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AUTOMATIC TWO-FINGERED GRIP SELECTION[1]

by

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1. INTRODUCTION

The importance of automatically determining gripping positions for robot hands on a variety of real objects has already been established [1] [2] [3]. Grip determination is essential to any task level planning process as well as being necessary for any situation in which the position and orientation of the part to be grasped is not known a priori. A robust, computational efficient technique for determining gripping positions of polyhedral objects described by Computer-Aided-Design (CAD) models is presented in [4]. The method presented in that paper achieves computationally efficiency in part through several approximations to the force equations of the grasp and a restriction of applied forces to, and moments in, the gripping plane. In this paper we present the complete force/moment equations involved, verify that the earlier method is in fact a good approximation, and extend the results to out-of-plane moments and forces applied to the object being grasped. The formulation presented here is exact for the general case, and both allows previous algorithms to be extended, and forms the basis for more general gripping situations as well.

The problem of determining grasping points for robots has been analyzed for a number of different situations and can be classified along several different dimensions: the types of gripper/object contact, the number of fingers, the techniques used for determining grip positions, the type of models used, consideration/lack of consideration of interference with other nearby objects. In early work [2] [5] simple heuristics like grasping as close as possible to the center of mass were proposed. Consideration has been given to non interference of the robot with other objects in the initial or final position of the object to be grasped [4] [6] [7] [8]. Object models determined by visual input [1], computer aided design models [4] [9] or unknown [10].

More recently, the emphasis on grasping research has focused on careful force and moment balances for different combinations of fingers and contact types for both two dimensional and three dimensional objects. Most investigations have considered point contact forces. For three fingered grippers, Hanafusa and Asada have considered a hand with three fingers placed in a ring with each finger exerting a force control by a spring and a stepping motor [11]. Baker, Fortune and Grosse [12] have extended that work to allow variable spacing between the fingers around the ring for both two dimensional polygons and three dimensional cylinders with polygonal cross sections. Abel, Holsman and McCarthy have analyzed two fingered grasping and planar objects and three fingered grasping on three dimensional objects with point contact forces including friction effects [13] [14]. Their techniques allow an analyses of potential grasping locations. Fearing [10] also analysis the forces and moments associated with grasping two dimensional polygonal objects and describes the use of tactile sensing to determine if greater forces should be applied to the grasp. Okada and Kanade [15] have used simulation to determine how multi jointed fingers can grasp an object with many points of contact.

One of the difficulties with the use of point contact forces is that there is no rotational stability in the contact i.e., yaw, pitch or roll rotations may occur, regardless of the applied force, unless further constraints are imposed by the hand interconnecting the contacting fingers. In a recently completed thesis, Jameson [16] has analysed multiple finger grasping for both point contacts and "soft" finger contacts. Both types of fingers supply forces normal to the contact surface. Soft finger contact can oppose yaw motions but offers no resistance to roll or pitch. As have a few other investigators, Jameson endeavors to find optimum gripping positions. Though modest success is obtained, for many problems the existence of a substantial number of local minima impedes effective solution techniques.

As pointed out in [4], however, not only is there possible resistance to roll, pitch and yaw through area contact between finger and object, but the friction forces resisting slippage in a gripper depend upon the shape of the contact between the fingers and the surface of the object. The case of surface contact has been considered by only a few others. One of the difficulties in solving the general problem is that the force and moment equations for the system under the condition that the object is just about to slip include moment terms naturally expressed about the center of the slip of the impending motion, and the center of slip is unknown. In [4] the center of slip is approximated by the centroid of a contact area, which under many circumstances is a good approximation. The problem was further simplified by consideration only of forces and moments in the gripping plane of a two fingered gripper.

In this paper the restriction of in plane forces and moments is eliminated and the approximation of the center of slip by the center of a contact area removed. Further, while [4] assumed a constant uniform pressure on the gripping surface, the analysis presented herein models the pressure as a linear distribution across the contact area. This allows a much more general solution to the problem to be obtained. Among other things, the inclusion of the out of plane forces and moments together with a pressure distribution allows impending twisting to be detected without the heuristics used in [4]. The resulting set of equations, however, cannot be explicitly solved. Instead, an implicit solution is obtained by generating a synthetic error function which has a value of zero at the solution point, and using an efficient conjugate direction iterative optimization for your minimization procedure proposed by Shanno [17].

2. GRIP EVALUATION FUNCTION

In the previous paper [4], we presented a method for selecting grips in which three quality measures were proposed for evaluation of proposed grips. These measures included resistance to rotational slippage, and two modes of twisting. An empirical weighting factor was applied to each of these criteria which gave an evaluation function of the form:

$$\frac{k_1 \Delta d + k_2 \tau_s / \mu_s N}{D + \epsilon} + k_3 \alpha \quad ,$$

where k_1, k_2 and k_3 are the weighting by factors for the three criteria; D is the distance of the center of slip, C, from the center of mass; α is the measure of the resilience to uncertainty in the relative position and orientation of the part and gripper during closing of the gripper; Δd is a measure of the resilience to twisting due to inertial forces during motion of the gripper and the object; τ_s is the in-plane torque that can be withstood before slippage occurs; N is the gripping force; and ϵ is an empirically determined parameter included to prevent the evaluation function from approaching infinity as the gripping position approaches the center of mass.

