Construction of Dynamically-Dependent Stochastic Error Models

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ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

UNIVERSITÉ
DE GENÈVE

PennState

IEEE/ION PLANS 2018, Monterey, California, US

Menu

➤ **Noise Characterization via Allan / Wavelet Variance**

➤ **Generalized Method of Wavelet Moments (GMWM)**

➤ **GMWM Extension for Covariate Dependencies**

➤ **Example with MEMS IMU**

Motivation

Stochastic noise characterization: *controlled* environment

- temperature (stable)
- dynamics (stable = at rest)

Example: data fusion

- Kalman Filter

Motivation

Stochastic noise characterization: *controlled* environment

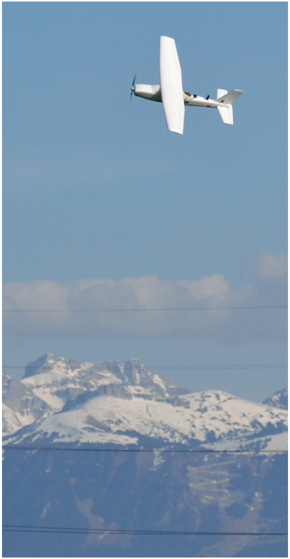
- temperature (stable)
- dynamics (stable = at rest)

Problem: *changing* environment

- environment (temperature)
- dynamics (acceleration / rotation)

Example: data fusion

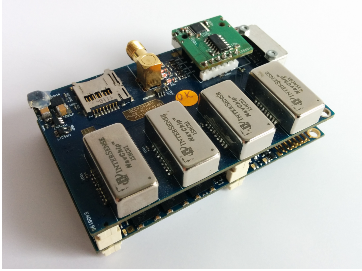
- Kalman Filter
- adaptive stochastic model

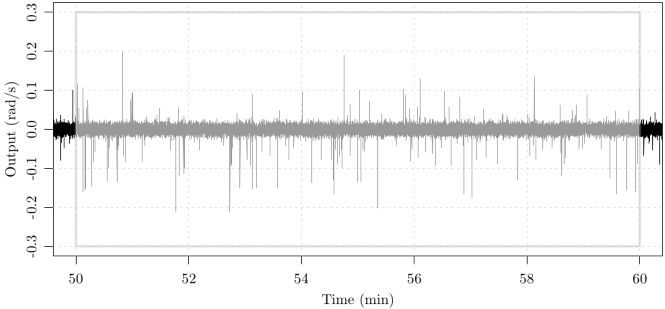


Analyzing a Signal

Types of signals

- Accelerometer
- Gyroscope
- Oscillator
- Gravimeter
- ...





Analysis of an Error Signal

State-space model

$$y_t = \omega t + u_t + \varepsilon_t$$

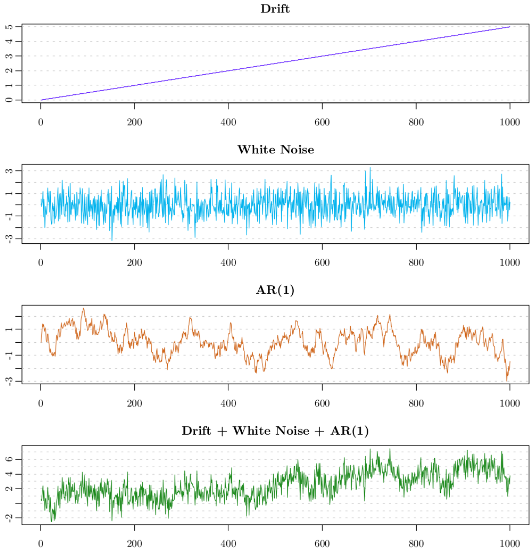
$$\varepsilon_t \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma^2_{\text{WN}})$$

$$u_t = \phi u_{t-1} + \eta_t$$

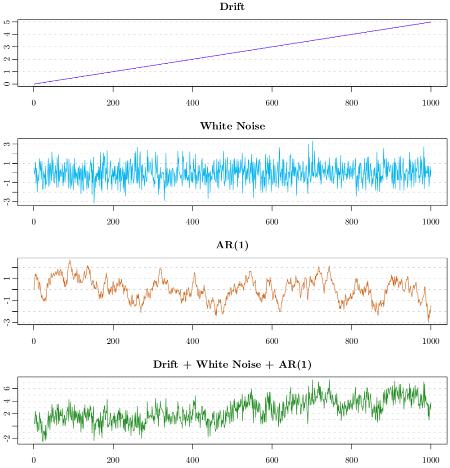
$$\eta_t \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma^2_{\text{AR(1)}})$$

