Dynamic Control of 3-D Rolling Contacts in Two-Arm Manipulation

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***Abstract*— When two or more arms are used to manipulate a large object, it is preferable not to have a rigid grasp in order to gain more dexterity in manipulation. It may therefore be necessary to control contact motion between the object and the effector(s) on one or more arms. This paper addresses the dynamic control of two arms cooperatively manipulating a large object with rolling contacts. In the framework presented here, the motion of the object as well as the loci of the contact point either on the surface of each effector or on the object can be directly controlled. The velocity and acceleration equations for three-dimensional rolling contacts are derived in order to obtain a dynamic model of the system. A nonlinear feedback control algorithm that decouples and linearizes the system is developed. This is used to demonstrate the control of rolling motion along each arm and the adaptation of grasps to varying loads.**

***Index Terms*— Dextrous manipulation, grasping, rolling contact, two-arm manipulation.**

I. Introduction

DEXTROUS ROBOTIC manipulation has been addressed from many different view points. The motivation for our work is the following: suppose we want to manipulate a large object (larger than the grasp of a single gripper) with no special feature (for example, a handle). It is unlikely that a single robot with gripper-like end-effector will be able to perform the task. We need multiple robots for such a task. When we coordinate many robots to manipulate such a large object, it may not be productive to hold the object rigidly at each grasp. A rigid grasp prevents fine manipulation and also severely restricts the workspace of the robot system. One solution to this problem is to allow relative motion at each robot-object contact.

In this paper, we address the manipulation of objects with two arms by explicitly controlling the interactions at the object-arm contact. Specifically, we maintain rolling contact and control the rolling motion at each contact. The advantage is that large objects of different shapes can be grasped, and the grasp can be adapted or modified by rolling the object along the arm(s) without necessitating regrasping. The objective therefore is to control the object position and orientation as well as the motion of the contact point along the surface of each arm and of the object during manipulation. Although the contact motion can be either rolling or sliding, we prefer rolling mainly because of two reasons. First, rolling is more energy efficient, and second, rolling does not have the nonlinearity that is associated with friction.

In the literature, *dynamics* and *rolling contact* have been addressed in great detail. However, the two subjects have never been integrated into a general maniplation via rolling contact technique. For example, in [21, Ch. 5] the kinematics of rolling contacts are discussed and dynamics and control in [21, Ch. 6]. But the constraints discussed in [21, Ch. 6] do not allow the contact points to move during the manipulation. This paper fills the gap. We introduce novel local contact coordinates. They allow us to formulate the dynamics and control of manipulation via rolling contacts in a general way, which admits motion of the contacts during the manipulation process.

The paper is organized as follows. We briefly discuss some previous work that is relevant to this study in Section II. We then present kinematic analysis of rolling contacts up to second order in Section III. This is followed by the development of a general framework for the constraint analysis of a two-arm system in Section IV. Section V is devoted to representing the motion equation of the system in state space in order to cast the problem into a standard affine nonlinear control problem. We then discuss feedback control of such a system in Section VI and present results from computer simulations to demonstrate the adaptation (reconfiguration) of two-arm grasps without regrasping and the ability to change the locations of contacts on the arms during a manipulation task in Section VII. Finally, we summarize our paper in Section VIII.

II. Previous Work

The kinematic constraint equations and transformations between Cartesian (task-space) and local coordinates are presented in [4], [11], and [19]. Montana [19] outlines a method for relating relative rigid body motion to the rates of change of contact coordinates. But his work is limited to the velocity analysis only. Further, his equations are not in a form that they can be differentiated for higher order kinematic analysis. Cai and Roth [4] adopt a more general approach and obtain expressions for all higher order derivatives, but they use only

Manuscript received November 10, 1995. This work was supported in part by the National Science Foundation under Grant MSS-91-57156, IRI-92-09880, IRI-95-96026, CDA-95-96021, BCS-92-16691, CDA-90-2253, and CDA 88-22719, by the ONR/DARPA Grant N00014-92-J-1647, by the Army/DAAL 03-89-C-0031PRI, the Whitaker Foundation, the University of Pennsylvania Research Foundation, and the NPS RIP Grant. This paper was recommended for publication by Associate Editor J. C. Trinkle and Editor A. Goldenberg upon evaluation of the reviewers' comments.

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Publisher Item Identifier S 1042-296X(97)01385-2.

a subset of the contact coordinates for each contact. The force analysis for such systems is discussed in [11].

It is well known that three-dimensional rolling constraint equations are nonholonomic [23]. Nonholonomic systems are controllable regardless of the structure of constraints [5]. Although it has been shown that a nonholonomic system cannot be feedback stabilized to a single equilibrium point by a smooth feedback, the system we are interested in can be shown to be small-time locally controllable [1]. Additionally, even though such a system is not input-state linearizable, the input-output linearization is still possible with properly chosen output equations [37]. Motion planning of nonholonomic systems has been extensively studied [14], [13]. However the work on dynamic control of such systems [7] is more relevant to this paper.

Force analysis of systems having multiple frictional point contacts has also been studied from the view point of multiarm coordination. Nakamura, Nagai, and Yoshikawa [22] divided the dynamical coordination problem into two phases: determining the resultant force by multiple robotic mechanisms and determining the internal force between the mechanisms. Unseren and Koivo [34] investigated the problem of two manipulators firmly holding an object by formulating the problem in the joint space and transforming it to a set of generalized coordinates. Tarn, Bejczy, and Yun [33] developed a closed kinematic-chain formulation and nonlinear control techniques for two-arm coordination problem. Yun and Kumar [36] formulated an algorithm that simultaneously controls the trajectory of the object and interaction forces for two-arm systems. But there was no contact motion allowed in those cases and therefore the grasp considered was rigid. Multifingered grasp with rolling contact was studied in [6] and [15]. The difference between the results of [6], [15] and this paper is that we are able to control the trajectory of contact points as well as the position/orientation of the object. Paljug *et al.* [25], [24] demonstrated the control of rolling and contact forces in multiarm manipulation for two dimensional objects. But since rolling in two dimensions introduces only holonomic constraints, the problem was much simpler.

III. Contact Kinematics

In this section we develop the contact kinematics of rigid bodies up to second order. These are explicit equations relating the velocities and accelerations of the contact points to those of the contacting rigid bodies. These equations depend on the local surface properties of each contacting body and are used later in the control of manipulation tasks. The detailed derivation is omitted here for the purpose of brevity but can be found in our recently published work [29] and in Sarkar's thesis [31]. First, we introduce some notation and definitions to facilitate the discussion.

A. Notation

The notation and framework for kinematic analysis are mostly borrowed from [19]. In Fig. 1, we consider two rigid objects (obj 1 and obj 2) contacting at a point. The contact point is the coincidence of two points p_1, fixed to obj 1 and

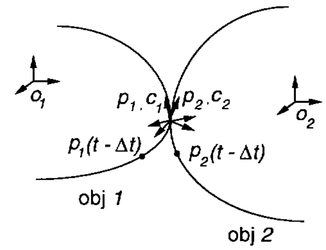


Fig. 1. Two rigid bodies with point contact.

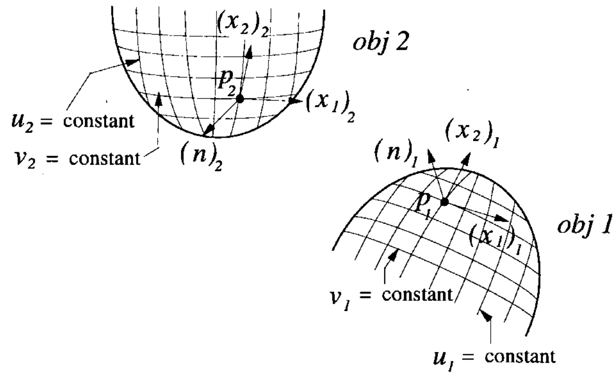


Fig. 2. Coordinate curves and contact frames.

p_2, fixed to obj 2 at time t. c_1 and c_2 are a pair of points, which do not belong to either body but move along the surface of obj 1 and obj 2, respectively, so that they are instantaneously at the point of contact. We choose reference frames on obj 1 and obj 2 at point o_1 and o_2, respectively. These reference frames are attached to the objects. We attach coordinate systems at points c_1 and c_2 which move with the contact points. Finally, we define a continuous family of object-fixed coordinate frames at points p_1 and p_2 in such a way that they coincide with c_1 and c_2 frames at time t.

We define five *contact coordinates* that characterize the motion of the point of contact. Each surface is parameterized by two coordinates [17]. The point of contact is characterized by the intersection of four coordinate curves (two on each surface). The corresponding coordinates $u_1,\ v_1,\ u_2$ and v_2 are the first four parameters (Fig. 2). The fifth parameter is ψ, the angle of contact which is the angle between the u_1 and u_2 curves [19]. In Fig. 3, it is the angle between $(x_1)_1$ (tangent to $v_1 = \text{constant curve}$) and $(x_1)_2$ (tangent to $v_2 = \text{constant curve}$). The sign of ψ is defined in such a way that a rotation of $(x_1)_1$ about the outward pointing unit normal $(n)_1$ to the surface at point p_1 through $-\psi$ aligns the axes $(x_1)_1$ and $(x_1)_2$.

Throughout the rest of this paper we will use various concepts of differential geometry. Detailed discussions on them can be found in any standard differential geometry text (see, for example, [18], [32], [16]), and therefore are omitted here. We will use the following notations for surface properties [17]: G and g_{ij} denote the metric tensor and its individual elements, respectively; Γ_{ij}^k and L_{ij} denote the Christoffel symbols of the second kind, and coefficients of the second fundamental form, respectively. We provide the other notations as and when they become necessary.