The normalization of τ_s by $(\mu_s * N)$ eliminated the need to know either the grasping force or the coefficient of friction. $\frac{\tau_s}{\mu_s * N}$ is an integral expression dependent only on the geometry of the grasp. The integral expression, however, is based on three approximations: 1) that the center of slip is located with the centroid of the contact between the part and the gripper; 2) that the applied torques are in the gripping plane (approximate worst case analysis) and 3) that the pressure applied by the gripper is uniform across the contact area.

This paper eliminates all three of the approximations listed above. In the process, the evaluation function is modified. Rather than find the largest torque that can be withstood (worst case analysis), the applied forces and torques are treated as inputs and the lowest coefficient of friction that can be used to withstand them is taken as a measure of the resilience to slippage. Here, the smaller the value, the better.

The term Δd in the original evaluation function is no longer necessary since twisting of the object can be brought into the evaluation more directly through the relaxation of the constant pressure approximation. In this paper a (piecewise) linear pressure variation is used. If the twisting forces on an object are close to causing the object to twist, the pressure will go to zero on a portion of the contact area. This can be detected and used to eliminate potential grips which would not be resilient to twisting under inertial forces.

The following section presents the force analysis of the grip, and section 4 discusses the technique used to solve the force and moment equations. Section 5 then compares the new results to previous results and formulates a new evaluation function using the current work.

3. ANALYSIS OF THE GRIPPING FORCES AND TORQUES

3.1. CONTACT PRESSURE DISTRIBUTION

As remarked above, a uniform pressure distribution between the gripper and the gripped body is insufficient to satisfy the conditions of static equilibrium except in certain special cases. In practice, a non-uniform pressure will generally be developed, the precise form of which depends upon the elastic deformation of the various load bearing members of the systems.

As considerable simplification is achieved if we can assume that the gripper surfaces are coated with a thin, rubber-like layer of elastic material whose thickness is linearly proportional to the local contact pressure. In this case, the gripper and the gripped body will be substantially stiffer than the layer and can reasonably be regarded as rigid (see figure 1)

The contact pressure distribution is then determined by the deformation of the elastic layer. We restrict attention to the case where two parallel plane faces

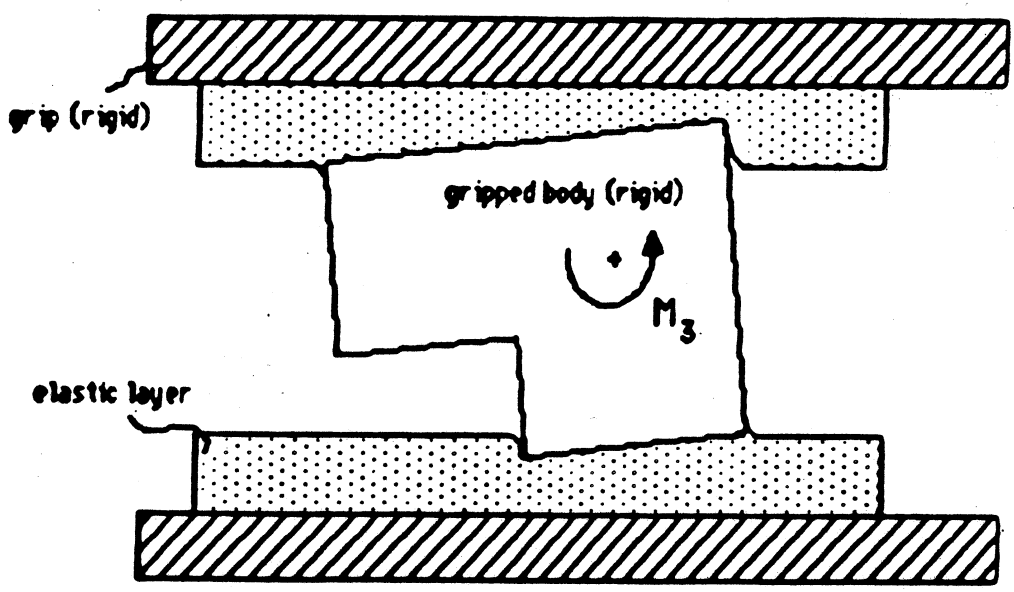

Figure 1

of the body are gripped by two parallel plane grips. If the body faces are oriented with the grip faces, the deformation at each face will be constant giving a uniform pressure distribution. Otherwise, if the body is rotated through some small angle, the deformation and hence the pressure distribution will vary linearly across the contact area as shown in figure 1. Notice that the coefficients of the linearly varying terms will be equal and opposite for the two opposed faces because of the parallelism of the gripped faces.

