

Managing Large-Scale Mapping and Localization for Pedestrians Using Inertial Sensors

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Abstract—Pedestrian navigation in indoor environments without a pre-installed infrastructure still presents many challenges. There are different approaches that address the problem using prior knowledge about the environment when the building plans or similar are available. Since this is not always the case, a family of technologies based on the principle of Simultaneous Localization and Mapping (SLAM) has been proposed. In this paper we will present some estimates on how a mapping process based on FootSLAM - a form of SLAM for pedestrians - might scale for a large-scale collaborative effort eventually encompassing most of our public indoor space, where the mapping entities are humans.

Our assumptions on pedestrian motion and area visiting rate together with calculations based on the computational requirements of pedestrian SLAM algorithms allow us to make estimates with regard to the feasibility, scalability and computational cost of wide-scale mapping of indoor areas by pedestrians.

Keywords-indoor navigation, localization, mapping, SLAM

I. INTRODUCTION

For a number of years location-based services (LBS) for pedestrians have been attracting considerable interest. LBSs allow a user or group of users to know their location and use this information to plan the next possible course of action. LBS applications range from entertainment and health applications - e.g. finding the closest cafeteria or monitoring the location of the elder - to security applications, for example to help coordinate a team of firefighters during a rescue mission.

Nevertheless, ubiquitous indoor navigation for pedestrians and robots without the use of expensive infrastructure remains a challenging goal. A number of solutions based on inertial sensors and pedestrian dead reckoning (PDR) have been studied [1][2][3][4]. However, these solutions work under the assumption that indoor plans are always available.

To address this, one family of proposed technologies is based on the principle of Simultaneous Localization and Mapping (SLAM) [5]. Currently many existing cleaning robots commence a cleaning cycle with no memory of previous operations and perform SLAM during the cleaning process. This is a robust and successful approach in most domestic environments where the robot is expected to clean all areas within a room or home. But for a pedestrian entering a large building in search of a specific destination

it is more appropriate to draw on an existing map. SLAM in this case might be used to generate the initial map and then only to update the map to account for future changes in the environment or to refine its accuracy. In this paper we will refer to SLAM as a problem statement pertaining to an algorithmic setting, not an actual application. In other words, a user contributing to a collaborative mapping process might or might not be simultaneously using a positioning service. However, the data collected in such a way will usually be processed by algorithms falling within the SLAM domain. Furthermore, SLAM might be conducted in an offline fashion many hours after the data had been collected.

In the domain of SLAM for pedestrians, a few approaches have been proposed [6][7][8][9][10]. In this contribution we will focus on FootSLAM [6] and present some estimates on how a mapping process based on it might scale for a large-scale collaborative mapping process eventually encompassing most of our public indoor space. To make this feasible, we are interested in possible collaborative approaches where a large number of pedestrians collaborate in the mapping activity to speed up the task of accurately mapping the environment. Different techniques to collaboratively map indoor environments can be found in [11][12][13], but the mapping role is played by moving robots that carry laser scanners and cameras to detect landmarks, features or other robots in the environment. There are also hybrid solutions in which humans and robots collaborate, such as in [14].

FeetSLAM [15] or collaborative FootSLAM was recently introduced to improve the accuracy and completeness of FootSLAM maps. Following the principle of crowdsourcing [16], the mapping entities are collaborating humans. In FeetSLAM, two or more maps obtained by one or more people walking within the same environment are combined in an iterative fashion. Over time, individual maps tend to become more and more accurate, and so does the total combined map that includes all visited areas.

Our vision is to use FeetSLAM to generate a map database of the indoor world, and this paper provides a context on how much time and resources would be necessary to achieve this.

The remainder of this paper is organized as follows: in Section II we present the methodology used. Section III

introduces FootSLAM maps and map-aided pedestrian dead reckoning based on these maps. We continue by estimating the size of the indoor world in Section IV and Section V introduces the concepts of visiting frequency and proportion of mapping individuals to compute the required time to map a given area. Next, Section VI calculates the effort in terms of computational and memory requirements needed to map the indoor world and briefly addresses privacy issues. Finally, we present the main conclusions of this paper.

