A REDUCED ORDER MODEL DERIVATION FOR LIGHTWEIGT ARMS WITH A PARALLEL MECHANISM

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Abstract

RALF (Robotic Arm, Large and Flexible) with a parallel link mechanism has been developed at School of Mechanical Engineering in Georgia Institute of Technology. The structure consists of two ten foot long links and a actuator link, and the upper link is driven using the parallel link mechanism.

In this paper, a derivation of reduced order model for RALF by the modal cost analysis method is shown. In order to derive the reduced order model, 2 analytical models with deferent kinds of mode shapes which have the first 5 component modes of each link are used as the original model.

The reduced order model which consists of the first 2 modes of each link is obtained from the control point of view. The evaluation of the reduced order model is made by the comparison between the frequency responses and the modal cost analysis results.

1. INTRODUCTION

Model reduction problems which derive an appropriate order model from a given large order system have been an important research issue in the structural dynamics field as well as in the control field.[1),2)]

The purpose of deriving a reduced order model are 1) reducing efforts for obtaining a controller which satisfies the desired performance, 2) obtaining a lower dimensional controller, 3) achieving simpler simulation of the given system, and 4) reducing simulation costs and time.

As is well known, a lot of model reduction techniques for linear systems have been developed. In those techniques, there are at least four important and popular state space based model reduction techniques for a flexible structure, namely, truncation of the internally balanced realization [3),4)], Hankel norm optimal approximation [5),6)], q-covariance equivalent approximation [7),8)], and modal cost analysis [9)-12)].

Internally balanced realization, which was developed by B.C.Moore, is a singular values based technique. The singular value is able to measure controllability and observability of the system. From controllability and observability points of view, the weaker portions of system, concerning both controllability and observability, are truncated in this technique.

The Hankel norm optimal approximation method is also a singular values based technique. The deference between this and the above is that the reduced order model obtained by this method minimizes Hankel norm error between the original system and the reduced order model.

The basic idea of q-covariance approximation is to approximate a low frequency characteristic of the original system by a reduced order model.

The model reduction techniques mentioned above deal with the over all system as an object of model reduction. They don't address the question of component model reduction. In a flexible manipulator model reduction, a component model reduction as well as a over all model reduction is an important research issue, because the manipulator dynamics is derived on the basis of a Lagrangian-assumed mode method.

R.E.Skelton, et al. proposed a modal cost analysis method and a component cost analysis method to solve LSS (Large Space Structure) model reduction problems.

The basic idea of these methods is to decompose a norm of the response into contributions from each coordinate of the original system and to find coordinates that contribute a small amount.

The modal cost analysis method is a special case of the component cost analysis method in the sense that system dynamics is represented in the modal coordinates.

In applying the modal cost analysis method to flexible structural systems, the damping ratio of each component mode plays an important role. The damping ratio of each system mode can be obtained by some vibration tests.

In this paper, reduced order models of RALF (Robotic Arm, Large and Flexible) by the modal cost analysis and evaluation of the reduced models are presented.

G.G.Hasting obtained a 6 order model for a 1 link flexible manipulator model using the internally balanced realization technique [13),15)] . As mentioned above, general model reduction methods are applicable to a single link flexible manipulator.

To achieve a reduced order model of RALF, the modal cost analysis method is used in this paper. The modal cost analysis method deals with the system described in modal coordinates, but the analytical model of RALF is represented in physical coordinates. In section 2, an overview of the modal cost analysis method and its application to RALF are presented. A brief outline the analytical model for RALF[14)] is shown in section 3. The reduced order model of RALF and its evaluation are discussed in section 4.

2. MODAL COST ANALYSIS METHOD

2.1 Background of modal cost analysis[9)10)]

The next paragraphs provide an overview of the modal cost analysis method.

Given the following linear second-order system which represents a typical mechanical system.

$$M \cdot \ddot{q}(t) + D \cdot \dot{q}(t) + K \cdot q(t) = D_W \cdot w(t) \qquad (2\text{-}1)$$

$$y(t) = P \cdot q(t) \qquad (2\text{-}2)$$

where, M, D, K, and P are the system inertia, damping, stiffness matrices and the output matrix, D_W is a noise distribution matrix, and q is an N dimensional vector, and w(t) is the white noise described as follows.

$$E[\ w(t)\ ] = 0 \qquad (2\text{-}3)$$

$$E[\ w(t) \cdot w(t)^T\ ] = W \cdot \delta(t) \qquad (2\text{-}4)$$

where, E[ ] and W represent the expectation operator and the intensity of the white noise, and $\delta(t)$ is Kronecker's δ function.

A response norm V is defined by eqn.(2-5).

$$V = \lim_{t \to \infty} E[\ y^T \cdot Q \cdot y\ ] \qquad (2\text{-}5)$$

where, Q is a weighting matrix.

There exists a transformation $q = T \cdot \eta$ that simultaneously diagonalizes M and K. Applying the transformation T to the system equations (2-1) and (2-2), we obtain the model expressed in the modal coordinates:

$$\ddot{\eta}(t) + \hat{D}\cdot\dot{\eta}(t) + \hat{K}\cdot\eta(t) = \hat{D}_W\cdot w(t) \qquad (2\text{-}6)$$

$$y(t) = \hat{P}\cdot\eta(t) \qquad (2\text{-}7)$$

where,

$$\hat{D} = \mathrm{diag}(\,0,\cdots,0,\ 2\zeta_1\omega_1,\cdots,\ 2\zeta_N\omega_N\,)$$

$$\hat{K} = \mathrm{diag}(\,0,\cdots,0,\ \omega_1,\cdots,\ \omega_N\,)$$

$$\hat{P} = [\ p_1,\ \cdots\cdot,p_N\ ]$$

ω_i is the natural frequency of i-th system mode, and ζ_i is the i-th mode damping ratio given by some vibration experiments.

If we assume the open loop system is lightly damped, the decomposition of the response norm V into contributions of each coordinate is given by eqn. (2-8).

$$V = \sum_{j=1}^{N} V_i$$

$$V_i = \frac{(\,p_i{}^T\cdot Q\cdot p_i\,)\cdot\sigma_i{}^2}{4\cdot\zeta_i\cdot\omega_i{}^3} \qquad (2\text{-}8)$$

where,

$$\sigma_i{}^2 = [\ \hat{D}_W\cdot W\cdot\hat{D}_W{}^T\ ]_{ii}$$

and the n eigenvalues of the system (2-6) are given by complex conjugate pair.

$$\lambda_i = -\zeta_i\cdot\omega_i + j\omega_i$$

$$\bar{\lambda}_i = -\zeta_i\cdot\omega_i - j\omega_i$$

$$0 < \zeta_i < 1 \quad (\ i = 1,\ 2,\ \cdots\ ,\ N\ ) \qquad (2\text{-}9)$$

2.2 Component cost derivation

The modal cost analysis method for the overall system model reduction is described in the above section.

