# An Interior Point Method Solving Motion Planning Problems with Narrow Passages

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Abstract—Algorithmic solutions for the motion planning problem have been investigated for five decades. Since the development of  $A^*$  in 1969 many approaches have been investigated, traditionally classified as either grid decomposition, potential fields or sampling-based. In this work, we focus on using numerical optimization, which is understudied for solving motion planning problems. This lack of interest in the favor of sampling-based methods is largely due to the non-convexity introduced by narrow passages. We address this shortcoming by grounding the solution in differential geometry. We demonstrate through a series of experiments on 3 Dofs and 6 Dofs narrow passage problems, how modeling explicitly the underlying Riemannian manifold leads to an efficient interior point non-linear programming solution. <sup>1</sup>

# I. INTRODUCTION

Autonomous motion planning is a central component of autonomous behavior. Hence, accuracy and effectiveness of motion planning algorithms can have dramatic impacts in terms of safety and acceptance of robots. Indeed, safety and more generally human-robot interaction constraints are often modeled as cost functionals [1], which are in turn optimized by a motion planning algorithm.

The motion planning community has focused on samplingbased approaches over the last two decades with a lot of success. The introduction of probabilistically complete planners [2] such as Probabilistic Road Maps (PRMs) and Rapidly Exploring Random Trees (RRTs) has allowed to solve virtually any problem in any dimension. However, when applied to real world robotics with many degrees of freedom in dynamic environments, sampling-based algorithms are typically too slow to be usable and many works usually resort to local methods (i.e., potential fields [3]).

For manipulation, Motion Optimization (MO) [4] is often preferred. In MO, motion planning is solved using trajectory optimization, i.e., gradient-based optimization in the full trajectory parameter space. MO does not suffer the myopic shortcomings of local methods as it considers the full horizon length, while also using gradient information provided by potential fields to converge rapidly to a local minimum. The disadvantage of MO is that it is remains local, and performs poorly when the problem is highly non-convex, where it

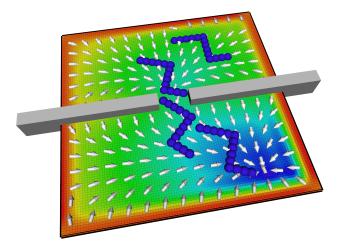


Fig. 1: Geodesic flow attractor in a planar environment, cool and hot colors are small and large geodesic distances resp.

often does not converge at all. Thus, MO is often used as a post-processioning step of an RRT search [5], [6].

The theory of motion planning is well developed and its connection to differential geometry is long standing. The geometry of the forward kinematics map is well understood, and the issues linked to sampling or interpolating SE(3) are well treated in classical text books [7], [2]. However due to the large interest in sampling-based approaches in the last two decades, little effort has been made to understand the differential structure of the workspace geometry, which, we argue, is essential for motion optimization. One notable exception is the work of Ratliff et al. [8], where the notion of Natural attractors and Riemannian metrics that we further improve in this work have been initially introduced.

In this work we introduce a formal treatment of the workspace geometry in terms of Riemannian geometry, and propose new terms to model the goal constraint, i.e., geodesic attractor (see Figure 1), and a geodesic flow agreement terms. We show experimentally that these terms lead to better convergence rates on three narrow passage environments, involving 3 and 6 Dofs. To our knowledge this work is the first to present the implementation of an Interior Point algorithm to solve complex path planning problems usually treated by sampling-based methods.

The remainder of this paper is structured as follows: In the next section we give a description of related work. In Section III, we formalize and discuss the notion of Workspace

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Riemannian Metrics. In Section IV, we describe the geodesic flow which is then used to introduce a new constraint model and an objective term. In Section V, we present empirical results on three narrow passage problems which demonstrate the efficacy of our approach.

## II. RELATED WORK

### A. Motion optimization

Motion optimization [4], [9], [10], [11] relates to functional optimization algorithms, which take gradient space in trajectory space. This approach was originally pioneered by Quinlan and Khatib [12], and further by Brock [13], with the aim to produce reactive robotic behaviors. These early works modeled the trajectory by a mass-spring system, and motion planning was solved by simulating the virtual elastic band system. In contrast to this initial approach, motion optimization models the problem through numerical optimization, leveraging the large body of work in the area.

