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Modeling Transmission Degradation on FSO Links Caused by Weather Phenomena for WMN Optimization

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***Abstract*—This work deals with wireless mesh networks (WMN) composed of FSO (free space optics) links. Although FSO links realize broadband transmission at low cost, their drawback is sensitivity to adverse weather conditions causing transmission degradation on multiple links. Hence, designing such FSO networks requires an optimization model to find the cheapest configuration of link capacities that will be able to carry an acceptable level of the demanded traffic in all weather states that can be foreseen in network operation. Such a model can be achieved using robust optimization techniques, and for that it is important to find a tractable way of characterizing possible link (capacity) degradation states corresponding to weather conditions not known in advance. In the paper we show how weather records can be translated to vectors of link capacity available (after degradation) in particular weather states, and how the resulting link degradation states may be represented mathematically in a compact and tractable way to be exploited in optimization. To solve the second task we will make use of a generalization of a combinatorial problem of finding a minimum hitting set to deduce a compact set approximating a given set of link degradation states. Finally, we illustrate the effectiveness of our model by means of a numerical study.**

***Index Terms*—free space optics, wireless mesh networks, robust optimization, uncertainty sets, hitting sets**

I. Introduction

Context and motivation. The FSO networks considered in this paper are characterized by fluctuations of capacity currently available on the links caused by adverse weather conditions. For this reason, optimization of FSO becomes complex as it has to cope with degradation of data transmission on the FSO links at the network design stage. As a continuation of our previous studies, this issue is undertaken below by introducing a new way of characterizing link degradation states for optimization purposes. Here, an important element is the use of proper, channel condition-dependent, signal modulation and coding schemes (MCS) at the transmitters. This allows affected link transmissions to remain operational, assuring proper data decoding at the receivers at the expense of decreased data transmission rate, i.e., link capacity. Thanks to that, the influence of degraded weather condition on link capacity (bit-rate) is kept under control but at some price that one needs to evaluate and optimize. In this study, the fraction of the maximum attainable (i.e., nominal) link capacity lost due to a change of its MCS is referred to as link degradation ratio. Typical values of such ratios are $0.50, 0.75$ and 1, and each particular weather condition that can occur defines a degradation state referred to as the link degradation state.

Problem studied. In our previous papers [1], [2] we presented optimization techniques addressing the problem of link capacity fluctuations in FSO networks caused by adverse circumstances, such as bad weather affecting the received signal strength in wireless networks. This problem is complex since it requires consideration of the state-dependent capacities for all network traffic demands in all possible network states caused by weather degradations. Closely related to that is the issue of preparing the data required to characterize the set of link degradation states for the dimensioning problem. Such data are obtained by translating weather states into network states characterized by link degradations ratios. We take into account how different weather factors as rain, snow, visibility, etc. impact the transmission quality measured by means of link margin (see [3]). Typically, a given weather condition affects a subset of network links, and each affected link loses

a portion (fraction) of its nominal capacity – the quantity called link degradation ratio above. Then, each particular weather condition that may occur defines a network state referred to as link degradation state. We will describe a full procedure to convert the weather data to the link degradation states set used in our optimization approach. This is the first question studied in this paper. On the other hand, this gives rise to a large number of such states to deal with, while their prediction is based solely on the weather conditions observed in the past. Thus, the set of the considered link degradation states, called the reference set of states, typically contains a large number of states of order of number of hours included in a given reference time horizon (i.e. $24 \times 365 = 8760$ for a period of one year). Furthermore, we need to dimension the network to deal with future weather states as well, which motivates the need for a compact characterization of the reference sets. A certain type of such compact sets (called uncertainty sets) was studied in [1], [2], [4]. In this paper we propose a version of uncertainty sets that estimate the original sets of degradation states more accurately. Our alternative characterization of the link degradation states captures the incidence of a group of links in several degradation states; more precisely, we find the most frequent subsets of links simultaneously affected by the weather phenomenona.