II. METHODOLOGY

We shall base our work in this paper on a number of assumptions:

- The majority of our society walks in some form or the other every day, or they might conduct comparable forms of motion such as using a wheelchair.
- Some of these people will contribute to a mapping process. This could be as active volunteers or simply by accepting that data are collected anonymously during usage of a device or service.
- People are most likely to need navigation services in unfamiliar environments, usually outside of private residences.
- The mapping process is largely governed by the normal usage of a building. In other words we propose that motion actively directed by the goal of mapping will remain the exception.
- We postulate that any successful positioning scheme drawing on maps will need to be able to coast through unmapped or poorly mapped areas. (Map-aided PDR fulfills this requirement and will be a special case discussed below).
- The more likely it is that people will desire or use a navigation service in a given area, the more it is frequented by people that can contribute to mapping.
- Much of our indoors is heavily frequented by people, at least part of the time.

We have chosen in this paper to approach the map generation problem from two directions. On the one hand we will look at small, individual areas of a building and look at the frequency with which they might be visited by humans. On the other hand we will look at the data that result directly from an average contributing person. By making assumptions about the proportion of actively contributing people within society we can make estimates of

- 1) how long it will take to map different areas of a building, and
- 2) the computational resources required to achieve this globally.

This fits with a possible introduction scenario where the only factor that changes significantly over time is the proportion of pedestrians (ρ) who at any point in time might actively or passively contribute to a collaborative

mapping effort. Coverage will grow most quickly where people tend to go, perhaps with a bias reflecting the social and technological background of the contributors (especially early adopters who might belong to specific social groups).

III. MAP-AIDED PDR BASED ON FOOTSLAM

A. FootSLAM Map Generation

FootSLAM maps are a probabilistic representation of human motion. In FootSLAM, measurements of a pedestrian's steps - what we call human odometry - are obtained for example by means of a foot-mounted inertial measurement unit (IMU) and pre-processed by a lower-level Kalman filter to obtain step vectors [4]. Subsequently, an upper-level particle filter is used to estimate possible errors in those step vectors [6]. Thus, each particle represents a possible hypothesis for the path followed by the pedestrian. A detailed explanation of the particle filter implementation can be found in Section II of [17].

FootSLAM discretizes the space using regular adjacent hexagonal prisms. The radius of the hexagonal base has been chosen to be 0.5 m, under the assumption that constraints and factors influencing human motion in buildings become decorrelated over spatial separations on the order of one meter. For now, the height of the prisms is chosen so that there is an integer number of prism layers (in our case 3) between two floors, assuming a uniform floor separation in the building.

During FootSLAM we count the number of times each particle crosses each one of the eight faces of the prisms it visits. Those particles that revisit similar face transitions are rewarded. Thus, for a reliable map to emerge, the pedestrian needs to revisit areas (loop closure). What could be seen as a constraint to FootSLAM's performance is addressed by FeetSLAM using other walks face transitions, helping reach convergence.

In FeetSLAM, at each iteration and for each dataset, the combined map of all other datasets is computed by finding the geometric transformations (translations along the x , y and z axes and rotations) that place them within the same coordinate system. This is needed because of the inherent property of SLAM algorithms where no absolute coordinate system is available and the map is then rotation and translation invariant. This combined map is used as a *prior* map during the FootSLAM estimation process of the given data set in the next iteration.

For example, we might have six datasets obtained from an office environment whose corresponding FootSLAM maps may or may not converge on their own. However, after jointly processing them with FeetSLAM, they reach convergence and their combined map represents all visited areas (Figure 1). Areas in which the pedestrians were walking more often (darker hexagons) help reach convergence and can be represented more accurately. An advantage of such a FootSLAM map is that in contrast to building plans where

only walls are shown, FootSLAM maps also reflect pieces of furniture and other obstacles that also channel human motion.

