SUB-OPTIMAL ALGORITHMS FOR FORCE DISTRIBUTION IN MULTIFINGERED GRIPPERS

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ABSTRACT

The work described in this paper addresses the problem of determination of the appropriate distribution of forces between the fingers of a multifingered gripper grasping an object. The system is statically indeterminate and an optimal solution for this problem is desired for force control. A fast and efficient sub-optimal method for computing the grasping forces is presented. This method is based on the superposition of finger-interaction forces on equilibrating forces. An interaction force is defined as the component of the vector difference of the finger contact forces at any two fingers along the line joining the two contact points. They are computed based on the assumption that the normals at the point of contact pass through the centroid of the contact points and are therefore independent of the actual geometry of the object. The contact interaction is modelled as a point contact. The problems associated with making the algorithm independent of the object geometry are explored.

1. Introduction

Dextrous, multifingered grippers have been the object of considerable research [1]-[4]. The kinematics and force control problems engendered by these devices have been analyzed in [5]-[8]. Force control of such a system requires the specification of contact forces between the fingers and the gripped object. The gripper-object system has a high degree of static indeterminacy [6], [8], [9] and the interaction between the gripper and the object is similar to the interaction of a legged locomotion system with the ground [10]-[12]. This paper outlines a method of computing the grasping forces for a general object.

In a legged locomotion system or a walking machine it is essential to compute the support forces required at the feet to maintain equilibrium with the force of gravity and the inertial forces [10], [12]. In the gripper-object system, in addition to computing the contact forces required to maintain the object in equilibrium (equilibrating forces), it is necessary to determine the finger-interaction forces to ensure that the friction angle at each contact point is within allowable limits. The interaction force between two fingers is defined as the component of the difference of the finger contact forces at the two fingers directed along the line joining the two contact points. (They are similar to the scalar internal forces which describe the pinch between two fingers [1], [8]). This problem lends itself to optimization of the contact forces through linear programming [6] but such techniques are very expensive in terms of computational time and are consequently unsuitable for real-time operation with currently available computer hardware.

A sub-optimal solution to this problem is proposed in this paper. This method is attractive in its speed and efficiency. Contacts are modelled as point contacts [8] (the point contact model is accurate when the finger tips are small compared to the object being held) and it is assumed that the object is stationary with respect to the gripper - manipulation issues are not in context here. A heuristic idea (based on this method) for selecting grasping postures is also presented.

2. Formulation

2.1 Coordinate systems:

Let X_E-Y_E-Z_E be a reference frame fixed with respect to the earth. Consider a reference frame X_B-Y_B-Z_B with the origin at the grasp centroid, the centroid of the support/contact points, $({}^E x_i, {}^E y_i, {}^E z_i)$ and the Z_B-axis parallel to the wrench axis [13], \$ (see figure 1). \$ is the axis of the wrench which is the resultant of the weight of the object and the inertial forces and moments. The leading superscripts E and B refer to the earth and body (object) fixed reference frames respectively. Let $({}^B x_\$, {}^B y_\$, 0)$ be the intersection of the wrench axis with the X_B-Y_B plane. From this point on, unless otherwise specified, all quantities are described in the body fixed reference frame. In figure 1,

$\underline{p}_i$ is the position of the i^{th} contact point,

n is the number of contact points,

$\underline{F}_i$ is the contact force at $\underline{p}_i$,

O is the centroid of the n contact points, and

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$(\underline{f},\underline{c})$ is the force-couple dyname [13] associated with the wrench about \$.

This problem can be decomposed into two sub-problems:

A. Determination of the forces required to maintain the equilibrium of the gripped body assuming that the finger interaction forces are absent.
B. Determination of the interaction forces needed to produce the finger forces computed in step A without violating the friction angle constraints.

The following subsections elaborate on procedures for A and B.