Contribution. The contribution of this paper is twofold. First, we first present a way how weather conditions can be converted to corresponding link capacity degradation vectors. We enumerate weather factors affecting the link margin, give an integrated formula for link margin, and explain how the value of link margin is related to the proper MCS choice. Second, we propose an enhanced characterization of link degradation states that captures everyday fluctuation of weather conditions. For that we define a special kind of uncertainty set, called a covering K-set, using a generalization of the minimum hitting set problem. This uncertainty set, which is new, comes as an alternative of two former uncertainty sets already proposed and investigated before in [1], [2], [4]. The presented considerations are illustrated by a numerical study. In our view the p-covering model is more natural as it better captures the specificities of the set $\mathcal{S}$ because it represents the frequency of subset of links used in it, contrary to previously proposed sets which capture only the topological aspects of the network.
Paper organization. In Section II we present a mathematical formulation of the problem of converting the data weather to an explicit list of link availability states (degradation states may be immediately obtained). In Section III we recall first our previous works in approximating the link degradation states set through the so called link K-set and node K-set, which are known as the uncertainty set. Then we introduce a new type of uncertainty set based on a generalization of the minimum hitting set problem. Next, in Section IV, we present a numerical study that illustrates effectiveness of the proposed uncertainty set representation. Finally, in Section V, we give concluding remarks.

II. Impact of weather conditions on link capacity

This section is dedicated to a general presentation of the impact of weather conditions in FSO data transmission performance and the concept of link margin. Further, we devote special attention to the problems associated with converting the link margin into availability ratio and the modulation schemes that better fit our system. Finally, we consider an example of FSO system in different weather condition and the modulation scheme that we apply.

A. Conversion formulae

The link margin of a particular FSO link is a function of the atmospheric conditions, as well as of the distance between the terminals of the link, the power of the transmitter, and the sensitivity of the receiver. Link margin can be considered as the extra-power available at the receiver (measured in milliwatts [mW]), compared to the minimum requirement of keeping the link active. Considering the technical data provided by the manufacturer and the weather conditions for a specific geographical zone, we can compute the link margin in order to obtain the availability ratio of the links using (after [3]) the following formula:

$$M_{Link} = P_e + |S_r| - Att_{Geo} - P_{Syst} - Att_{Fog} - Att_{Rain} - Att_{Snow} \quad (1)$$

where M_{Link} is the link margin (in dB), P_e is the emitted power, expressed in decibels per mW, i.e., in dBm, S_r is the receiver sensitivity in decibels per milliwatt (in dBm), Att_{Geo} is the geometric attenuation (in dB), P_{Syst} is the equipment loss (in dB) given by the manufacturer (possibly multiplied by 2), Att_{Fog} is the fog attenuation (in dB), Att_{Rain} is the rain attenuation (in dB), Att_{Snow} corresponds to the snow attenuation (in dB).

Note that in formula (1) one group of the factors are equipment dependent, while the others are weather dependent. We will not discuss the particular factors here (for details see [3]) but focus on the modulation and coding schemes used to evaluate link degradation ratios for different weather scenarios, see Tables I and II.

B. Adaptation of modulation and coding schemes

Bad weather conditions deteriorate the FSO link, and in severe cases the transmission capacity of a link can be lost even entirely. In order to keep the link active, the data rates of the affected links can be increased by changing the signal modulation scheme, depending on the receiver sensitivity.

The advantage of using QAM is that it is a higher order form of modulation and as a result the data rate of a link can be increased. While higher order modulation rates are able to offer much faster data rates and higher levels of spectral efficiency for the radio communications system, this comes at a price. As a result, many radio communications systems now use dynamic adaptive modulation techniques. They sense the channel conditions and adapt the modulation scheme to obtain the highest data rate for the given conditions. Selecting

the right order of QAM modulation for any given situation, and having the ability to dynamically adapt it can enable the optimum throughput to be obtained for the link conditions for that moment. Reducing the order of the QAM modulation enables lower bit error rates to be achieved and this reduces the amount of error correction required. In this way the throughput can be maximised for the prevailing link quality [5], [6].

If we note M_{link} (the link margin of link i in dB) and $M^*_{link_{max}}$(the $maximum$ link margin of our system in the best conditions in dB), in general we can do the correspondence shown in Table I between link margin of a FSO system and the modulation scheme that we apply.

TABLE I: Link margin and corresponding modulation schemes.