B. Map Characteristics of Map Aided PDR and FootSLAM

We recall that PDR drawing on maps for positioning is able to coast through unmapped or poorly mapped areas. Indeed, from a probabilistic perspective a FootSLAM map, in the absence of any observations, is equivalent in its mapping characteristics to an entirely open area that exerts no influence on human motion within it. Hence a PDR segment in an unmapped area will deteriorate in accuracy just as it would were a person to walk in a (known, i.e. mapped) nonrestrictive region. It is the strong restrictions of walls that maintains the position accuracy while a pedestrian walks in a building; after entering a room through a door the accuracy might be on the order of 1 meter, deteriorating while the user is in a large space until the next constrictive opening is passed.

It is very likely that a typical building contains areas of varying visiting frequency, perhaps spanning several orders of magnitude. A pedestrian in such a building will usually enter through areas with (relatively) high visiting frequency and cover longer distances in areas of average to high visiting frequency. Similarly, the areas in the building that strongly channel pedestrians' motion are more likely to be visited more frequently in the first place (for example the main corridors in Figure 1). This advantageous situation will mean that during the course of FootSLAM mapping those areas that are most beneficial towards achieving positioning accuracy are usually mapped more quickly than the less relevant areas.

IV. HOW BIG IS INDOORS

With a world population of approximately 7 billion humans as of 2012, we will approximate the total number of pedestrians N_p we may have to localize to be $10 \cdot 10^9$.

In Europe (EU 27 plus Norway and Switzerland) there are approximately 512 million inhabitants spread over ca. 25 billion square meters [18], which means that there is a ratio of ca. 50m^2 per person. If we assume that this number provides an estimate for the extent of indoor area per person, we can postulate that there are $S_w = 5 \cdot 10^{10} \text{ m}^2$ of indoor area in the world to be potentially mapped.

However, one must note that only 25% of the indoor areas in the EU correspond to non-residential areas, i.e. public/open spaces where users could strongly benefit from LBSs. Nevertheless, our goal here is to estimate the costs of mapping *all* kinds of indoor areas, without further differentiation of application domains.

V. RATE OF COVERAGE

A. Walking Speed

Pedestrians show different walking speeds depending on their gender, age and weather conditions among others.

Speeds vary, on average, between 1.16 and 1.56m/s [19]. Using one single foot-mounted IMU to obtain a pedestrian's step measurements we can assume that the average step rate v is roughly one step per second.

B. Visiting Frequency of Two-dimensional Space

We define the visiting frequency f of an area to be the number of people passing through it in a given space of time. We do not distinguish between the direction of travel or speed and we assume no pedestrian sources or drains. For the sake of simplicity and because of the discretization of current implementations of FootSLAM we will typically refer to an area of one square meter.

When quantifying the length of time needed to map indoor areas it is clear that we shall have to take into account the large differences in the number of times these areas are visited by people. But we have not found a source or process that could quantify the probability distribution of the visiting frequency in our very diverse indoor world. One might speculate that the visiting frequency distribution approximately follows a structure such as a Zipf distribution [20]. One approach to arrive at at least a rough approximation might be to model a typical day-in-my-life of a representative set of people and estimate where they spend their time (e.g. working in an office, going out, traveling, shopping, etc). To help understand and quantify the variability in visiting frequency, we shall define six profiles of visiting frequency of one square meter, in decreasing order of visiting frequency:

- 1) Maximal frequency: 2 people passing through the square meter per second: 172800 visits per day (for example the area around a turnstile before entering a platform in an underground station of a big city).
- 2) High frequency: $\frac{0.1}{s}$ (one person every ten seconds): 8640 visits per day (perhaps a busy museum or shop entrance, with roughly 3 million visitors per year).
- 3) Medium frequency: $\frac{0.01}{s}$ (one person every 100 seconds): 864 visits per day (area in front of a busy ATM; entrance to an elevator).
- 4) Low frequency: $\frac{0.001}{s}$ (one person every 1000 seconds, i.e. just over 3 per hour): 86.4 visits per day (low frequented region of a typical office corridor).
- 5) Very low frequency: $\frac{0.0001}{s}$ (one person every 10000 seconds, i.e. roughly ten persons per day): 8.64 visits per day (entrance of a residential building).
- 6) Minimal frequency: $\frac{0.00001}{s}$ (one person every 100000 seconds, i.e. roughly one person per day): 8.64 visits in 10 days (a storage room that is not frequently visited, roughly once a day).