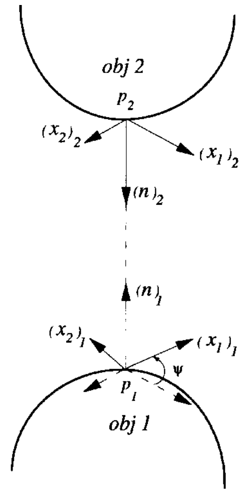


Fig. 3. Definition of angle ψ.

B. Contact Kinematic Equations

Using the notations described in the preceding subsection, we may present the equations for contact kinematics. Let $U_1 = [u_1 \quad v_1]^T$ and $U_2 = [u_2 \quad v_2]^T$, together with ψ, be the contact coordinates. Let the relative linear and angular velocities of the contacting rigid bodies observed in the frame p_2 (also in the frame c_2) be

$$^{p_2}V_{p_1p_2} = \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} \quad ^{p_2}\omega_{p_1} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}.$$

And let the relative linear and angular accelerations of the contacting rigid bodies in the frame c_2 be

$$^{p_2}a_{p_1p_2} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} \quad ^{p_2}\dot{\omega}_{p_1} = \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{bmatrix}.$$

The first-order contact kinematics relating the rate of change of contact coordinates to the rigid body velocities are [19], [29], [31]

$$\dot{U}_1 = (\sqrt{G_1})^{-1} R_\psi (\tilde{H}_1 + H_2)^{-1} \left[-\begin{pmatrix} \omega_y \\ -\omega_x \end{pmatrix} - H_2 \begin{pmatrix} V_x \\ V_y \end{pmatrix} \right] \tag{1}$$

$$\dot{U}_2 = (\sqrt{G_2})^{-1} (\tilde{H}_1 + H_2)^{-1} \left[-\begin{pmatrix} \omega_y \\ -\omega_x \end{pmatrix} + \tilde{H}_1 \begin{pmatrix} V_x \\ V_y \end{pmatrix} \right] \tag{2}$$

$$\dot{\psi} = \sigma_1 \Gamma_1 \dot{U}_1 + \sigma_2 \Gamma_2 \dot{U}_2 - \omega_z \tag{3}$$

$$V_z = 0 \tag{4}$$

where $\sigma_i = (\frac{(g_{22})_i}{(g_{11})_i})^{\frac{1}{2}}$, $H_i = (\sqrt{G_i})^{-1} L_i (\sqrt{G_i})^{-1}$, $\tilde{H}_1 = R_\psi H_1 R_\psi$, and

$$R_\psi = \begin{bmatrix} \cos\psi & -\sin\psi \\ -\sin\psi & -\cos\psi \end{bmatrix}.$$

In the above equations, G_i is the metric tensor of obj i, and $\sqrt{G_i}$ is the square root[1] of G_i. See [20] for a different treatment where contact kinematics are formulated as a virtual kinematic chain.

[1] If a matrix A is positive definite, it can be factorized as $A = PP^T$ for some matrix P, which is called the square root of A.

Let U be a 5×1 vector of contact coordinates, $U = [u_1, v_1, u_2, v_2, \psi]^T$, and q be a 12×1 vector of Lagrangian coordinates [26] which includes the position and orientation coordinates for each of the two rigid bodies. We can write the above first-order contact kinematics equations in a form that explicitly shows the first-order relationship between the contact coordinates and the Lagrangian coordinates

$$\dot{U}_{5\times 1} = C_1(q, U)_{5\times 12} \dot{q}_{12\times 1}. \tag{5}$$

The second-order contact kinematics that relate the second-order derivatives of the contact coordinates to the relative acceleration of the rigid bodies are given by [29], [31]

$$\begin{aligned} \begin{bmatrix} \ddot{U}_1 \\ \ddot{U}_2 \end{bmatrix} = &\begin{bmatrix} R_\psi\sqrt{G_1} & -\sqrt{G_2} \\ R_\psi E_1 H_1 \sqrt{G_1} & -E_1 H_2 \sqrt{G_2} \end{bmatrix}^{-1} \\ &\times \left\{ \begin{bmatrix} R_\psi \sqrt{G_1} \bar{\Gamma}_1 \\ R_\psi E_1 \sqrt{G_1}\, \bar{\bar{L}}_1 \end{bmatrix} W_1 + \begin{bmatrix} \sqrt{G_2}\bar{\Gamma}_2 \\ E_1 (\sqrt{G_2})^{-1}\, \bar{\bar{L}}_2 \end{bmatrix} W_2 \right. \\ &+ \begin{bmatrix} -2\omega_z E_1 R_\psi \sqrt{G_1} & 0 \\ -\omega_z R_\psi H_1 \sqrt{G_1} & -\dot{\psi} H_2 \sqrt{G_2} \end{bmatrix} \begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{bmatrix} \\ &\left. - \begin{bmatrix} 0_{2\times 1} \\ \sigma_1 \Gamma_1 \dot{U}_1 \begin{pmatrix} \omega_y \\ -\omega_x \end{pmatrix} \end{bmatrix} + \begin{bmatrix} 0_{2\times 1} \\ \begin{pmatrix} \alpha_x \\ \alpha_y \end{pmatrix} \end{bmatrix} - \begin{bmatrix} \begin{pmatrix} a_x \\ a_y \end{pmatrix} \\ 0_{2\times 1} \end{bmatrix} \right\} \end{aligned} \tag{6}$$

$$\begin{aligned} \ddot{\psi} = &\sigma_1(\Gamma_1 \ddot{U}_1 + \bar{\bar{\Gamma}}_1 W_1) + \sigma_2(\Gamma_2 \ddot{U}_2 + \bar{\bar{\Gamma}}_2 W_2) \\ &+ \begin{pmatrix} \omega_y \\ -\omega_x \end{pmatrix}^T R_\psi E_1 (\sqrt{G_1})^{-1} L_1 \dot{U}_1 - \alpha_z \end{aligned} \tag{7}$$

$$a_z = \bar{L}_1 W_1 + \bar{L}_2 W_2 + 2 \begin{pmatrix} \omega_y \\ -\omega_x \end{pmatrix}^T R_\psi \sqrt{G_1} \dot{U}_1 \tag{8}$$

where

$$E_1 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad W_i = \begin{bmatrix} \dot{u}_i^2 & \dot{u}_i \dot{v}_i & \dot{v}_i^2 \end{bmatrix}^T. \tag{9}$$

In the above equations, the matrices Γ, L, $\bar{\Gamma}$, $\bar{L}$, $\bar{\bar{\Gamma}}$ and $\bar{\bar{L}}$ describe local differential geometric properties of the contacting surfaces. Their detailed forms are provided in Appendix A.

Similar to (5), the above second-order contact kinematics equations can be written in a form that explicitly shows the second-order relationship between the contact coordinates and the Lagrangian coordinates

$$\ddot{U}_{5\times 1} = C_1(q, U)_{5\times 12} \ddot{q}_{12\times 1} + C_2(q, U, \dot{q}, \dot{U})_{5\times 1}. \tag{10}$$

The first- and second-order contact kinematic equations, as described here, allow us to relate the relative motion of the contacting bodies to the velocities and accelerations of the contact point. Therefore, by knowing the motion of the contacting bodies we can predict the locus of the contact point over time using the above equations. And this allows us to explicitly control the motion of the contact point through a state-space formulation which we develop in subsequent sections.

C. Kinematics of Rolling Contact

Two bodies are said to be in a condition of rolling contact if the relative velocity of the contact points or the sliding velocity [3], [4] is zero, i.e.,

$$ {}^{p_2}V_{p_1p_2}(t) = \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} = 0. \tag{11} $$

Because this definition imposes a condition on velocities (or on the first-order derivative and not on higher order derivatives), this is referred to as the first-order condition for rolling.

Further, if the time derivative of the relative velocity is also zero, i.e., (12), as shown at the bottom of the page, two bodies are said to be in a condition of second-order rolling. Note that, $p_1(t)$ and $p_1(t-\Delta t)$ are two different points fixed to obj 1, and likewise $p_2(t)$ and $p_2(t-\Delta t)$ are two points fixed to obj 2 (see Fig. 1). Equation (12) can be rewritten as in (13) shown at the bottom of the page. The first term in (13) is nothing but ${}^{p_2}a_{p_1p_2}(t)$. The second term can be expressed as (detailed proof can be found in [31]) ${}^{p2}\omega_{p1} \times {}^{o1}V_{c1o1}$. Therefore, the second-order condition for rolling is given by

$$ {}^{p_2}a_{p_1p_2} + {}^{p2}\omega_{p1} \times {}^{o1}V_{c1o1} = 0. \tag{14} $$

Using contact kinematic equations, the above condition can be expressed in the form, as shown in (15) at the bottom of the page. For a detailed discussion on higher order conditions for rolling, refer Cai and Roth [3], [4]. It is seen from the above equation that unlike contact velocity, the contact acceleration is not automatically zero for second-order condition for rolling. However, if we impose another condition known as the no-spin condition, we achieve what is called pure rolling [10, p. 242]. The pivoting component of the angular velocity is zero for pure rolling ([23, p. 18]). Thus we define the first-order no-spin condition as follows:

$$ {}^{p_2}\omega_{p_1} \cdot (n)_2 = 0 $$

where $(n)_2$ is the outward pointing surface normal for object 2 at the contact point. In the contact frame c_2,

$$ \omega_z = 0. \tag{16} $$

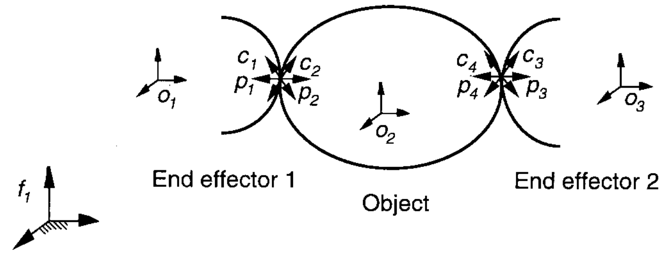


Fig. 4. Schematic of the two arm manipulation system.