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link margin	modulation scheme	degradation ratio
$M_{link} > 0.5M^*_{link_{max}}$	16-QAM	0%
$0.25M^*_{link_{max}} < M_{link} \leq 0.5M^*_{link_{max}}$	4-QAM	50%
$1 < M_{link} \leq 0.25M^*_{link_{max}}$	BPSK	75%
otherwise	loss of communication	100%

C. An example

In order to explain the elements that determine the link margin we consider the following example. We assume an FSO transmission system with the distance between the transmitter and receiver equal to 7 km, which uses a wavelength of 1550 nm for data transmission. Four different weather conditions are taken into account: sunny, heavy rainfall, light snowfall, and heavy snowfall. Table II illustrates the link margin and the modulation and coding scheme applied to this system. For this example we have assumed that the $M^*_{link_{max}}$ is the maximum links margin reached for a visibility of 15 km and it corresponds to 49.39 dB.

TABLE II: Link margin for 4 typical weather conditions.

	sunny day	heavy rain [10 mm/h]	moderate snow [2.2 mm/h]	heavy snow [10 mm/h]
emitted power [dBm]	51	51	51	51
visibility [km]	15	5.5	3.0	0.2
receiver sensitivity [dBm]	-38.23	-38.23	-38.23	-38.23
beam divergence [mrad]	2.5	2.5	2.5	2.5
aperture surface [m^2]	0.025	0.025	0.025	0.025
Att_{Geo} [dB]	39.83	39.83	39.83	39.83
Att_{Fog} [dB]	0.00026	0.0013	0.059	594.33
Att_{Rain} [dB]	0	35.23	0	0
Att_{Snow} [dB]	0	0	47.10	144.89
system losses [dB]	0	0	0	0
link margin [dB]	49.39	14.17	0.63	-689.83
modulation scheme	16-QAM	4-QAM	BPSK	communication lost

III. Compact approximation of the set of degradation states

A. Optimization problem

In our considerations we assume an undirected network graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ composed of the set of nodes $\mathcal{V}$ and the set of undirected links $\mathcal{E} \subseteq \mathcal{V}^{|2|}$, where $\mathcal{V}^{|2|}$ is the set of all 2-element subsets of $\mathcal{V}$, i.e., each link $e \in \mathcal{E}$ represents and unordered pair $\{v, w\}$ of nodes for some $v, w \in \mathcal{V}$ ($v \neq w$). The unit capacity cost on a given link e is denoted by $\xi(e)$. As the network links are exposed to adverse weather conditions, we consider an explicit list of *link degradation states* $\mathcal{S}$ induced by considered set of weather conditions. For each state $s \in \mathcal{S}$ the link degradation ratios $\beta(e, s)$ are computed for all links using the formulae described in Section II (and summarized in Table I) applied to the weather condition corresponding to state s.

The considered optimization (network dimensioning) problem consists of finding the values of nominal link capacities y_e, $e \in \mathcal{E}$, such that their total cost $C(y) := \sum_{e \in \mathcal{E}} \xi(e) y_e$ is minimized, and in all states the same (given) traffic demand matrix is realized — this implies 100% traffic protection. We assume the global rerouting mechanism [7] to protect the traffic flows — this in turn means that in each state $s \in \mathcal{S}$ the traffic matrix can be realized from scratch in available link capacities which are given by $\beta(e, s) \cdot y_e$, $e \in \mathcal{E}$. A mathematical formulation of this problem, not given here for the space reason, can be found in [1].

In fact, for given link degradation state list $\mathcal{S}$, the considered problem is not too difficult to solve by the commercial solvers (like CPLEX). However, an issue appears here since when the network is to be optimized, the list of states is not known and can only be estimated based on historical weather records for the network area. Such an estimation can be done for, say, a coming year by using the data for the past year but this does not guarantee a reliable estimations due to weather variations and enormous number of possible link degradation states implied by future weather conditions.

To deal with this problem the so called uncertainty sets can be used [1]. Such sets, described in a compact way, can be used instead of the explicit list of link degradation states $\mathcal{S}$, and when properly constructed they can provide solutions that cover most of the real weather conditions with a reasonable total capacity cost $C(y)$. To apply this approach the problem formulation must be adjusted in order to use an uncertainty set instead of the explicit list $\mathcal{S}$ to characterize link degradation states; such formulations for certain types of uncertainty sets (the so called K-sets), together with a specialized solution algorithm, are given in [1] and [2]. As compared to $\mathcal{S}$, these K-sets contain a limited number of link degradation sets, and are compactly described through a system of inequalities. They approximate $\mathcal{S}$ through an uncertainty set including all states with up to K links or K nodes degraded with the same link degradation ratio $\beta(e)$, $e \in \mathcal{E}$; in the latter case, all links incident to the degraded nodes are assumed to be affected. Hence, we may say that parameter K specifies the maximum number of areas (each of such area is composed of one or several links), that can be simultaneously affected.