As mentioned above, the component cost analysis method[11),12)] is effective in the derivation of each component cost which expresses a contribution of each component mode to the response norm, V. The disadvantage of this technique is that the damping ratio to each component mode is required. From a practical point of view, giving an appropriate damping ratio to each component mode is not easily accomplished. On the other hand, the damping ratio of each system mode can be obtained by some vibration tests for use in calculating the modal cost. Since the damping ratio of the component modes is not the same as the damping ratio of the system modes, a component cost derivation technique based on the modal cost analysis is shown in the following section.

Consider the following second order system with a set of m holonomic constraint equations as a RALF model.

$$M \cdot \ddot{q}(t) + D \cdot \dot{q}(t) + K \cdot q(t) + f + \Phi_q{}^T \cdot \lambda = D_w \cdot w(t) \qquad (2\text{-}10\ a.)$$

$$y(t) = P \cdot q(t) \qquad (2\text{-}10\ b.)$$

$$\Phi(q) = 0 \qquad (2\text{-}10\ c.)$$

where, Φ_q is the constraint Jacobian matrix, λ is the vector of Lagrange multipliers, and f indicates the vector including nonlinear coupling terms and their derivatives.

Using the singular value decomposition technique (14), a transformation, $q = V_2 \cdot z$, is found which transforms the system equations (2-10 a.) and (2-10 b.) into

$$V_2^T \cdot M \cdot V_2 \cdot \ddot{z}(t) + V_2^T \cdot D \cdot V_2 \cdot \dot{z}(t) + V_2^T \cdot K \cdot V_2 \cdot z(t) + V_2^T \cdot f$$

$$= V_2^T \cdot Dw \cdot w(t) + V_2^T \cdot M \cdot \Phi_q^+ \cdot \dot{\Phi}_q \cdot V_2 \cdot \dot{z}(t) \qquad (2\text{-}11 \text{ a.})$$

$$y(t) = P \cdot V_2 \cdot z(t) \qquad (2\text{-}11 \text{ b.})$$

where, Φ_q^+ is the pseudo-inverse of Φ_q.

Equation (2-11) is linearized about zero velocity to get

$$V_2^T \cdot M \cdot V_2 \cdot \ddot{z}(t) + V_2^T \cdot D \cdot V_2 \cdot \dot{z}(t) + V_2^T \cdot K \cdot V_2 \cdot z(t)$$

$$= V_2^T \cdot Dw \cdot w(t) \qquad (2\text{-}12 \text{ a.})$$

$$y(t) = P \cdot V_2 \cdot z(t) \qquad (2\text{-}12 \text{ b.})$$

where,

$$\Phi_q = [\ U_1 \quad U_2\ ]\ [\ \Sigma_m \quad 0\ ] \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} \qquad (2\text{-}13)$$

$$\Sigma_m = \mathrm{diag}(\sigma_1, \sigma_2, \cdots, \sigma_m) \qquad (2\text{-}14)$$

Here, the σ_i's are called the singular values of matrix Φ_q, ordered $\sigma_1 \geqq \sigma_2 \geqq \cdots \geqq \sigma_m > 0$.

There exists a transformation $z = T' \cdot \xi$ that transforms equations (2-12) to the expression in the modal coordinates.

Applying the transformation T' to the system (2-12), we obtain equations (2-15):

$$T'^T \cdot V_2^T \cdot M \cdot V_2 \cdot T' \cdot \ddot{\xi}(t) + T'^T \cdot V_2^T \cdot D \cdot V_2 \cdot T' \cdot \dot{\xi}(t)$$
$$+ T'^T \cdot V_2^T \cdot K \cdot V_2 \cdot T' \cdot \xi(t)$$
$$= T'^T \cdot V_2^T \cdot Dw \cdot w(t) \qquad (2\text{-}15\ a.)$$
$$y(t) = P \cdot V_2 \cdot T' \cdot \xi(t) \qquad (2\text{-}15\ b.)$$

where,

$$T'^T \cdot V_2^T \cdot M \cdot V_2 \cdot T' = I$$
$$T'^T \cdot V_2^T \cdot D \cdot V_2 \cdot T' = \mathrm{diag}(0, \cdots, 0, 2\zeta_1 \omega_1, \cdots, 2\zeta_{N-m} \omega_{N-m})$$
$$T'^T \cdot V_2^T \cdot K \cdot V_2 \cdot T' = \mathrm{diag}(0, \cdots, 0, \omega_1, \cdots, \omega_{N-m}),$$

ω_i is the natural frequency of i-th system mode, and ζ_i is the i-th mode damping ratio given by some vibration experiments.

Applying the modal cost analysis method to the system (2-15), we obtain the modal cost V_α attributable to the system mode α .

To obtain the component cost V_{ci} for each component mode from the modal cost V_α , the following transformation is used.

$$V_{ci} = | V_2 \cdot T' | \cdot [V_1, \cdots, V_{N-m}]^T \qquad (2\text{-}16)$$

where, $|\ |$ denotes the absolute value operator.

The reader is reminded that V_{ci} as given by eqn.(2-16) is not always equivalent to the component cost analysis [11),12)] results.

3. ANALYTICAL MODEL OF RALF

3.1 Overview of analytical model

The schematic drawing of a 2 link manipulator RALF (Robotic Arm , Large and Flexible) with a parallel mechanism is shown Fig. 3.1. In order to derive an analytical model of RALF, the reference frame is defined as shown in Fig. 3.2. Lagrange's equations and the assumed mode method is used for deriving the equations of motion of this flexible structure.

The absolute position vectors of an arbitrary point on each link are defined by the following :

$$r_i = R_i + U_{ri} + U_{fi} \qquad (i=1,2,3) \qquad (3\text{-}1)$$

where, R_i is the position vector of the origin of the reference body with respect to the global frame, U_{ri} is the undeformed position vector, and U_{fi} is the elastic deflection vector.

U_{fi} is expressed in linear combination form as follows.

$$U_{fi}(x,t) = \sum_{j=1}^{n} \Psi_{ij}(x)\cdot q_{fij}(t) \qquad (3\text{-}2)$$

where, $\Psi_{ij}(x)$ and $q_{fij}(t)$ denote an admissible shape function and time dependent modal coordinates, respectively.