2.2 Force distribution for maintaining equilibrium:

The force distribution must satisfy the six equations of equilibrium enumerated below.

$$\Sigma F_{ix} = 0 \qquad (1.1)$$

$$\Sigma F_{iy} = 0 \qquad (1.2)$$

$$\Sigma F_{iz} = f \qquad (1.3)$$

$$\Sigma (y_i F_{iz} - z_i F_{iy}) = 0 \qquad (1.4)$$

$$\Sigma (z_i F_{ix} - x_i F_{iz}) = 0 \qquad (1.5)$$

$$\Sigma (x_i F_{iy} - y_i F_{ix}) = c \qquad (1.6)$$

There are two ways this force distribution can be arrived at. The first procedure is based on the zero finger force interaction principle but the second method differs slightly in that the interaction forces are zero only if the points of contact are considered to be projections of the actual points of contact on the X_B-Y_B plane.

2.2.1 Method I

Equation (1) can be decomposed into two sub-problems. Three out of the six equations of equilibrium, (1.1), (1.2), and (1.6) are written as:

$$\begin{bmatrix} 1 & 0 & 1 & 0 & \cdots & 1 & 0 \\ 0 & 1 & 0 & 1 & \cdots & 0 & 1 \\ -y_1 & x_1 & -y_2 & x_2 & \cdots & -y_n & x_n \end{bmatrix} \begin{bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ \cdot \\ \cdot \\ \cdot \\ F_{nx} \\ F_{ny} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ c \end{bmatrix} \qquad (2)$$

where F_{ix} and F_{iy} are the x and y components of $\underline{F}_i$ and the x_i and y_i coordinates refer to the i^{th}

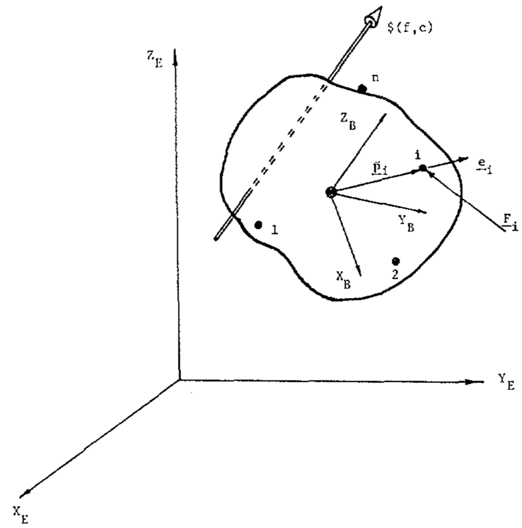


Figure 1: Reference frames for the gripper-object system.

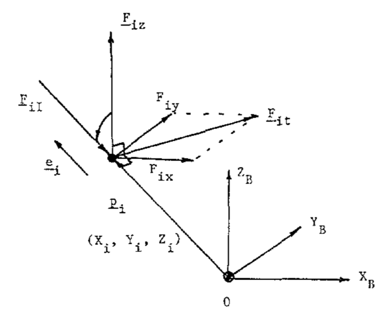


Figure 2: Finger contact forces at the i^{th} contact point.

finger contact point (from this point on, all references are to the body reference frame and the superscript B is deleted for convenience). The matrix equation (2) represents an undetermined set of equations with only 3 equations in 2n unknowns. There are clearly 2n-3 degrees of freedom in this system. However in accordance with the zero force interaction principle [12], the vector difference between any two contact forces should have no component along the line joining the two contact points. Mathematically, this condition is expressed as:

$$(\underline{F}_i - \underline{F}_j) \cdot (\underline{p}_i - \underline{p}_j) = 0 \;, \quad i,j = 1,..,n. \qquad (3)$$

The matrix equation (2) can now be solved subject to the restriction (3) and this yields a simple solution given by:

$$F_{ix} = \frac{c\,(-y_i)}{(nI)} \qquad (4.1)$$

$$F_{iy} = \frac{c\,(x_i)}{(nI)} \qquad (4.2)$$

where $I = \frac{1}{n} \sum_{i=1}^{n} [x_i^2 + y_i^2]$, and n is the number of contact points.

The forces on the X-Y plane are thus given by equations (4.1) and (4.2) and it is easy to verify by substitution that they satisfy equation (3). If $\underline{F}_{it}$ is the resultant of $\underline{F}_{ix}$ and $\underline{F}_{iy}$, then $\underline{F}_{it}$ is perpendicular to the corresponding position vector and

$$(F_{ix}\hat{\underline{i}} + F_{iy}\hat{\underline{j}}) \cdot \underline{p}_i = 0 \tag{5}$$

This force field is analogous to the velocity field of a rigid body where the velocity of any point is perpendicular to the position vector if the origin is coincident with the velocity center [12].