It follows that the most general form of the linear contact pressure distribution is

$$p_I = a_{I0} + a_1x_1 + a_2x_2 \tag{1}$$

$$p_{II} = a_{II0} - a_1x_1 - a_2x_2 \tag{2}$$

where x_1, x_2 are co-ordinates in the plane of the gripper face and suffices, I, II refer to the two opposed contact areas.

3.2. FRICTIONAL TRACTIONS

We are concerned with finding the minimum coefficient of friction f which is sufficient to prevent the body from slipping in the grip and hence we consider the limiting case where slip is about to occur. Kinematically, the incipient motion must consist of rotation about some axis perpendicular to the gripped surfaces, but the location of this slip axis - defined by the co-ordinates c_1, c_2 at which it cuts the contact faces - is unknown. However, if it were known, the direction of slip would be kinematically determined at all points in the contact areas and hence the frictional tractions would be completely defined both in magnitude ($f p_I$, $f p_{II}$) and direction (opposing the slip direction).

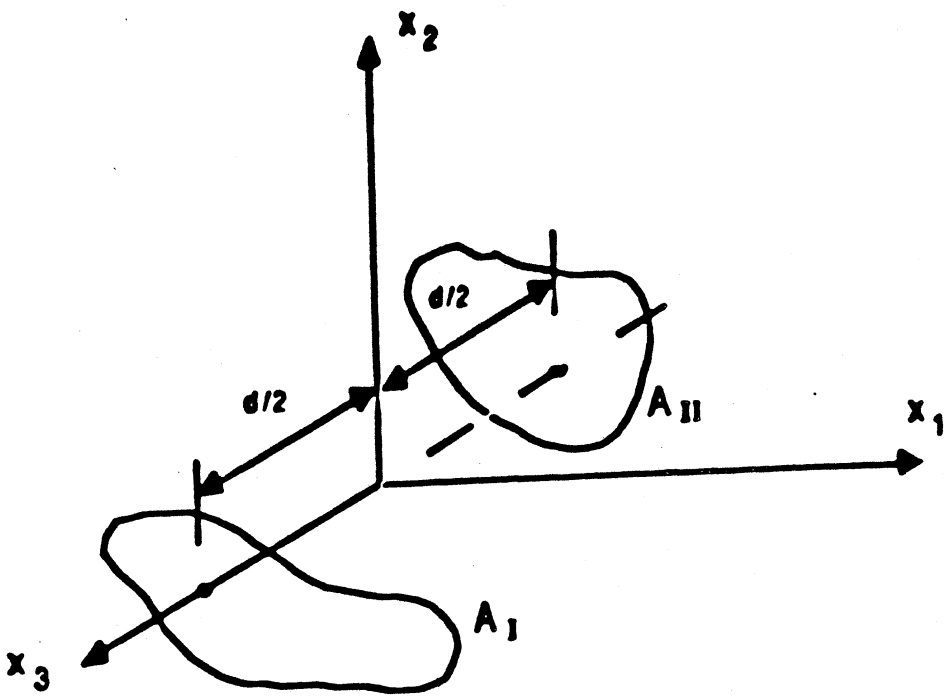

Figure 2

The requirement that the gripped body be in equilibrium under the influence of prescribed external forces and tractions at the gripped faces provides six equations for the unknowns $c_1, c_2, a_{I0}, a_{II0}, a_1, a_2$ and the unknown limiting coefficient of friction μ_s. One extra equation is obtained by the specified gripping force f (see 3.4 below).

Note, incidentally, that the limiting case of pure translational slippage corresponds to the center of slip being at infinity.

3.3. EQUATIONS OF EQUILIBRIUM

The co-ordinate system and the location of the two contact areas A_I, A_{II} is shown in figure 2. Notice that the origin of co-ordinates is taken midway between the two contact areas, so that A_I, A_{II} lie in the planes $x_3 = d/2$, $x_3 = -d/2$, respectively. We assume that the body is on the point of rotating in a clockwise sense about the slip axis, when viewed along the positive x_3 axis. (This is not a restrictive assumption - the opposite direction of slip would be indicated by a negative value of the limiting coefficient of friction.)

In-plane Forces

Figure 3 shows the in-plane external forces, F_1, F_2, M_3 acting on the gripped body and the fictional tractions exerted on the body by the gripper in contact area A_I (the forces on A_{II} are omitted for clarity). Notice that the incipient rotation will appear anti-clockwise in this view (negative x_3 direction) and it is opposed by the frictional tractions μ_s.