For pure rolling up to the second-order, the following condition must be satisfied [31]:

$$ {}^{p_2}\dot{\omega}_{p_1} \cdot (n)_2 = 0 $$

or equivalently, in the contact frame c_2,

$$ \alpha_z = 0. \tag{17} $$

It should be noted at this point that for pure rolling, from (15) and (16), we get $a_x = a_y = 0$. In other words, the tangential components of the relative linear rigid body accelerations are zero for pure rolling.

IV. Constraint Equations of the Two-Arm Systems

In this section we derive the constraint equations for the two-arm systems shown schematically in Fig. 4. We assume that each arm has a single point contact with the object which is held at the end-effectors. We first introduce the notation for two-arm system, followed by a discussion on constraint analysis and contact models. Finally, we present the constraint equations for pure rolling contact.

A. Coordinates and Reference Frames

We assume that each arm has six degree-of-freedom. These arms manipulate a passive object through frictional point contacts. The contact can occur at any point on the surface of the most distal link. For simplicity, we assume that each arm contacts the object at one point only.

We use subscripts 1, 2, and 3 to denote the first arm, the object and the second arm, respectively (Fig. 4). We attach frames o_i to each arm and the object. The origins of the frames o_1 and o_3 are chosen at any convenient point on the contacting surfaces belonging to the respective arms. The origin of the frame o_2 is chosen at the center of mass of the object.

$$ \lim_{\Delta t \to 0} \frac{{}^{p_2(t)}V_{p_1(t)p_2(t)}(t) - {}^{p_2(t-\Delta t)}V_{p_1(t-\Delta t)p_2(t-\Delta t)}(t-\Delta t)}{\Delta t} = 0 \tag{12} $$

$$ \lim_{\Delta t \to 0} \frac{{}^{p_2(t)}V_{p_1(t)p_2(t)}(t) - {}^{p_2(t)}V_{p_1(t)p_2(t)}(t-\Delta t)}{\Delta t} + \lim_{\Delta t \to 0} \frac{{}^{p_2(t)}V_{p_1(t)p_2(t)}(t-\Delta t) - {}^{p_2(t-\Delta t)}V_{p_1(t-\Delta t)p_2(t-\Delta t)}(t-\Delta t)}{\Delta t} = 0. \tag{13} $$

$$ \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} -(\sqrt{g_{11}}\dot{u}_1 \sin\psi + \sqrt{g_{22}}\dot{v}_1 \cos\psi)\omega_z \\ -(\sqrt{g_{11}}\dot{u}_1 \cos\psi - \sqrt{g_{22}}\dot{v}_1)\omega_z \\ (\sqrt{g_{11}}\dot{u}_1 \sin\psi + \sqrt{g_{22}}\dot{v}_1 \cos\psi)\omega_x + (\sqrt{g_{11}}\dot{u}_1 \cos\psi - \sqrt{g_{22}}\dot{v}_1 \sin\psi)\omega_y \end{bmatrix} = \begin{bmatrix} (-E_1 R_\psi \sqrt{G_1}\dot{U}_1)\omega_z \\ \begin{pmatrix} \omega_y \\ -\omega_x \end{pmatrix}^T R_\psi \sqrt{G_1}\dot{U}_1 \end{bmatrix}. \tag{15} $$

We specify the position and orientation of each arm and the object by specifying the position of the origin of frame o_i and the orientation of the frame o_i with respect to an inertial frame f_1. We define the position of the origin of the frame o_i by the vector $X_i = [x_i \quad y_i \quad z_i]^T$, and the orientation of the frame o_i by a vector of Roll-Pitch-Yaw Euler angles (see [27] for a definition), $\Phi_i = [\zeta_i \quad \theta_i \quad \phi_i]^T$, with respect to the inertial frame f_1. We define $q = [X_1 \quad \Phi_1 \quad X_2 \quad \Phi_2 \quad X_3 \quad \Phi_3]^T$ to be an 18 × 1 vector of Lagrangian coordinates.

Of the two contact points, the contact point which is between the arm 1 and the object is called *contact 1*, and the other which is between the arm 2 and the object is called *contact 2*. At *contact 1*, c_1 and c_2 are the contact frames on the arm 1 and on the object, respectively. Similarly, c_3 and c_4 are the respective contact frames on the arm 2 and on the object at *contact 2*. p_1 is the frame attached to the arm 1 and p_2 to the object. p_1 and p_2 coincide with c_1 and c_2, respectively, at time t. Similarly, p_3 and p_4 are defined for arm 2 and the object, respectively.

At each contact point there are five contact coordinates. Therefore, the total set of contact coordinates U is a 10 × 1 vector and is given by $U = [U_1 \quad U_2 \; \psi_1 \quad U_3 \quad U_4 \quad \psi_2]^T$.

The relative rigid body angular and linear velocities are defined in the following manner:

$$\omega_{\mathrm{rel},1} = \omega_1 - \omega_2 \tag{18}$$

$$\omega_{\mathrm{rel},2} = \omega_3 - \omega_4 \tag{19}$$

$$V_{\mathrm{rel},1} = V_{p_1} - V_{p_2} \tag{20}$$

$$V_{\mathrm{rel},2} = V_{p_3} - V_{p_4} \tag{21}$$

where ω_1 and ω_2 are the angular velocitities of the first end-effector and the object, respectively. Similarly, ω_3 and ω_4 are the angular velocitities of the second end-effector and the object, respectively. Evidently, ω_2 and ω_4 are same. Note that $\omega_{\mathrm{rel},1}$ and $V_{\mathrm{rel},1}$ are relative velocities expressed in c_2 while $\omega_{\mathrm{rel},2}$ and $V_{\mathrm{rel},2}$ are relative velocities in c_4.

B. Constraint Analysis

In order to maintain contact between the arms and the object we need to ensure that the normal components of the relative linear velocities between the arm and the object at each contact point are zero, that is

$$V_{\mathrm{rel},1} \cdot (n)_2 = 0 \tag{22}$$

$$V_{\mathrm{rel},2} \cdot (n)_4 = 0 \tag{23}$$

where $(n)_2$ and $(n)_4$ are the two outward pointing unit normals on the object at *contact 1* and *contact 2*, respectively. These constraints are holonomic constraints. we can write (22)–(23) in the following form [31]:

$$A_1(q, U)\dot{q} = 0 \tag{24}$$

where

$$A_1 = \begin{bmatrix} E_2T_1 & -E_2N_1 & -E_2T_2 & E_2N_2 & 0 & 0 \\ 0 & 0 & -E_2T_4 & E_2N_4 & E_2T_3 & -E_2N_3 \end{bmatrix}_{2\times 18}$$

$$E_2 = [0 \quad 0 \quad 1]$$

where N_i's and T_i's are given in Appendix B.

Once the contact is maintained, we are interested in imposing rolling constraints at each contact point. In order to ensure rolling motion, by definition, the tangential components of the relative linear velocities between the arms and the object at each contact point must be zero, that is

$$V_{\mathrm{rel},1} \cdot (x_1)_2 = 0 \tag{25}$$

$$V_{\mathrm{rel},1} \cdot (x_2)_2 = 0 \tag{26}$$

$$V_{\mathrm{rel},2} \cdot (x_1)_4 = 0 \tag{27}$$

$$V_{\mathrm{rel},2} \cdot (x_2)_4 = 0. \tag{28}$$

Additionally, if the component of the relative angular velocity along the contact normal is zero, we achieve what is called pure rolling. If we impose this *no-spin* condition on the relative angular velocity we get

$$\omega_{\mathrm{rel},1} \cdot (n)_2 = 0 \tag{29}$$

$$\omega_{\mathrm{rel},2} \cdot (n)_4 = 0. \tag{30}$$

Equations (25)–(30) are nonholonomic constraints.

Substituting the rolling conditions, (25)–(28) and the no-spin conditions, (29–(30), into the first-order contact kinematics, (1)–(3), we obtain [31]

$$A_2(U)\dot{U} = 0 \tag{31}$$

where $A_2(U)$ is a 6 × 10 matrix given by

$$A_2(U) = \begin{bmatrix} \sqrt{G_1} & -R_{\psi_1}\sqrt{G_2} & 0 & 0 & 0 & 0 \\ -\sigma_1\Gamma_1 & -\sigma_2\Gamma_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{G_3} & -R_{\psi_2}\sqrt{G_4} & 0 \\ 0 & 0 & 0 & -\sigma_3\Gamma_3 & -\sigma_4\Gamma_4 & 1 \end{bmatrix}.$$

These constraint equations are expressed entirely using the contact coordinates. From (5), $\dot{U}$ and $\dot{q}$ are linearly related by

$$\dot{U} = A_3(U)\dot{q} \tag{32}$$

where the matrix $A_3(U)$ is 10 × 18 dimensional and is given by

$$A_3(U) = \begin{bmatrix} K_8 & K_9 & K_{10} & K_{11} & 0_{2\times 3} & 0_{2\times 3} \\ K_{12} & K_{13} & K_{14} & K_{15} & 0_{2\times 3} & 0_{2\times 3} \\ K_{16} & K_{17} & K_{18} & K_{19} & 0_{1\times 3} & 0_{1\times 3} \\ 0_{2\times 3} & 0_{2\times 3} & K_{20} & K_{21} & K_{22} & K_{23} \\ 0_{2\times 3} & 0_{2\times 3} & K_{24} & K_{25} & K_{26} & K_{27} \\ 0_{1\times 3} & 0_{1\times 3} & K_{28} & K_{29} & K_{30} & K_{31} \end{bmatrix}.$$

Here K_8–K_{15} and K_{20}–K_{27} are 2 × 3 matrices, and K_{16}–K_{19} and K_{28}–K_{31} are 1 × 3 matrices. These are functions of local surface properties and of the positions and orientations of the contacting rigid bodies. The detail expressions for each matrix is given in Appendix B.