If (2) is rewritten as

$$\underline{Gr} = \underline{w} \tag{6}$$

where $\underline{G}$ is the 3×2n coefficient matrix, $\underline{r}$ is the 2n×1 unknown force vector and $\underline{w}$ is a 3×1 load vector, then this system of equations can also be solved by taking the Moore-Penrose Generalized Inverse (or the pseudo-inverse) of $\underline{G}$ [14]. If $\underline{G}^+$ is the pseudo-inverse of $\underline{G}$, then for a full rank matrix (one in which the rank of $\underline{G}$ is the minimum of p and q), then

$$\underline{G}^+ = \underline{G}^T(\underline{GG}^T)^{-1} \tag{7.1}$$

and $\underline{r}$ can be found from the equation

$$\underline{r} = \underline{G}^+\underline{w} \tag{7.2}$$

(In the event $\underline{G}$ is not of full rank, the pseudo-inverse can still be found by using the LU decomposition scheme or the Householder algorithm [14]). The pseudo-inverse can be analytically derived and it is interesting to note that the solution thus obtained from equations (7.1) and (7.2) is identical to equations (4.1) and (4.2).

This in fact provides a physical interpretation of the pseudo-inverse. The null-space of the coefficient matrix, $\underline{G}$, consists of all possible interaction force vectors and the row-space of $\underline{G}$ comprises of all the equilibrating force vectors with zero interaction force components. The pseudo-inverse seeks the solution vector with the least Euclidean norm (length) and hence the force vector which lies completely in the row-space of the coefficient matrix (which has no interaction force components).

Having found the finger forces in the x and y directions the three remaining equations of equilibrium, (1.3), (1.4) and (1.5) can be applied to solve the second sub-problem :

$$\begin{bmatrix} 1 & 1 & \cdots & 1 \\ (x_1-x_\$) & (x_2-x_\$) & \cdots & (x_n-x_\$) \\ (y_1-y_\$) & (y_2-y_\$) & \cdots & (y_n-y_\$) \end{bmatrix} \begin{bmatrix} F_{1z} \\ F_{2z} \\ \vdots \\ F_{nz} \end{bmatrix} = \begin{bmatrix} f \\ \Sigma z_i F_{ix} \\ \Sigma z_i F_{iy} \end{bmatrix} \tag{8}$$

This can be solved again by using the pseudo-inverse which serves to minimize the norm of the $\underline{F}_z$ vector. (A physical interpretation for the pseudo-inverse can be made in terms of the fingers having equal compliances in the Z-direction). The zero force-interaction principle can not used here as it would necessitate the equality of all the $\underline{F}_{iz}$ components and thus over-constrain the problem. The $\underline{F}_{iz}$ force field obtained by the pseudo-inverse is described by a planar force distribution. This completes one method for step A. It should be noted that equation (2) has to be solved before the right hand side of equation (8) is known - the two equations are not completely decoupled.

2.2.2 Method II

This method decouples the sub-problems of finding forces parallel to the x_B-y_B plane and forces parallel to the wrench axis completely. This time, equations (1.1) through (1.5) are used to solve for F_{ix} and F_{iy}.

$$\begin{bmatrix} 1 & 0 & 1 & 0 & \cdots & 1 & 0 \\ 0 & 1 & 0 & 1 & \cdots & 0 & 1 \\ -y_1 & x_1 & -y_2 & x_2 & \cdots & -y_n & x_n \\ z_1 & 0 & z_2 & 0 & \cdots & z_n & 0 \\ 0 & z_1 & 0 & z_2 & \cdots & 0 & z_n \end{bmatrix} \begin{bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ \cdot \\ \cdot \\ \cdot \\ F_{nx} \\ F_{ny} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ c \\ 0 \\ 0 \end{bmatrix} \tag{9}$$

