Gaussian Boson Sampling for binary optimization

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Abstract—In this study, we consider a Gaussian Boson Sampler for solving a Flight Gate Assignment problem. We employ a Variational Quantum Eigensolver approach using the Conditional Value-at-risk cost function. We provide proof of principle by carrying out numerical simulations on randomly generated instances.

Index Terms—Gaussian Boson Sampling, Variational Quantum Eigensolver, Combinatorial Optimization

I. INTRODUCTION

The Variational Quantum Eigensolver (VQE), a variational approach for finding the ground state energy of Hamiltonians, makes use of the quantum device to prepare an ansatz state in the form of a parametric quantum circuit. This ansatz can be represented by $\boldsymbol{\theta} \in \mathbb{R}^p \mapsto |\varphi(\boldsymbol{\theta})\rangle$. Then, the goal is to minimize the Hamiltonian expectation value in a hybrid quantum-classical loop. VQE has recently been proposed to address combinatorial optimization problems [1].

In this paper, we explore a VQE-based approach, already presented in [2], to address the Flight-Gate Assignment (FGA) problem. As in [3], we use a Gaussian Boson Sampler (GBS) instead of a digital quantum computer. In contrast, our approach employs a more general GBS ansatz, along with a cost function that is computed analytically in the case of VQE.

II. GAUSSIAN BOSON SAMPLING

A GBS is a non-universal photonic quantum platform and one of the prominent candidates for demonstrating quantum advantage in the near future [4], [5].

The GBS ansatz consists of pure N-modes Gaussian states without displacement sampled using threshold detectors (TD) [6]. The quantum state is prepared from the vacuum by applying squeezing gates with parameters $(r_1, \ldots, r_N) \in \mathbb{R}^N_+$

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$$\hat{\Pi}_{j}^{(0)} = |0_{j}\rangle\langle 0_{j}| \quad ext{and} \quad \hat{\Pi}_{j}^{(1)} = \mathbb{I} - |0_{j}\rangle\langle 0_{j}| \,,$$

where \mathbb{I} is the identity operator and $|0_j\rangle$ the vacuum on the optical mode j. The probability to detect a pattern is proportional to the *Torontonian* of the submatrix of $O = \mathbb{I} - \Sigma^{-1}$ corresponding to the pattern.

III. THE FLIGHT-GATE ASSIGNMENT PROBLEM

The FGA problem aims at optimizing the assignment of flight activities F to airport gates G. In this work, we consider a scenario where the total transfer time of passengers has to be minimized, subject to two linear constraints 1) each flight can be assigned to at most one gate 2) two flight activities cannot be assigned to the same gate simultaneously.

This problem can be formulated as a Quadratic Unconstrained Binary Optimization (QUBO) problem [7]

$$Q(\mathbf{x}) = T(\mathbf{x}) + \lambda^{\text{one}} \sum_{i} \left(\sum_{\alpha} x_{i\alpha} - 1 \right)^{2} + \lambda^{\text{not}} \sum_{(i,j) \in P} \sum_{\alpha} x_{i\alpha} x_{j\alpha}.$$

Here $\mathbf{x} = (x_{i\alpha})_{i \in F, \alpha \in G} \in \{0, 1\}^{|F| \times |G|}$ is a set of binary variables representing an assignment $(x_{i\alpha} = 1 \text{ if and only if} flight <math>i \in F$ is assigned to gate $\alpha \in G$), $T(\mathbf{x})$ is a quadratic functional accounting for the total passenger travel time and the last two terms encode the two constraints, with $P \subset F \times F$ listing the incompatible pairs of flights and $\lambda^{\text{one}}, \lambda^{\text{not}} > 0$ are constants chosen large enough.

We can embed this problem into GBS with $N = |F| \times |G|$ optical modes as follows: the QUBO is turned into a Hamiltonian \hat{Q} by replacing each binary decision variable $x_{i\alpha}$ with the projection $\hat{\Pi}_{i\alpha}^{(1)}$. The ground state of \hat{Q} then encodes the optimal solution corresponding to the QUBO minimizer.