In our approach, we have assumed global rerouting, that is we are allowed to reroute all traffic, affected or not, for each network state. Then, we solve the link dimensionning problem for various weather scenarios. More precisely, we

first optimize link capacity (i.e., we solve the network dimensionning problem) taking into account all weather states for the time horizon, for which we produce the reference set of states. Then we dimension the network assuming a special (uncertainty) set of link degradation states. To solve the optimization problem we have used a cut generation algorithm. It is an iterative algorithm which results from application of the classical Bender's decomposition method to our optimization problem. In each iteration the master problem involving only the capacity variables y is solved and then its optimal solution y^* is tested for feasibility with respect to the chosen uncertainty set. If the test is positive, the algorithm is stopped and y^* is optimal for the main problem. If not, new inequalities deduced from the feasibility tests are added to the master problem and the algorithm is reiterated.

B. Uncertainty sets

Let $\mathcal{F} \subseteq 2^{\mathcal{E}}$ be a given family of subsets of the set of links $\mathcal{E}$ and let $\Delta(e, f)$ denote the binary incidence coefficient equal to 1 if, and only if, $e \in f$ for $f \in \mathcal{F}$. Now, for a given integer parameter $K \geq 1$, consider a set $\mathcal{U}(K, \mathcal{F}) \subset \{0,1\}^{|\mathcal{E}|+|\mathcal{F}|}$ described by the following system of inequalities in binary variables $u = (u_e : e \in \mathcal{E})$ and $y = (y_f : f \in \mathcal{F})$:

$$\sum_{f \in \mathcal{F}} y_f \leq K \tag{2a}$$
$$u_e \leq \sum_{f \in \mathcal{F}} \Delta(e, f) y_f, \quad e \in \mathcal{E} \tag{2b}$$
$$u_e \in \{0,1\},\ e \in \mathcal{E};\ \ y_f \in \{0,1\},\ f \in \mathcal{F}. \tag{2c}$$

The purpose of introducing $\mathcal{U}(K, \mathcal{F})$ is to define the so called *uncertainty set*, which is a set of u vectors $\mathcal{U}'(K, \mathcal{F})$ that describe a list of degradation states. The definition is as follows:

$$\mathcal{U}'(K, \mathcal{F}) := \{u : \exists y \in \{0,1\}^{|\mathcal{F}|},\ (u, y) \in \mathcal{U}(K, \mathcal{F})\}. \tag{3}$$

The idea is that each vector $u \in \mathcal{U}'(K, \mathcal{F})$ defines a set $s(u) := \{e \in \mathcal{E} : u_e = 1\}$ of degraded links. Assuming that each link is in all states degraded with the same link degradation ratio $\beta(e)$, we get a full description of the link degradation states determined by a given uncertainty set.

In fact, the so introduced notion of uncertainty set is a generalization of some previously used notions. To illustrate this point let us first consider the family $\mathcal{F} = \{\{e\} : e \in \mathcal{E}\}$. Since in this case we can identify the singletons $\{e\}$ composing family $\mathcal{F}$ with the corresponding links, the set $\mathcal{U}(K, \mathcal{F}) \subset \{0,1\}^{2|\mathcal{E}|}$ is defined by:

$$\sum_{e \in \mathcal{E}} y_{\{e\}} \leq K \tag{4a}$$
$$u_e \leq \sum_{e' \in \mathcal{E}} \Delta(e, \{e'\}) y_{\{e'\}}, \quad e \in \mathcal{E} \tag{4b}$$
$$u_e \in \{0,1\},\ e \in \mathcal{E};\ \ y_{\{e\}} \in \{0,1\},\ e \in \mathcal{E}. \tag{4c}$$

Clearly, the resulting uncertainty set $\mathcal{U}'(K, \mathcal{F})$ is defined simply like this:

$$\sum_{e \in \mathcal{E}} u_e \leq K \tag{5a}$$
$$u_e \in \{0,1\},\ e \in \mathcal{E}. \tag{5b}$$

The above set describes in a compact way the list of states $s(u)$, $u \in \mathcal{U}'(K, \mathcal{F})$, containing all the states corresponding to simultaneous degradation of a subset of k links (there are $\binom{|\mathcal{E}|}{k}$ of such states) for all $k = 0, 1, \ldots, K$. The so defined uncertainty set corresponds to the notion of *link K-set* considered in [1].