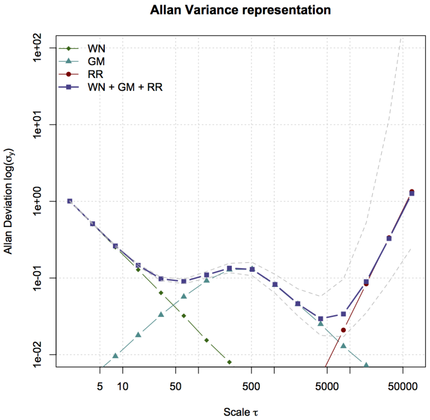
Noise parameters

$$\boldsymbol{\theta} = (\omega, \phi, \sigma^2_{\text{AR(1)}}, \sigma^2_{\text{WN}})$$



Analysis of an Error Signal via Allan Variance





Challenges

Existing methods

- "Graphical" Allan Variance
  - Limited to a few models
  - (Proven as) not consistent in general
  - "Inefficient" (non-automated)
- MLE (with EM algorithm)
  - Computationally intensive
  - Diverges with "complex" models

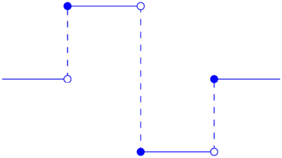
Need of an estimator

- Complex time-series model
- Computational efficiency
- Robust to outliers

Generalized Method of Wavelet Moments

Main idea

- Usage of the Wavelet Variance (WV)
- Filter of the signal with the Wavelet Function
- Exploitation of the relationship existing between a model $\boldsymbol{\theta}$ and its WV $\boldsymbol{\nu}(\boldsymbol{\theta})$ (i.e. **mapping** $\boldsymbol{\theta} \mapsto \boldsymbol{\nu}(\boldsymbol{\theta})$).
- "Inverse" this mapping by minimizing some discrepancies between empirical (i.e. observed WV/AV $\hat{\boldsymbol{\nu}}$) and the theoretical WV for a model $\boldsymbol{\nu}(\boldsymbol{\theta})$.



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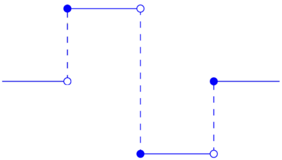
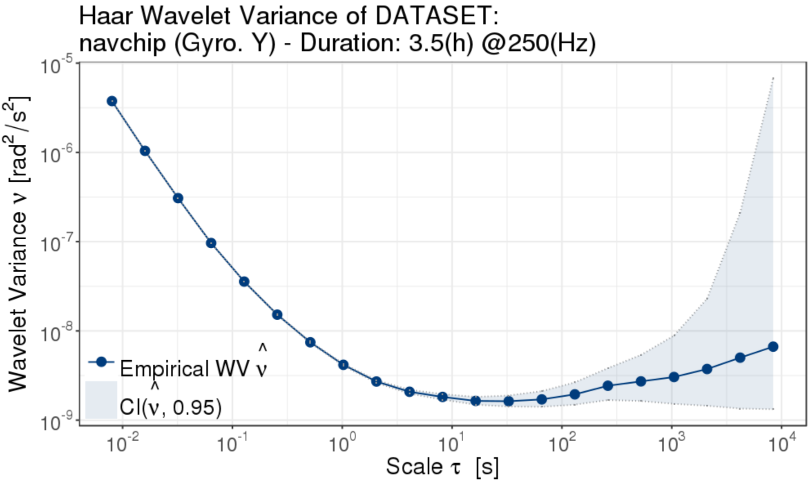
Definition

Solution of the following optimization problem with weighting matrix $\boldsymbol{\Omega}$:

