, as well as for the PDF and the outage probability of the composite IG/FTR wireless channel model defined in the previous section. Results are validated by Monte Carlo simulations, showing an excellent agreement.

In Fig. 1, the PDF of the FTR fading model is plotted for two values of the m parameter: one integer (m = 3) and one non-integer m = 1.5 which has been obtain from (7). Simulation results show a perfect match to the analytical results in all cases.

Fig. 2 depicts the PDF of the received signal amplitude for the IG/FTR channel model, for different values of the gamma distribution parameter λ and \overline{Z} . The theoretical plots perfectly agreed with the Monte-Carlo simulation. Independently of the FTR model parameters, it can be observed that smaller values of λ make PDFs more sparse, and larger values of λ decrease the shadowing severity. This effect is shown for different parameters of $\overline{Z} = 1, 5$.

Finally, Fig. 3 plots the outage probability over IG/FTR fading in terms of the normalized threshold for different values of the FTR parameters (m and K). It can be observed that the asymptotic expression in (21) perfectly fits the theoretical curves as $\gamma_{th}/\gamma_z \rightarrow 0$. As expected, increasing the K



Fig. 2. Analytical and simulation results for the signal amplitude PDF of composite IG/FTR model with parameters $\bar{\gamma} = 1$, K = 4, $\Delta = 0.2$, and m = 2.



Fig. 3. Analytical, asymptotic and simulation results for the outage probability of composite IG/FTR vs. γ_{th}/\bar{Z} with parameters $\bar{\gamma} = 1$, $\Delta = 0.3$, and $\lambda = 2$.

parameter, representing the power ratio of the specular to diffuse components, decreases the outage probability. Also, the outage probability declines for light (m = 10) fluctuations in comparison to strong (m = 2) fluctuations of the specular components. On the other, the diversity order remains the same independently of the FTR parameters, as shown by the same slope in the high SNR regime (high values of \overline{Z}).

VI. CONCLUSION

We presented a flexible connection that describes the relationship between the Fluctuating Two-Ray (FTR) and Rician Shadowed fading models. Based on this novel formulation, we provided for the first time a closed-form expression of the GMGF for FTR fading. As an application of these results, the chief statistical functions as well as the exact and asymptotic outage probability of a composite IG/FTR wireless channel model have been provided in closed-form.

APPENDIX Solving Integral I1

The integral I1 for positive integer P1, arbitrary positive P2, arbitrary α and $|\beta| < 1$ can be computed as

$$I1 \triangleq \int_{0}^{\pi} \frac{(1+\alpha\cos(\theta))^{P_{1}}}{(1+\beta\cos(\theta))^{P_{2}}} d\theta$$
(22)
$$\stackrel{(A)}{=} \frac{1}{(1-\beta)^{P_{2}}} \int_{0}^{1} (1-\alpha+2\alpha x)^{P_{1}} \left(1+\frac{2\beta}{1-\beta}x\right)^{-P_{2}} \times (-1)x^{-\frac{1}{2}}(1-x)^{-\frac{1}{2}} dx$$
(23)

$$\overset{(B)}{=} \frac{-1}{(1-\beta)^{P2}} \sum_{q=0}^{P1} \binom{P1}{q} (2\alpha)^q (1-\alpha)^{P1-q} \\ \times \int_0^1 x^{q-\frac{1}{2}} (1-x)^{-\frac{1}{2}} \left(1+\frac{2\beta}{1-\beta}x\right)^{-P2} dx$$
(24)

$$\stackrel{(C)}{=} \frac{-1}{(1-\beta)^{P_2}} \sum_{q=0}^{P_1} \binom{P_1}{q} (2\alpha)^q (1-\alpha)^{P_1-q} \frac{\sqrt{\pi}\Gamma\left(q+\frac{1}{2}\right)}{\Gamma(q+1)} \\ \times {}_2F_1\left(P_2, q+\frac{1}{2}; q+1; -\frac{2\beta}{1-\beta}\right),$$
(25)

where (A) followed from the change of variables $\cos(\theta) = 2x - 1$, yielding $d\theta = \frac{-dx}{\sqrt{x}\sqrt{1-x}}$, (B) followed by using the binomial theorem $(a+b)^n = \sum_{q=0}^n {n \choose q} a^{n-q} b^q$ for positive integer P1, and (C) is obtained from the integral representation of the Gauss hypergeometric function [9, eq. (9.111)]

$${}_{2}F_{1}(a,b;c;z) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)} \int_{0}^{1} t^{b-1} (1-t)^{c-b-1} (1-tz)^{-a} dt \quad (26)$$

where a = P2, $b = q + \frac{1}{2}$, c = q + 1 and $z = -\frac{2\beta}{1-\beta}$.

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