In this paper, an analytical model which has five component modes of each link is assumed as the RALF original model. A linearized RALF model is given by eqn. (3-3).

$$V_2^T\cdot M\cdot V_2\cdot \ddot{z}(t) + V_2^T\cdot D\cdot V_2\cdot \dot{z}(t) + V_2^T\cdot K\cdot V_2\cdot z(t)$$
$$= V_2^T\cdot Dw\cdot w(t) \qquad (3\text{-}3)$$

where, V_2 is given by eqn (2-13).

3.2 Natural frequencies and Mode shapes

For numerical analysis, selection of the mode shape functions is necessary and may greatly influence the results.

The following 2 sets of boundary conditions for each link in Table 3.1 are considered in this paper.

	case 1	case 2
Lower link	clamped - mass	pinned - pinned - mass
Upper link	clamped - free	pinned - pinned - free
Actuator link	pinned - pinned	pinned -pinned

Table 3.1 Boundary conditions

The natural frequencies for the above 2 cases are in Table 3.2.

	case 1 (Hz)	case 2 (Hz)
1st mode	6.21	5.62
2nd mode	16.90	14.40
3rd mode	30.73	30.70
4th mode	95.61	68.47
5th mode	104.65	86.47
6th mode	120.73	120.68

Table 3.2 Natural frequencies

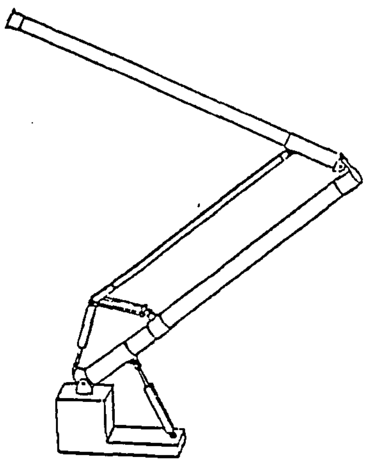


Fig. 3.1 RALF

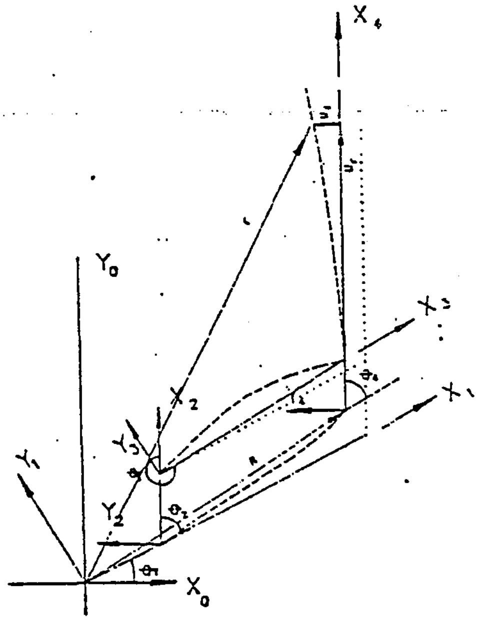


Fig. 3.2
Coordinate systems for the assumed mode method

4. REDUCED ORDER MODEL

4.1 Component cost

The inputs to RALF are forces by hydraulic actuators mounted at the lower link and the actuator link as in Fig. 3.1, and the outputs are the tip position of the upper link, the strain at the center of each link, the strains at base of the lower link and the upper link, and the joint angles θ_1 and θ_4 as shown in Fig. 3.2.

To obtain the component cost for each component mode, the weighting matrix Q in eqn. (2-5) is chosen to be the identity matrix. The intensity of white noise W in eqn. (2-4) is chosen to be 1, because there is no data about the noise in hydraulic actuator output that would allow selection of an appropriate value of the intensity W. The damping ratios which are used in the modal cost calculation are given in Table 4.1.

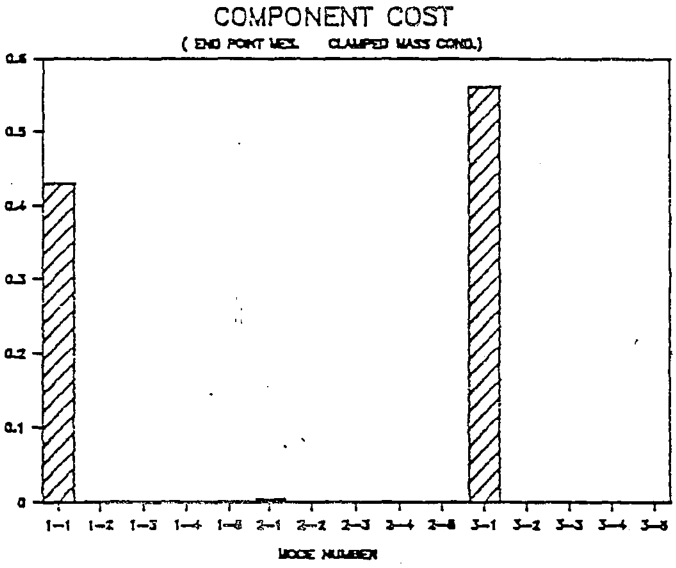
The resultant component costs for the case 1 model in Table 3.1 are shown in Fig. 4.1. Every cost V^*_{cij} is normalized by the following equation.

$$V^*_{cij} = V_{cij} \Big/ \sum_{i=1}^{3} \sum_{j=1}^{5} V_{cij} \qquad (\ 4\text{-}1\ )$$

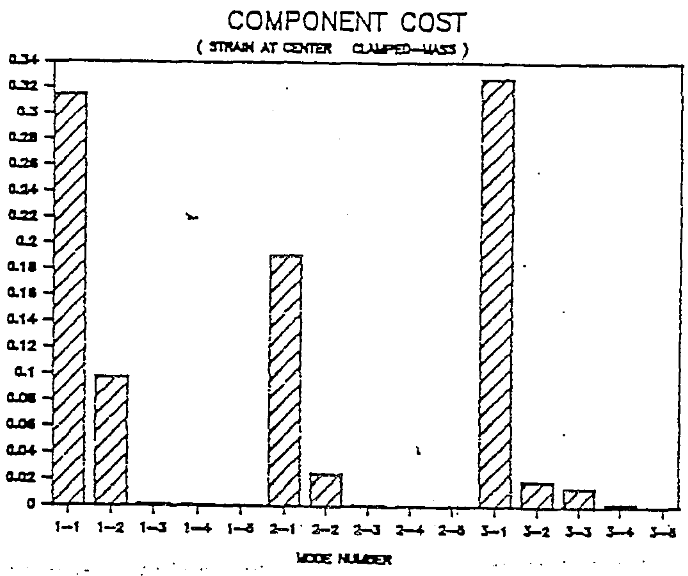
where, i and j indicate the component number and the mode number as shown in Table 4.2.