In [14], Ratliff and Toussaint, have proposed to minimize the geodesic distance of the body part instead of the Cartesian space arc length proposed in [4]. These details are crucial for achieving convergence of the non-linear program for motion planning. In [15], we have proposed a Riemannian metric for handling arbitrary workspace geometries that go beyond primitive shapes such as circles and boxes by defining harmonic potentials. In this work we go a step further and address the problem of geodesic distance to the goal, by leveraging the heat method [16].

#### B. Harmonic functions

Originally artificial potential fields were used for the obstacle avoidance function within the robot workspace [3]. This allowed to get realtime robot behavior by using the operational space formalism. Potential fields are known to be prone to local minima. Thus functions that limit local minima by satisfying Laplace's Equation have been investigated [17] (i.e., harmonic functions).

Laplace's equation is a partial differential equation, so numerically solving for a harmonic potential field can be computationally challenging and suffers from the curse of dimensionality when applied to the robot configuration space. Recently [18], this method has been extend for computing robot navigation paths by leveraging GPUs. In this work we combine such functions with motion-optimization techniques. For this we make use of the heat method [16], which diffuses heat in the workspace leading to harmonic function at convergence.

#### C. Numerical Optimization

The motion objective is generally optimized using an augmented Lagrangian formulation, which uses generic constrained optimization solvers, recomputing the Lagrange multipliers in the outer loops and constructing a series of unconstrained objectives for the inner loop optimizers. These unconstrained objectives encode the violated constraints as shifted penalties. In this work we instead use an Primaldual interior-point method [19] for nonlinear optimization based on a Gauss-Newton approximate of the Hessian. This algorithm handles better inequality constraints by using log barrier terms and forcing the solution to remain within the feasibility region.

## III. WORKSPACE RIEMANNIAN METRICS

The whole problem of path planning is to take into account the obstacles  $\mathcal{O}_i$  that populate the workspace  $\mathcal{W} \subset \mathbb{R}^3$ . Obstacles regions are subsets of  $\mathcal{W}$ . Obstacles are define to have non empty interior and have a smooth boundary  $\partial \mathcal{O}_i$ .

The freespace is the set difference of  $\mathcal{W}$  and the union of the obstacles:

$$\mathcal{F} := \mathcal{W}_{free} = \mathcal{W} \setminus \bigcup_{i \in \mathcal{O}} \mathcal{O}_i.$$

The freespace is a smooth compact manifold with a smooth boundary  $\partial \mathcal{F}$ . It is associated with an atlas of a single chart, which is the global Euclidean coordinate system.

A metric tensor  $g_p$  is a smooth map defined on the tangent bundle of a manifold  $\mathcal{M}$ . The map  $g_p$  is defined for each point  $p \in \mathcal{M}$ , and associates a real number given two vectors  $g(p) : T_p(\mathcal{M}) \times T_p(\mathcal{M}) \to \mathbb{R}$ . A smooth manifold equipped with a positive definite metric tensor, i.e.,  $\forall (u,v) \in T_p(\mathcal{M})^2, g_p(u,v) > 0$ , is called a Riemannian manifold.

For points  $p \in \mathcal{W}$ , the tangent space  $T_p(\mathcal{W})$  is the Euclidean space. If  $\mathcal{F}$  is equipped with the usual Euclidean metric  $\|.\|_2$ , it defines a Riemannian manifold  $(\mathcal{F}, g = I)$ . Here, we aim to characterize Riemannian metrics tensors g for  $\mathcal{F}$ , which make geodesics wrap around obstacles regions  $\mathcal{O} = \bigcup_{i \in \mathcal{O}} \mathcal{O}_i$ . The following sections will identify a class of functions that induces such metrics.