Another kind of a known uncertainty set is obtained for $\mathcal{F} = \{\delta(v) : v \in \mathcal{V}\}$, where $\delta(v)$ denotes the set all links in $\mathcal{E}$ incident with node $v \in \mathcal{V}$. Then the set is as follows:

$$\sum_{v \in \mathcal{V}} y_{\delta(v)} \leq K \tag{6a}$$
$$u_e \leq \sum_{v \in \mathcal{V}} \Delta(e, \delta(v)) y_{\delta(v)}, \quad e \in \mathcal{E} \tag{6b}$$
$$u_e \in \{0,1\},\ e \in \mathcal{E};\ \ y_{\delta(v)} \in \{0,1\},\ v \in \mathcal{V}. \tag{6c}$$

Simplifying the notation by using variables y_v instead of $y_{\delta(v)}$, we can rewrite (6) like this:

$$\sum_{v \in \mathcal{V}} y_v \leq K \tag{7a}$$
$$u_{\{v,w\}} \leq y_v + y_w, \quad \{v, w\} \in \mathcal{E} \tag{7b}$$
$$u_e \in \{0,1\},\ e \in \mathcal{E};\ \ y_v \in \{0,1\},\ v \in \mathcal{V}. \tag{7c}$$

The set $\mathcal{U}'(K, \mathcal{F})$ resulting from (7) corresponds to the notion of *node K-set* considered in [2].

C. Covering K-sets

Below we will show a way to find families $\mathcal{F}$ for constructing uncertainty sets for a given (possibly very large) state list $\mathcal{S}$.

Let us assume that all states s in $\mathcal{S}$ (each representing a set of simultaneously degraded links) have cardinality greater or equal to a given integer parameter p. Now, let $X^{|p|}$ denote the family of all p-element subsets of a given set X, and consider the following binary programming problem:

$$\min \quad \sum_{f \in \mathcal{E}^{|p|}} x_f \tag{8a}$$
$$\sum_{f \in s^{|p|}} x_f \geq K, \quad s \in \mathcal{S} \tag{8b}$$
$$x_f \in \{0,1\}, \quad f \in \mathcal{E}^{|p|}. \tag{8c}$$

Finally we define the family $\mathcal{F} := \{f \in \mathcal{E}^{|p|} : x^*_f = 1\}$ (where x^* is an optimal solution of (8)) and call the so obtained uncertainty set $\mathcal{U}'(K, \mathcal{F})$ the *p-covering K-set* of the list of states $\mathcal{S}$. Note that formulation (8) is actually a generalization of *minimum hitting set problem* — a well known NP-hard combinatorial problem [8].

The so obtained family $\mathcal{F}$ may be seen as containing those subsets (of reasonable size p) of links that are frequently simultaneously affected, and we assure protection against link degradation states that contain all links included in up to K of such subsets.

To illustrate the concept of p-covering K-set, consider a simply network instance with 11 links ($\mathcal{E} = \{e_1, e_2, \ldots, e_{11}\}$) and the set $\mathcal{S} = \{s_1, s_2, \ldots, s_7\}$ composed of 7 states given in Table III.

Solving the binary problem (8) for $p = 2, K = 1$, we obtain the following family $\mathcal{F}$ (composed of three 2-link subsets of $\mathcal{E}$) defining the corresponding 2-covering 1-set: $\{\{e_2, e_4\}, \{e_6, e_8\}, \{e_7, e_9\}\}$. For $p = 2$ and $K = 2$, the

TABLE III: An example of 2-covering K-sets.