In Figures 4.1, the mode number i-j denotes j-th component mode of i-th component. The modeling error for some reduced order models shown in Table 4.3 vs. the attitude of RALF which is defined by θ_1 and θ_2 are given in Fig. 4.2.

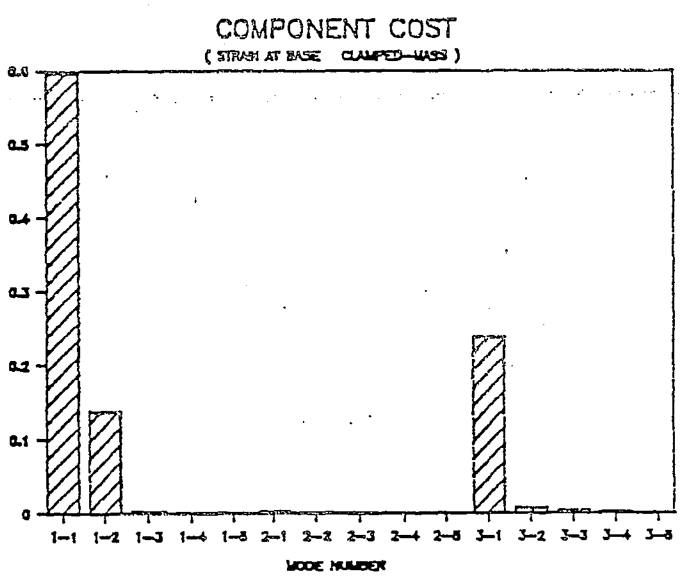
a.



b.



c.



d.

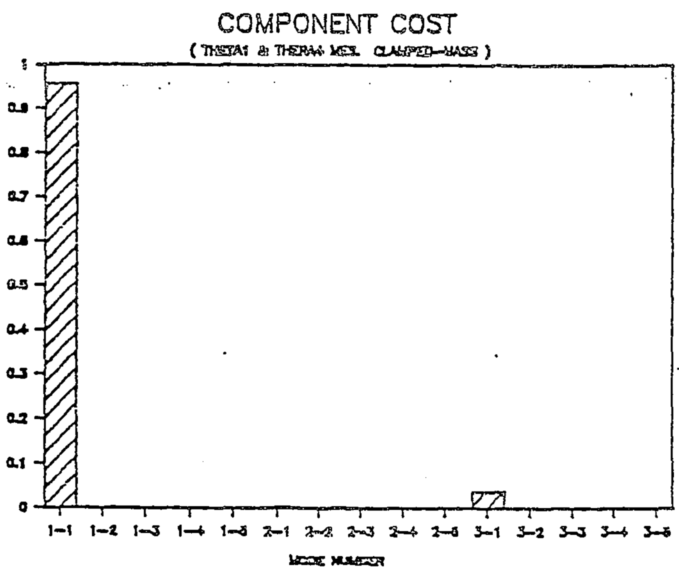


Fig. 4.1 The component cost

a. Tip position of the upper link measurement

b. Strain measurement at center of each link

c. Strain measurement at base of lower link and upper link

d. θ_1 and θ_4 measurement

ζ_1	ζ_2	$\zeta_3, \cdots\cdots, \zeta_{15}$
0.0142	0.0085	0.007

Table 4.1 The damping ratio

i		j	
1	Lower link	1	1st mode
2	Actuator link	2	2nd mode
3	Upper link	3	3rd mode
		4	4th mode
		5	5th mode

Table 4.2 contents of i, j

reduced order model	included component mode
1	1-1, 2-1, 3-1
2	1-1, 1-2, 2-1, 2-2 3-1, 3-2
3	1-1, 1-2, 2-1, 2-3 3-1, 3-2, 3-3
4	1-1, 1-2, 2-1, 3-1

Table 4.3 Reduced order models

The modeling error ε is defined by eqn. (4-2).

$$\varepsilon = 1 - \left(\sum_{\alpha} \sum_{\beta} V_{c\alpha\beta} \Big/ \sum_{i} \sum_{j} V_{cij} \right) \qquad (4\text{-}2)$$

where, $V_{c\alpha\beta}$ (the β-th component mode of the α-th component) indicates the component cost of the component mode included in the reduced order model.

The big change of the modeling error vs. the change in the angles is found in the reduced order model 1. The modeling error for both the model 2 and the model 3 are satisfied less than 5%.

The result of the component cost calculation for the case of the pinned-pinned-mass boundary condition mentioned in section 3.2 is shown in Fig. 4.3. The component cost distribution has the same characteristics in both Fig.4.1 and Fig. 4.3.