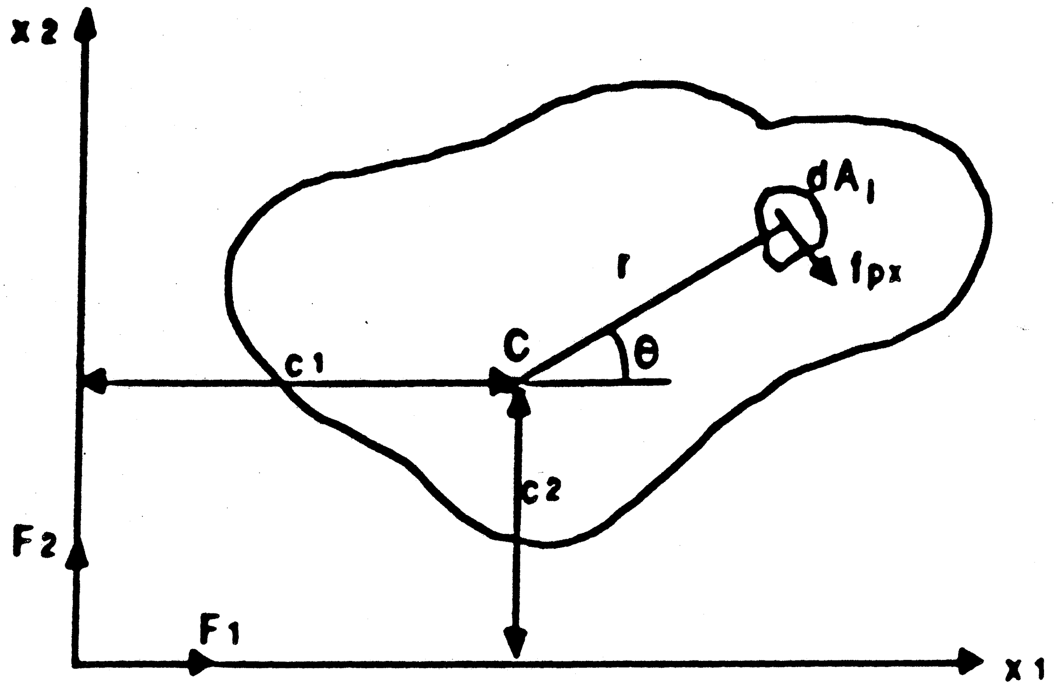

Figure 3

Summing the forces in the x_1, x_2 directions, we obtain the equations

$$F_1 + \int_{A_I} \mu\, p_I \sin\theta\, dA_I + \int_{A_{II}} \mu\, p_{II} \sin\theta\, dA_{II} = 0 \tag{3}$$

$$F_2 - \int_{A_I} \mu\, p_I \cos\theta\, dA_I - \int_{A_{II}} \mu\, p_{II} \cos\theta\, dA_{II} = 0 \tag{4}$$

and, taking moments about the x_3 axis through C,

$$\begin{aligned} M_3 + F_1 c_2 - F_2 c_1 - \int_{A_I} \mu\, p_I\, r\, dA_I \\ - \int_{A_{II}} \mu\, p_{II}\, r\, dA_{II} = 0 \end{aligned} \tag{5}$$

Out-of-plane Forces

Summing the moments about the x_1, x_2 axes respectively, we find

$$M_1 + \left\{ \int_{A_I} \mu\, p_I \cos\theta \; dA_I - \int_{A_{II}} p_{II} \cos\theta \; dA_{II} \right\} d/2$$
$$- \int_{A_I} p_I \; x_2\, dA_I + \int_{A_{II}} p_{II} \; x_2\, dA_{II} = 0 \tag{6}$$

$$M_2 + \left\{ \int_{A_I} \mu\, p_I \sin\theta \; dA_I - \int_{A_{II}} \mu\, p_{II} \sin\theta \; dA_{II} \right\} d/2$$
$$+ \int_{A_I} p_I \; x_1\, dA_I - \int_{A_{II}} p_{II} \; x_1\, dA_{II} = 0 \tag{7}$$

Where M_1, M_2 are the external moments about the x_1, x_2 axes. Also, summing the forces in the x_3 direction, we obtain

$$F_3 - \int_{A_I} p_I \; dA_I + \int_{A_{II}} p_{II}\, dA_{II} = 0 \tag{8}$$

where F_3 is the external force in the x_3 direction.

3.4. THE GRIPPING FORCE

If there were no external force F_3, the gripper would exert equal and opposite normal forces, F_n, to the two surfaces A_I, A_{II} of the body (represented by the last two terms in equation (8)). The force F_n is a measure of the tightness of the grip and must be specified. For the more general case where F_3 is not zero, we can define

$$2F_n = \int_{A_I} p_I \; dA_I + \int_{A_{II}} p_{II}\, dA_{II} \tag{9}$$

On substitution for p_I, p_{II} from (1,2) into (3 through 9), we obtain a set of 7 equations for the unknowns listed in 3.2 above.

3.5. EVALUATIONS OF INTEGRALS

As in [4], the contact areas A_I, A_{II} are defined (or approximated) as polygons, which are specified through the coordinates of the vertices. These polygons can then be regarded as the sum of a set of triangles by joining each vertex to the center of slip. The integrals in equations (3 through 9) can now be evaluated as the algebraic sum of a series of integrals over the component triangles, each of which can be obtained in closed form.