state set	composition	$p=2, K=1$	$p=2, K=2$
s_1	$\{e_2, e_3, e_5, e_7, e_9, e_{11}\}$	$\{e_7, e_9\}$	$\{e_7, e_9\}, \{e_2, e_3\}$
s_2	$\{e_2, e_4, e_6, e_7, e_8\}$	$\{e_2, e_4\}$	$\{e_2, e_4\}, \{e_6, e_8\}$
s_3	$\{e_3, e_5, e_6, e_8, e_{10}, e_{11}\}$	$\{e_6, e_8\}$	$\{e_6, e_8\}, \{e_5, e_{10}\}$
s_4	$\{e_1, e_2, e_4, e_7, e_8, e_9, e_{10}\}$	$\{e_7, e_9\}$	$\{e_7, e_9\}, \{e_2, e_4\}$
s_5	$\{e_2, e_3, e_6, e_7, e_9, e_{10}\}$	$\{e_7, e_9\}$	$\{e_7, e_9\}, \{e_2, e_3\}$
s_6	$\{e_1, e_2, e_4, e_6, e_8, e_{11}\}$	$\{e_2, e_4\}$	$\{e_2, e_4\}, \{e_6, e_8\}$
s_7	$\{e_2, e_4, e_5, e_6, e_9, e_{10}\}$	$\{e_2, e_4\}$	$\{e_2, e_4\}, \{e_5, e_{10}\}$

resulting family defining the 2-covering 2-set has 2 additional subsets: $\{e_2, e_3\}, \{e_5, e_{10}\}$. In the above table, columns 3 and 4 illustrate how the sets in $\mathcal{S}$ are actually covered by the sets from the corresponding families $\mathcal{F}$.

Note that an advantage of using covering K-sets is that they are defined using the information of the actual list of states $\mathcal{S}$ that is to be approximated; this feature is not present when specifying link or node K-sets.

IV. Numerical results

Below we present results of a numerical study illustrating the performance of our optimization approach for various weather scenarios. All numerical calculations were performed on an Intel (R) Core (TM) i7-4610M, 3.0 GHz computer with 16 GB of RAM. The algorithms were coded in Java and the optimization problems were solved with IBM ILOG CPLEX 12.6 (using Concert Technology) running with the default setting.

In our numerical tests, we have dimensionned the network assuming a special (uncertainty) set of link degradation states represented by the so-called link, node K-set or p-covering K-set where K specifies the number of areas (represented by network links, nodes or group of links) that can be simultaneously affected by weather conditions. For all the considered K-sets the degradation ratio $\beta(e) = 0.50$ was assumed uniformly for all affected links in the network. This value represents the degradation ratio of more than 80% of affected links all over the degradation states (not considering the normal states, i.e. corresponding to sunny days with no link affected), while two-third of such states contain only links with the same degradation ratio $\beta(e) = 0.50$. We run the optimization approach for a selected set of values of parameter K, checking to what extent the demand traffic matrix is satisfied by the K-set based on solution in all states in the reference degradation set. Finally, we have examined the percentage of carried traffic achieved for different K-sets as compared to carried traffic observed when all states in the reference set are tested for the obtained dimensionned network.

A. Network nobel-germany

We have tested our optimization approach on a moderate-size network example derived from the instance called *nobel-germany* that can be found in SNDlib [9]. The instance is composed of 17 nodes and 26 undirected links, and there are 120 undirected traffic demands. We have used three different weather scenarios constructed for the purpose of this study.

Indeed, using the weather records from [10], [11], we have built up three different weather scenarios: the first scenario is typical for southern Europe (called "Mediterranean climate" scenario), the second one is typical for middle Europe (called "moderate continental climate" scenario), and the third is typical for northern Europe (called "temperate continental climate" scenario). These scenarios were then used to estimate link degradation state for each hour of a typical year for *nobel-germany*. A detailed description of such an estimation process is given in [12], while other details on the obtained network instances are given in [13].

Tables IV, V and VI show the results for different types of uncertainly sets (link K-sets, node K-sets) compared to p-covering K-sets approach described above (for $p = 2$). Such sets have been computed in advance with respect to the reference set of states. Typically, we have used formulation (8) with $p = 2$ and $K \in \{2, 3\}$ to compute such subsets. Although the problem is NP-hard, the computation time for the considered instances does not exceed 3 minutes. The number of subsets are for $K = 2$ and $K = 3$ respectively 18 and 20 for "cold continental climate" scenario, 45 and 60 for "temperate continental climate" scenario, and finally 67 and 113 for "Mediterranean climate" scenario. In each table, the uncertainty set used is pointed in the first column where $link(K)$ represents a link K-set, $node(K)$ a node K-set, and $set(K)$ a covering K-set. The second column gives the optimal capacity cost $C^*(K)$. The third column represents the average carried traffic $AT(K)$. The last column shows the computation time in [s]. For our results we have considered only the states where the network is connected. (It happens that in 10-15% of the time the weather conditions are extremely bad so that can cause the disconnection of the network.)