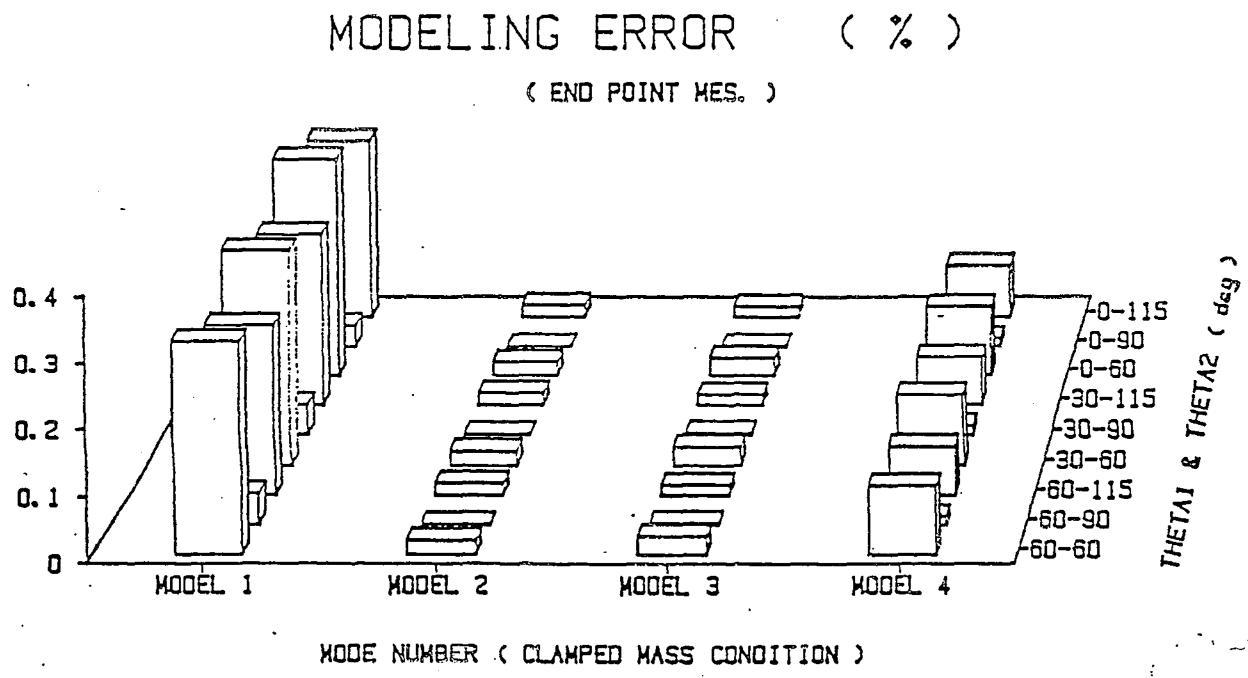


Fig 4.2a The modeling error vs. angles $\theta_1 \theta_2$
Tip position of the upper link measurement

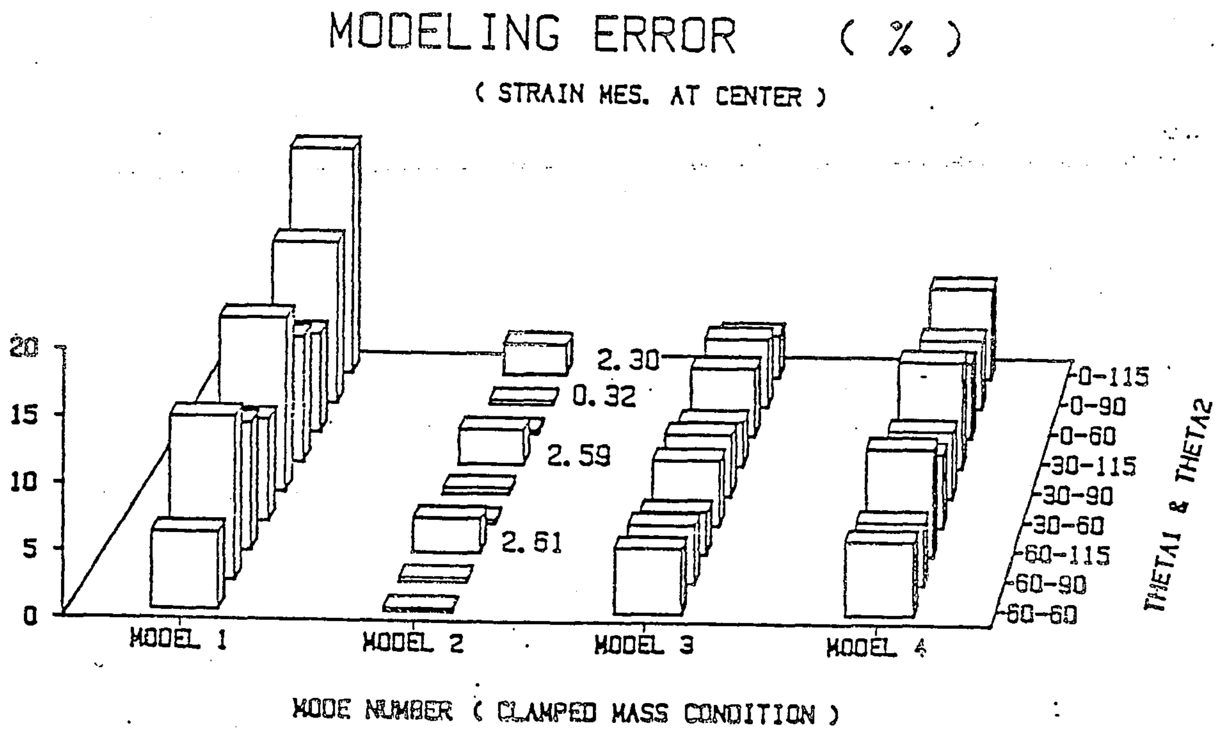


Fig 4.2b The modeling error vs. angles $\theta_1 \theta_2$
Strain measurement at center position of each link

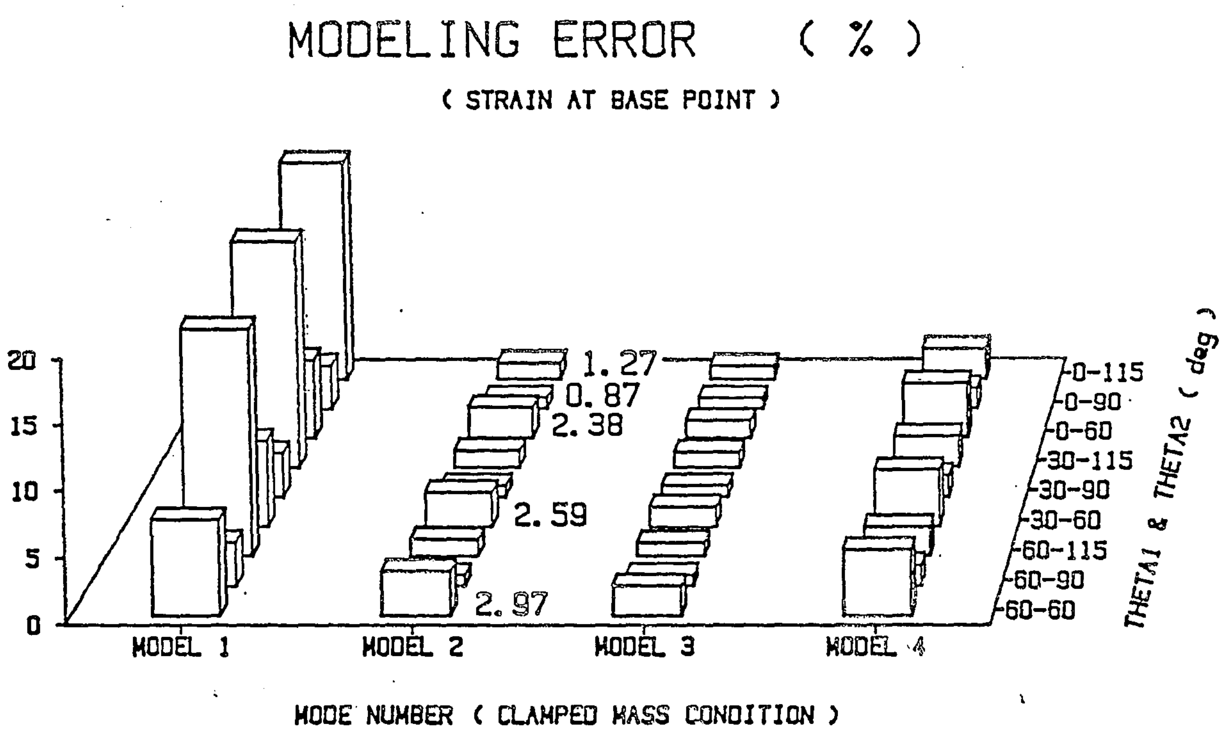


Fig 4.2c The modeling error vs. angles $\theta_1 \theta_2$
Strain measurement at base of lower link and upper link