Substituting (32) into (31) we get

$$A_4(U)\dot{q} = 0 \tag{33}$$

where $A_4(U) = A_2(U)A_3(U)$ is a 6 × 18 matrix.

Finally, combining (24) and (33) we write

$$A(q, U)\dot{q} = 0 \tag{34}$$

where $A(q, U) = [A_1^T(q, U) \quad A_4^T(U)]^T$ is an 8 × 18 matrix which characterizes both holonomic and nonholonomic constraints.

V. Motion Equations

In the preceding section we have developed the constraint equations for the two-arm system. We now derive the motion equations and represent them in the state space in such a way that the system becomes a standard affine nonlinear control system.

A. State Space Representation

The equations of motion of each arm (denoted by $i = 1$ and $i = 3$) can be written in operational space [12] in the form

$$M_i(X_i, \Phi_i)\begin{bmatrix}\ddot{X}_i \\ \ddot{\Phi}_i\end{bmatrix} + V_i(X_i, \Phi_i, \dot{X}_i, \dot{\Phi}_i) = J_i^{-T}(X_i, \Phi_i)\tau_i - Q_{ai}(X_i, \Phi_i, U_i)\lambda_i \qquad (35)$$

where M_i is the inertia matrix, V_i is the vector of the position and velocity dependent forces, J_i is the Jacobian and τ_i is the 6 × 1 vector of joint torques for arm i. λ_i is the 4 × 1 vector of contact constraint forces and moment at *contact* i. Here $\lambda_i = [F_{ix} \quad F_{iy} \quad F_{iz} \quad M_{iz}]^T$, where F_{ix}, F_{iy} and F_{iz} are the components of the contact force (exerted by the arm on the object) and M_{iz} is the contact moment in the contact frame (c_2 for arm 1 and c_4 for arm 3), $Q_{ai}(X_i, \Phi_i, U_i)$ is a 6×4 Jacobian matrix which relates the constraint forces to the generalized forces corresponding to the coordinates (X_i, Φ_i).

Similarly, following the same notation we write the dynamic equations for the object

$$M_2(X_2, \Phi_2)\begin{bmatrix}\ddot{X}_2 \\ \ddot{\Phi}_2\end{bmatrix} + V_2(X_2, \Phi_2, \dot{X}_2, \dot{\Phi}_2) = Q_{o1}(X_2, \Phi_2, U_2)\lambda_1 + Q_{o3}(X_2, \Phi_2, U_4)\lambda_3 \qquad (36)$$

where Q_{o1} and Q_{o3} are two 6 × 4 Jacobian matrices, one for each contact point, which relate the constraint forces to the generalized forces corresponding to the coordinates (X_2, Φ_2).

We can combine (35) and (36) to get,

$$M(q)\ddot{q} + V(q, \dot{q}) = E(q)\tau - A^T(q, U)\lambda \qquad (37)$$

where

$$M(q) = \begin{bmatrix} M_1(X_1, \Phi_1) & 0 & 0 \\ 0 & M_2(X_2, \Phi_2) & 0 \\ 0 & 0 & M_3(X_3, \Phi_3)\end{bmatrix}$$

$$V(q, \dot{q}) = \begin{bmatrix} V_1(X_1, \Phi_1, \dot{X}_1, \dot{\Phi}_1) \\ V_2(X_2, \Phi_2, \dot{X}_2, \dot{\Phi}_2) \\ V_3(X_3, \Phi_3, \dot{X}_3, \dot{\Phi}_3)\end{bmatrix}$$

$$E(q) = \begin{bmatrix} J_1^{-T}(X_1, \Phi_1) & 0 \\ 0 & 0 \\ 0 & J_3^{-T}(X_3, \Phi_3)\end{bmatrix}$$

$$\tau = \begin{bmatrix}\tau_1 \\ \tau_3\end{bmatrix} \quad \lambda = \begin{bmatrix}\lambda_1 \\ \lambda_3\end{bmatrix}$$

$$A^T(q, U) = \begin{bmatrix} Q_{a1}(X_1, \Phi_1, U_1) & 0 \\ Q_{o1}(X_2, \Phi_2, U_2) & Q_{o3}(X_2, \Phi_2, U_4) \\ 0 & Q_{a3}(X_3, \Phi_3, U_3)\end{bmatrix}.$$

It should be noted from the *Principle of Virtual Work* [28] that the matrix $A(q, U)$ in (37) are the same as the matrix $A(q, U)$ in (34).

We now proceed to represent (37) in the state space in order to facilitate the controller design. It should be noted that, because of the presence of the nonholonomic constraints in the two-arm system, we may not simply use q and $\dot{q}$ as a state vector. The problem is further complicated by the presence of the local contact coordinates U.

From the constraint equation (34) we note that $\dot{q}$ always belongs to the null space of $A(q, U)$. Let $S_1(q, U)$ be an 18 × 10 dimensional full-rank matrix such that

$$A(q, U)S_1(q, U) = 0. \qquad (38)$$

That is, the columns of $S_1(q, U)$ are in the null space of $A(q, U)$. It follows that $\dot{q}$ can be represented as a linear combination of the columns of $S_1(q, U)$, i.e.,

$$\dot{q} = S_1(q, U)\nu \qquad (39)$$

where ν is a 10 × 1 dimensional vector. Substituting (39) into (32) we express U also in terms of ν

$$\dot{U} = A_3(U)\dot{q} = A_3(U)S_1(q, U)\nu = S_2(q, U)\nu \qquad (40)$$

where $S_2 = A_3 S_1$. The detailed derivation of S_1 and S_2 matrices for the two-arm system is deferred until the next subsection.

Premultiplying (37) by S_1^T, we obtain

$$S_1^T M\ddot{q} + S_1^T V = S_1^T E\tau - S_1^T A^T\lambda. \qquad (41)$$

Noting (38), the term involving the constraint force λ in (41) vanishes. Differentiating (39) once with respect to the time, we have

$$\ddot{q} = S_1(q, U)\dot{\nu} + \dot{S}_1(q, U)\nu. \qquad (42)$$

Substituting (42) into (41) we obtain

$$S_1^T M S_1\dot{\nu} + S_1^T M\dot{S}_1\nu + S_1^T V = S_1^T E\tau \qquad (43)$$

Therefore,

$$\dot{\nu} = \left(S_1^T M S_1\right)^{-1}\left(S_1^T E\tau - S_1^T M\dot{S}_1\nu - S_1^T V\right). \qquad (44)$$

Choosing the following state vector:

$$x = \begin{bmatrix} q \\ U \\ \nu\end{bmatrix}$$

we may represent the motion equation (37) and the constraint equation (34) in the following state space form:

$$\dot{x} = \begin{bmatrix}\dot{q} \\ \dot{U} \\ \dot{\nu}\end{bmatrix} = \begin{bmatrix} S_1(q, U)\nu \\ S_2(q, U)\nu \\ f_3(q, U, \nu)\end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ g_3(q, U)\end{bmatrix}\tau$$

or equivalently

$$\dot{x} = f(x) + g(x)\tau \qquad (45)$$

where f_3 and g_3 are obtained from (44) and are given by

$$f_3(q, U, \nu) = -\left(S_1^T M S_1\right)^{-1} S_1^T\left(M\dot{S}_1\nu + V\right)$$
$$g_3(q, U) = \left(S_1^T M S_1\right)^{-1} S_1^T E.$$

Here we note that we have been able to describe the two-arm system as an affine nonlinear system as evident from (45).

B. Derivation of S_1 and S_2

In this subsection we derive the explicit expression for $S_1(q,U)$ and $S_2(q,U)$ matrices introduced above. We note that the choice for $S_1(q,U)$ is not unique. We choose $S_1(q,U)$ in such a way that ν corresponds to certain meaningful velocities.

It is physically meaningful to choose the object velocities and the derivatives of contact coordinates as ν. Therefore we decide

$$\nu = \begin{bmatrix} \dot{X}_2^T & \dot{\Phi}_2^T & \dot{U}_1^T & \dot{U}_3^T \end{bmatrix}_{10\times 1}^T. \tag{46}$$

In order to obtain an expression for S_1 we first note that for pure rolling $V_x = V_y = \omega_z = 0$. Substituting $V_x = V_y = 0$ in the (1) we obtain

$$\begin{pmatrix} \omega_y \\ -\omega_x \end{pmatrix} = -(\tilde{H}_1 + H_2)R_{\psi_1}\sqrt{G_1}\dot{U}_1. \tag{47}$$

Using (18) and substituting ω_1 and ω_2 in terms of the Euler angles, we get

$$\omega_{\text{rel},1} = F_1\dot{\Phi}_1 - F_2\dot{\Phi}_2 \tag{48}$$

where F_1 and F_2 are given in Appendix B.

From the above two equations and noting that $\omega_z = 0$, we can express $\dot{\Phi}_1$ in terms of ν as

$$\begin{aligned} \dot{\Phi}_1 &= F_1^{-1}F_2\dot{\Phi}_2 - F_1^{-1}\begin{bmatrix} E_1(\tilde{H}_1 + H_2)R_{\psi_1}\sqrt{G_1} \\ 0_{1\times 2} \end{bmatrix}\dot{U}_1 \\ &= K_{32}\dot{\Phi}_2 + K_{33}\dot{U}_1 \end{aligned} \tag{49}$$

where $E_1 = \left[\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right]$ as defined before.