## A. Define g over $\mathcal{W}$ or $\mathcal{F}$ ?

The obstacle region  $\mathcal{O}$  defines disconnected subsets of  $\mathcal{W}$ , which can be viewed as topological holes. In other words, the freespace is itself a Riemannian manifold  $\mathcal{F}_{rie}$  in which all geodesics naturally avoid  $\mathcal{O}$ . Recall that a geodesic on a Riemannian manifold  $\mathcal{M}$  is defined to be the a curve  $\gamma$  :  $\mathbb{R}^+ \to \mathcal{M}$ , which "length" is measured as:

$$L(\gamma) = \int_0^T \sqrt{g_{\gamma(t)}(\dot{\gamma}(t),\dot{\gamma}(t))} dt$$

where  $\forall t, \gamma(t) \in \mathcal{M}$  and is continuously differentiable. Geodesics correspond to curves for which  $\nabla_{\dot{\gamma}}\dot{\gamma} = 0$ , where  $\nabla$  is an affine connection (intuitively this means that the acceleration is either 0 or orthogonal to the tangent plane, it generalizes the notion of straight line to curved spaces). Geodesics are invariant to affine re-parametrization (t' = at + b), but not to arbitrary re-parametrizations.

There may exist multiple geodesics linking two points in space  $p_1, p_2 \in \mathcal{M}$ . Minimizing geodesics define a metric over Riemannian manifolds:  $d(p_1, p_2) = \operatorname{argmin} L(\gamma)$ . These curves can be obtained by the calculus of variation on the energy functional because:

$$L(\gamma) \le 2(p_1 - p_2)E(\gamma).$$

Hence by definition, minimizing geodesic are curves that only take value in the manifold. This implies that we can consider two types of problems to have minimizing geodesic wrap around obstacles:

- a) Topological: g defined over  $\mathcal{W}$ , such that  $\forall p_1, p_2 \in \mathcal{F} \implies \forall t \in [p_1, p_2], \gamma(t) \in \mathcal{F}$
- b) Behavioral: g defined over  $\mathcal{F}$ , with  $\gamma(t) \in \mathcal{F}$  by definition

Finding a satisfying definition for each of these conditions is different. For a) the problem is to fine the constraints in g such that the implication is true. For b) we need to define what a good metric g would be.

# B. The Obstacle-based Riemannian metric

To define a metric g in the line of a):

" g defined over  $\mathcal{W}$ , such that  $\forall (p_1, p_2) \in \mathcal{F}^2 \implies \forall t \in [p_1, p_2], \gamma(t) \in \mathcal{F}$ "

poses constraints on the geodesic flow inside  $\mathcal{F}$  induced by the metric. Generally, this is enforced by having  $\forall (p_1, p_2) \in \mathcal{O}^2$ ,  $\lim_{p_1 \to p_2} L(\gamma) > \operatorname{diam}(\mathcal{F})$ . This will ensure that any minimizing geodesic in  $\mathcal{F}$  is shorter than the minimizing geodesics in  $\mathcal{O}$ .

We define the following properties of the geodesics in a Riemannian manifold equiped with such a metric:

- i) *Non-penetrating:* At the boundary, the geodesics should be parallel to the boundary. Thus if  $\hat{n}$  is normal to  $p \in \partial \mathcal{F}$  pointing inward and  $\mathcal{V} = \{v | \hat{n}^T v > 0\}$  is the set of all vectors penetrating the  $\mathcal{O}$ , then  $\forall v \in \mathcal{V}, g_p(v, v) = 0$ . In other words, the null space of  $g_p$  should contain all vectors with positive dot product with boundary normal vectors.
- ii) Blending to Euclidean: Away from the boundary geodesics should obey Euclidean geometry.  $\lim_{d(p)\to\infty} g_p = \mathbb{I}$ , where d(p) is the minimal distance to the boundary at p.
- iii) *Multi-resolution:* The influence of the detail geometry of the boundary should vanish with distance from the boundary.
- iv) *Ordering:* On a planar section of the workspace, a geodesic between two points farther from the boundary than points closer should not cross.

