THE UNIFIED FORMULATION OF CONSTRAINED ROBOT SYSTEMS[1]

Han-Pang Huang

Department of Mechanical Engineering
National Taiwan University
Taipei, Taiwan, R.O.C.

ABSTRACT

This paper presents a unified formulation for both externally and internally constrained robot systems. Furthermore, we relate the constrained robot formulation to singular systems of differential equations.

1. INTRODUCTION

In many industrial applications of robots, such as deburring, grinding, and scribing, the robot end effector is in contact with its external environment. The contact feature results in a closed loop structure of robot. In certain cases, the contact occurs at places other than the end effector; a robot with a jig hand [6] is a typical example. Another case of robot with closed loop structure is two robots carrying a common load. This application is particularly important when the object to be carried is big or heavy. In the above cases, the constraints are externally imposed on the robot end effector. However, the constraints may be internal so that the closed kinematic chain of the robot is formed by the robot links. This inner closed chain primarily consists of four-bar or five-bar linkages. The ITRI[2] U-type robot, Cincinnati T3, and pantograph robot are of this type. A robot with closed chain structure is regarded as a constrained robot. It will be shown that the externally constrained and internally constrained robots share the same mathematical formulation.

This paper gives a unified formulation for constrained robot systems. Several cases will be discussed: the robot in contact with its environment, the robot with a jig hand, two robots carrying a common load, and the robot with internal constraints. Finally, we relate the formulation of the constrained robot systems to singular systems of differential equations.

2. ROBOT IN CONTACT WITH ITS ENVIRONMENT

A schematic diagram of a robot with n links, whose end effector is in contact with its environment, is shown in Fig.1. It is assumed that the contact between the end effector and the constraint surface occurs at a point. A closed kinematic chain of the robot is formed through the constraint surface. Let $p \in R^n$ denote the position vector of the end effector in cartesian coordinates. Define $q \in R^n$ as a position vector denoted in joint space. The kinematic relation between cartesian coordinates and joint coordinates can be expressed as

$$p = H(q) \tag{1}$$

where $H(q)$ is a vector function. We further assume the constraint surface is rigid and frictionless and it has form

$$\phi(p) = 0 \tag{2}$$

where $\phi(p)$ is a scalar function with continuous gradient. Then the equations of motion of robot, taking into account the contact force, can be expressed in matrix form as [4]

$$\begin{bmatrix} M(q) & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} \tau - F(q,\dot{q}) + J^T(q) D^T(H(q))\lambda \\ \phi(H(q)) \end{bmatrix} \tag{3}$$

where $M(q)$ is an nxn inertial matrix; $F(q,\dot{q})$ is an n-dimensional vector defining coriolis, centrifugal and gravity terms; τ denotes the n-dimensional joint torque ; λ is a scalar and has the meaning of contact force; $J(q) = \frac{\partial H(q)}{\partial q}$ and $D(p) = \frac{\partial \phi(p)}{\partial q}$ are Jacobian matrices with orders of nxn and lxn, respectively. The detailed derivation of the above equations has been given in [4].

Clearly, the coefficient matrix multiplying the derivatives in Eq.(3) is singular; those equations represent a singular system of differential equations. In other words, the variable λ is not explicitly governed by a differential equation.

3. ROBOT WITH A JIG HAND

A schematic diagram of a robot with a jig hand is given in Fig.2. The closed chain structure is formed through the contact at the jig hand. This structure may increase the stiffness and improve the accuracy of the end effector at the expense of constraining the arm motion. We will assume the jig hand can slide on the constraint surface and the contact between the jig hand and the constraint surface occurs at a point.

Let $p_c \in R^n$ be the position vector at the contact point of the jig hand. Suppose the jig hand is extended from the link m. Then, we can obtain the relation between p_c and q as

1. This work was partially supported by the National Science Council, R.O.C., under the contract number NSC76-0401-E002-14.

2. ITRI is the abbreviation of Industrial Technology Research Institute in Taiwan.

$$p_c = H_c(q) \tag{4}$$

and the relation between p and p_c as

$$p = Q(p_c) \tag{5}$$

where H_c and Q are vector functions.

