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Yang, Zhenyu; Podlech, Steffen

Published in:
Proceedings of the 2008 IEEE International Conference on Robotics and Biomimetics

Publication date:
2009

Document Version
Publisher's PDF, also known as Version of record

Proceedings of the 2008 IEEE
International Conference on Robotics and Biomimetics
Bangkok, Thailand, February 21 - 26, 2009

Theoretical Modeling Issue in Active Noise Control for a One-Dimensional Acoustic Duct System

Zhenyu Yang and Steffen Podlech
Esbjerg Institute of Technology
Aalborg University
Niels Bohrs Vej 8, DK-6700 Esbjerg, Denmark
Email: yang,sp@aaue.dk

***Abstract*—The theoretical modeling of active noise control for an acoustic duct system is discussed. A Partial Differential Equation (PDE) model with external inputs for the acoustic duct is first developed by following the physical acoustic propagation principles. Then this PDE model is converted into an infinite dimensional complex-valued state space model using the variable separation technique under the assumption that the acoustic terminal impedance is constant. The assumed acoustic impedance can be estimated using the experimental system identification method. By combining the acoustic duct model with the model of the canceling loudspeaker module, in which the acoustic coupling between the acoustic duct and the loudspeaker is also explicitly modeled, a mathematical model for the entire one-dimensional active duct noise control system is obtained and validated. The developed model provides a solid and sufficient platform for design of active noise control using advanced filtering technique, advanced control techniques and development of virtual sensors as well.**

***Index Terms*—Theoretical modeling, parameter identification, active noise control, virtual sensor**

I. Introduction

Due to the dramatically rapid development of the microprocessor technology in recent decades, the Active Noise Control (ANC) technique which is regarded as one of effective and efficient ways to handle low-frequency noises has been attracting more and more attention from the theoretical perspective and applicable perspective as well [3], [5], [9], [10], [11], [21].

Along with any kind of ANC development, a mathematical model of the considered system in some kind of precision needs to be obtained beforehand, such as the impulse-response model [9], [11], the transfer function model [12], [15] or the state space model [7], [18], [17] etc.. On one hand, the model could be constructed purely based on experimental data, such as the impulse-response model or transfer function model. This method is often referred to as the so-called *black-box modeling*. This kind of modeling methods is relatively easy in terms of model construction and precision control, however, the main drawback of this method is that the developed model has no physical meanings and the modeling procedure has to be repeated once the physical system/enviroment changes (e.g., loudspeaker/microphone shifts position). On the other hand, the model could be developed pure based on the physical principles and calculations, and it is often referred to as the so-called *white-box modeling*. This kind of modeling method gets rid of the drawbacks caused by the black-box method, however, in many cases this modeling method may leads to an endless procedure or a so complicated model that it might not be usable for the further purpose, due to the complicated physics involved. Furthermore, many theoretical values/calculations which usually are under some ideal assumptions may not leads to a model precise enough to fit to the reality. Thereby in most realistic situations, these two kind of modeling methods are combined for model development. This leads to the so-called *grey-box modeling*. How to balance the theoretical modeling part and the experimental modeling part in the grey-box modeling method depends on the modeling purpose, the known knowledge of the system, and available testing facilities etc..

In the following, the grey-box modeling method will be employed for an ANC modeling problem based on a one-dimensional acoustic duct system. The strategy is to keep as much physical modeling part as possible, under the condition that the developed model is a type of state-space description. This idea is motivated by the following considerations:

- The model developed in this way consists of the understandings of the physical system to the maximal extent without losing its flexibility, so that it can be used for a class of systems instead of a specific setup;
- The state space model has a good orientation for applying advanced filtering techniques (e.g., Kalman filtering techniques, virtual sensor concepts [1], and advanced control techniques as well [12], [8], [19]; Furthermore,
- The models used by most of existing work are obtained mainly using the black-box method.