Once again, the pseudo-inverse is used as a tool to keep the magnitude of the forces small. With some algebraic manipulation this can be solved and an analytical expression for the forces can be found.

$$F_{xi} = \frac{c \times (-y_i\overline{z}^2 + z_i\overline{yz})}{n(I\overline{z}^2 - \overline{yz}^2 - \overline{xz}^2)} \tag{10}$$

$$F_{yi} = \frac{c \times (x_i\overline{z}^2 - z_i\overline{xz})}{n(I\overline{z}^2 - \overline{yz}^2 - \overline{xz}^2)} \tag{11}$$

where, $\overline{z}^2 = \frac{1}{n}\sum_{i=1}^{n} z_i^2$,

$\overline{xz} = \frac{1}{n}\sum_{i=1}^{n} (x_i z_i)$,

and $\overline{yz} = \frac{1}{n}\sum_{i=1}^{n} (y_i z_i)$.

Now the z-components of the forces are found by writing equations (1.3) through (1.5) to get equation (8), but now the terms $\Sigma z_i F_{ix}$ and $\Sigma z_i F_{iy}$ are zero because of the way the F_{ix} and F_{iy} were found. This time analytical inversion for equation (8) is not as cumbersome and the $\underline{F}_{iz}$ are given by:

$$F_{iz} = (f/D) \times (1 - A\,(x_i - x_\$) - B\,(y_i - y_\$)) \quad (12)$$

where,

$x_i' = (x_i - x_\$)$,

$y_i' = (y_i - y_\$)$,

$$A = \frac{\Sigma x_i' \; \Sigma y_i'^2 - \Sigma x_i' y_i' \; \Sigma y_i'}{\Sigma x_i'^2 \Sigma y_i'^2 - (\Sigma x_i' y_i')^2},$$

$$B = \frac{(\Sigma y_i')/D - A(\Sigma x_i' y_i')}{\Sigma y_i'^2},$$

$$D = n - \frac{(\Sigma y_i')^2}{\Sigma y_i'^2} - A\left(\Sigma x_i' - \frac{\Sigma x_i' y_i' \; \Sigma y_i'}{\Sigma y_i'^2}\right).$$

Equations (10) and (11) do not describe a force field with zero interaction forces (equation (5) does not hold) - unless the contact points are all on the X-Y plane. But, on the other hand, the complete decoupling of the equations (9) and (12) which are required to find the forces is an advantage. It is difficult to say which of the two methods is better as in either case, the solution is sub-optimal. However, method I involves 11n+16 multiplications and 11n+5 additions and method II needs 15n+13 multiplications and 16n-3 additions. If parallel processing is available then method II cuts the time requirement to that needed for 9n+3 multiplications and 7n-4 additions. Clearly, method II is superior in a parallel processing environment (it takes 15-20 % less time) but otherwise, method I is faster.

2.3 Interaction Forces:

The forces computed by either of the two methods described earlier are F_{iz} ($i = 1,\dots,n$) which are parallel to the wrench axis, \$ and F_{ix}, F_{iy} ($i=1,\dots,n$) which lie on planes perpendicular to the wrench axis. It is assumed that method I is used and the resultant of the forces F_{ix} and F_{iy} is perpendicular to the vector $\underline{p}_i$ (see figure 2). Let the total interaction force exerted by the i^{th} finger on the object be $\underline{F}_{iI}$. Then

$$\sum_{i=1}^{n} \underline{F}_{iI} = 0 \quad (13)$$

$$\sum_{i=1}^{n} (\underline{p}_i \times \underline{F}_{iI}) = 0 \quad (14)$$

The resultant of the interaction forces has to be zero for equilibrium to be maintained (equation (13)). The geometric significance of (14) is that the lines of action of $\underline{F}_{iI}$ have to pass through a point of concurrence. In an ideal situation, the lines of action would be along the normals to the surface of the gripped object at the contact points. In a practical situation, the normals at the contact points are unknown and, in general, they are not concurrent. It is proposed that the point of concurrence be chosen as the centroid of the contact of points, namely the origin. This choice is merely a convenience and in principle, any other point could be chosen. The reader is requested to bear with this gross assumption - its validity is discussed later. Now the unit normals, $\underline{e}_i$, at all the n contact points are given by

$$e_{ix} = x_i/d_i, \quad e_{iy} = y_i/d_i, \text{ and } \quad e_{iz} = z_i/d_i \quad (15)$$

where $\quad d_i = \sqrt{(x_i^2 + y_i^2 + z_i^2)}$.