TABLE IV: Mediterranean climate.

uncertainty set	cost $C^*(K)$	carried traffic $AT(K)$[%]	computational time [s]
$link(1)$	149	99.07	75.41
$node(1)$	179	99.57	66.21
$link(2)$	188	99.64	73.61
$set(2)$	207	99.72	152.31
$link(3)$	208	99.76	102.38
$link(4)$	221	99.79	95.45
$node(2)$	221	99.81	692.24
$set(3)$	223	99.79	171.06
$link(5)$	228	99.76	112.56
$link(6)$	229	99.82	114.34
$link(7)$	230	99.82	134.05
$link(8)$	230	99.82	163.04
$link(9)$	231	99.82	137.26
$link(10)$	231	99.82	108.76

When comparing the total capacity cost and the average carried traffic percentage for different types of uncertainty sets, namely for link K-sets, node K-sets, and 2-covering K-

TABLE V: Temperate continental climate.

uncertainty. set	cost $C^*(K)$	carried traffic $AT(K)$ [%]	computational time [s]
$link(1)$	149	92.11	65.03
$node(1)$	179	94.50	70.11
$link(2)$	188	94.51	82.56
$set(2)$	203	95.41	140.02
$link(3)$	208	95.87	109.06
$set(3)$	218	96.71	123.11
$node(2)$	221	96.90	729.97
$link(4)$	221	96.67	103.95
$link(5)$	228	96.47	122.43
$link(6)$	229	96.94	129.25
$link(7)$	230	97.03	155.91
$link(8)$	230	97.02	166.05
$link(9)$	231	97.02	136.18
$link(10)$	231	97.15	111.53

TABLE VI: Cold continental climate.

uncertainty. set	cost $C^*(K)$	carried traffic $AT(K)$[%]	computational time [s]
$link(1)$	149	81.06	68.58
$node(1)$	179	84.82	72.38
$link(2)$	188	84.83	80.61
$set(2)$	190	85.22	96.95
$set(3)$	205	87.35	120.74
$link(3)$	208	87.19	110.31
$link(4)$	221	88.85	110.68
$node(2)$	221	89.46	742.51
$link(5)$	228	88.79	118.61
$link(6)$	229	89.73	122.52
$link(7)$	230	89.97	150.59
$link(8)$	230	89.92	168.46
$link(9)$	231	90.07	135.62
$link(10)$	231	90.08	103.61

sets, we observe that in all tables, using 2-covering K-sets is competitive as compared to using link K-sets. In particular, consider the results for 2-covering 3-set given in Tables V and VI. (Recall that using this set, i.e., $set(3)$, protects the traffic against all combinations of simultaneous link degradation in three sets of two links each; these sets compose the family obtained in the construction of this set, see Section III-C.) For this set we achieve costs (218, 205) carried traffic percentages (96.71%, 87.35%). Considering results for $link(4)$ in Table V and for $link(3)$ in Table VI, we observe higher costs (221, 208) and lower carried traffic percentage (96.67%, 87.19%). Yet, although not always comparable, in some cases (as in Table IV), node K-sets perform better than the other two K-set types approaches; for instance see the $node(2)$ result and compare it with $set(3)$ and $link(4)$). Note that as compared to node K-sets, 2-covering 3-sets ($set(3)$) solutions are cheaper for temperate and cold continental climate at the same time achieving competitive percentages of carried traffic.

B. Paris Metropolitan Area network

In this section we will study of a realistic FSO network instance with 12 nodes and 21 links. We call this instance PMAN (Paris Metropolitan Area network) since it was created using the real data for the Paris Metropolitan Area. It should be noted that although all the data used to construct the PMAN instance are real, the instance itself does not exist in reality and was elaborated only for our own purpose. A full characterization of PMAN is available in [1].

We examined the weather conditions in Paris Metropolitan Area observed during one year period from January 1, 2016 until December 31, 2016 available in [10] and extracted a set $\mathcal{W}$ of all $366 \times 24 = 8784$ hourly weather states. Then we translated each such obtained weather state to the corresponding link degradation states. We observed that among all 8784 states there are 7531 connected states (85.74%) and 1253 (14.26%) disconnected states. Among the 7531 connected states, 68.46% are *nominal states* with no degraded links.