From the practical point of view, the one-dimensional acoustic duct system can be regarded as a simplification of some ventilation system for large-sized buildings. By following the acoustic propagation principle [2], a Partial Differential Equation (PDE) model is firstly obtained. Then this PDE model is converted into a infinite dimensional complex-valued state space model using the variable separation method. By combining the model of the canceling loudspeaker module, where the acoustic coupling between the acoustic duct and the loudspeaker is also explicitly modeled, a mathematical model for the entire system is obtained. The key system parameter - acoustic duct terminal impedance can be estimated using the experimental method by checking the eigenvalue's

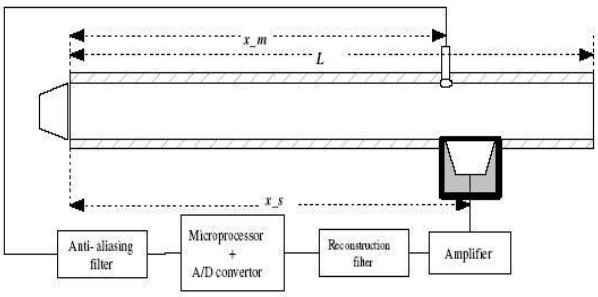

Fig. 1. Schematic Diagram of the considered system

compatibility [6], [20]. The developed model provide a solid and sufficient platform for using advanced filtering techniques, advanced control techniques and development of virtual sensors as well.

The rest of the paper is organized as: Section 2 briefs the considered one-dimensional ANC system; Section 3 focuses on the modeling of the acoustic duct system; Section 4 presents the modeling of the canceling loudspeaker module and afterwards the entire system model; Section 5 discusses the black-box identification of the system parameter; and finally, we conclude the paper in Section 6.

II. Considered Acoustic Duct System

Consider an acoustic duct which diameter is significantly smaller than its length. As shown in Fig.1, at one end of the duct one loudspeaker is installed to emulate the primary noise source. Another loudspeaker which acts as the secondary acoustic resource and one microphone which measures the attenuated residual are used in the considered system. The measured residual goes into an anti-aliasing filter after passing the microphone's amplifier circuit. The filtered signal feeds into the microprocessor/PC which acts as the ANC controller. The output signal of the controller goes through a reconstruction filter and then the amplifier circuit for loudspeaker, so as to drive the canceling loudspeaker to generator a proper anti-noise signal. From the practical point of view, the duct should be sealed properly at all openings (microphone ports, sound sources, etc.) in order to achieve a low background SNR level. Meanwhile, the microphone must mounted flush with the inside wall of the tube and isolated from the tube to minimize the sensitivity to vibration. All relevant system parameters, values and variables are listed in the following Table.1.

III. Modeling the Acoustic Duct System

A. One-Dimensional PDE Model

According to acoustic principles [2], one dimensional hard-walled duct excited by a pressure input at one end and a mass flow input in the domain can be described by a linear second-order PDE:

$$\frac{\partial^2(u(x,t))}{\partial t^2} - c^2\frac{\partial^2(u(x,t))}{\partial x^2} =$$

TABLE I
Variable/Parameters of the considered acoustic duct system

Parameter	Notation	Value	Unit
Duct length	L	1.70	m
Duct radius	a	0.052	m
Intersection area	S	8.5 * 10^{-3}	m^2
Sound speed	c	343	m/s
medium density	ρ	1.21	kg/m^3
Loudspeaker location	x_s	variable	m
Microphone location	x_m	variable	m
Terminal impedance	K	variable	-
Number of modes	N	variable	-
Particle displacement	$u(x,t)$	variable	m
Particle spatial location	x	variable	m
Pressure excitation($x=0$)	$p(t)$	variable	N/m^2
Mass flow	$M(t)$	variable	kg/s

$$-\frac{\partial}{\partial x}(\frac{\delta(x)p(t)}{\rho}) - \frac{\partial}{\partial t}(\frac{\delta(x-x_s)M(t)}{\rho S}) \quad (1)$$

The terminal at $x = L$ is assumed as a partially reflective boundary condition. At the open end, some of the acoustic signal transmits through boundary and part of it will be back reflected into the duct due to the sudden geometry change from a well defined environment (inside the duct) to the undefined ambient space around the duct. It can be formulated as

$$c\frac{\partial u(L,t)}{\partial x} = -K\frac{\partial u(L,t)}{\partial t} \quad (2)$$

where K is the acoustic terminal impedance at the open end, which is complex-valued and will be discussed in the following section. The duct at $x = 0$ is modeled as a totally reflective end, i.e.,

$$\frac{\partial u(0,t)}{\partial x} = 0. \quad (3)$$

The acoustic pressure inside the duct is related to $u(x,t)$ as

$$P(x,t) = -\rho c^2\frac{\partial u(x,t)}{\partial x}. \quad (4)$$

Equation (1), (2), (3) and (4) consist of the model of the acoustic propagation dynamic in the one-dimensional duct system [7], [17].