These profiles of visiting frequency will be later used to compute an estimation of the required time to map a given area.

C. Proportion of mapping individuals

Mobile phone penetration has reached almost 100% in modern society and it is to be expected that the proportion

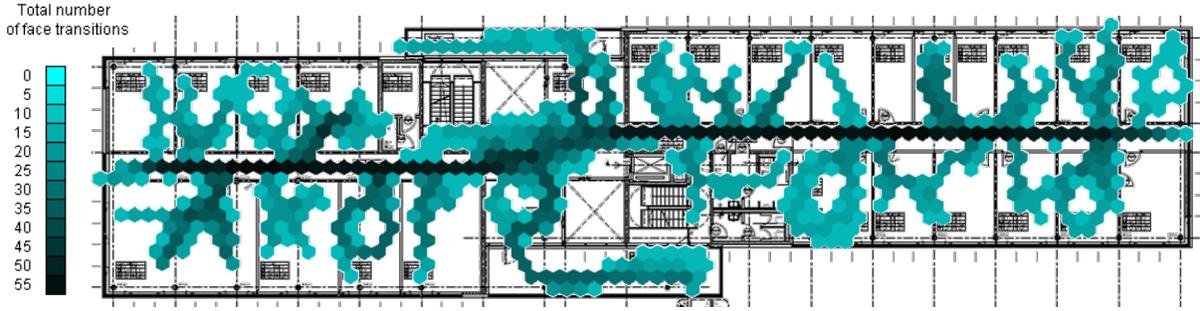


Figure 1. Combined FootSLAM map obtained from six walks in an office environment (in aquamarine) after 10 iterations using the FeetSLAM algorithm. The total duration of these 6 data sets was roughly 66 minutes. Darker hexagons represent hexagonal prisms with more face transitions counts that correspond to more frequented areas. The building layout was not used during FeetSLAM, but it is shown as a reference.

of user devices with high computing power resources will increase and eventually lead to a dominance of devices with a least smart-phone computing capabilities. One can only speculate, however, with regards to the proportion of pedestrians ρ who at any point in time might actively or passively contribute to a collaborative mapping effort whilst going about their daily lives.

D. Time-to-map

FootSLAM as well as other many forms of Simultaneous Localization and Mapping will require several visits to a certain area in order to achieve a map with some degree of reliability or local coverage. As far as FootSLAM is concerned we can achieve a high accuracy of a local map after roughly 10 to 100 visits. Using these two limits we can use the above quantitative definitions and assumptions to compute the time-to-map of areas corresponding to each of the different profiles of visiting frequency (Figure 2). The red rectangle shows the cases that are of particular interest and relevance. The required time to map an area ranges from 20 seconds for the turnstile type of area ($\rho = 0.25, f = 2 \text{ s}^{-1}, N_{vis} = 10$) to roughly one year for those areas that are less frequently visited, such as a residential entrance ($\rho = 0.0025, f = 10^{-4} \text{ s}^{-1}, N_{vis} = 10$ and $\rho = 0.025, f = 10^{-4} \text{ s}^{-1}, N_{vis} = 100$).

VI. EFFORT

A. Computational Effort

1) *FootSLAM*: We will assume that a healthy and active person takes about $N_s = 10000$ steps per day. We can also assume that 50% of this activity takes place in open/public spaces, with FootSLAM activated, i.e. $N_s^{FS} = 5000$ steps.

Thus, at a roughly walking rate v of one step per second the data collected during one day have a duration of $d = N_s^{FS}/v = 5000$ seconds. If we process these data on a single core processor at 100 times faster-than-realtime rate, then the required CPU time to process the data of a single walk is: $T_{CPU} = d/100 = 50$ seconds.

2) *FeetSLAM*: As stated before, the maps obtained by different walks within the same building can be merged using the FeetSLAM algorithm to generate a more accurate and complete map of the indoor environment. For the following calculations, we will assume that an average building has a floor surface area of $S_b = 10^4 \text{ m}^2$.