Notice that if the center of slip lies outside the contact area, some of these triangles will make a negative contribution to the area - however, this can be accommodated algebraically by defining the resulting expressions such that the co-ordinates of the polygon vertices are taken in cyclic order.

The resulting equations are non-linear in the co-ordinates c_1, c_2 of the slip axis and cannot be inverted algebraically. Instead, an implicit, iterative approach is used, as described in section 4 below.

3.6. NEGATIVE PRESSURES AND SEPARATION

As so far developed, there is no guarantee that the pressures defined by equations (1) and (2) will be positive throughout the respective contact areas - indeed, for sufficiently large out-of-plane moments, M_1, M_2, regions of negative pressure can always be generated.

Since the pressure distribution is linear and the nominal contact area is polygonal, the pressure can only be negative within the nominal area if it is negative at one or more of the vertices. It is therefore a simple matter to check for negative pressures during the solution procedure. One approach would be to simply exclude gripping positions which require negative pressures on the grounds that they are unlikely to be optimal.

However, it is a relatively straightforward matter to generalize the algorithm to deal with this eventuality. First, we note that a more general statement of the relationship for the pressure, p, is

$$p = C\,u \quad ; \quad u \geq 0 \tag{19}$$

$$p = 0 \quad ; \quad u < 0 \tag{20}$$

where C is some positive constant and u is the deformation of the elastic surface layer. This relation - which merely states that there will be zero contact pressure if there is a gap between the surfaces - makes explicit the unilateral nature of the contact process.

When negative pressures are indicated at one or more vertices in the unmodified algorithm, we anticipate that part of the nominal contact area will separate. In this case, the boundary between contact and separation regions has to be found as part of the solution.

We note from equations (19) and (20) that if u is a continuous function, the pressure must tend to zero at the boundary. It follows from equations (1) and (2) that the boundary must be a straight line and hence that the actual contact area will be a new polygon obtained by cutting off some corner of the nominal area. (Notice that the pressure does not have to be zero at those boundaries which are common to the actual and nominal contact areas, since at those points, u is discontinuous.)

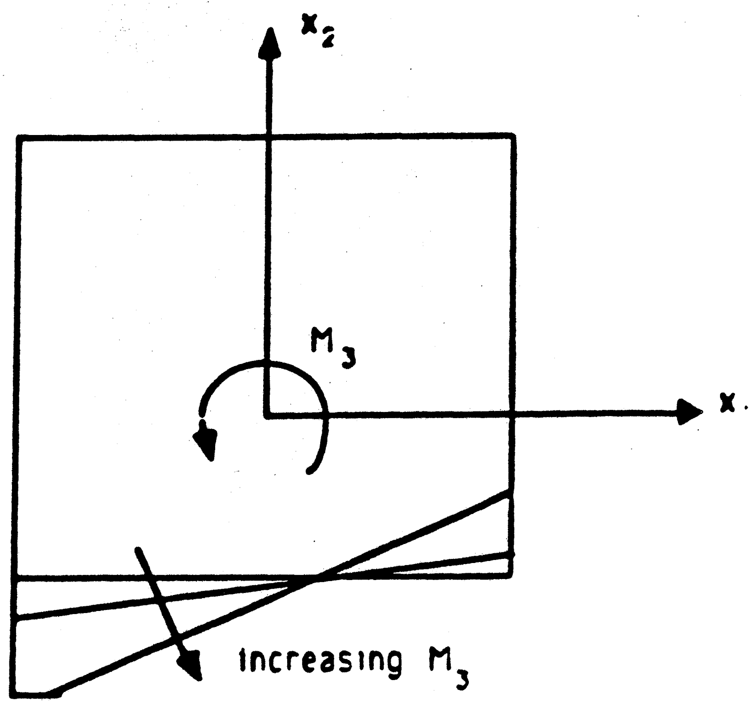

Figure 4

These conclusions can be implemented in the iterative solution as follows: At each stage of iteration, a check is made for negative pressures at the vertices of the nominal contact area. If any are found, the equation of the zero pressure line is determined from equations (1) and (2) and its intersection with the nominal contact area defines a new polygon which is used for the next iteration. The process is found to converge well, the contact area approaching its final value monotonically, as would be anticipated from experience with similar routines in elastic contact problems.

The results obtained indicate loss of contact at opposite sides of the two gripped areas in response to a sufficiently large out-of-plane moment, as anticipated. A less obvious result is that coupling occurs between out-of-plane and in-plane moments. The separation due to the out-of-plane moment displaces the line of action of the frictional forces and hence generates an orthogonal out-of-plane moment. This makes the line of separation inclined to the original moment, the effect becoming more pronounced as the in-plane moment, M_3, increases (see figure 4).

As the out-of-plane moment increases, the actual contact areas shrink until one of them becomes a line or a point. Further increase of moment will cause the body to be twisted out of the grip, unless the gripping force, P, increases in response to separation of the gripping surfaces. The present algorithm can therefore be used to evaluate grips from the point of view of resisting twisting moments - a feature which was treated by a separate procedure in [4].