B. Variable Separation Method

The PDE-based model obtained from the above section can be converted into a state-space model description using the variable separation method [7]. In order to clearly and simply illustrate the idea. Hereby we consider the homogenous wave equation (5) and boundary conditions (2) and (3), where

$$\frac{\partial^2(u(x,t))}{\partial t^2} = c^2\frac{\partial^2(u(x,t))}{\partial x^2}. \quad (5)$$

According to the variable separation method, the function $u(x,t)$ can expressed as a product of a spatial domain function and a temporal domain function, i.e.,

$$u(x,t) = X(x)T(t). \quad (6)$$

By applying (6) into (5) and using the separation factor λ, it can be see that the spatial domain function $X(x)$ should satisfy the following equation:

$$X^{"}(x) - \lambda^2 X(x) = 0. \tag{7}$$

The general solution of (7) can be written as:

$$X(x) = Ae^{\lambda x} + Be^{-\lambda x}, \tag{8}$$

where A and B are some constants. By applying the boundary condition (3), there is $A = -B$. Without triviality, we assume $A = -B = 1$.

Meanwhile, we can observe that the time domain function $T(t)$ should satisfy:

$$\ddot{T}(t) - \lambda^2 c^2 T(t) = 0. \tag{9}$$

Accordingly, the general solution of (9) can be be written as:

$$T(t) = Ce^{\lambda ct} + De^{-\lambda ct}, \tag{10}$$

here C and D are some constants need to be determined by specific boundary condition.

Substituting (6) into the boundary condition (2), there is

$$X^{'}(L)T(t) = -\frac{K}{c}\left(X(L)\dot{T}(t)\right). \tag{11}$$

At this point the constant D in (10) can be assumed to be zero, as it is connected to the part of the signal, which is transmitted through the partial reflective boundary and therefore does not give any further contribution to the conditions within the duct part. Thereby we focus on the remaining terms in (11). Substituting (8) and (10) into (11), there is

$$\lambda = \frac{1}{2L} ln\left(\frac{1-K}{1+K}\right). \tag{12}$$

With respect to the Euler equation for the specific value $1 = e^{2n\pi i}$ for $n = \pm 0; \pm 1; \pm 2; \ldots$, we have

$$e^{2\lambda L} e^{2n\pi i} = \left(\frac{1-K}{1+K}\right) \quad \text{for } n = \pm\,0;\ \pm\,1;\ \pm\,2;\ \ldots. \tag{13}$$

By rewriting above equation, then a complex expression of eigenvalues for different wave modal can be expressed as:

$$\lambda_n = \frac{1}{2L} ln\left(\frac{1-K}{1+K}\right) - \frac{n\pi i}{L}\,; \tag{14}$$

for $n = \pm\,0;\ \pm\,1;\ \pm\,2;\ \ldots$. The imaginary part of $c\lambda_n$ corresponds the *n-th mode resonance frequency* of the considered acoustic duct. Thereby, according to different K value, there are

(i) for	$K = 1$	permeable duct end
(ii) for	$K = 0$	perfect reflected duct end
(iii) for	$0 < K < 1$	partial reflective duct end

By summarizing above results, a homogenous state-space model can be defined as

$$\begin{cases} \dot{X}_a(t) = A_a X_a(t) \\ y_a(t) \hat{=} p(x_m, t) = C_a X_a(t) \end{cases} \tag{15}$$

where the infinite-dimensional state vector $X_a(t) \hat{=} [\cdots\ \ a_{-1}(t)\ \ a_0(t)\ \ a_1(t)\ \ \cdots]^T$ consists of modal wave amplitudes, and system matrices are

$$A_a \hat{=} diag(c\lambda_n) = \begin{bmatrix} \ddots & 0 & 0 & 0 & \cdots \\ \cdots & c\lambda_{-1} & 0 & 0 & \cdots \\ \cdots & 0 & c\lambda_0 & 0 & \cdots \\ \cdots & 0 & 0 & c\lambda_1 & \cdots \\ \cdots & 0 & 0 & 0 & \cdots \end{bmatrix}$$