From (20) and expressing the linear velocities in terms of the Lagrange coordinates, we get

$$V_{\text{rel},1} = T_1\dot{X}_1 - T_2\dot{X}_2 - N_1\dot{\Phi}_1 + N_2\dot{\Phi}_2 \tag{50}$$

where T_1, T_2, N_1 and N_2 are given in Apeendix B. Since $V_{\text{rel},1}$ is zero for rolling motion, from the above equation after substituting $\dot{\Phi}_1$ from (49) we obtain the following expression for $\dot{X}_1$ in terms of ν

$$\dot{X}_1 = K_{34}\dot{X}_2 + K_{35}\dot{\Phi}_2 + K_{36}\dot{U}_1 \tag{51}$$

where the expressions for K_{34}, K_{35}, and K_{36} are given in Appendix B. Similarly, following the same procedure for *contact 2* we arrive at the following equations:

$$\dot{\Phi}_3 = K_{37}\dot{\Phi}_2 + K_{38}\dot{U}_3 \tag{52}$$

$$\dot{X}_3 = K_{39}\dot{X}_2 + K_{40}\dot{\Phi}_2 + K_{41}\dot{U}_3 \tag{53}$$

where the expressions for K_{37} to K_{41} are provided in Appendix B.

We now construct S_1 matrix from the above equations. It is an 18×10 matrix which satisfies the equation $\dot{q} = S_1(q,U)\nu$ and has the following form:

$$S_1 = \begin{bmatrix} (K_{34})_{3\times 3} & (K_{35})_{3\times 3} & (K_{36})_{3\times 2} & 0_{3\times 2} \\ 0_{3\times 3} & (K_{32})_{3\times 3} & (K_{33})_{3\times 2} & 0_{3\times 2} \\ I_{3\times 3} & 0_{3\times 3} & 0_{3\times 2} & 0_{3\times 2} \\ 0_{3\times 3} & I_{3\times 3} & 0_{3\times 2} & 0_{3\times 2} \\ (K_{39})_{3\times 3} & (K_{40})_{3\times 3} & 0_{3\times 2} & (K_{41})_{3\times 2} \\ 0_{3\times 3} & (K_{37})_{3\times 3} & 0_{3\times 2} & (K_{38})_{3\times 2} \end{bmatrix}_{18\times 10}.$$

We now express $\dot{U}$ as a function of the same ν such that

$$\dot{U} = S_2(U)\nu. \tag{54}$$

In order to construct S_2 matrix we note that for pure rolling (3) yields

$$\dot{\psi} = \sigma_1\Gamma_1\dot{U}_1 + \sigma_2\Gamma_2\dot{U}_2. \tag{55}$$

Using (47) and (55) for *contact 1* and similar equations for *contact 2* we construct S_2, a 10×10 matrix, as follows:

$$S_2 = \begin{bmatrix} 0_{2\times 3} & 0_{2\times 3} & I_{2\times 2} & 0_{2\times 2} \\ 0_{2\times 3} & 0_{2\times 3} & K_{42_{2\times 2}} & 0_{2\times 2} \\ 0_{1\times 3} & 0_{1\times 3} & K_{43_{1\times 2}} & 0_{1\times 2} \\ 0_{2\times 3} & 0_{2\times 3} & 0_{2\times 2} & I_{2\times 2} \\ 0_{2\times 3} & 0_{2\times 3} & 0_{2\times 2} & K_{44_{2\times 2}} \\ 0_{1\times 3} & 0_{1\times 3} & 0_{1\times 2} & K_{45_{1\times 2}} \end{bmatrix}_{10\times 10}$$

where the expressions for individual elements are provided in Appendix B. Here K_{42} and K_{44} are 2×2 matrices while K_{43} and K_{45} are 1×2 matrices and are functions of the local surface properties.

VI. Feedback Control

Here we first discuss the choice of output equations which is important in the formulation of the nonholonomic control problem. We then develop a feedback linearization framework which not only linearizes the nonlinear system but also decouples it.

A. Output Equations

We want to control both the gross motion of the object needed to perform the task (which may be, for example, to move the object from one point in space to a different point following a desired trajectory), and the fine motion at the point of contacts needed for readjustment of the grasp without releasing the object. Therefore, the output equations should be functions of both Lagrangian coordinates and the contact coordinates and can be written as

$$y = h(q,U) = [h_1(q)^T \quad h_2(U)^T]^T. \tag{56}$$

Since the system has 10 degrees-of-freedom, we can control 10 independent variables. It is important to control the position and orientation of the object since, in general, those are the variables needed to be controlled in order to perform a manipulation task. Therefore the first block of our output equations becomes

$$h_1(q) = \begin{bmatrix} X_2^T & \Phi_2^T \end{bmatrix}_{6\times 1}^T. \tag{57}$$

The remaining four variables can be chosen to be functions of the contact coordinates. They can either be the contact coordinates of both arms, that is, U_1 and U_3, or they can be the contact coordinates on the object at both contact points, that is, U_2 and U_4. It should be noted at this point that since we are imposing rolling constraints at the contact points, we can only control either U_1 or U_2 at *contact 1* and similarly, either U_3 or U_4 at *contact 2*. This is because U_1 and U_2 (and similarly, U_3 and U_4) are no longer independent. We therefore choose the following two sets of output equations to demonstrate the methodology.

Output-I: We choose the following output equations when we are specifically interested in controlling the loci of the contact points on the arms

$$y = \begin{bmatrix} X_2^T & \Phi_2^T & U_1^T & U_3^T \end{bmatrix}^T. \tag{58}$$

Output-II: We choose the following output equations when we are specifically interested in controlling the loci of the contact points on the object at both contacts

$$y = \begin{bmatrix} X_2^T & \Phi_2^T & U_2^T & U_4^T \end{bmatrix}^T. \tag{59}$$

B. Feedback Linearization

At this point it is clear from the foregoing discussion that we have formulated the control problem of the two-arm system as the standard controller design of a nonlinear system characterized by the state equation (45) and the output equation (56). Since the system is nonlinear, we will utilize the differential geometric control method to linearize the system [9]. Because the system is subject to nonholonomic constraints, it is not input-state linearizable [2]. Therefore we pursue input-output linearization.

To compute the nonlinear feedback for input-output linearization, we note the following Lie derivatives:

$$L_g h = \frac{\partial h}{\partial x} g = \begin{bmatrix} \frac{\partial h_1}{\partial q} & 0 & 0 \\ 0 & \frac{\partial h_2}{\partial U} & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ g_3 \end{bmatrix} = 0$$

$$L_f h = \begin{bmatrix} \frac{\partial h_1}{\partial q} S_1 \nu \\ \frac{\partial h_2}{\partial U} S_2 \nu \end{bmatrix}$$

$$L_g L_f h = \begin{bmatrix} \frac{\partial h_1}{\partial q} S_1 g_3 \\ \frac{\partial h_2}{\partial U} S_2 g_3 \end{bmatrix}.$$

Because $L_g L_f h$ is nonzero, the relative degree of the system for each component of the output is two [9]. The feedback for achieving input-output linearization is then given by [9]

$$\tau = -(L_g L_f h)^{-1} L_f^2 h + (L_g L_f h)^{-1} u$$

where u is the new input.

Applying the above nonlinear feedback, the closed-loop system becomes

$$\ddot{y} = u. \tag{60}$$

A linear feedback can be applied in addition to the nonlinear feedback to properly place the poles of the overall system.

VII. Simulation Results

We consider two 6-degree-of-freedom manipulators each with a flat effector on the sixth link. The object is spherical with a radius, $\rho = 0.1$ m. The coordinate curves on the object and the effectors are shown in Fig. 5.

We define the following coordinate system for a plane (which represents the flat effectors):

$$f : U \subset \Re^2 \to \Re^3 : (\xi^1, \xi^2) \mapsto (\xi^1, \xi^2, 0).$$

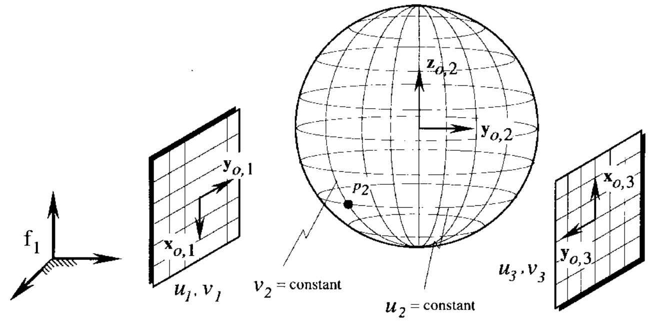


Fig. 5. Coordinate curves on the effectors and the object.

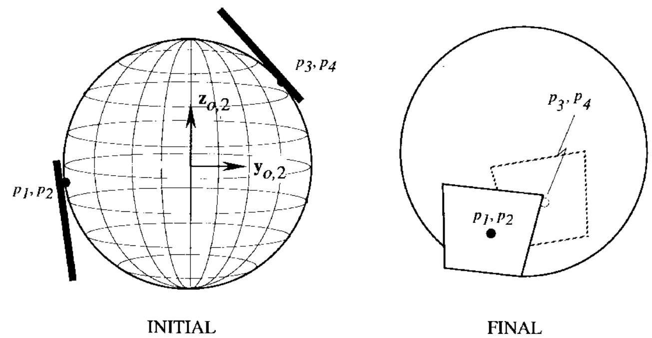


Fig. 6. Centering the contact.