#### C. The Eigen spectrum of the metric tensor

In order to gain intuition on what the obstacle-based metric tensor represent we can look at the behavior of its eigen spectrum. The tensor stretches space along particular dimensions. A Riemannian metric tensor g operates on the tangent space  $T_p(\mathcal{M})$  (i.e., the space of velocities) of a given manifold point  $p \in \mathcal{M}$ .

If the manifold  $\mathcal{M} \subset \mathbb{R}^3$ , and the tensor g is a scaling of the dot product:

$$g_p(u,v) = u^T A_p v$$

where  $v, w \in T_p(\mathcal{M})$ . Since the matrix A is semi-positive definite by definition, the singular value decomposition has the following form:

$$A_p = U\Sigma U^T = U \begin{pmatrix} \lambda_1 & 0 & 0\\ 0 & \lambda_2 & 0\\ 0 & 0 & \lambda_3 \end{pmatrix} U^T$$

where  $\lambda_i$  are the eigenvalues of A. The matrix U is orthogonal, it operates a change of coordinates that preserves scaling. The matrix  $\Sigma$  scales each dimension in that coordinate system. Thus the eigen values describe how the space is warped locally to p.

In the case where  $\mathcal{M}$  is a subset of Euclidean space, the tangent space is  $T_p(\mathcal{M}) = \mathbb{R}^3$ . The metric tensor operates a scaling of euclidean space, define by the eigen spectrum of  $g_p$ . At the boundary  $\partial \mathcal{F}$  the inverse of the metric tensor would loose rank, effectively removing volume in the tangent space going through the obstacle surface. Equivalently,  $g_p$  measures velocity in the direction of the surface infinitly.

Now that we have defined what a good Workspace Riemannian metric might be and understand how they operate in the workspace, we can introduce two key elements that we use in our experiments. First the workspace geometry map  $\phi_{ws}(p)$ , which allows to define a metric tensor g and the geodesic flow, which encodes the geodesic distance to the goal.

#### D. The workspace geometry map

Any mapping of the form  $z = \phi(p)$  into Euclidean space defines a Pullback metric  $A(p) = J_{\phi}^T J_{\phi}$ . In fact, we can represent any metric A(p), as the pullback of a mapping  $\phi(p)$  to some higher dimensional space [14]. This is the famous Nash embedding theorem. This observation means that the generalized velocity term of the form  $\dot{p}^T A(p) \dot{p}$  can be equally well described as a Euclidean velocity through the map's co-domain.

The metric A(p) can be used to generate terminal potentials that follow geodesic contours of the workspace under the metric. We denote this as the Natural attractor, as the gradient of this attractor is the Natural gradient. Using a map  $\phi(p)$  allows a more convenient representation that is easier to use to form attractors for the terminal potential than directly specifying A(p).

In [15], we make use of the electric potential proposed in [20]. The potential field emanating from an object surface  $\Omega$  can be used to define a coordinate system around the object. Three values are used to define a coordinates, i.e., the potential and two coordinates for the field lines.

This coordinate system would provide a good candidate for the map  $\phi$ , however the metric induced by such a map contains a seam (i.e., a point where the *u* and *v* coordinates wrap from 0 to  $2\pi$ ). Additionally, potential-based metrics induce lower dimensional  $\phi$  maps which are less expensive to compute. Thus our workspace geometry map is of the form:

$$\phi_{\rm ws}(p) = \begin{bmatrix} \alpha_1 \phi_1(p) \\ \vdots \\ \alpha_{d+1} \phi_{d+1}(p) \end{bmatrix}, \tag{1}$$

where  $\phi_i$  are the individual potential values computed for each object in the scene.  $\phi_{d+1}$  is the 3D identity map and  $\alpha_i$  are proximity functions, which is constant for i = d + 1. In our experiments, we simply use  $\phi_i \sim \exp(-\sigma(p))$ , where  $\sigma : \mathbb{R}^N \to \mathbb{R}$ , is a signed distance function, negative inside obstacles and positive outside.