4. IMPLICIT SOLUTION TO GRIPPING EQUATIONS

The gripping equations developed in the previous section are solved implicitly by forming a single error function having the property that it achieves a minimum value of zero for the values of c_1, c_2, a_{I0}, a_{II0}, a_1, a_2, and μ_s which simultaneously satisfy equations (3)-(9). A robust conjugate direction iterative minimization technique is then applied to the error function derived to determine the solution.

The number of unknowns can be reduced to six by slightly rearranging equations 3-9 as follows:

$$\mu_s = \frac{F_1}{h_4} = \frac{g_4}{h_4} \tag{10}$$

$$\mu_s = \frac{F_2}{h_5} = \frac{g_5}{h_5} \tag{11}$$

$$\mu_s = \frac{M_3}{h_3} + \frac{F_1c_2 - F_2c_1}{h_3} = \frac{g_3}{h_3} \tag{12}$$

$$\begin{aligned} \mu_s &= \frac{\mu_1}{h_1} - \frac{\int_{AI} p_I\, z_2 dA_I + \int_{AII} p_{II}\, z_2 dA_{II}}{h_1} \\ &= \frac{g_1}{h_1} \end{aligned} \tag{13}$$

$$\begin{aligned} \mu_s &= \frac{M_2 = \int_{AI} p_I\, z_1 dA_I - \int_{AII} p_{II}\, z_1 dA_{II}}{\mu_{M2}} \\ &= \frac{g_2}{h_2} \end{aligned} \tag{14}$$

$$1 = \frac{F_3}{h_6} = \frac{g_6}{h_6} \tag{15}$$

$$1 = \frac{2F_n}{h_7} = \frac{g_7}{h_7} \quad , \tag{16}$$

where the functions $g_1 \cdots g_7$ and $h_1 \cdots h_7$ are directly obtainable from equation (3)-(9). Obviously, the coefficient of friction, μ_s, can be eliminated from the first five equations leaving six equations and six unknowns. The remaining equation can be used to determine μ_s after the remaining unknowns are determined.

>From equation (10)-(16) one can form a set of error terms are follows:

$$e_i = \begin{cases} (g_i\, h_{i+1} - g_{i+1} h_i\,)^2 & i = 1, \ldots, 4 \\ (g_{i+1} - h_{i+1})^2 \quad , & i = 5, 6 \end{cases} \quad . \tag{17}$$

Clearly, if $e_i = 0$ for $i = 1 \cdots 6$ and μ_s is determined by any of the equations (10)-(14), the entire set of equations will be satisfied.

We can form a single error figure, E, by taking a weighted sum of the individual error components,

$$E = \sum_{i=1}^{6} k_i \; e_i \tag{18}$$

the weighting factors k_i are included only to be completely general. The individual e_i terms differ dimensionally, and it is possible for individual error terms to differ by many orders of magnitude simply by choosing different scales for the unit of the independent variables. The actual choice of the values and dimensions for the coefficients e_i is not terribly important. Each weighted error term will have a threshold below which it may be deemed that the error is sufficiently small. As long as the scale factors have been chosen such that these thresholds are within an order of magnitude or two of each other and within the computational accuracy of the arithmetic being used the formulation will be adequate. In practice it has been found that for modest size industrial parts described in terms of typical units, i.e. using inches, centimeters or meters instead of light years to measure distance, weighting coefficients of unity magnitude and dimensioned appropriately are satisfactory.

The conjugate direction algorithm proposed by Shanno [17] has been programmed in Pascal to execute on both Apollo and VAX computers. It has been used to perform minimization of the variable E. Depending upon the particular problem being solved convergence times have ranged in the 1-10 second region.

There is one obvious numerical problem which must be separately tested for and taken into account. Certain applied forces result in an impending slip which would be purely translational with no rotational components. In this case the center of clip is theoretically at infinity and the optimization algorithm will eventually encounter numerical difficulties. However, this condition can easily be detected by observing that in this case trial solutions with the center of slip will eventually be located a large distance from the contact position of the gripper and may be fixed at this large distance with the error functions subsequently being minimized with respect to the remaining four unknowns.

5. APPLICATION OF NEW RESULTS TO GRIP SELECTION

This section applies the techniques derived above to several interesting cases and examines some of the less obvious effects. It also compares the approximations used in the previous grip selection with the exact solutions obtained here. And, the new techniques are incorporated into a modified evaluation function for selecting grips, which is then compared to the one used in [4].

The new equations derived above are used to compute both the value of the pressure distribution over the contact area between a given object and the gripper, and the coefficient of friction, μ_s, at which the object just begins to slip. The forces and moments acting on the object must be specified to obtain these values. The values for μ_s and pressure can be used to rate the grip. A low coefficient of friction corresponds to a good grip, and a loss of contact area corresponds to a grip on an object that is beginning to twist out of the gripper.