The natural basis and the corresponding unit normal for a plane with the above coordinate system are

$$x_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad x_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad n = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

For a sphere with radius ρ, let us define a coordinate system

$$\begin{aligned} f : U \to \Re^3 &: (u_2, v_2) \\ &\mapsto (\rho \sin u_2 \cos v_2,\ \rho \sin u_2 \sin v_2,\ \rho \cos u_2). \end{aligned}$$

The natural basis for this coordinate system and the corresponding unit normal are

$$x_1 = \begin{bmatrix} \rho \cos u_2 \cos v_2 \\ \rho \cos u_2 \sin v_2 \\ -\rho \sin u_2 \end{bmatrix} \quad x_2 = \begin{bmatrix} -\rho \sin u_2 \sin v_2 \\ \rho \sin u_2 \cos v_2 \\ 0 \end{bmatrix}$$

$$n = \begin{bmatrix} \sin u_2 \cos v_2 \\ \sin u_2 \sin v_2 \\ \cos u_2 \end{bmatrix}.$$

Various local surface quantities are shown in Appendix C.

The transformation matrices for *contact 1* are

$${}^{o_1}R_{c_1} = I_{3\times 3}$$

$${}^{o_2}R_{c_2} = \begin{bmatrix} \cos u^2 \cos v^2 & -\sin v^2 & \sin u^2 \cos v^2 \\ \cos u^2 \sin v^2 & \cos v^2 & \sin u^2 \sin v^2 \\ -\sin u^2 & 0 & \cos u^2 \end{bmatrix}.$$

Similarly, ${}^{o_3}R_{c_3}$ and ${}^{o_4}R_{c_4}$ are defined for *contact 2*.

We assume that the torsional coefficient of friction is high so that the no-spin condition is maintained. This may not be the case with contacts between perfectly rigid bodies. It is, however, typical of objects with some compliance at the surface for which the rigid body kinematic model would still be a good nominal model. When the no-spin condition is maintained we achieve pure rolling and we substitute $\omega_z = \alpha_z = a_x = a_y = 0$ in addition to $V_x = V_y = 0$ in the contact equations. We consider the case of pure rolling at the contact points. Therefore, for *contact 1*, using the contact equations developed in Section III-B [see (1)–(3)], we get

$$\begin{aligned} \dot{u}_1 &= \rho(\omega_y \cos\psi + \omega_x \sin\psi) \\ \dot{v}_1 &= \rho(-\omega_y \sin\psi + \omega_x \cos\psi) \\ \dot{u}_2 &= \omega_y \\ \dot{v}_2 &= -\omega_x \csc \mathrm{u}_2 \\ \dot{\psi}_1 &= -(\omega_x \cot u_2). \end{aligned} \tag{61}$$

From (6) and (7), we get

$$\begin{aligned} \ddot{u}_1 &= \rho\dot{\psi}_1(\dot{u}_2 \sin\psi_1 + \dot{v}_2 \sin u_2 \cos\psi_1) - \rho\dot{u}_2\dot{v}_2 \cos u_2 \sin\psi_1 \\ &\quad - \rho(\dot{v}_2)^2 \sin u_2 \cos u_2 \cos\psi_1 + \rho(\alpha_x \sin\psi_1 + \alpha_y \cos\psi_1) \\ \ddot{v}_1 &= \rho\dot{\psi}_1(\dot{u}_2 \cos\psi_1 - \dot{v}_2 \sin u_2 \sin\psi_1) - \rho\dot{u}_2\dot{v}_2 \cos u_2 \cos\psi_1 \\ &\quad + \rho(\dot{v}_2)^2 \sin u_2 \cos u_2 \sin\psi_1 + \rho(\alpha_x \cos\psi_1 - \alpha_y \sin\psi_1) \\ \ddot{u}_2 &= \dot{v}_2\dot{\psi}_1 \sin u_2 + \alpha_y \end{aligned} \tag{62}$$

$$\begin{aligned} \ddot{v}_2 &= \dot{u}_2\dot{\psi}_1 \csc u_2 - \dot{u}_2\dot{v}_2 \cot u_2 - \alpha_x \csc u_2 \\ \ddot{\psi}_1 &= \ddot{v}_2 \sin u_2 - \dot{u}_2\dot{v}_2 \cos u_2. \end{aligned} \tag{63}$$

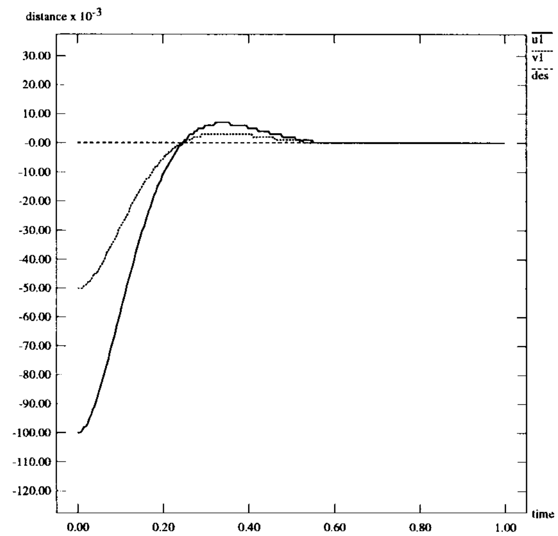
Similarly, we get contact equations for *contact 2*. In the simulations, the linear system represented by (60) is designed in such a way that each second-order system has a small overshoot with a damping ratio and settling time of 0.65 and 0.5 s, respectively. We have simulated two types of tasks as described below.

A. Centering the Contact

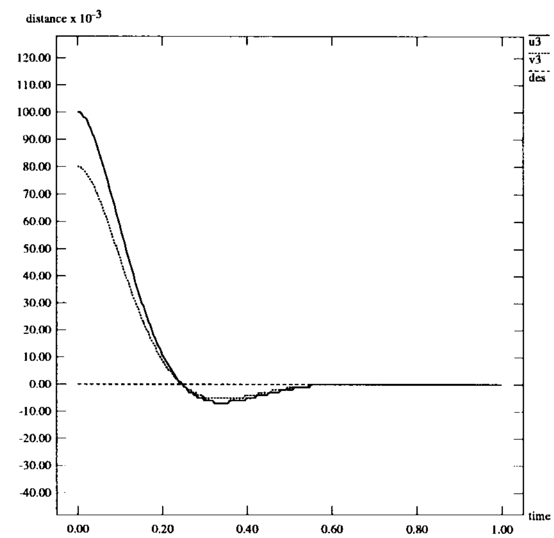
Here we control the loci of the contact points on the end effectors while simultaneously moving the object in a vertical straight line. We start with a configuration where the contact points on the end effectors are not at their centers (reference points) but at some distance away from them (see Fig. 6). Our objective is to roll the effectors in such a way that the contact points are brought to the centers of the respective effectors while the object follows the desired trajectory. We use the first set of output equations described by (58). The initial and final configuration of the system for this task is shown in Fig. 6. Fig. 7 shows how the contact points on the effectors converge to the desired points. Fig. 8(a) shows the convergence of the motion of the object. The motion of the contact points on the object which are not explicitly controlled in this case are shown in Fig. 8(b). Although these variables appear to drift, they are stable in the sense each variable converges to some value. This represents the zero dynamics of the system. The zero dynamics of the system is Lagrange stable which is typical for nonholonomic system [1], [35].

B. Grasp Adaptation

In this example we start with a contact configuration that requires large internal forces to stably hold the object weight. The end effectors/object are rolled to the desired contact configuration so that the arms have a better mechanical advantage while simultaneously trying to move the object along a horizontal straight line. Here we use the output equations described by (59). The initial and final configuration of the system for this task is shown in Fig. 9. Fig. 10 shows the motion of the contact points on the object. Fig. 11(a) shows how the object moves along the prescribed horizontal line. The object motion and the motion of the contact points on the object are asymptotically stable and the response is that of a typical, underdamped second-order system. The motion of the contact points on the effectors, [Fig. 11(b)] on the other hand, is stable in the Lyapunov sense (that is, Lagrange stable)—this tendency to "drift" is typical of nonholonomic systems [1], [35].



(a)



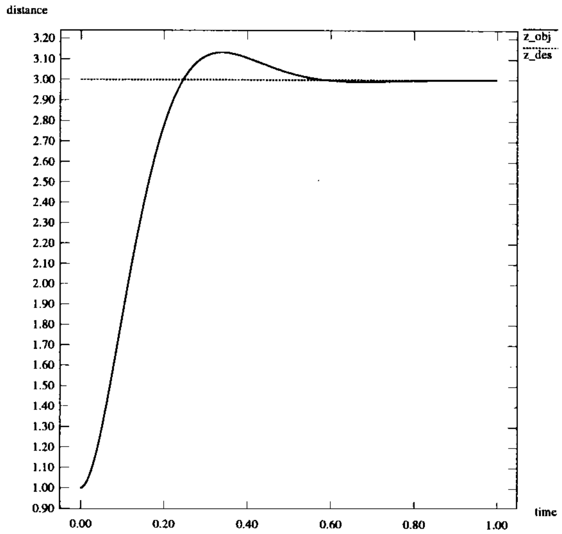
(b)

Fig. 7. The motion of the contact point on the first end effector (a) and at the second end effector (b).