#### IV. GEODESIC FLOW

At a particular point p on the manifold, the geodesic flow is defined as

$$G^t(V) = \dot{\gamma}_V(t)$$

where  $V \in T_p(\mathcal{W})$  is a vector on the tangent space. Hence it defines all the points on the manifold that can be reached in length t. Computing the geodesic flow seems prohibitive as they would generally require either shooting geodesics from the source point p or solving many shortest path problems.

However geodesic flows can be helpful in solving motion optimization problems as they provide real geodesic distance, which would wrap correctly around obstacles, informing the optimizer about the true nature of the underlying manifold.

Hence in this work, we compute this flow using the heat method [16], where heat is diffused on a regular grid according to the heat equation:

$$\frac{\partial \phi}{\partial t} = \Delta \phi = \sum_{i=1}^{N} \frac{\partial \phi}{\partial x_N}$$

where  $\Delta$  is the Laplace operator and  $\phi$  is some potential function, i.e., heat. It is possible to either solve the heat equation in closed form, resulting in a matrix inversion operation, or to iterate over the grid, similar to policy evaluation in dynamic programming. In either case only small incremental steps can be taken in time.

The gradient of the resulting potential function agrees with the geodesic distance gradient, which stems from the property of diffusion processes. Note that at convergence  $\Delta \phi = 0$ , which means that  $\phi$  is a harmonic function, comprising a single maximum, i.e., the heat source.

Figure 2 shows a heat diffusion process calculated in a two dimension workspace populated by three circular obstacles. Here the heat source is only set at the first time step and then left out. When computing a geodesic flow the heat source is kept at a fixed temperature.

In order to retrieve a geodesic distance we solve for the potential by inverting a matrix relating the potential to its spacial derivative. The geodesic attractor is then obtained we invert the resulting vector field to point towards the source and interpolating that vector field on a regular grid using bicubic and tri-cubic splines in the planar and Cartesian case respectively. The resulting vector field can be seen for the planar case in Figure 1.

Note that we performed diffusion on low resolution grid in all our experiments in a time negligible compared to the optimization time. The sink of the attractor is often inaccurate leading to bad convergence of the interior point algorithm. Hence we linearly blend the geodesic attractor to a euclidean attractor close to the goal.

#### V. RESULTS

Our experiments are conducted to highlight the impact of considering explicitly the Riemannian manifold structure of the workspace in motion optimization.

## A. Setup

We first defined three environments that exhibit narrow passage to a certain degree. We planed motions using freeflying robots with 3 Dofs in the planar case and 6 Dofs in the Cartesian case. The forward kinematics maps  $x : q \mapsto p \in \mathbb{R}^N$  are defined using a single homogeneous transform, for which we define analytically the Jacobian. In the Cartesian case, i.e., 3D, we parametrize the robot orientation with Euler's angles and use a rotation matrix internally. Each robot's body is represented by  $\approx 10$  keypoints with radius, as can be seen in Figure 3.

In Table I we report the different objective terms used in our experiments. The acceleration terms favors shorter and smoother paths. The joint postural serves to resolve the redundancy at the goal configuration. Additionally to these objectives, we define two types geodesic terms, one based on the workspace geometry map  $\phi_{ws}$ , which we used in our previous work [15], to implicitly define the metric tensor. In this work we additionally introduce the objective term "geodesic flow" which measures the agreement between the motion of the keypoints  $\dot{x}_t^i$  and the geodesic flow. The equation presented here is a simplified version of the objective term which uses a linearization of arccos.

In Table II we introduce the two types of constraint functions that we make use of for goal and obstacle avoidance. The signed distance functions  $\sigma$  are used to define collision constraint. All our obstacles are boxes represented by an exact box distance, for which we define gradient and hessian. We bound the hessian near the edges of the box for numerical stability. We also define a softmin function which allows to define one collision constraint per configuration rather than

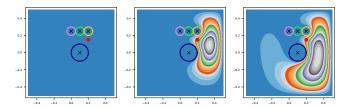


Fig. 2: Heat diffusion process in an environment with circular obstacle. The heat source is initially set to the red dot (left) and flows through the environment by following the heat equation.