$$C_a \hat{=} row\_vector(-\rho c^2 \tfrac{d(\varphi_n(x_m))}{dx})_{n=0,\pm1,\cdots}$$

$$= \begin{bmatrix} \cdots & -\rho c^2 \frac{d(\varphi_{-1}(x_m))}{dx} & -\rho c^2 \frac{d(\varphi_0(x_m))}{dx} & \cdots \end{bmatrix}. \tag{16}$$

where λ_n is expressed in (14), x_m represents the position of the measurement microphone, and

$$\varphi_n(x) = e^{\lambda_n x} + e^{-\lambda_n x}, \text{ for } n = 0, \pm1, \pm2, \cdots, \tag{17}$$

which is derived from (8).

When the system has external inputs/distubances, such as the system described by (1), the similar procedure can be used to obtain a standard state-space model which is described in the following subsection.

C. State-Space Model

The infinite-dimensional state-space model can be expressed as

$$\begin{cases} \dot{X}_a(t) = A_a X_a(t) + B_a u_a(t) + B_p p(t) \\ y_a(t) = C_a X_a(t) \end{cases} \tag{18}$$

where the state $X_a(t) \hat{=} [\cdots\ \ a_{-1}(t)\ \ a_0(t)\ \ a_1(t)\ \ \cdots]^T$ is the vector of modal wave amplitude. The control input

$$u_a(t) \hat{=} \frac{d(M(t))}{dt} \tag{19}$$

is the mass flow rate generated by the canceling loudspeaker. The input $p(t)$ is the air pressure generated by the original noise resource. The output

$$y_a(t) \hat{=} p(x_m, t) \tag{20}$$

is the air pressure measured by the residual microphone at location x_m. Matrices A_a and C_a are defined in (16),

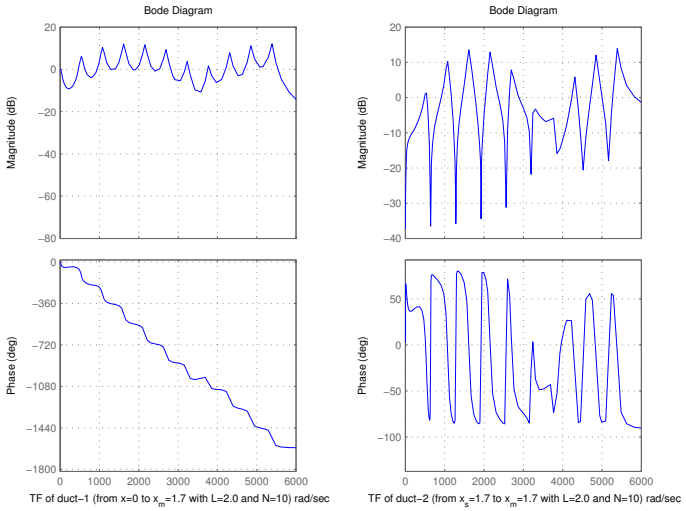

Fig. 2. Frequency Property of the Acoustic Duct

respectively. The other system matrices are defined as

$$B_a \hat{=} column\_vector(-\frac{1}{4c\lambda_n^2 L\rho S}\frac{d(\varphi_n(x_s))}{dx})_{n=0,\pm1,\cdots} = \begin{bmatrix} \vdots \\ -\frac{1}{4c\lambda_{-1}^2 L\rho S}\frac{d(\varphi_{-1}(x_s))}{dx} \\ -\frac{1}{4c\lambda_0^2 L\rho S}\frac{d(\varphi_0(x_s))}{dx} \\ -\frac{1}{4c\lambda_1^2 L\rho S}\frac{d(\varphi_1(x_s))}{dx} \\ \vdots \end{bmatrix}$$

$$B_p \hat{=} column\_vector(\frac{1}{2c\lambda_n L\rho})_{n=0,\pm1,\cdots} = \begin{bmatrix} \vdots \\ \frac{1}{2c\lambda_{-1} L\rho} \\ \frac{1}{2c\lambda_0 L\rho} \\ \frac{1}{2c\lambda_1 L\rho} \\ \vdots \end{bmatrix} \quad (21)$$

where function $\varphi_n(x)$ and coefficient λ_n are defined as in (14) and ()17, respectively.