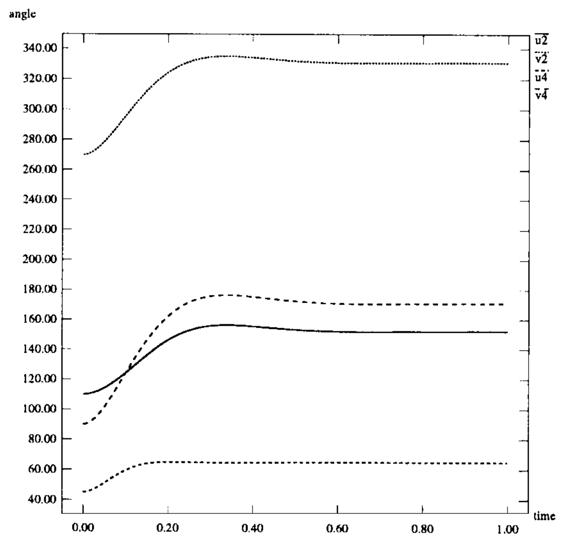
VIII. Concluding Remarks

A. Summary

We have presented a new theoretical framework for the coordinated control of two arms manipulating a large ob-

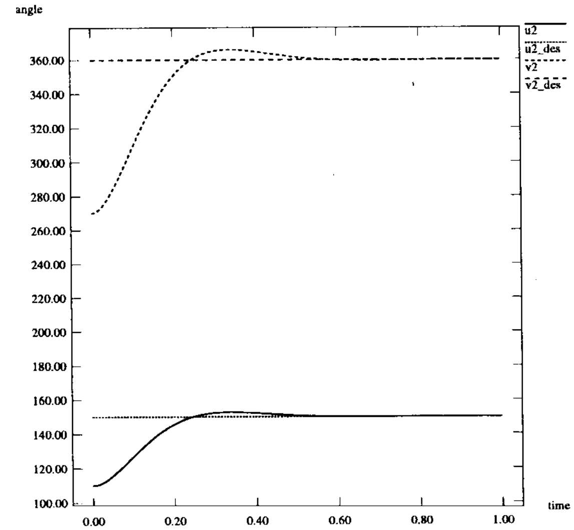


(a)

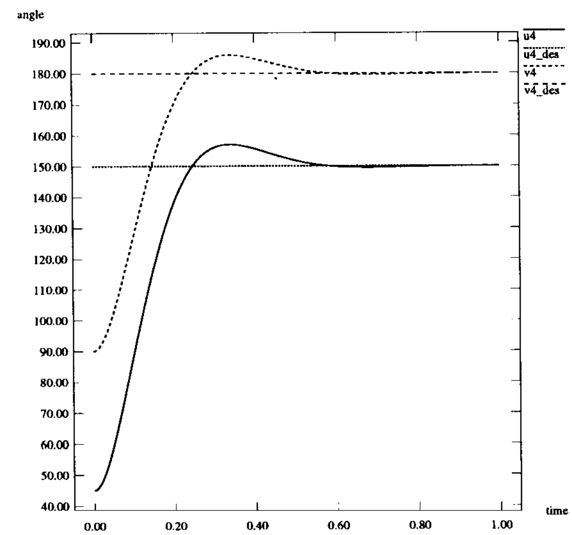


(b)

Fig. 8. Vertical motion of the object (a) and motion of the contact points on the object at both contact points (b).



(a)



(b)

Fig. 10. The motion of the contact point on the object at *contact* 1 (a) and at *contact* 2 (b).

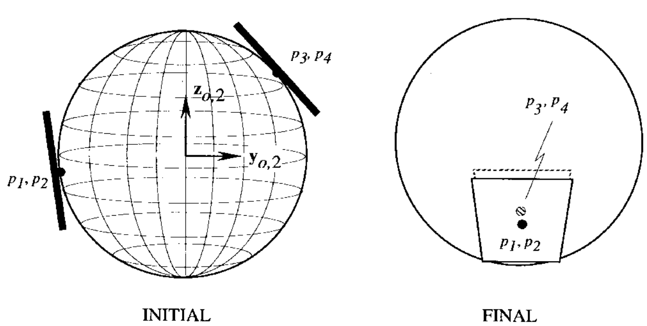
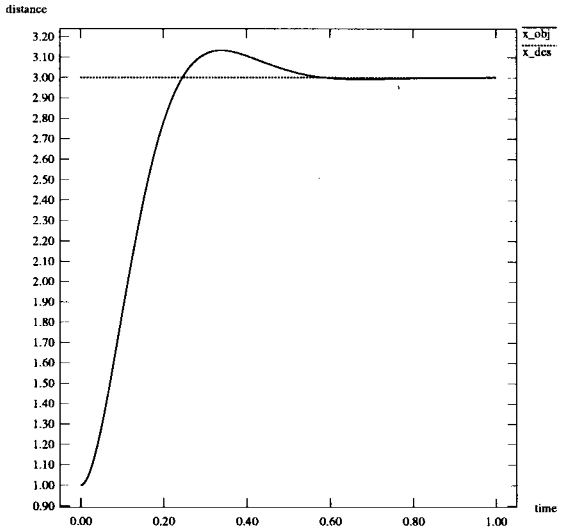


Fig. 9. Grasp adaptation.

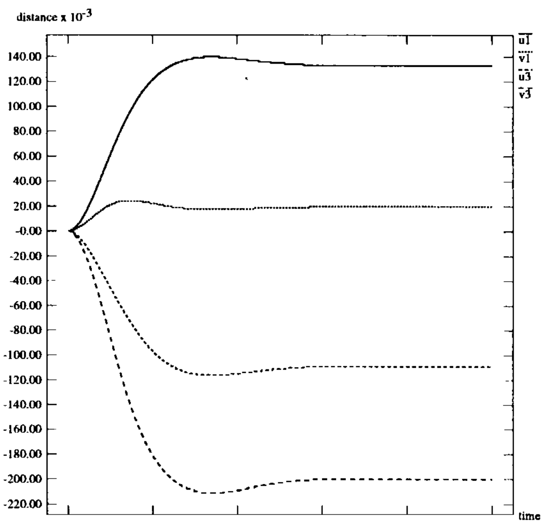
ject. This framework allows explicit control not only of the object motion but also of the contact locations between the arms and the object through controlling rolling motion at the contacts. The control algorithm is a nonlinear feedback which cancels the dynamics and decouples the outputs. Since the control model requires the use of first- and second-order contact kinematics equations, we have developed and presented contact kinematics up to second order. Although the first-order contact kinematics was previously developed [19], second-order contact kinematics utilizing the complete five-dimensional contact space is new. Also, our control algorithm that explicitly uses such contact equations by enhancing the state space with contact coordinates is a new contribution. It enables one to control the contact coordinates directly. Finally, we have demonstrated the efficacy of the control scheme via computer simulations.

B. Discussion

There are several aspects of this problem that have not been reported in detail in this paper. First, the validity of our



(a)



(b)

Fig. 11. Horizontal motion of the object (a) and the motion of the contact points on each end effectors (b).

equations requires that the grasp is always force closed. In this paper, we have not presented the control of contact forces. Neither have we discussed the conditions for maintaining force closure. These issues are dealt with in [30], [8]. Second, we have only considered the soft contact model in our simulations and examples, although it can be argued that the point contact (without torsional moments) model is more appropriate for robot-object contacts. However it is easy to incorporate point contact models as shown in the paper. The increase in difficulty comes from the fact that we now need at least three robot-object contacts (and therefore three arms) to maintain force closure.

The main disadvantage of the proposed control scheme is that it is model-based. Another disadvantage is that it relies on feedback of the position and velocity of the contact point moving over the robot effector. This underscores the importance of developing good tactile sensors. Although the main objective of this work was to lay down the theoretical foundation for coordinated manipulation, it is clear that experimental validation and testing is an important direction for future work.

Appendix A
Definitions of Matrices in the Second-Order Contact Kinematics

$$
\begin{aligned}
\Gamma &= \begin{bmatrix} \Gamma^2_{11} & \Gamma^2_{12} \end{bmatrix} \\
L &= \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \\
\bar{\Gamma} &= \begin{bmatrix} \Gamma^1_{11} & 2\Gamma^1_{12} & \Gamma^1_{22} \\ \Gamma^2_{11} & 2\Gamma^2_{12} & \Gamma^2_{22} \end{bmatrix} \\
\bar{L} &= \begin{bmatrix} L_{11} & 2L_{12} & L_{22} \end{bmatrix} \\
\bar{\bar{\Gamma}} &= \begin{bmatrix} (\Gamma^2_{21} - \Gamma^1_{11})\Gamma^2_{11} + \frac{\partial \Gamma^2_{11}}{\partial \xi^1} \\ (\Gamma^2_{21} - \Gamma^1_{11})\Gamma^2_{12} + (\Gamma^2_{22} - \Gamma^1_{12})\Gamma^2_{11} + \frac{\partial \Gamma^2_{12}}{\partial \xi^1} + \frac{\partial \Gamma^2_{11}}{\partial \xi^2} \\ (\Gamma^2_{22} - \Gamma^1_{12})\Gamma^2_{12} + \frac{\partial \Gamma^2_{12}}{\partial \xi^2} \end{bmatrix}^T \\
\bar{\bar{L}} &= \begin{bmatrix} \begin{pmatrix} \Gamma^1_{11} L_{11} - \frac{\partial L_{11}}{\partial \xi^1} \\ \Gamma^1_{11} L_{12} + \Gamma^1_{12} L_{11} - \frac{\partial L_{12}}{\partial \xi^1} - \frac{\partial L_{11}}{\partial \xi^2} \\ \Gamma^1_{12} L_{12} - \frac{\partial L_{12}}{\partial \xi^2} \end{pmatrix}^T \\ \begin{pmatrix} \Gamma^2_{21} L_{21} - \frac{\partial L_{21}}{\partial \xi^1} \\ \Gamma^2_{21} L_{22} + \Gamma^2_{22} L_{21} - \frac{\partial L_{22}}{\partial \xi^1} - \frac{\partial L_{21}}{\partial \xi^2} \\ \Gamma^2_{22} L_{22} - \frac{\partial L_{22}}{\partial \xi^2} \end{pmatrix}^T \end{bmatrix}.
\end{aligned}
$$

For each matrix, we use the subscript i when we refer to obj i. For example, Γ_1 is the Γ matrix for obj 1 and $(\Gamma^k_{ij})_1$ is its ijkth component.