Objective Terms	Mathematical Expressions
Squared-norm of C-space accelerations	$c_{t1}(q_t, \dot{q}_t, \ddot{q}_t) = \ \ddot{q}_t\ ^2$
Geodesic Term	$c_{t2}(q_t, \dot{q}_t, \ddot{q}_t) = \ \frac{d}{dt}\phi_{ws}(x(q_t))\ ^2$
Geodesic Flow Term	$c_{t3}(q_t, \dot{q}_t, \ddot{q}_t) = \arccos(\dot{x}_t^T \frac{\partial \phi_{F}(x(q_t))}{\partial x})$
Joint postural	$c_{t4}(q_t,\dot{q}_t,\ddot{q}_t) = \ q_t - q_{ ext{default}}\ ^2$

TABLE I: Elementary objective terms

Constraint Terms	Mathematical Expressions
Signed distance function in workspace geometry	$c_{t5}(q_t) = \operatorname{softmin}_i \sigma[(x(q_t))]$
Euclidean goal	$c_{t6}(q_T) =   x(q_T) - x_{\text{goal}}  $
Natural goal	$\begin{vmatrix} c_{t6}(q_T) = \ x(q_T) - x_{\text{goal}}\  \\ c_{t7}(q_T) = \ \phi_{\text{ws}}((x(q_T))) - \phi_{\text{ws}}(x_{\text{goal}})\  \end{vmatrix}$
Geodesic goal	$c_{t8}(q_T) = \phi_{F}((x(q_T)))$

TABLE II: Elementary constraint terms

one constraint per keypoint and configuration. We found this to make the algorithm more stable in practice. The temperature parameter of the softmin is set very aggressive to approximate a min function very closely.

Finally we use three different goal equality constraints for comparison. The goal constraints are all defined with respect to a single keypoint serving as end-effector of the free-flying robot. We define a vanilla Euclidean attractor that simply computes euclidean distance to the goal. We then define a Natural attractor that defines the euclidean distance in the workspace map  $\phi_{ws}$ . Finally we define our geodesic distance attractor  $\phi_F$ , as depicted Figure 1.

We make use of the interior point algorithm IPOPT [19] for optimizing the objective functional. The functional is defined similarly to the KOMO objective introduced in [11]. We define a clique at each time step, for which we compute velocity and accelerations by finite differences.

#### B. Statistical study

In order to assess the influence of the different terms on the ability of the optimizer to find feasible motion plans we conducted an ablation study. In each case we start from a single configuration which is depicted in strong blue in the motion traces of Figure 3. We then sampled a goalset on the other side of the narrow passage on a regular grid.

## TABLE III: Goals sampled per environments

Planar Narrow	28
Cartesian Narrow	32
Cartesian Maze	36

Each environment is tested with 9 conditions except for the planar case where we also report using the geodesic flow objective. This results in about 250 trajectories per environment, which took under one hour distributed over three cores. All our implementation relies on c++, but it is not optimized for efficiency, we do not report times as our focus is to asses success rates among different modeling paradigms. We stopped the optimizer after 20 seconds if not converged. Success is defined as reaching the goal and being collision free. We also report the break down in Table IV. The best success rate for each setup is highlighted in bold.

## C. Comparing different goal constraints

We first compare the different goal constraints. The geodesic flow attractor which we introduce in this paper outperforms Euclidean and Natural on the three benchmarks.

The Natural attractor performs the worst. The gradient of the attractor vanishes due to the flat geometry of the boxes obstacles. In environments populated with more round obstacles Natural attractors can perform significantly better than Euclidean attractors pulling the motion towards the goal by wrapping around obstacles. This problem is highlighted by the *goal reached* rate which are low with all Natural attractors. On the other hand the geodesic attractor is able to pull correctly the freeflyers into the passage leading to *goal reached* rates of near 100% in both Cartesian case.

### D. Comparing with and without flow

A narrow passage forces the solution motion to coordinate the different DoFs of the robot to navigate the passage. We test a heuristic forcing the motion to follow the geodesic flow towards the narrow passage on the planar case, which presents a more challenging case as one can see by the low success rates in this case.