The initial condition for (18) can be determined through [7]

$$a_n(0) = \frac{1}{4c\lambda_n L}\int_0^L \frac{\partial u(x,0)}{\partial t}\varphi_n(x)dx - \frac{1}{4\lambda_n^2 L}\int_0^L \frac{\partial u(x,0)}{\partial t}\frac{d\varphi_n(x)}{dx}dx. \quad (22)$$

Using the data from Table I, the considered system have the frequency property as shown in Fig.2.

IV. Entire System Model

A. Modeling the Loudspeaker Module

The basic structure of a typical low-frequency loudspeaker can be found in [4]. With respect to a typical voice-coil loudspeaker, at the small amplitude displacement of the diaphragm-coil assembly, the dynamic of the loudspeaker can be approximated by a linear model. System parameters, values and variables of our considered system are listed in the following table II.

TABLE II
Variables/Parameters of the Canceling Loudspeaker

Parameter	Notation	Value	Unit
Assembly mass	m_s	$2.8*10^-3$	Kg
Viscous friction	f_s	5.2	Ns/m
Suspension stiffness	k_s	$12*10^3$	N/m
Voice coil resistance	R_s	4	Ohm
Voice coil inductance	L_s	1.1	mH
Force factor	Bl	3.9	N/An
Effective radius	r_s	0.07	m
Assem. displacement	$x(t)$	variable	m
Assem. velocity	$\dot{x}(t)$	variable	m/sec
Assem. acceleration	$a_s(t)$	variable	m/sec^2
coil current	$i_s(t)$	variable	An
Input voltage	$u_{in}(t)$	variable	Volt
EMF voltage	$u_{emf}(t)$	variable	Volt

From the electromagnetic analysis, there are

$$\dot{i}_s(t) = -\frac{R_s}{L_s}i_s(t) - \frac{Bl}{L_s}\dot{x}(t) + \frac{1}{L_s}u_{in}(t), \quad (23)$$

$$F_s(t) - F_{ext}(t) = m_s\ddot{x}(t) + f_s\dot{x}(t) + k_s x(t), \quad (24)$$

where $F_s(t)$ is the electromagnetic force generated by the voice coils by following principle $F_s(t) = Bli_s(t)$, and F_{ext} represents all the external forces acting on the assembly. For example, if the speaker is mounted in a cavity and faced a duct in front, then, there is

$$F_{ext}(t) = F_{rear}(t) + F_{front}(t).$$

The force $F_{rear}(t)$ generated by the air pressure inside the rear enclosure can be calculated as

$$F_{rear}(t) = \frac{\rho c^2 S_d^2}{V_0}x(t). \quad (25)$$

where V_0 is the volume capacity of the cavity, and S_d is the effective area of the speaker, i.e., $S_d = \pi r_s^2$.

Denote the air pressure in front of the speaker as $p(x_s,t)$, then, the front external force F_{front} can be calculated as

$$F_{front}(t) = p(x_s,t)S_d. \quad (26)$$

Combining (23),(25) and (26), a state space model for the speaker with coupled acoustic dynamics can be obtained as

$$\begin{cases} \dot{X}_{spk}(t) = A_{spk}X_{spk}(t) + B_{spk}U_{spk}(t) \\ Y_{spk}(t) = C_{spk}X_{spk}(t) \end{cases} \quad (27)$$

where the state vector is $X_{spk}(t)\hat{=}[i_s(t)\ \ x(t)\ \ \dot{x}(t)]^T$ and the input vector is $U_{spk}(t)\hat{=}[u_{in}(t)\ \ p(x_s,t)]^T$, and system matrices are

$$A_{spk} = \begin{bmatrix} -\frac{R_s}{L_s} & 0 & -\frac{Bl}{L_s} \\ 0 & 1 & 0 \\ \frac{Bl}{m_s} & -\frac{k_s V_0 + \rho c^2 S_d^2}{m_s V_0} & -\frac{f_s}{m_s} \end{bmatrix}$$

$$B_{spk} = \begin{bmatrix} \frac{1}{L_s} & 0 \\ 0 & 0 \\ 0 & -S_d \end{bmatrix},$$