Equation (13) can be rewritten as:

$$\begin{bmatrix} e_{1x} & e_{2x} & \cdot & \cdot & \cdot & e_{nx} \\ e_{1y} & e_{2y} & \cdot & \cdot & \cdot & e_{ny} \\ e_{1z} & e_{2z} & \cdot & \cdot & \cdot & e_{nz} \end{bmatrix} \begin{bmatrix} F_{1I} \\ F_{2I} \\ \cdot \\ \cdot \\ \cdot \\ F_{nI} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (16)$$

For n=3, equations (16) comprises of three homogeneous linear equations in 3 unknowns, the only solution seems to be a trivial one. This is not the case since the rank of the system of 3 equations in (16) is only 2. This is because any three points are coplanar, and the determinant of the coefficient matrix formed by the $\underline{e}_i$'s is always zero for n=3. Thus, (16) has only one degree of freedom. If n=4 there are 3 independent equations (in general the points are not coplanar, unless the grip is planar, and the rank is 3) and again there is one degree of freedom. For $n \geq 4$ there are n-3 degrees of freedom. In figure 2, the net contact force, $\underline{F}_i$ is given by

$$\underline{F}_i = \underline{F}_{it} + \underline{F}_{iz} + \underline{F}_{iI}$$

and can be resolved along the normal $\underline{e}_i$ (postulated normal) to get $\underline{F}_{in}$ and on to the plane perpendicular to the normal to get $\underline{F}_{if}$. Let η_i be the angle between $\underline{F}_{iz}$ and $\underline{e}_i$ (i.e. $\cos \eta_i = e_{iz}$). Then,

$$F_{in} = F_{iI} - F_{iz} \cos \eta_i \quad (17)$$

$$F_{if} = \sqrt{(F_{iz}^2 \sin^2 \eta_i + F_{it}^2)} \quad (18)$$

and the friction angle, ϕ_i, at the i^{th} contact point is defined as

$$\tan \phi_i = (F_{if} / F_{in}) \qquad (19)$$

In order to prevent the finger from slipping, it is essential that the friction angle be within a prescribed limit. If μ_i is the coefficient of friction between the i^{th} finger and the surface of the object then the maximum value of ϕ_i is $\tan^{-1}\mu_i$. This yields n inequality constraints:

$$F_{iI} \geqq \max\ (F_{if}/\mu_i + F_{iz}\cos \eta_i,\ 0) \qquad (20)$$

The interaction forces should be made as low as possible to minimize the isometric work and to prevent crushing the object. The interaction forces can be determined by minimizing the largest of the F_{iI} satisfying the equalities in (16) and the inequalities in (20) --- this is a classic linear programming exercise. The problem is easily solved this way but clearly this method is not viable for a real-time control algorithm. An alternative faster algorithm is presented below.

n = 2 is a trivial case where $\underline{F}_{1I} = -\underline{F}_{2I}$. For n = 3 or 4 there is generally only one degree of freedom in equation (16). Setting $F_{1I} = 1$ the other interaction forces can be found. Then the F_{iI} are scaled with the smallest multiplicative factor which satisfies the constraints in (20). In general if the rank of the coefficient matrix in (16) is k, then if n-k=1 the $\underline{F}_{iI}$ can be found by elimination followed by the scaling process. For n-k > 1, if F_{1I} is set to 1, the other F_{iI} can be obtained by using the pseudo-inverse technique (instead of Gaussian elimination). However, unlike the other instances (equations (9) and (12)), an analytical inverse is not as simple. An alternative method is proposed to suit a parallel-processing environment. With any (n-k) of the interaction forces set to their minimum values (as defined in equation (20)) the remaining interaction forces can be found from eq (16) by elimination. This process can be repeated for all possible pairs of interaction forces to obtain ${}^{n}C_{n-k}$ possible solution vectors. Not of all of these will satisfy the constraint in equation (20). From the valid solutions, the solution with the smallest maximum interaction force is selected as a 'best' solution. All the ${}^{n}C_{n-k}$ solutions can be computed independently thus facilitating implementation on parallel processors.