We assume the constraint surface is given by Eq.(2) and the constraint surface is rigid and frictionless. Following the same procedure as section I, the equations of motion of the jig hand system can be expressed in matrix form as

$$\begin{bmatrix} M(q) & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} \tau - F(q,\dot{q}) + J_c^T(q) D^T(Q(p_c))\lambda_c \\ \phi(Q(p_c)) \end{bmatrix} \tag{6}$$

where $J_c(q) = \frac{\partial H_c(q)}{\partial q}$ is an nxn Jacobian matrix; λ_c is a scalar; the other notation is the same as before. Again it is a singular system of differential equations. In case the robot end effector is also in contact with the constraint surface, two closed loops are formed through the contact at the jig hand and the end effector. The equations of motion of the robot system should be slightly modified to include the contact force at the end effector.

4. TWO ROBOTS CARRYING A COMMON LOAD

A schematic diagram of two robots carrying a common load is shown in Fig.3. A typical case of this system is that the object to be carried is too big or too heavy for handling by a single robot [5]. Let $q_1 \varepsilon R^n$ and $q_2 \varepsilon R^{n'}$ denote the vectors of the first and second robot joint angles, respectively. The equations of motion of each robot, taking into account the reaction forces (and moments) on them, are given by

$$M_i(q_i)\ddot{q}_i + F_i(q_i,\dot{q}_i) = \tau_i + J_i^T(q_i) D_i^T \lambda_i$$

$$i = 1,2 \tag{7}$$

where M_i, F_i, and τ_i have the same definitions as before; $J_i(q_i) = \frac{\partial H_i(q_i)}{\partial q_i}$ is a Jacobian matrix; $\lambda_i^T = [-{}^cF_i \ \ -{}^cN_i]$ denotes a 6x1 reaction force vector;

$$D_i^T = \begin{bmatrix} {}^0_cR & 0 \\ {}^0_cRK_i & {}^0_cR \end{bmatrix}, \quad K_i = \begin{bmatrix} 0 & -d_{iz} & d_{iy} \\ d_{iz} & 0 & -d_{ix} \\ -d_{iy} & d_{ix} & 0 \end{bmatrix}, \quad i=1,2 \tag{8}$$

where ${}^c_nR\ {}^nd_c = [d_{1x}\ d_{1y}\ d_{1z}]^T$, ${}^c_{n'}R\ {}^{n'}d_c = [d_{2x}\ d_{2y}\ d_{2z}]$
Note that ${}^cF_i$ and ${}^cN_i$ are the reaction force and moment with respect to the center of mass of the carried object; ${}^c_nR$ is a 3x3 rotation matrix; ${}^nd_c$ is a pure position vector.

Let $p_c \varepsilon R^6$ be the position vector of the center of mass of the object with respect to the world coordinates. Suppose the object is rigidly gripped by the robot end effectors, the direct kinematic relations of the two robots must satisfy constraints

$$p_c = H_1(q_1) \tag{9}$$

and

$$p_c = H_2(q_2) \tag{10}$$

where H_1 and H_2 are two vector functions.

By applying the Euler's equations, the equations of motion of the object, taking into account the reaction forces, are

$$M_c(p_c)\ddot{p}_c + F_c(p_c,\dot{p}_c) = -B_1 D_1^T \lambda_1 - B_2 D_2^T \lambda_2 \tag{11}$$

where $p_c = [p_x\ p_y\ p_z\ \psi\ \theta\ \phi]^T$ is the 6-dimensional position vector and

$$M_c(p_c) = \left[\begin{array}{c|ccc} mI_{3x3} & & 0_{3x3} & \\ \hline & -I_{xx}S\theta & & I_{xx} \\ 0_{3x3} & I_{yy}c\theta s\phi & I_{yy}c\phi & 0 \\ & I_{zz}c\theta c\phi & -I_{zz}S\phi & 0 \end{array}\right] \quad \begin{array}{l} S\theta = \sin\theta \\ c\theta = \cos\theta \end{array}$$