$$C_{spk} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$

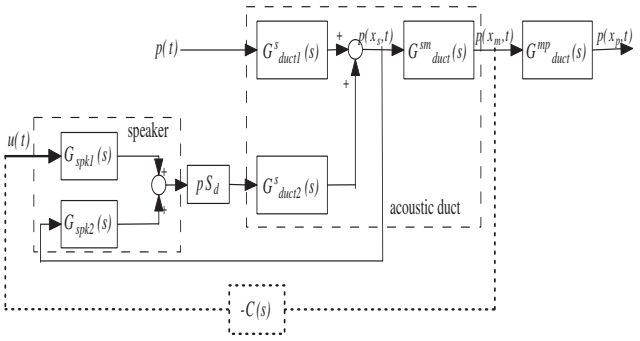

Fig. 3. Block diagram of the entire ANC system

This model is a two-input one-output LTI state space description. Comparing with the existing models [7], [8], [14], this model consider the coupling acoustic dynamics coming from the front duct as well as the rear enclosure. The usage of rear enclosure is quite important to avoid the "acoustic short-circuit" from the practical point of view [17]. In the entire system diagram as shown in Fig.3, the loudspeaker model is represented by two transfer functions: one denoted as $\bar{G}_{spk1}(s)$ is from $u_{in}(t)$ to $\dot{x}(t)$, another one denoted as $\bar{G}_{spk2}(s)$ is from $p(x_s,t)$ to $\dot{x}(t)$.

B. Entire System Model

The entire system model can be obtained by combining the loudspeaker model (27) and the acoustic model (18) together. However, the output of the loudspeaker model is the assembly velocity $\dot{x}(t)$, while the controllable input of the acoustic model (18) is the mass flow rate $u_a(t)$. Nevertheless, there is the following relationship linking these two parts, i.e.,

$$M(t) = \rho S_d \dot{x}(t) \Rightarrow u_a(t) = \frac{d(M(t))}{dt} = \rho S_d a_s(t). \quad (28)$$

Therefore, the following two transfer functions which denote the output of the speaker model as $a_s(t)$ are used:

$$\begin{aligned} G_{spk1}(s) &= \bar{G}_{spk1}(s)s, \\ G_{spk2}(s) &= \bar{G}_{spk2}(s)s, \end{aligned} \quad (29)$$

where $\bar{G}_{spk1}(s)$ and $\bar{G}_{spk2}(s)$ are defined according to (27).

There are two pressure measurements at locations x_s and x_m, respectively, need to be known in order to construct the physical/control feedback mechanism. The pressure $p(x_s,t)$ is required by the loudspeaker model, and pressure $p(x_m,t)$ represents the pressure measurement by the microphone and it will be used by the ANC controller later. Furthermore, in order to evaluate the ANC performance at other spatial operating points, a movable *performance point* x_p can also be defined [12]. For simulation purpose, all these pressures can be calculated based on the acoustic duct model (18). For instance, we can define the following transfer functions [17]:

$$\begin{aligned} G^s_{duct1}(s) &= \quad G_{duct1}(s) \text{ when } x_m = x_s, \\ G^s_{duct2}(s) &= \quad G_{duct2}(s) \text{ when } x_m = x_s, \\ G^{sm}_{duct}(s) &= \quad \frac{G^m_{duct1}(s)}{G^s_{duct1}(s)} = \frac{n^m_{a1}(s)}{n^s_{a1}(s)}, \\ G^{mp}_{duct}(s) &= \quad \frac{G^p_{duct1}(s)}{G^m_{duct1}(s)} = \frac{n^p_{a1}(s)}{n^s_{a1}(m)}, \end{aligned} \quad (30)$$

From characteristics of matrices (16) it can be noticed that in the calculation of pressures, different spatial point, such as x_s, x_m or x_p, only appear in vector C_a. Therefore, $G^{sm}_{duct}(s)$ and $G^{mp}_{duct}(s)$ are complex constants w.r.t. x_s, x_m and x_m, x_p, respectively.

The entire system block diagram is shown in Fig.3. Even though this developed model theoretically is infinite dimensional, in the practical design and simulation, this model can be truncated by taking several acoustic modes though selecting a proper N. This simplification won't cause serious problem to ANC design due to the fact that ANC system is mainly used to deal with low frequency noises. The neglected modes can be regarded as modeling uncertainties [12], [18].

V. Identification of Parameter K

It is obvious that the acoustic terminal impedance K plays a critical role in determining system's properties. In principle, K is a complex and frequency-dependent parameter [2], [13], [12]. However, some simplification or approximation of this parameter could make the developed model (18) standard. For instance,in case that K is assumed constant, then system (18) can be simplified to a two-input one-output LTI system [8], [15], [17].