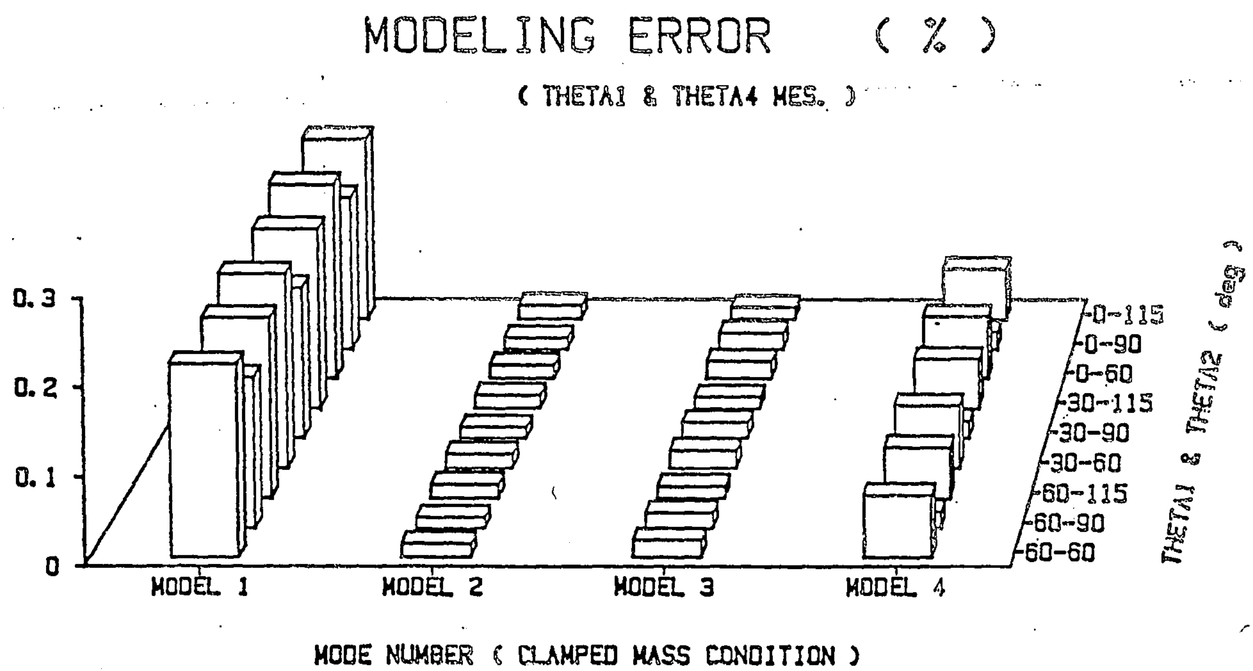


Fig 4.2d The modeling error vs. angles $\theta_1 \theta_2$
θ_1 and θ_4 measurement