In case a large number of individual maps is available, they can be combined using a tree structure, whereby smaller groups of maps first combined. Later on, the combined maps of each group are merged until one single final map is available. If we choose to combine maps in groups of $N_m = 4$ maps, then one iteration of the FeetSLAM algorithm takes about 10 minutes in a single core processor and the time to run 4 iterations - at which point we have shown good convergence results - will be bounded by half an hour (later iterations reduce the area where translations and rotation values are searched for and take less time). If we assume that $N_M = 64$ maps of an indoor environment are enough to achieve great accuracy (areas are revisited between 10 and 100 times) and that we can execute $N_g = N_M/N_m = 16$ FeetSLAM processes in parallel, we could obtain the total combined map of the building in $T_b = \log_{N_m}(N_M) \cdot 0.5 \text{ hours} = 3 \cdot 0.5 \text{ hours} = 1.5 \text{ hours}$.

As a consequence, running FeetSLAM for our $S_w = 5 \cdot 10^{10} \text{ m}^2$ of estimated indoor area in groups of $S_b = 10^4 \text{ m}^2$ and using 1000 cores of standard desktop performance (ca. 3GHz) operating in parallel, we could have the *entire* indoor world mapped in $T_w = S_w/S_b \cdot T_b \approx 313$ days, i.e. in less than 1 year.

B. Memory Requirements

Uncompressed, a simple 3D FootSLAM map using hexagonal prisms requires 8 counters per hexagon prism. Assuming that the prisms that lie in between floor levels can be represented efficiently (as they represent a prism without any face transitions) we approximately need to represent prisms covering the floor surface area. We assume that we can represent most occurring hexagon prisms with 4 byte values. As a result, the map of a building with a floor surface area of $S_b = 10^4 \text{ m}^2$ can be stored using $M_b = 40 \text{ KB}$.

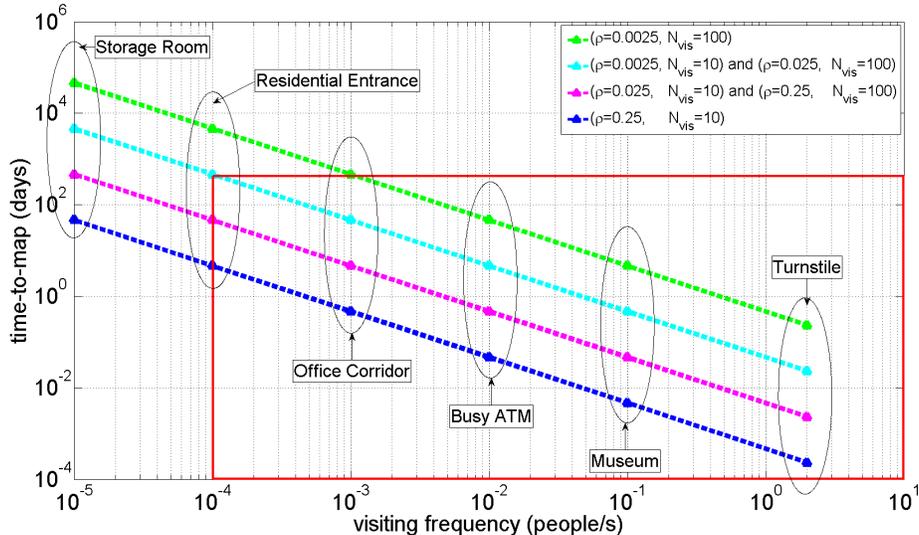


Figure 2. Time-to-map an area of one square meter given the frequency of visiting pedestrians f assuming different values for the density of collaborating pedestrians $\rho = \{0.0025, 0.025, 0.25\}$ and different needs of number of visits $N_{vis} = \{10, 100\}$. The red rectangle shows the cases that are relevant.

Assuming $S_w = 5 \cdot 10^{10}$ m² of indoor area we require roughly $M_w = S_w/S_b \cdot M_b = 2 \cdot 10^{11} = 200$ gigabytes of memory to store the maps of the indoor world. However, we postulate that we can compress this with loss-less source encoding by at least a factor of four, and much higher compression factors might be achieved if small amounts of distortion can be tolerated.