For n = 3, the process of determining interaction forces requires 38 multiplications, 20 additions and 6 square root operations and these figures are 58, 32 and 8 for n = 4. These numbers have to be added to those obtained for calculating equilibrating forces to arrive at the total computational cost for the algorithm - 87 multiplications, 58 additions and 6 square roots for n = 3 and 118 multiplications, 81 additions and 8 square roots for n = 4 (using method I on a single processor).

3. Examples

Example (a): Hex-nut (see figure 3 (a))

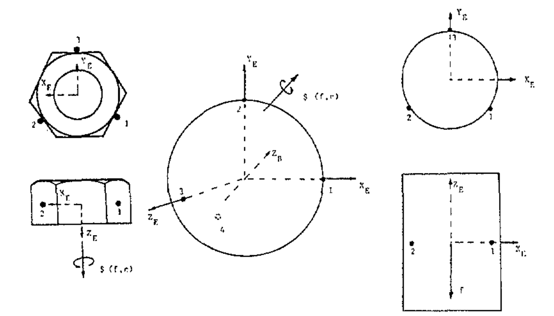
No. of fingers: 3
Coordinates of contact points (in meters):
1 - (0.0216,-0.0125,0.0)
2 - (-0.0216,-0.0125,0.0)
3 - (0.0000, 0.0250,0.0)
Load force (N) : $10\underline{k}$ applied at (0.0, 0.0, 0.0)
Load couple (N-m): $1\underline{k}$
Equilibrating forces (N) : $\underline{F}_1 = 6.66\underline{i}+11.55\underline{j}+3.33\underline{k}$
$\underline{F}_2 = 6.66\underline{i}-11.55\underline{j}+3.33\underline{k}$
$\underline{F}_3 = -13.33\underline{i}+3.33\underline{k}$
Interaction forces (N) : $\underline{F}_{1I} = -47.61\underline{i}+27.48\underline{j}$
$\underline{F}_{2I} = 47.61\underline{i}+27.48\underline{j}$
$\underline{F}_{3I} = -54.98\underline{j}$
Predicted friction angle : $\mu_1 = \mu_2 = \mu_3 = 0.25$
Actual friction angle : $\mu_1 = \mu_2 = \mu_3 = 0.25$

Example (b) sphere : (see figure 3 (b))

No. of fingers: 4
Coordinates of contact points (m):
1 - (1.0,0.0,0.0)
2 - (0.0,1.0,0.0)
3 - (0.0,0.0,1.0)
4 - (-.5774,-.5774,-.5774)
Load force (N): $1\underline{i}+1\underline{j}+1\underline{k}$ applied at (0.5,1.0,0.5)
Load couple : $1\underline{k}$

Finger	Predicted friction angle	Actual friction angle
1	0.25	0.43
2	0.16	0.32
3	0.08	0.23
4	0.00	0.00

In this section some examples of application of the proposed method have been presented. In all these examples, it is assumed that the desired coefficient of friction is 0.25. All the quantities are described in the earth coordinate system (unless otherwise specified). Example (a) illustrates a common assembly operation of screwing a nut onto a threaded bolt. Usually such an operation involves a small thrust as well as a moment to thread it in - the weight is assumed to be neglible compared to these forces. With a three fingered grip, as described in figure 3 (a), the assumption about the concurrence of the normals at the centroid of the contact points is definitely valid. That is why the predicted (modelled) and actual friction angles are identical for all fingers. Figure 3 (c) illustrates another example of a "correct" three fingered grip for a cylindrical object. The gripper (algorithm) does not distinguish between this situation and the one in case (a). To this extent the algorithm operates independent of exact object geometries.



a. HexNut b. Sphere c. Cylinder

Figure 3: Examples.

A more complicated problem is presented in figure 3 (b), where the assumption about the normals is clearly not satisfied. The problem with the predicted and actual friction angles is apparent. Even with a simple geometry such as that of a spherical object, a bad choice of grip postures results in large friction angles.