Appendix B
Expressions for Different Variables

The expressions for F_i's, T_i's, and N_i's are

$$
\begin{aligned}
F_1 &= {}^{c_2}R_{c_1}\,{}^{c_1}R_{o_1}\,{}^{o_1}R_{f_1} K_1 \\
F_2 &= {}^{c_2}R_{o_2}\,{}^{o_2}R_{f_1} K_2 \\
T_1 &= {}^{c_2}R_{c_1}\,{}^{c_1}R_{o_1}\,{}^{o_1}R_{f_1} \\
N_1 &= \mathrm{Dual}\{{}^{c_2}R_{c_1}\,{}^{c_1}R_{o_1}\,{}^{o_1}r_{p_1 o_1}\} F_1 \\
T_2 &= {}^{c_2}R_{o_2}\,{}^{o_2}R_{f_1} \\
N_2 &= \mathrm{Dual}\{{}^{c_2}R_{o_2}\,{}^{o_2}r_{p_2 o_2}\} F_2.
\end{aligned}
$$

In the above expressions, the term $\mathrm{Dual}\{\cdot\}$ represents the skew-symmetric matrix representation of the vector $\{\cdot\}$. For example, if $r = [r_x \quad r_y \quad r_z]^T$ is a 3×1 vector, then

$$
\mathrm{Dual}\{r\} = \begin{bmatrix} 0 & -r_z & r_y \\ r_z & 0 & -r_x \\ -r_y & r_x & 0 \end{bmatrix}.
$$

The individual elements of the A_3 matrix of (32) and other related expressions are as follows:

$$
\begin{aligned}
K_4 &= (\sqrt{G_1})^{-1} R_{\psi_1} (\tilde{H}_1 + H_2)^{-1} \\
K_5 &= (\sqrt{G_2})^{-1} (\tilde{H}_1 + H_2)^{-1}
\end{aligned}
$$

$$
\begin{aligned}
K_6 &= (\sqrt{G_3})^{-1} R_{\psi_2} (\tilde{H}_3 + H_4)^{-1}\\
K_7 &= (\sqrt{G_4})^{-1} (\tilde{H}_3 + H_4)^{-1}\\
K_8 &= -K_4 H_2 E_4 T_1\\
K_9 &= -K_4 (E_3 F_1 - H_2 E_4 N_1)\\
K_{10} &= K_4 H_2 E_4 T_2\\
K_{11} &= K_4 (E_3 F_2 - H_2 E_4 N_2)\\
K_{12} &= K_5 \tilde{H}_1 E_4 T_1\\
K_{13} &= -K_5 (E_3 F_1 + \tilde{H}_1 E_4 N_1)\\
K_{14} &= -K_5 \tilde{H}_1 E_4 T_2\\
K_{15} &= K_5 (E_3 F_2 + \tilde{H}_1 E_4 N_2)\\
K_{16} &= \sigma_1 \Gamma_1 K_8 + \sigma_2 \Gamma_2 K_{12}\\
K_{17} &= \sigma_1 \Gamma_1 K_9 + \sigma_2 \Gamma_2 K_{13} + E_2 F_1\\
K_{18} &= \sigma_1 \Gamma_1 K_{10} + \sigma_2 \Gamma_2 K_{14}\\
K_{19} &= \sigma_1 \Gamma_1 K_{11} + \sigma_2 \Gamma_2 K_{15} - E_2 F_2\\
K_{20} &= K_6 H_4 E_4 T_4\\
K_{21} &= K_6 (E_3 F_4 - H_4 E_4 N_4)\\
K_{22} &= -K_6 H_4 E_4 T_3\\
K_{23} &= -K_6 (E_3 F_3 - H_4 E_4 N_3)\\
K_{24} &= -K_7 \tilde{H}_3 E_4 T_4\\
K_{25} &= K_7 (E_3 F_4 + \tilde{H}_3 E_4 N_4)\\
K_{26} &= K_7 \tilde{H}_3 E_4 T_3\\
K_{27} &= -K_7 (E_3 F_3 + \tilde{H}_3 E_4 N_3)\\
K_{28} &= \sigma_3 \Gamma_3 K_{22} + \sigma_4 \Gamma_4 K_{26}\\
K_{29} &= \sigma_3 \Gamma_3 K_{23} + \sigma_4 \Gamma_4 K_{27} - E_2 F_4\\
K_{30} &= \sigma_3 \Gamma_3 K_{20} + \sigma_4 \Gamma_4 K_{24}\\
K_{31} &= \sigma_3 \Gamma_3 K_{21} + \sigma_4 \Gamma_4 K_{25} + E_2 F_3\\
E_3 &= \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \quad E_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.
\end{aligned}
$$

The expressions for each element of $S_1(q, U)$ matrix are

$$
\begin{aligned}
K_{32} &= F_1^{-1} F_2\\
K_{33} &= -F_1^{-1} \begin{bmatrix} E_1 (\tilde{H}_1 + H_2) R_{\psi_1} \sqrt{G_1} \\ 0_{1\times 2} \end{bmatrix}\\
K_{34} &= T_1^{-1} T_2\\
K_{35} &= T_1^{-1} (N_1 K_{32} - N_2)\\
K_{36} &= T_1^{-1} N_1 K_{33}\\
K_{37} &= F_3^{-1} F_4\\
K_{38} &= -F_3^{-1} \begin{bmatrix} E_1 (\tilde{H}_3 + H_4) R_{\psi_2} \sqrt{G_3} \\ 0_{1\times 2} \end{bmatrix}\\
K_{39} &= T_3^{-1} T_4\\
K_{40} &= T_3^{-1} (N_3 K_{37} - N_4)\\
K_{41} &= T_3^{-1} N_3 K_{38}.
\end{aligned}
$$

Recall that $E_1 = \left[\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right]$.

The elements of $S_2(U)$ are as follows:

$$
\begin{aligned}
K_{42} &= (\sqrt{G_2})^{-1} R_{\psi_1} \sqrt{G_1} & K_{44} &= (\sqrt{G_4})^{-1} R_{\psi_2} \sqrt{G_3}\\
K_{43} &= \sigma_1 \Gamma_1 + \sigma_2 \Gamma_2 K_{42} & K_{45} &= \sigma_3 \Gamma_3 + \sigma_4 \Gamma_4 K_{44}.
\end{aligned}
$$

APPENDIX C
LOCAL SURFACE PROPERTIES OF A PLANE AND A SPHERE

A. Plane

The component of metric tensor is given by

$$
\begin{aligned}
g_{11} &= 1 & g_{12} &= 0\\
g_{21} &= 0 & g_{22} &= 1.
\end{aligned}
$$

Since the natural basis vectors and the components of the metric tensor are constant for a plane, Christoffel symbols, coefficients of the second fundamental forms, and all other higher order surface properties are zero. Therefore, for a plane $\Gamma = L = \bar{\Gamma} = \bar{L} = \bar{\bar{\Gamma}} = \bar{\bar{L}} = 0$.

B. Sphere

The components of the metric tensor G are given by

$$
\begin{aligned}
g_{11} &= \rho^2 & g_{12} &= 0\\
g_{21} &= 0 & g_{22} &= \rho^2 \sin(\xi^1)^2.
\end{aligned}
$$

G is diagonal because the coordinate system is orthogonal.

There are eight Christoffel symbols of each kind. They are

$$
\begin{aligned}
&[11,1] = 0 && [11,2] = 0\\
&[12,1] = 0 && [12,2] = \rho^2 \sin\xi^1 \cos\xi^1\\
&[21,1] = 0 && [21,2] = \rho^2 \sin\xi^1 \cos\xi^1\\
&[22,1] = -\rho^2 \sin\xi^1 \cos\xi^1 && [22,2] = 0\\
&\Gamma_{11}^1 = 0 && \Gamma_{11}^2 = 0\\
&\Gamma_{12}^1 = 0 && \Gamma_{12}^2 = \cot\xi^1\\
&\Gamma_{21}^1 = 0 && \Gamma_{21}^2 = \cot\xi^1\\
&\Gamma_{22}^1 = -\sin\xi^1 \cos\xi^1 && \Gamma_{22}^2 = 0.
\end{aligned}
$$

The coefficients of the second fundamental form L_{ij} are

$$
\begin{aligned}
L_{11} &= -\rho & L_{12} &= 0\\
L_{21} &= 0 & L_{22} &= -\rho \sin(\xi^1)^2.
\end{aligned}
$$

The derivatives of the Christoffel symbols of the second kind that are not zero are

$$
\begin{aligned}
\Gamma_{12,1}^2 &= -\csc^2\xi^1\\
\Gamma_{21,1}^2 &= -\csc^2\xi^1\\
\Gamma_{22,1}^1 &= -\cos(2\xi^1).
\end{aligned}
$$

Finally, the only nonzero derivative of the coefficients of the second fundamental form is

$$
L_{22,1} = -2\rho \sin\xi^1 \cos\xi^1.
$$

Therefore for a sphere with the above coordinate system we have

$$
\begin{aligned}
\Gamma &= [0 \quad \cot\xi^1]\\
\bar{\Gamma} &= \begin{bmatrix} 0 & 0 & -\sin\xi^1 \cos\xi^1 \\ 0 & 2\cot\xi^1 & 0 \end{bmatrix}\\
\bar{\bar{\Gamma}} &= [0 \quad -1 \quad 0]\\
L &= \begin{bmatrix} -\rho & 0 \\ 0 & -\rho(\sin\xi^1)^2 \end{bmatrix}\\
\bar{L} &= [-\rho \quad 0 \quad -\rho(\sin\xi^1)^2]\\
\bar{\bar{L}} &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & \rho \sin\xi^1 \cos\xi^1 & 0 \end{bmatrix}.
\end{aligned}
$$

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