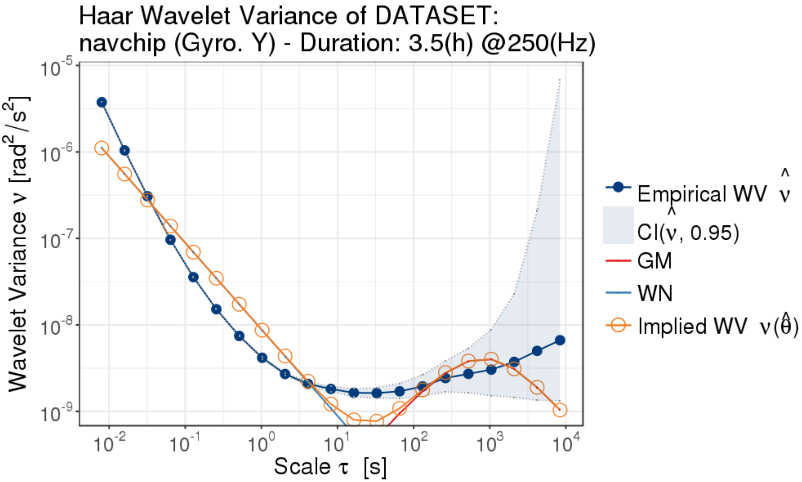
$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta} \in \boldsymbol{\Theta}}{\operatorname{argmin}} \; \left(\hat{\boldsymbol{\nu}} - \boldsymbol{\nu}(\boldsymbol{\theta})\right)^T \boldsymbol{\Omega} \left(\hat{\boldsymbol{\nu}} - \boldsymbol{\nu}(\boldsymbol{\theta})\right)$$