Fig. 1: Fraction of successful runs for threshold t = 0.1.

IV. VQE USING THE CONDITIONAL VALUE-AT-RISK

The *Conditional Value-at-Risk* (CVaR) has been proposed to improve the VQE approach to combinatorial optimization problems by restricting the search space to the x's with the lowest energy [8]. Here, the cost function to minimize reads

$$\mathcal{C}(\boldsymbol{\theta}) = \mathrm{CVaR}_{\alpha}(X(\hat{Q}, \boldsymbol{\theta})), \quad \alpha \in (0, 1],$$
(1)

where $X(\hat{Q}, \theta)$ is the distribution of the observable \hat{Q} in the quantum state $|\varphi(\theta)\rangle$ and $\text{CVaR}_{\alpha}(X) = \mathbb{E}[X \mid X \leq F_X^{-1}(\alpha)]$ the CVaR with tail α -left of a random variable X, and F_X is the cumulative density function of X.

In the experiments, the cost function can be estimated by performing K measurements of $|\varphi(\theta)\rangle$ and averaging over the $\lceil \alpha K \rceil$ lowest energy values. When $\alpha = 1$, we recover the VQE cost function.

V. METHOD

We assess the performance of the GBS ansatz for solving the QUBO-FGA problem using the CVaR-VQE approach. The GBS is parameterized by the symmetric matrix $\boldsymbol{\theta} = U \operatorname{diag}(r_1, \ldots, r_N) U^T$. For fastening the training procedure, we train only the real parts of the 3N parameters corresponding to the smallest entries of the QUBO Hamiltonian.

The classical minimization is performed with constrained optimization by linear approximation (COBYLA), with at most 50N function evaluations. We consider the randomly generated non-trivial and classically-hard problem instances in [2]. For each problem sizes $N \in \{6, 8, 10, 12, 14, 16\}$, we consider 50 different instances and run 5 training with random initialization. When $\alpha = 1$, the cost function in (1) admits an analytical expression with respect to θ . Therefore, we can use the highly efficient ADAM optimizer routines available in TensorFlow to minimize it.

We say an instance is *successful* if one of the five runs results in a fidelity of the quantum state $|\varphi(\theta_*)\rangle$ after training with the ground state of \hat{Q} higher than a threshold $t \in \{0.1, 0.01\}$.



Fig. 2: Fraction of successful runs for threshold t = 0.01.

VI. NUMERICAL RESULTS AND COMMENTS

The fraction of successful runs for fidelity threshold of 0.1 and 0.01 are presented respectively in Fig. 1 and Fig. 2.

We observe that employing CVaR outperforms VQE significantly, achieving almost perfect success rates for $\alpha = 0.01$ or 0.1. The only exception is when $(\alpha, t) = (0.01, 0.1)$, which is attributed to the CVaR cost function's inability to reward fidelity higher than α [8]. For $\alpha = 0.25$ and $N = \{12, 16\}$, the fraction of successful runs decreases. In these cases, the failing instances typically involve 3 or 4 flight activities. The algorithm appears here to be trapped in local minima.

VII. CONCLUSION AND OUTLOOK

In this study, we have tackled the FGA problem by employing GBS in combination with a CVaR-VQE approach, which significantly enhances performance compared to VQE. Despite focusing on small-sized instances, this work serves as an initial proof of principle for this new approach.

However, we believe that there is substantial room for improvement in the algorithm, for instance through careful choice of parameters to optimize. Another direction would use the binary encoding of the FGA problem, as proposed in [2], which directly represents the first constraint.

Lastly, it would be beneficial to compare the use of the GBS ansatz with other ansatz types, specifically the one proposed in [2]. This comparison could offer valuable insights and further enhance our understanding of the problem-solving capabilities of different ansatz strategies.

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