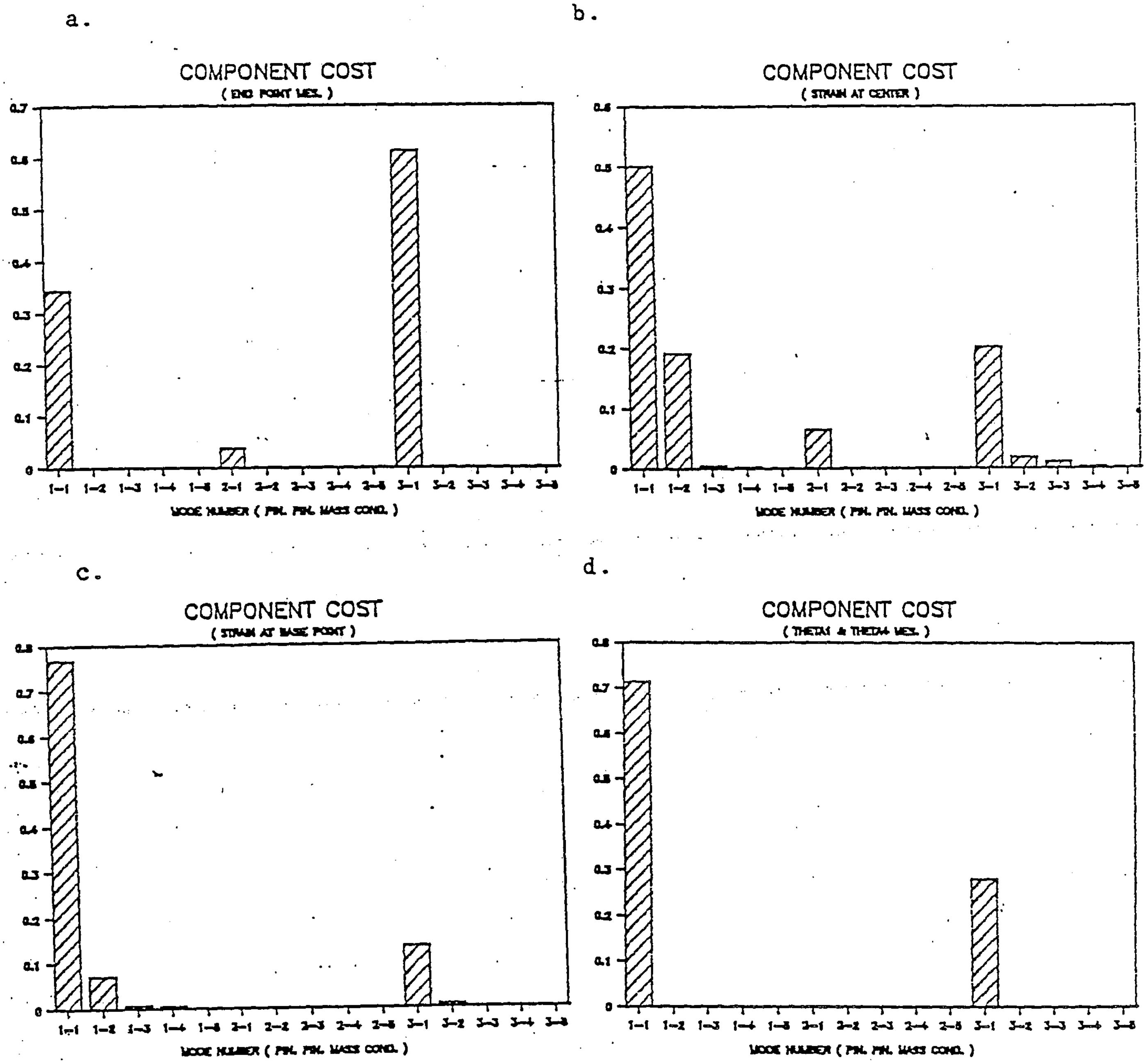


Fig. 4.3 The component cost (single output case)

a. Tip position of the upper link measurement

b. Strain measurement at center of each link

c. Strain measurement at base of lower link and upper link

d. θ_1 and θ_4 measurement

4.2 Discussion of component cost calculation results

The results indicate that the selection of the output has an effect on the model order estimated with the modal cost analysis approach, because the modal cost analysis method relates to the system controllability and observability. And changing the angles θ_1 and θ_2 has little effect in the reduced order model 2 which includes the first 2 modes of each component is found in results. In the strain measurement at base, the variations of the modeling error are comparatively small.

Figure 4.4 and 4.5 represent the experimental data of RALF in the case of the strain measurement at base point and at center point, respectively. In Fig.4.4 and 4.5. the input is the step signal to the hydraulic actuators. Each figure has the time response data and the spectrum data. The first (5Hz), second (9.4Hz) and third (30.8Hz) system modes are excited by the actuators. The spectrum ratio of the first mode and the second mode is 10 : 1, and that of the second mode and the third mode becomes almost 20 : 1.

The calculating modal cost of RALF in the strain measurement cases are given in Fig.4.6 and 4.7. The modal cost ratio between the first mode and the second mode coincides roughly with the spectrum data.

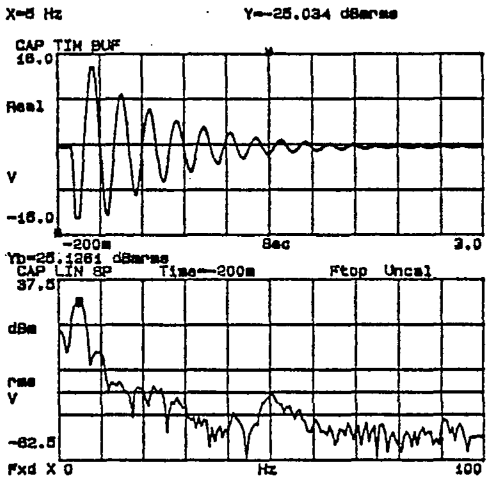
Since the time response is represented by the combination of component mode, the component cost derived by the coordinate transformation will represent the importance of the component mode.

From these points of view, the reduced order model 2 in Table 4.3 which includes the first 2 modes of each component is recommended for RALF model. Furthermore, in the control of RALF, it is desirable to measure the strain at base of the lower link and upper link.

In the previous work done by J.W.Lee and J.D.Huggins [14], the comparison between this model and the experimental system was shown to have agreement in the general trend of the vibration of the reduced order model and the experimental system .

Changing the boundary conditions does not have a significant effect on the model order estimation. The reason is that there is little difference in the natural frequency as shown in Table 3.2.

a.



b.

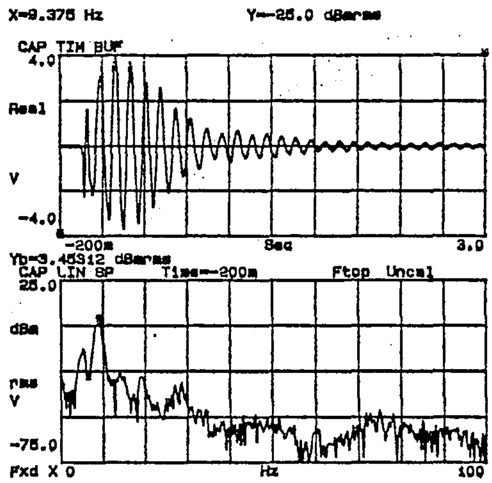
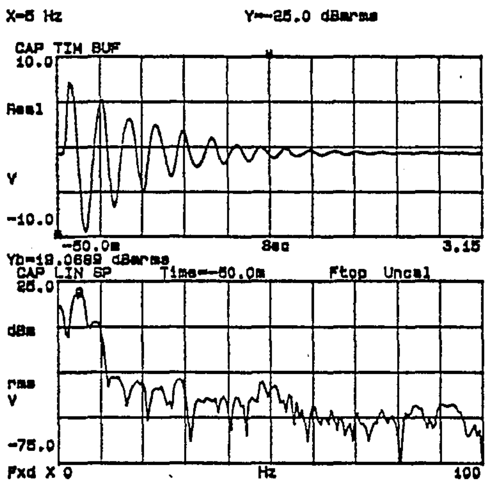
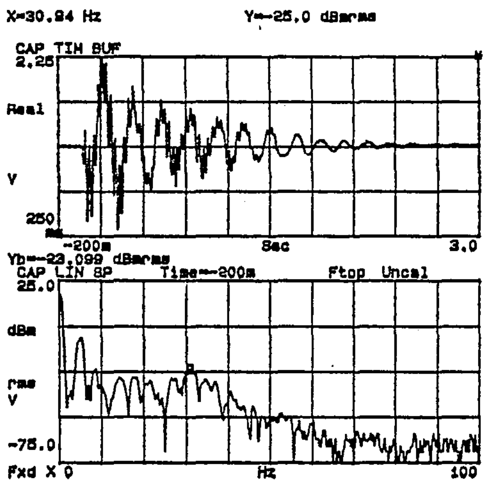


Fig. 4.4 Spectrum at the base strain measurement
a. Lower beam base strain
b. Upper beam base strain

a.



b.



c.