Forcing the motion to agree with the geodesic flow enhances very significantly the case where the goal constraint does not allow to find good passages through the narrow passage, which is especially the case with the Euclidean attractor.

## E. Riemannian metrics

We also study the geodesic term based on the workspace geometry map. This term was introduced in [14], but no formal experiments were conducted to measure the impact of this term.

Here we can see quite clearly that it helps finding collision free paths in all examples and using all goal constraints models. It is worth noting that the best success rates are obtained when we use a geodesic attractor with a strong geodesic term, confirming the importance of modeling the motion optimization problem with Riemannian geometry.

## CONCLUSION

We presented a methodology for modeling motion optimization problems. We showed that these models allow to solve challenging path planning problem with an interior point method. We draw key insights from Riemannian geometry to introduce a new objective and a constraint terms based on the geodesic flow that lead to higher success rates on planning problems with narrow passages.

One way to view our contribution is to see it as a way to integrate a dynamic programming precomputation (i.e., the geodesic flow), within a gradient-based optimization framework. In future work we aim to further remove this need for solving a diffusion process and rather decompose the workspace in primitive shapes that analytically provide a solution to the geodesic flow such as proposed in [21].

TABLE IV: Simulation results on the three problems of Figure 3. All numbers are rates averaged over a goal set.

Attractor	Euclidean Natural				Geodesic Flow			Euclidean			Natural			Geodesic Flow				
Geod. Term	0	10	50	0	10	50	0	10	50	0	10	50	0	10	50	0	10	50
Problem	Planar Narrow								Cartesian Narrow									
success	0.00	0.14	0.04	0.07	0.00	0.00	0.04	0.18	0.29	0.00	0.62	0.62	0.00	0.06	0.28	0.00	0.88	0.91
collision free	0.00	0.14	0.04	0.57	0.57	0.71	0.04	0.25	0.54	0.03	0.72	0.69	0.69	0.81	0.94	0.00	0.88	0.97
goal reached	1.00	1.00	1.00	0.39	0.36	0.21	0.96	0.89	0.71	0.97	0.88	0.94	0.31	0.25	0.34	1.00	1.00	0.94
Problem	Planar Narrow (with flow)							Cartesian Maze										
success	0.00	0.18	0.32	0.00	0.04	0.14	0.00	0.14	0.32	0.00	0.06	0.42	0.00	0.03	0.19	0.00	0.31	0.53
collision free	0.04	0.29	0.46	0.18	0.18	0.54	0.00	0.14	0.32	0.00	0.08	0.50	0.56	0.64	0.83	0.00	0.31	0.69
goal reached	0.93	0.86	0.82	0.68	0.57	0.43	0.75	0.86	0.96	0.97	0.88	0.94	0.31	0.25	0.34	1.00	1.00	0.94

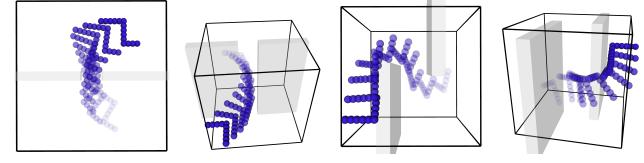


Fig. 3: 2D planar environment with SE(2) configuration space, translation and rotation (left), and two 3D Cartesian environments (right) with free-flying SE(3) configuration spaces used in the experiments (one view angle of Cartesian Narrow and two view angles Cartesian Maze). The trajectories show time frames as color fading. All environments present a narrow passage, which requires to find a motion coordinating translation and orientation DoFs to find a collision free motion.