The individual traffic demands in PMAN (the demands are directed and there are 132 of them) are of the order of several Gbps, except for the traffic between Paris 1 and Paris 2 with much higher demand of 74.38 Gbps (in each direction). The full traffic matrix for PMAN can be found in [1]. An $M = 10$ Gbps FSO module is assumed, as for example the module of the LightPointe's AireLink 80 10Gbps system [14]; the cost of one such module on each link is equal to 1. In our example we have considered all states (8784 states) including 1252 cases when the network is disconnected.

TABLE VII: PMAN network.

uncertainty set	cost $C^*(K)$	carried traffic $AT(K)$[%]	computational time [s]
$link(1)$	45	93.39	31.76
$link(2)$	48	93.63	39.76
$set(2)$	50	93.70	93.47
$link(3)$	51	93.67	47.24
$set(3)$	51	93.64	86.51
$node(1)$	52	93.51	87.23
$link(4)$	54	93.71	50.06
$node(2)$	54	93.48	435.24
$link(5)$	54	93.45	49.01
$link(6)$	54	93.54	52.74
$link(7)$	54	93.45	51.41
$link(8)$	54	93.47	72.86
$link(9)$	54	93.47	77.84
$link(10)$	54	93.47	72.50

As before we have tested our optimization approach for the three uncertainty sets on PMAN as shown in Table VII. Comparing to *nobel-germany* instances, the results for PMAN pleads clearly in favor of the new uncertainty set proposed in this study. One may note that $set(2)$ has a lower cost compared to $link(3)$ or $node(1)$ while giving one of the highest traffic satisfaction for all uncertainty scenarios. We can observe that in some cases the obtained results don't exhibit the expected

relation between the cost and the traffic satisfaction, that is, for the same uncertainty set type, higher is the cost higher the traffic satisfaction should be. The reason for that stands in the fact that these criteria are optimized separately. In our optimization approach we build a link cost-efficient network for some given uncertainty set and we measure what is the highest traffic satisfaction this network can provide for each state of the reference set. Hence, it may happen that one network instance with a given dimensioning distribution performs better than another instance with a higher dimensioning cost but with a different distribution of capacities. This may be observed in Table VII for cases $link(3)$ and $link(5)$ or $set(2)$ and $set(3)$.

V. Concluding remarks

In the paper we have presented some new results on a robust optimization problem related to FSO network dimensioning resilient to link capacities degradation caused by weather conditions. Our solution approach optimizes link capacity taking into account all weather states for a given time period, for which we determine a reference set of (link capacity degradation) states. One of the main issues here is potential (large) size of the reference set, which may make the optimization problem computationally untractable for large networks. Furthermore, we need to construct such a reference set of states using historical weather data records, which may not fit the future states for which the network is being optimized.

To handle this, in our previous work we used the so called link K-sets (where up to K arbitrary links can be degraded simultaneously) and node K-sets (where up to K arbitrary nodes can be degraded simultaneously). In this paper we have investigated a more general notion of K-set, called p-covering K-set, where the corresponding link groups are more appropriate for the reference set approximation, and show how to build such groups of links.

In contrast to previous uncertainty sets, the new set is composed of subsets of links being the most frequently met in the states of the reference set. This is a qualitative step forward in achieving a more natural compact approximation of the reference set. Unlikely to the link and node K-sets, the new set is built by examining explicitly all subsets of the reference set. Next, we test these new uncertainty sets for a selected set of values of parameters p and K ($p = 2$ and $K = 2, 3$) and check to what extent the demand traffic matrix is satisfied by the obtained solution in all states in the reference set of states $\mathcal{S}$, and at what cost.

The presented approach has been illustrated by means of a moderate-size network instance (*nobel-germany*) for three different weather scenarios, as well as using a realistic instance representing the Paris Metropolitan Area Network. The numerical results show that using p-covering K-sets for state modeling gives encouraging results when compared to the previously considered link and node K-set uncertainty sets. This is in particular true for PMAN, a realistic network instance.

As far as the future work is concerned, we will investigate a natural extension of the above-described p-covering set model admitting multiple link degradation ratios.

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