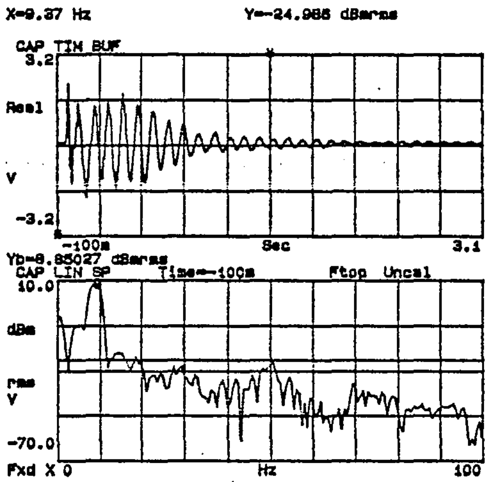


Fig. 4.5 Spectrum at center strain measurement

a. Lower beam center strain

b. Actuator link center strain

c. Upper link center strain

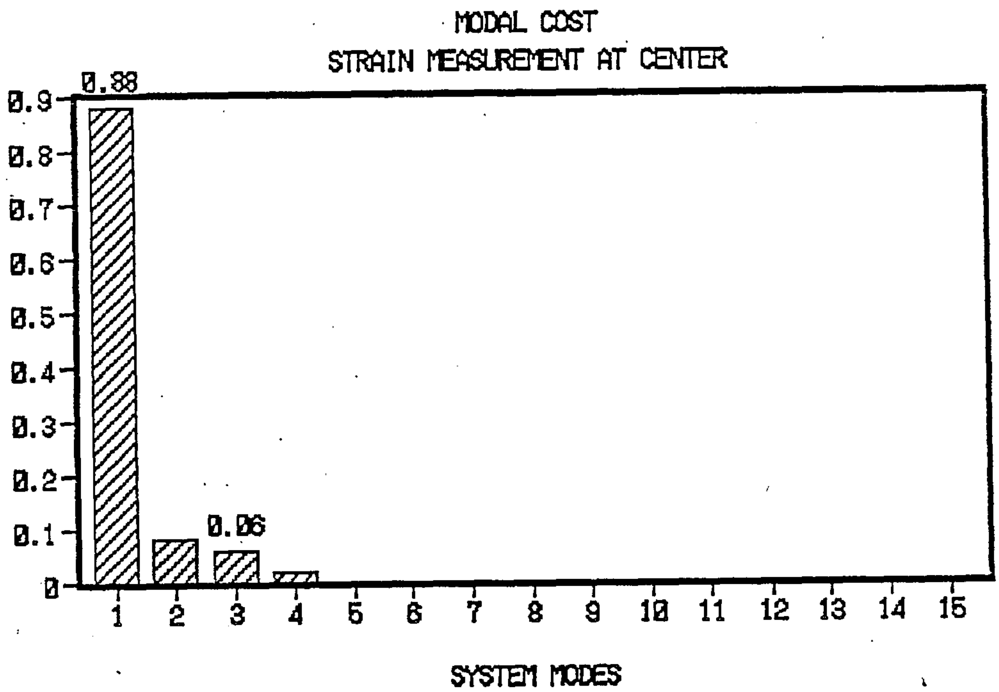


Fig. 4.6 Modal cost of strain measurement at base

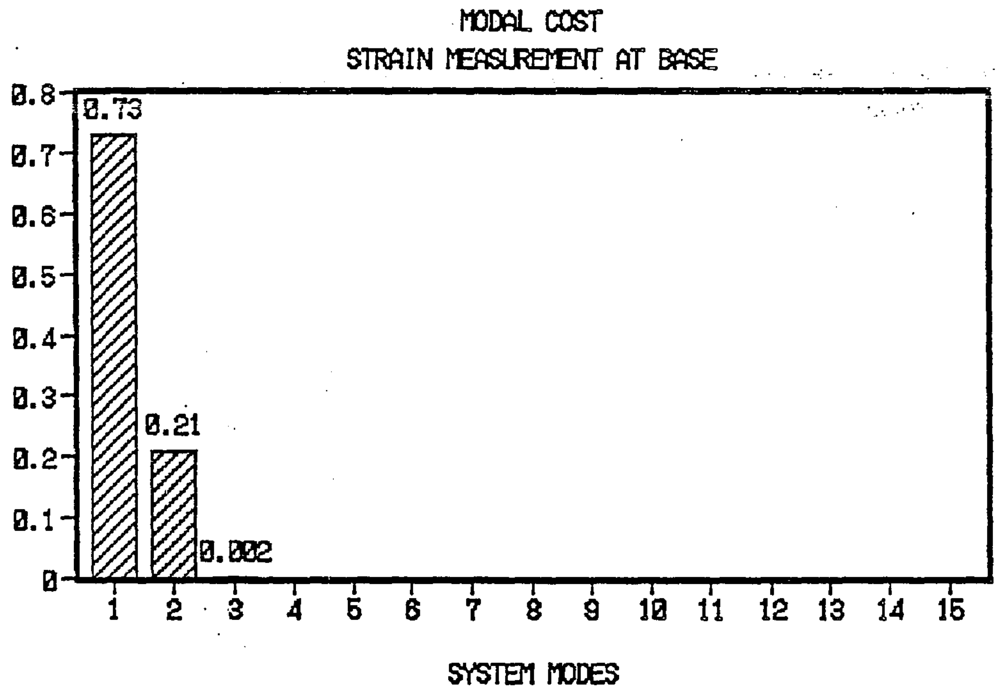


Fig. 4.7 Modal cost of strain measurement at center

5. CONCLUSION

A reduced order model of RALF system which consists of the first 2 component modes of each link was derived using the modal cost analysis approach. From the component cost calculation results, the modeling error ratio of this model has little variation, in spite of the change in angles of the joints.
Additionally, the strain measurement at base has the narrowest variation width in the modeling error ratio. As discussed in section 4, it is desirable to measure the strain at base in the control of RALF.

Because of non-linearity of RALF, in the derivation of the reduced order model of RALF, the change of the modeling error ratio versus the change of the joint angles of RALF was considered. Furthermore, the component cost were considered for 2 RALF models which have different mode shapes. As described in section 4, changing the boundary conditions did not have a significant effect on the model order estimation.

In order to avoid the complexity in the designation of the weighting matrix Q, the Q was selected as the identity matrix, when calculating the modal cost. In the case of measuring several kinds of outputs, the Q should be selected appropriately.

Using the transformation matrix, the modal cost was transformed to the component cost. It is not theoretically evident whether the resultant component cost is equivalent to the results by the component cost analysis theory. However, the comparison between the spectrum data and the modal cost shows that the modal cost relates to the frequency response strongly. Therefore, the component cost derived by the coordinate transformation will represent the importance of the component mode.

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