C. Communication Effort

We assume that a pedestrian either contributing to the mapping task or with localization needs will compute his step vectors [21] on his mobile device. A server will be in charge of processing these step vectors using FootSLAM. If we can quantize each step vector to 4 bytes, then roughly 4 bytes per second need to be uploaded to the server.

The map merging process (FeetSLAM) is also performed at the server. Later on, pedestrians can access/download the combined maps of the buildings they visit, with a size of the order of tens of kilobytes.

D. Privacy Issues

Technologies that help determine the precise position of people allow for an immense number of location-based services. Nevertheless, these technologies also open the door to mischievous usage of these data. Having accurate information about the places that an individual visits and the times when these visits took place can reveal aspects of his private life, for example disabilities, likes and dislikes, place of residence, etc. and could be used by third parties for unwanted advertising purposes among others. Such privacy issues are not within the scope of this paper, but we suggest that the same regulations that apply for other location sensors (e.g. GPS) may be applied to the inertial sensors data (Section IV-A.2 of [22]).

In the personal domain we believe using FootSLAM as a base for localization is clearly an acceptable compromise for indoor navigation since the data collected by inertial sensors do not reveal sensitive information of the environment - as opposed to visual SLAM approaches - such as other people's location.

VII. CONCLUSION AND FURTHER WORK

This paper proposed an approach called FootSLAM to generate maps of our indoor world with the mapping entities being walking people. FootSLAM maps are of great value since building plans may be unavailable, incomplete, outdated or subject to privacy rights. These maps can be used as a basis for pedestrian dead reckoning, enabling a number of location-based applications with high accuracy requirements to develop.

In this paper we have estimated the time-to-map a given area as a function of the proportion of contributing pedestrians, the frequency of visits of that area and a required number of visits (related to the required accuracy of the map). We have shown that areas that are frequently visited could be mapped in less than a day and areas that are hardly ever visited would take years to be mapped. Nevertheless, we believe that those areas that are frequently visited are the areas that represent possible targets for location-based applications, e.g. airports, underground stations, shopping centers, etc.

FeetSLAM or collaborative FootSLAM has been presented as a form of crowdsourcing to help the accuracy and completeness of FootSLAM maps, combining different datasets obtained from different walks within the same environment. We have estimated that the maps of the entire indoor world could be stored using roughly 200 gigabytes

and that the time to compute these maps using tens of walks per area would take ca. one year on 1000 cores.

These initial estimates suggest the feasibility of our proposed mapping technique to build an indoor map database within an affordable period of time and with relatively sparse user penetration. Further work will focus on experimentally validating these estimates within a large building, given different numbers of collaborating pedestrians and different number of visits per area.

In this paper we have also introduced the idea of 3D FootSLAM based on hexagonal prism grid instead of the flat 2D representation based on hexagonal polygons that has been used until now. However, we are still addressing its implementation and many challenges that have arisen.

Future work needs to address the enhancement of the performance of both FootSLAM and FeetSLAM to reduce their time and memory requirements.