$$F_c(p_c,p_c) = \begin{bmatrix} -mg_x \\ -mg_y \\ -mg_z \\ (I_{zz}-I_{yy})c^2\theta S\phi c\phi\dot{\psi}^2 - [I_{xx}+(I_{yy}-I_{zz})cs\phi]c\dot{\psi}\dot{\phi} \\ \quad +(I_{yy}-I_{zz})S\phi c\phi\dot{\theta}^2 \\ (I_{zz}-I_{xx})S\theta c\theta c\phi\dot{\psi}^2 - (I_{yy}+I_{zz}-I_{xx})S\theta S\phi\dot{\psi}\dot{\theta} - I_{yy}S\phi\dot{\theta}\dot{\psi} \\ \quad +(I_{yy}-I_{zz}+I_{xx})c\theta c\phi\dot{\psi}\dot{\phi} \\ (I_{xx}-I_{yy})S\theta c\theta S\phi\dot{\psi}^2 - (I_{zz}-I_{xx}+I_{yy})S\theta c\phi\dot{\psi}\dot{\theta} \\ \quad -(I_{zz}+I_{xx}-I_{yy})c\phi\theta\phi - (I_{zz}+I_{xx}-I_{yy})c\theta s\phi\dot{\psi}\dot{\phi} \end{bmatrix}$$

$$B_i = \begin{bmatrix} {}^0_cR & 0 \\ -K_i & I \end{bmatrix}, \quad i = 1,2$$

Thus, the complete equations of motion of the two-robot system, taking into account the object dynamics, can be conviently represented in the form of a singular system:

$$\begin{bmatrix} M_1(q_1) & & 0 & & \\ & M_2(q_2) & & & \\ & & M_c(p_c) & & \\ & 0 & & 0 & \\ & & & & 0 \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{p}_c \\ \dot{\lambda}_1 \\ \dot{\lambda}_2 \end{bmatrix} = \begin{bmatrix} \tau_1 - F_1(q_1,\dot{q}_1) + J_1^T(q_1) D_1^T \lambda_1 \\ \tau_2 - F_2(q_2,\dot{q}_2) + J_2^T(q_2) D_2^T \lambda_2 \\ -F_c(p_c,\dot{p}_c) - B_1 D_1^T \lambda_1 - B_2 D_2^T \lambda_2 \\ p_c - H_1(q_1) \\ p_c - H_2(q_2) \end{bmatrix} \tag{12}$$

Not that Eqns.(9)(10) are treated as constraints rather than simply the kinematic relations.

5. ROBOT WITH INTERNAL CONSTRAINTS

In this section, a robot with inner closed chain will be discussed. Several types of robots possess the inner closed chain structure, such as ITRI U-type robot and Cincinnati Milacron T3 robot. We focus on the analysis of ITRI U-type robot, as shown in Fig.4. The ITRI U-type robot has five degree of freedoms and three closed chains. These three closed chains are formed by a four-bar linkage, ball screw A, and ball screw B, respectively. Furthermore, they do not lie in the same plane.

Let θ_1, ..., θ_5 be the robot joint angles which are actually driven by motors. Define an 11-dimensional vector q as $q = [\theta_1\ \theta_2\ \theta_3\ \theta_4\ \theta_5\ \theta_2'\ \theta_3'\ \Omega_2\ \Omega_3\ \phi_2\ \phi_3]^T$, where θ_2' and θ_3' are two virtual joints; Ω_2 and Ω_3 characterize the motion of ball screw A; ϕ_2 and ϕ_3 characterize the motion of ball screw B. During robot motion, the three closed chains should satisfy the following position constraints

$$c_1(q) = \cos\theta_{2'} + \cos(\theta_2+\theta_3) + 3\left[\cos(\theta_{2'}+\theta_{3'}) - \cos\theta_2\right] = 0 \tag{13}$$

$$c_2(q) = \sin\theta_{2'} + \sin(\theta_2+\theta_3) + 3\left[\sin(\theta_{2'}+\theta_{3'}) - \sin\theta_2\right] = 0 \tag{14}$$

$$c_3(q) = -\overline{EF} + d_A \sin\Omega_2 - \ell_{2'}\cos\theta_{2'} = 0 \tag{15}$$