The parameter K can be estimated using some system identification method (*black-box method*) [6], [20]. The challenge here is that the theoretically developed model is infinite dimensional and complex-valued with some specific structure [6], while the model identified through system identification purely based on measured input and output data is a "black-box" product and it is usually real-valued and finite dimensional. Therefore, a compatible principle is found in [20], which states the positive imaginary parts of eigenvalues of the theoretical model should be identical to those of the identified model. Then the terminal impedance can be estimated by taking average of its estimations corresponding to each resonance frequency.

By using one estimated K based on our physical setup, the developed model is validated through experiments. From Fig. 4 it can be observed that the resonance frequency of the developed model is almost consistent with the practical test. The slight deviation is mainly due to the limitation that K can only be a constant [20].

In order to get a more precise estimation of K, [20] proposed a "peak-by-peak" identification method. The main idea is that for each identification iteration, select the considered frequency range covering only one resonance peak, and meanwhile limit the estimated system order to be two in the system identification procedure. Finally the K is taking

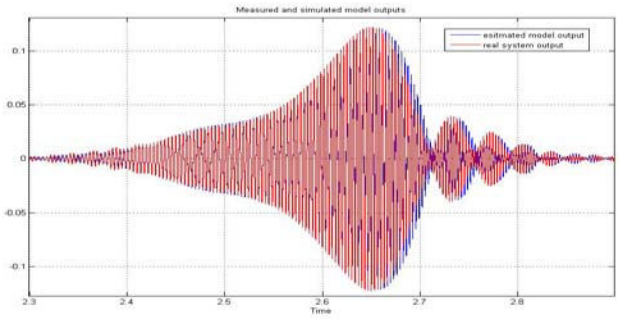

Fig. 4. Comparison of the estimated model and the real system

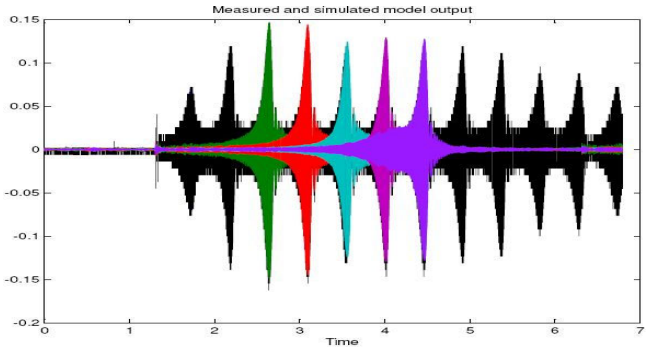

Fig. 5. Comparison of estimated models ("peak-by-peak") and the real system

average of its estimations corresponding to each resonance frequency. The main benefit of this method is that the the complexity and computation load for system identification procedure is significantly reduced, meanwhile a more precise estimation of each K can be obtained due to the uncorrelated data derived by artificial peak-by-peak selection. One output comparison of the estimated model and real system is shown in Fig.5.

VI. Discussions and Conclusions

The developed model serves a solid and sufficient basis for ANC design using advanced control techniques, such as the pole-placement design [8], the lead-lag compensators [18], the mode predictive control [19]. The global attenuation [8], the selection of best actuatoring/measureing position [15] can also be efficiently analyzed by simply define the performance point x_p or x_s and x_m as (sliding) variables in the model [16]. The developed model is also suitable for applying advanced filtering techniques, such as Kalman filters, for ANC design. Meanwhile, this model also gives a good orientation for virtual sensor development [1]. Actually, the virtual sensor concept is already employed in this model, i.e., the pressure estimation at the performance point x_p. The ANC control system uses the measured signal $p(x_m, t)$ as the feedback signal to controller, so as to attenuate/minimize the noise pressure at the performance point, i.e., the pressure $p(x_p, t)$ is not measurable to the controller, even though it is the control task.

The development of a theoretical model for a class of ANC system is discussed. The developed model contains the physical meanings to the maximal extent comparing with all existing relevant models. The variable separation method is employed to convert the original PDE model into a state space description. The experimental modeling is only used to estimate the key system parameter - acoustic terminal impedance. The developed model can serves as the solid and sufficient platform for applying advanced techniques for ANC design.

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