4. Conclusion

A fast and efficient sub-optimal method to compute the grasping forces for a multifingered gripper is described. A salient feature is the decomposition of the contact forces into equilibrating forces and interaction forces. The interaction forces are along the vector emanating from the grasp centroid through the corresponding contact point. Another attractive simplification is the decomposition of the equilibrating forces parallel and perpendicular to the wrench axis. The solution obtained is optimal to the extent that every component is independenlty minimized by the least-squares minimization technique. A better solution is, obviously, one which minimizes the maximum net finger contact force. The tradeoff between computational simplicity and optimality is evident.

This algorithm works well if the contact normals are along the lines joining the contact points with the centroid. The performance is then independent of the task forces (load wrench). It is important to note that the use of more fingers does not necessarily lead to a better grip. The best grip is one which satisfies the assumptions about the contact normals and the grip is symmetrical about the wrench axis. (The grasp centroid then lies on the wrench axis.) This concept of a 'best configuration' could be used as a heuristic idea in evaluating and selecting grip postures. With the state of the art in robot vision and image processing, it is possible to locate axes of symmetry for the object and accordingly select a grip posture.

5. Acknowledgements

The support of DARPA contract no. DAAE 07-84-K-R001 is gratefully acknowledged.

6. References

[1] Cutkosky, M.R. Robotic grasping and fine manipulation. Kluwer academic publishers, Boston, 1985.

[2] Jacobsen, S.C., E.K. Iversen, D.F. Knutti, R.T. Johnson, and K.B. Biggers. Design of the Utah/M.I.T. dextrous hand. Proc. of IEEE conf. on Robotics and Automation. San Fransisco, pp. 1520-1532, March 1986.

[3] Kobayashi, Hiroaki. On the articulated hands. Robotics Research: The Second International Symposium. Ed. H. Hanafusa and H. Inoue. Kyoto, pp. 293-299. Aug. 20-23,1984.

[4] Okada, T. Computer control of multijointed finger system for precise handling. IEEE Trans. on Systems, Man and Cybernetics. Vol. SMC-9 no. 2, Feb. 1979.

[5] Hollerbach, J. and S. Narasimhan. Finger force computation without the grip Jacobian. Proc. of the 1986 IEEE conf. on Robotics and Automation. pp. 871-875. San Fransisco, 1986.

[6] Kerr, J. and Roth, B. Analysis of multifingered hands. Int. J. of Robotics Research. pp. 3-17. 4(4), 1986.

[7] Salisbury, J.K. and J.J. Craig. Articulated hands: Force control and kinematic issues. Int. J. of Robotics Research. 1(1), pp. 4-17. 1982.

[8] Salisbury, J.K. and B. Roth. Kinematic and force analysis of articulated mechanical hands. J. of Mechanisms, Transmissions and Automation in Design. 105, pp. 35-41. March 1983.

[9] Holzmann, W. and J.M. McCarthy. Computing the friction forces associated with a three fingered grasp. Proc. of the 1985 IEEE conf. on Robotics and Automation. pp. 594-600. St. Louis, 1985.

[10] Klein, C.A. K.W. Olson and D.R. Pugh. Use of force and altitude sensors for locomotion of a legged vehicle over irregular terrain. Int. J. of Robotics Research. Vol. 2 (2).pp. 3-17, 1983.

[11] McGhee, R.B. and D.E. Orin. A mathematical programming approach to control of joint positions and torques in legged locomotion systems. Theory and Practice of Robots and Manipulators. Ed. Morecki, A. and K. Kedzior. Elsevier, pp. 225:232. 1977.

[12] Waldron, K.J. Force and motion management in legged locomotion. To appear in IEEE Trans. on Robotics and Automation.

[13] Hunt, K.H. Kinematic Geometry of Mechanisms. Oxford: Clarendon Press, 1978.

[14] Noble, B. Methods for computing the Moore-Penrose Generalized Inverse, and related matters. Proc. of an Advanced Seminar sponsored by the Mathematics Research Center, U. of Wisconsin. Ed. M.Z. Nashed. Academic Press. 1976.