5.1. DIRECT CALCULATION OF μ_s, AND PRESSURE DISTRIBUTION

One of the most interesting results of applying the complete force equations is that for a fixed out-of-plane moment applied to the object being grasped, increasing the distance between the two faces of the gripper decreases μ_s. This is

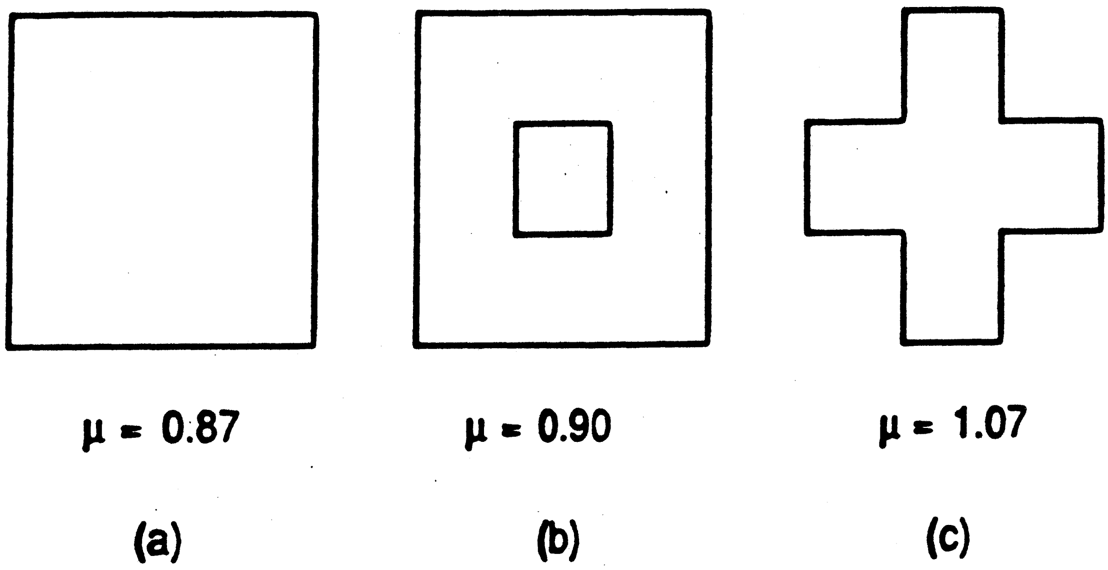

Figure 5.

because the forces within each face that oppose the applied out-of-plane moment are increased as the distance between these faces increases. This also reduces the tendency to lose contact area(pressure going to zero over a portion of the contact area). Thus the overall grip is improved.

If we examine the μ_s for the contact shapes in Figures 5a and 5b, a square and a "hollow" square, under a moment normal to the gripping plane, we find that they are nearly identical, whereas a cross obtained by removing the corners of the square requires a considerable larger μ_s (see figure 5c). This suggests that the maximizing overall contact area is not as important as maximizing the average distance over all the segments of the contact shape.

5.2. COMPARISON WITH PREVIOUS METHOD

The new method can be compared to the method in [4] by examining the difference in μ_s computed by the two methods. The calculation by the method