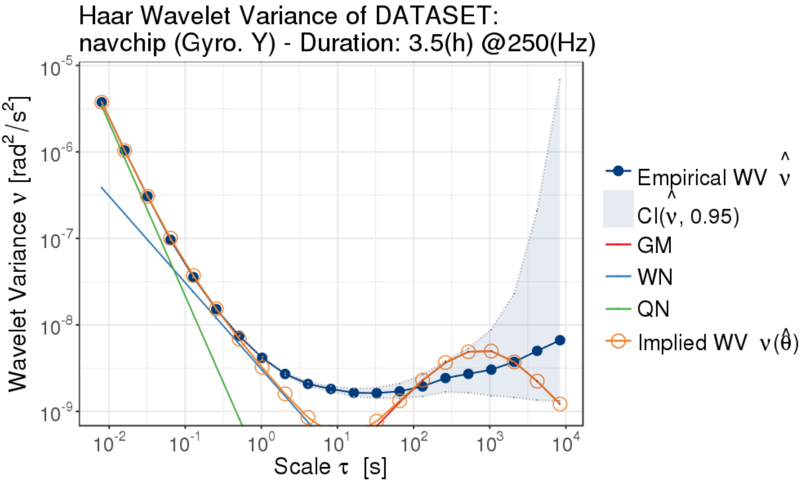
GMWM Example: Empirical WV



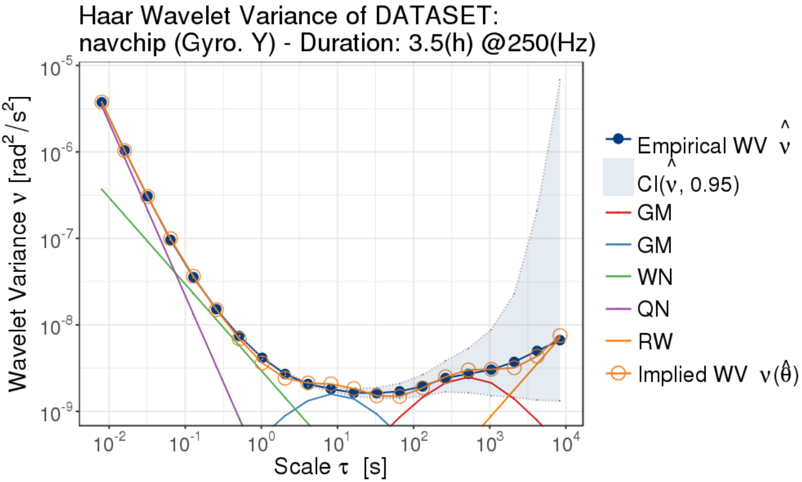
GMWM Example: underfitted model



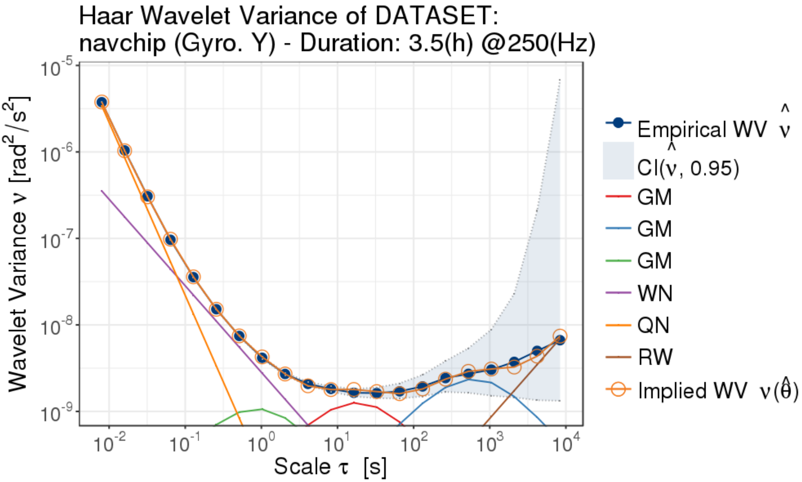
GMWM Example: underfitted model cont.



GMWM Example: suitable model

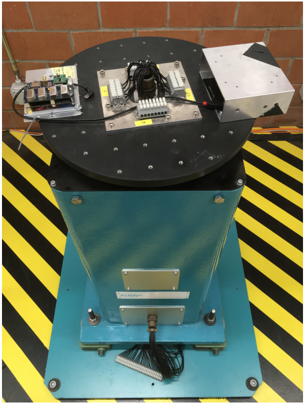


GMWM Example: overfitted model, WVIC

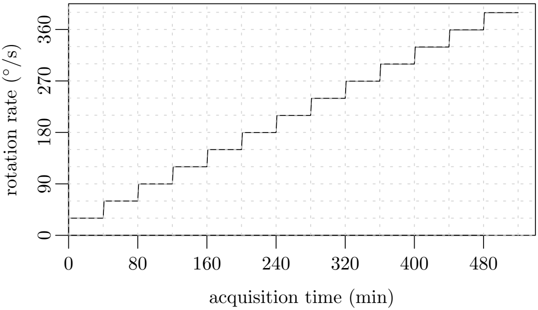


Experimental setup

Rotation table



Procedure



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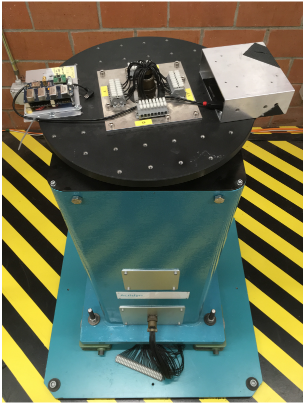

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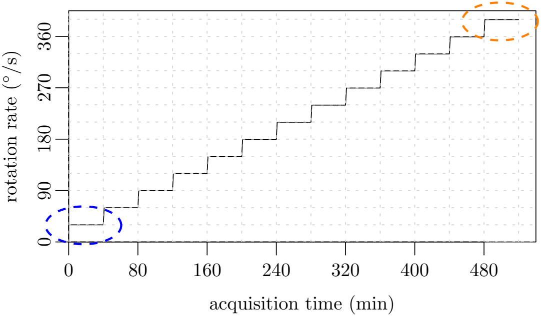
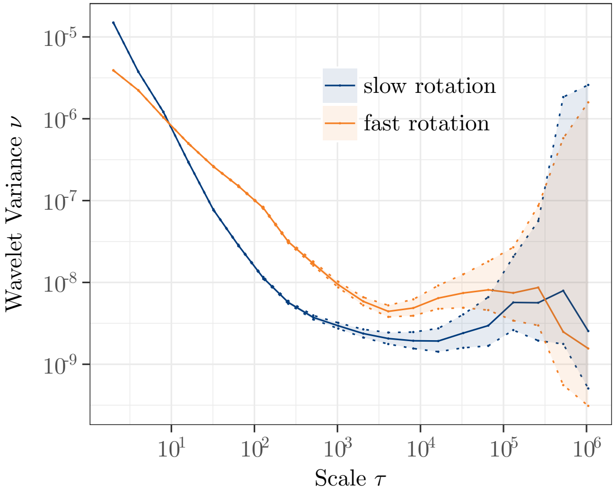