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#### REFERENCES

- T. Kruse, A. K. Pandey, R. Alami, and A. Kirsch, "Human-aware robot navigation: A survey," *Robotics and Autonomous Systems*, vol. 61, no. 12, pp. 1726–1743, 2013.
- [2] S. M. LaValle, *Planning algorithms*. Cambridge university press, 2006.
- [3] O. Khatib, "Real-time obstacle avoidance for manipulators and mobile robots," *The Int. Journal of Robotics Research*, vol. 5, no. 1, pp. 90–98, 1986.
- [4] M. Zucker, N. Ratliff, A. D. Dragan, M. Pivtoraiko, M. Klingensmith, C. M. Dellin, J. A. Bagnell, and S. S. Srinivasa, "Chomp: Covariant hamiltonian optimization for motion planning," *The Int. Journal of Robotics Research*, vol. 32, no. 9-10, pp. 1164–1193, 2013.
- [5] K. Hauser, "Semi-infinite programming for trajectory optimization with nonconvex obstacles," in *International Workshop on the Algorithmic Foundations of Robotics*. Springer, 2018, pp. 565–580.
- [6] A. Kuntz, C. Bowen, and R. Alterovitz, "Fast anytime motion planning in point clouds by interleaving sampling and interior point optimization," in *Robotics Research*. Springer, 2020, pp. 929–945.
- [7] J.-C. Latombe, *Robot motion planning*. Springer Science & Business Media, 1992.
- [8] N. Ratliff, M. Toussaint, and S. Schaal, "Understanding the geometry of workspace obstacles in motion optimization," in *IEEE Int. Conf. Robotics And Automation (ICRA)*. IEEE, 2015.

- [9] M. Kalakrishnan, S. Chitta, E. Theodorou, P. Pastor, and S. Schaal, "STOMP: Stochastic trajectory optimization for motion planning," in *IEEE Int. Conf. Robotics And Automation (ICRA)*, 2011.
- [10] J. Schulman, J. Ho, A. X. Lee, I. Awwal, H. Bradlow, and P. Abbeel, "Finding locally optimal, collision-free trajectories with sequential convex optimization." in *Robotics: Science and Systems (RSS)*, 2013.
- [11] M. Toussaint, "Newton methods for k-order markov constrained motion problems," arXiv preprint arXiv:1407.0414, 2014.
- [12] S. Quinlan and O. Khatib, "Elastic bands: Connecting path planning and control," in *IEEE Int. Conf. Robotics And Automation (ICRA)*, 1993.
- [13] O. Brock and O. Khatib, "Elastic strips: A framework for integrated planning and execution," in *Experimental Robotics VI*. Springer, 2000, pp. 329–338.
- [14] N. Ratliff, M. Toussaint, and S. Schaal, "Understanding the geometry of workspace obstacles in motion optimization," in *IEEE Int. Conf. Robotics And Automation (ICRA)*, 2015.
- [15] J. Mainprice, N. Ratliff, and S. Schaal, "Warping the workspace geometry with electric potentials for motion optimization of manipulation tasks," in 2016 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS). IEEE, 2016, pp. 3156–3163.
- [16] K. Crane, C. Weischedel, and M. Wardetzky, "Geodesics in heat: A new approach to computing distance based on heat flow," ACM Transactions on Graphics (TOG), vol. 32, no. 5, pp. 1–11, 2013.
- [17] C. I. Connolly and R. A. Grupen, "The applications of harmonic functions to robotics," *Journal of robotic Systems*, vol. 10, no. 7, pp. 931–946, 1993.
- [18] K. H. Wray, D. Ruiken, R. A. Grupen, and S. Zilberstein, "Log-space harmonic function path planning," in 2016 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS). IEEE, 2016, pp. 1511–1516.
- [19] A. Wächter and L. T. Biegler, "On the implementation of an interiorpoint filter line-search algorithm for large-scale nonlinear programming," *Mathematical programming*, vol. 106, no. 1, pp. 25–57, 2006.
- [20] H. Wang, K. A. Sidorov, P. Sandilands, and T. Komura, "Harmonic parameterization by electrostatics," ACM Transactions on Graphics (TOG), vol. 32, no. 5, p. 155, 2013.
- [21] L. Huber, A. Billard, and J.-J. Slotine, "Avoidance of convex and concave obstacles with convergence ensured through contraction," *IEEE Robotics and Automation Letters*, vol. 4, no. 2, pp. 1462–1469, 2019.