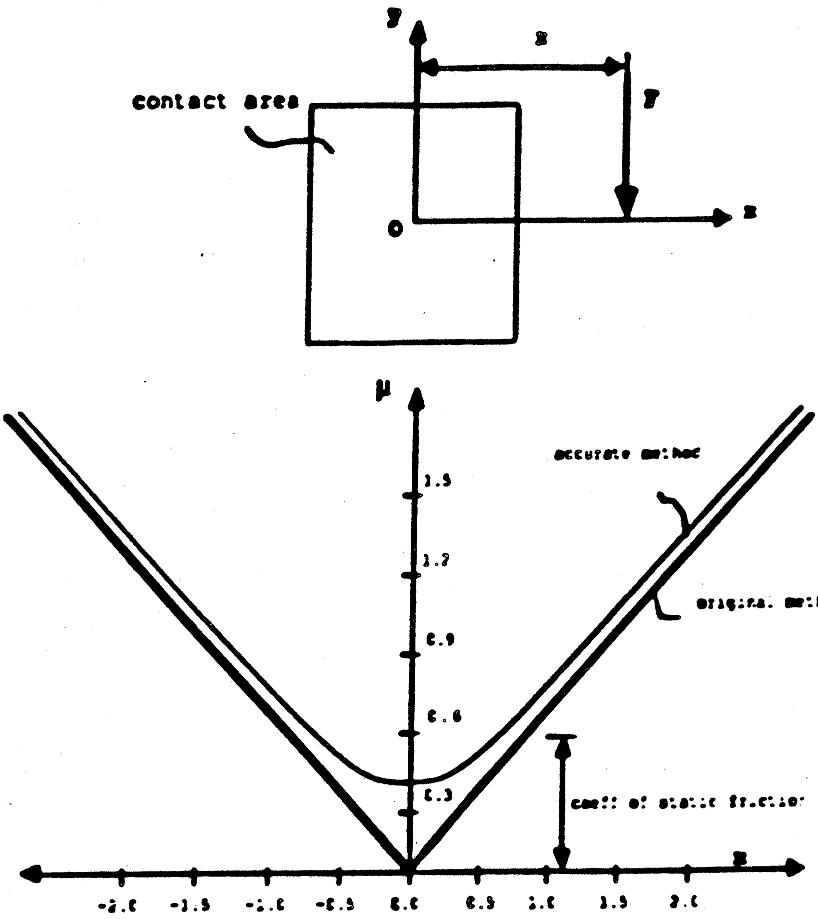

Figure 6

in [4] assumed that the center of slip was at the centroid of the contact area, and requires some rearrangement of terms to obtain μ_s, but this is easily accomplished. Figure 6 shows the difference between μ_s calculated by the two methods for a long bar, with μ_s plotted versus the distance of the grip from the center of mass. The thick straight line beginning at the origin represents the original approximation. The model used in [4] actually added a small empirically determined ϵ to raise this line and make the model more accurate. The thinner curved line represents the new model where the true value of the center of slip is computed. The approximation in is good as long as the gripping position is not too near the center of mass of the bar.

5.3. REVISED GRIP SELECTION METHOD

The grip selection procedure used in [4] requires an evaluation function by which potential grips can be compared. The original evaluation function included a term reflecting resilience to twisting during motion of the robot arm. In the current scheme, the tendency toward twisting during motion is detected by loss of contact area with the gripper. In the new scheme, such potential grips are simply are discarded. This removes the need for an explicit term (Δd) in the evaluation function to account to reflect resilience to twisting. Our evaluation function then simply becomes:

$$(k_1/\mu_s)+k_2\alpha$$

where α has the same meaning as in the original evaluation function, i.e., it reflects resilience to twisting due to misalignment during the closing of the gripper.

The evaluation function requires that it be presented with a set of applied forces on the object. In particular, several current path planning methods [18] [19] can provide all inertial forces applied to the part in the gripper as a by-product of the trajectory calculations by converting the acceleration information they provide into inertial force information. A more general form of grip selection would use this applied force time history as an input rather than making a worst case approximation. If a force trajectory is not readily available, the grip can be tested for samples of unit forces and moments applied in many directions. A "ball" of unit forces directed at the object is applied one force at a time and their corresponding friction coefficients, μ_s, are computed. This is equivalent to taking the tessellation of a sphere and individually applying forces normal to each of the tessellated surfaces. The maximum μ_s obtained from this procedure corresponds to the worst-case situation of applied forces that could cause the object to slip, and is used in the evaluation function.

To use the rating procedure for selecting grips, a set of proposed grips must be presented to the evaluation function. The method in [4] can be used to propose a suitable set of potential grips. In short, this method proposes a dense set of grips on the surface of the object to be grasped and checks them for interference between the gripper and the part. Grips with interference are thrown out, and the others are passed to the rating procedure we have described. The grips are evaluated and sorted in the order of desirability of rank. The rated grips can be passed to a run-time system that will pick the first grip that is allowed under run-time interference constraints. The new evaluation function can be used in the same manner.

6. SUMMARY AND CONCLUSIONS

In the development of a previous grip selection method for two fingered grippers, several approximations were made. The center of impending slip was approximated by the centroid of the contact area. Only forces and moments in the gripping plane were considered, and a constant pressure was assumed on the contact between the gripper and the object being gripped. This paper has eliminated all of these approximations.

The difficulty in using the true center of slip in the force and moment equations which describe a grip is that one cannot find an explicit solution to the problem. This difficulty has been overcome by using an implicit solution technique, i.e., forming a non-negative error function which takes on a value of zero at the solution of the equations and then using an iterative optimizer to determine the solution. In so doing, the applied forces and moments were treated as inputs rather than the objective function. This is consistent with the possibility of obtaining applied force and moment trajectories from path planners in the future. Now, instead of working with a fixed coefficient of friction and trying to maximize the force which can be withstood, the forces are treated as inputs and the coefficient of friction is taken as the figure of merit, lower coefficients be better.

Finally, the constant gripping pressure approximation was replace with a linear pressure variation approximation. As well as provide a more accurate solution, this gives a better indication of impending twist of an object in a grip during motion of the arm. The pressure will go to zero over a portion of the contact area before twisting will occur.

The present work is based upon polyhedral models of the objects to be grasped. This is believed to be a reasonable approximation in most cases. There is one situation, however, which is not adequately modelled by the polyhedral approximation. The situation of a torque vector in the gripping plane with a cylindrical object being held is not adequately modelled by our polyhedral approximation. That is, it is not necessarily the case that limits obtained under successively finer polyhedral approximations to the cylinder are the correct solution to the curvilinear case. Additional work is required to address this situation.

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