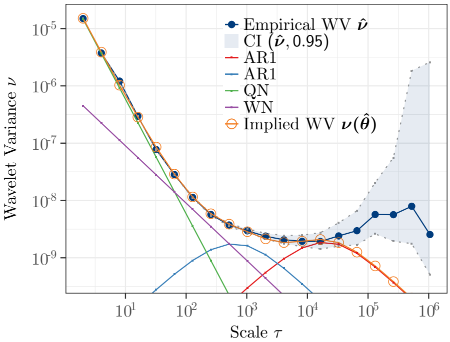
GMWM Analysis

MEMS IMU Gyroscope rotating at 30 °/s and 390 °/s

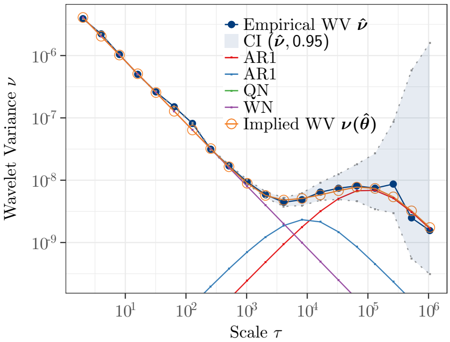


GMWM Analysis cont.

rotating at 30 °/s



rotating at 390 °/s



Process	Units	Datasheet	rotation rate 30 °/s	rotation rate 390 °/s
WN	$°/\sqrt{\text{hr}}$	0.18	0.146	0.439
QN	$(\text{rad/s})^2$	-	9.756e-06	2.220e-16

Extension for Covariate Dependency

Definition

- external process: $X_t, t \in \mathbb{N}$
  ➤ previous example: rotational speed
- *White Noise* process:

$$V_t \overset{iid}{\sim} \mathcal{N}\left(0, \gamma^2\right)$$

- *Auto-Regressive* process of order 1:

$$\begin{aligned} u_t &= \phi \; u_{t-1} + \varepsilon \\ \varepsilon &\overset{iid}{\sim} \mathcal{N}\left(0, \eta^2\right) \end{aligned}$$

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Extended parameter vector

$$\boldsymbol{\theta} = \begin{bmatrix} \boldsymbol{\varsigma}^T & \boldsymbol{\varphi}_1^T & \dots & \boldsymbol{\varphi}_d^T & \boldsymbol{\upsilon}_1^T & \dots & \boldsymbol{\upsilon}_d^T \end{bmatrix}^T \in \boldsymbol{\Theta}$$

Extension

Dynamic GMWM estimator

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta} \in \boldsymbol{\Theta}}{\operatorname{argmin}} \frac{1}{K} \sum_{k=1}^{K} \left\| \hat{\boldsymbol{\nu}}_k - \boldsymbol{\nu}(\boldsymbol{\theta}, c_k) \right\|^2_{\hat{\boldsymbol{\Omega}}_k}$$

c_k explains the covariate influence on the WV of bin k

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c_k explains the covariate influence on the WV of bin k

And the properties?

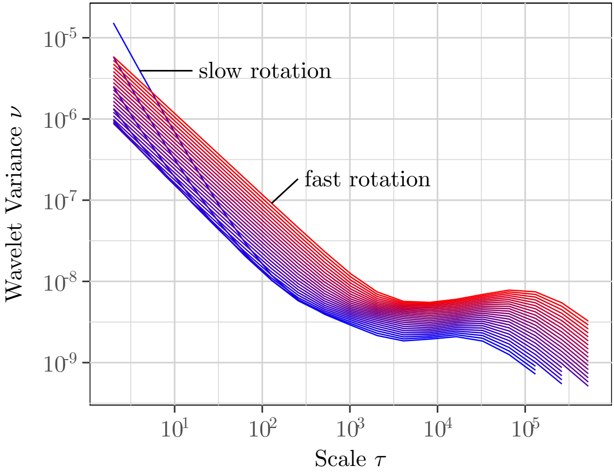
Identifiable ✓

Consistent ✓

Asymptotically Normal ✓

GMWM Example: covariate influence

MEMS IMU Gyroscope stochastic noise as a function of rotational speed



Conclusions

Properties

- Numerically stable
- Computationally efficient
- Covariate dependency (example: rotational speed)

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- Computationally efficient
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And more...

- Implementation
  - opensource package in statistical tool *R*
  - online webbrowser tool on `ggmwm.smac-group.com`
- Mathematical derivations in an upcoming publication
- Application examples
  - rotational speed
  - temperature: to be published with proofs

Thank you

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- samuel.orso@unige.ch
- stephane@psu.edu
- `github.com/SMAC-group/GMWM`

References

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