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REFERENCES

- [1] E. Foxlin, "Pedestrian tracking with shoe-mounted inertial sensors," in *IEEE Computer Graphics and Applications*, vol. 25, no. 6, 2005, pp. 38–46.
- [2] S. Beauregard, Widyawan, and M. Klepal, "Indoor PDR performance enhancement using minimal map information and particle filters," in *Proc. of the IEEE/ION PLANS 2008, Monterey, USA, 2008*.
- [3] A. Brajdic and R. Harle, "Particle filtering on GPU for indoor pedestrian localization," in *Indoor Positioning and Indoor Navigation (IPIN), 2012 International Conference on*, November 2012.
- [4] B. Krach and P. Robertson, "Integration of foot-mounted inertial sensors into a bayesian location estimation framework," in *WPNC. IEEE, 2008*, pp. 55–61.
- [5] H. Durrant-Whyte and T. Bailey, "Simultaneous localization and mapping (SLAM): Part I the essential algorithms," *IEEE Robotics and Automation Magazine*, vol. 2, p. 2006, 2006.
- [6] P. Robertson, M. Angermann, and B. Krach, "Simultaneous localization and mapping for pedestrians using only foot-mounted inertial sensors," in *In Proc. UbiComp 2009, ACM, 2009*, pp. 93–96.
- [7] M. Hardegger, D. Roggen, S. Mazilu, and G. Troester, "ActionSLAM: Using location-related actions as landmarks in pedestrian SLAM," in *Indoor Positioning and Indoor Navigation (IPIN), 2012 International Conference on*, November 2012.
- [8] S. Grzonka, A. Karwath, F. Dijoux, and W. Burgard, "Activity-based estimation of human trajectories," *Robotics, IEEE Transactions on*, vol. 28, no. 1, pp. 234–245, feb. 2012.
- [9] B. Cinaz and H. Kenn, "HeadSLAM - simultaneous localization and mapping with head-mounted inertial and laser range sensors," in *Wearable Computers, 2008. ISWC 2008. 12th IEEE International Symposium on*, 28 2008-oct. 1 2008, pp. 3–10.
- [10] E. Kaiser and M. Lawo, "Wearable navigation system for the visually impaired and blind people," in *Computer and Information Science (ICIS), 2012 IEEE/ACIS 11th International Conference on*, 30 2012-june 1 2012, pp. 230–233.
- [11] A. Howard, "Multi-robot simultaneous localization and mapping using particle filters," *Int. J. Rob. Res.*, vol. 25, no. 12, pp. 1243–1256, Dec. 2006.
- [12] A. León, R. Barea, L. M. Bergasa, E. López, M. Ocaña, and D. Schleicher, "Multi-robot SLAM and Map Merging," in *IX Workshop of Physical Agents (WAF 08)*, sep. 2008, pp. 171–176.
- [13] H.-C. Lee, S.-H. Lee, S.-H. Lee, T.-S. Lee, D.-J. Kim, K.-S. Park, K.-W. Lee, and B.-H. Lee, "Comparison and analysis of scan matching techniques for cooperative-SLAM," in *Ubiquitous Robots and Ambient Intelligence (URAI), 2011 8th International Conference on*, nov. 2011, pp. 165–168.
- [14] A. Kleiner, C. Dornhege, and S. Dali, "Mapping disaster areas jointly: RFID-coordinated SLAM by humans and robots," in *Safety, Security and Rescue Robotics, 2007. SSR 2007. IEEE International Workshop on*, sept. 2007, pp. 1–6.
- [15] P. Robertson, M. Garcia Puyol, and M. Angermann, "Collaborative pedestrian mapping of buildings using inertial sensors and FootSLAM," in *ION GNSS 2011, Portland, Oregon, USA*, september 2011.
- [16] J. Howe, "The rise of crowdsourcing," *Wired*, vol. 14, no. 6, 2006.
- [17] M. Garcia Puyol, P. Robertson, and O. Heirich, "Complexity-reduced FootSLAM for indoor pedestrian navigation," in *Indoor Positioning and Indoor Navigation (IPIN), 2012 International Conference on*, November 2012.
- [18] O. Rapf, "Europes's buildings under the microscope," in *Buildings Performance Institute Europe (BPIE)*. BPIE, 2011. [Online]. Available: http://www.europeanclimate.org/documents/LR_\%20CbC_study.pdf
- [19] R. Knoblauch, M. Pietrucha, and M. Nitzburg, "Field studies of pedestrian walking speed and start-up time," *Transportation Research Record: Journal of the Transportation Research Board*, vol. 1538, no. -1, pp. 27–38, 1996.
- [20] G. Zipf, "Human behaviour and the principle of least-effort." Cambridge, MA: Addison-Wesley, 1949.
- [21] B. Krach and P. Roberston, "Cascaded estimation architecture for integration of foot-mounted inertial sensors," in *Position, Location and Navigation Symposium (PLANS), 2008 IEEE/ION*, may 2008, pp. 112–119.
- [22] M. Angermann and P. Robertson, "FootSLAM: Pedestrian simultaneous localization and mapping without exteroceptive sensors - hitchhiking on human perception and cognition," *Proceedings of the IEEE*, vol. 100, no. Special Centennial Issue, pp. 1840–1848, 5 2012.