$$c_4(q) = -\overline{OF} + d_A \cos\Omega_2 + \ell_{2'}\sin\theta_{2'} = 0 \tag{16}$$

$$c_5(q) = -\overline{CD} + d_B \sin\phi_2 + \overline{OB}\sin\gamma\,\sin\theta_2 - \overline{OB}\cos\gamma\,\cos\theta_2 = 0 \tag{17}$$

$$c_6(q) = -\overline{OD} + d_B \cos\phi_2 + \overline{OB}\cos\gamma\,\sin\theta_2 + \overline{OB}\sin\gamma\,\cos\theta_2 = 0 \tag{18}$$

where d_A, d_B, $\ell_{2'}$, γ, $\overline{EF}$, $\overline{OF}$, $\overline{OB}$, $\overline{OD}$, and $\overline{CD}$ are known quantities.

Applying the Euler's equations, we can derive the equations of motion of the robot system, taking into account multiple constraints, as

$$\begin{bmatrix} M(q) & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \ddot{q} \\ \dot{f} \end{bmatrix} = \begin{bmatrix} \tau - F(q,\dot{q}) + J^T(q) f \\ c(q) \end{bmatrix} \tag{19}$$

$c(q) = \left[c_1(q)\ \ c_2(q)\ \ c_3(q)\ \ c_4(q)\ \ c_5(q)\ \ c_6(q)\right]^T$; $J(q) = \frac{\partial c(q)}{\partial q}$; f is a 6-dimensional reaction force vector. The detailed derivation of Eq.(19) is provided in our previous report [1].

6. CONCLUSION

In this paper, we have analyzed each type of constrained robot and have presented a unified formulation for constrained robot systems. Since the equations of motion of constrained robots consist of a set of differential equations and a set of algebraic equations, we feel the singular system of differential equations is an appropriate model to describe the constrained robot systems. However, it also poses difficult problems in controller design, trajectory planning and system simulation. Research in these areas is under way [2, 3].

7. REFERENCE

[1] W.H. Cheng, H.P. Huang, Y.C. Gwe, "Dynamic Analysis of ITRI U-Type Robot," National Science Council report, No.NSC75-0611-E002-04, 1987.

[2] H.P. Huang, Constrained Manipulators and Contact Force Control of Contour Following Problems, Ph.D. dissertation, Dept. of Electrical Engineering and Computer Science, The University of Michigan, Ann Arbor.

[3] H.P. Huang, N.H. McClamroch, "Time-Optimal Control for a Robotic Contour Following Problem," to appear in 1987 IEEE J. of Robotics and Automation.

[4] N.H. McClamroch, H.P. Huang, "Dynamics of a Closed Chain Manipulator," Proc. of American Control Conference, Boston, 1985.

[5] N.H. McClamroch, "Singular Systems of Differential Equations as Dynamic Models for Constrained Robot Systems," IEEE Intl. Conf. on Robotics and Automation, 1986.

[6] H. West, H. Asada, "A Method for the Design of Hybrid Position/Force Controllers for Manipulators Constrained by Contact with the Environment," IEEE Intl. Conf. on Robotics and Automation, 1985.

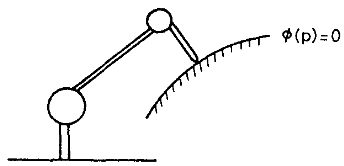

Fig. 1

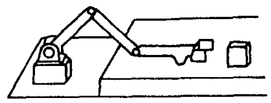

Fig. 2

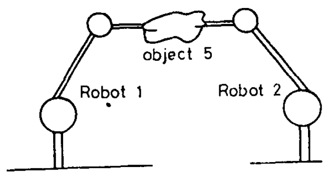

Fig. 3

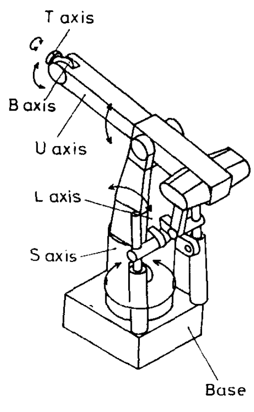

Fig. 4