Pseudo-Holonomic Behavior of Planar Space Robots

Ranjan Mukherjee and Mary Zurowski
Mechanical Engineering Department
Naval Postgraduate School
Monterey, CA 93943

Abstract

The angular momentum of a free-flying space robot is a conserved quantity. This conservation law acts as a nonholonomic constraint and manifests itself when cyclic motion of the joints of the space robot produces a change in the orientation of the whole system. Consequently closed trajectories in the joint space of the robot usually fail to produce closed trajectories in the workspace. In this paper we first show that there exist some closed paths that are "holonomic loops" in the sense that the nonholonomic space robot exhibits holonomic behavior globally on these trajectories. When the joints of the space robot move along these trajectories, the end-effector traces a closed curve in the workspace. In this paper, we present a simple method to plan such trajectories that produce repeatable motion of the space robot.

1 Introduction

A free-flying space robot is a nonholonomic mechanical system. The nonholonomy in its mechanical structure is due to the conservation of the total angular momentum of the system. The motion planning and control of such systems have been studied extensively by a number of researchers. Some of the early work in this field was done by Alexander and Cannon [1], Vafa and Dubowsky [13], Longman et.al. [5], Umetani and Yoshida [12], Nakamura and Mukherjee [8], Miyazaki, et.al. [6], Papadopoulous and Dubowsky [9], Sreenath [11], etc. A more complete list of references in this area can be obtained from the references within the above references. Some of the above mentioned researchers addressed the problem of reorientation of the space vehicle by using the joint motion of the robot. The motivation was to utilize the nonholonomic property of the space robotic system and achieve reorientation in the absence of external generalized forces.

With growing interest in the area of motion planning of multibody systems in space, the problem of reorientation of a space robot using internal motion has been revisited. Some of the recent work in the area of motion planning are due to Reyhanoglu and McClamroch [10], Yamada and Yoshikawa [16], Walsh and Sastry [15], and Mukherjee and Zurowski [7]. In these works, the approach to the nonholonomic motion planning problem has been based on the theory of integration on manifolds where a closed path in the joint space of the robot was appropriately chosen to produce a desired change in the orientation of the space vehicle. It is possible to find these closed paths in the space of the independent variables that produce a change in the dependent variables because of the nonholonomic nature of the constraint.

The configuration variables of a holonomic system evolve in a way that when the independent variables move along closed trajectories, the dependent variables also move along closed trajectories. This property of repeatability is ensured if the differential constraints of motion of a system are integrable, and naturally holonomic systems exhibit this property. However, it should be realized that systems with nonintegrable differential constraints may also exhibit this property for certain closed spatial trajectories of the independent variables. In other words, the integrability of the differential constraints is only a sufficient condition for repeatability, it is by no means a necessary condition. In the case of the space robot, closed paths in the joint space do not usually produce closed paths in the workspace because of the associated change in the orientation of the space vehicle. But it can be shown that there exist closed paths in the joint space such that the space vehicle does not undergo any net change in its orientation as the joints move along these closed paths. These closed paths are like "holonomic loops" on which the nonholonomic space robot system exhibit holonomic behavior globally. It is important to find such closed paths because a space robot may be required to perform a task repeatedly in its workspace without any drift in its configuration variables.

In section 2 of this paper we derive a necessary condition for the repeatability in the motion of free-flying planar space robots. All nonholonomic systems do not exhibit repeatability or pseudo-holonomic behavior; we show this with an example in section 2. In section 3, we discuss a simple method for planning repeatable trajectories for space robots. Section 4 contains some of the results that we have obtained from simulations.

2 Necessary Condition for Repeatability

We consider a space robot with two links mounted on a space vehicle as shown in Fig.1. The Cartesian coordinates of the end-effector x_E, y_E have a functional dependence of the form

$$\begin{aligned} x_E &= f_1(x_0, y_0, \theta_0, \theta_1, \theta_2) \\ y_E &= f_2(x_0, y_0, \theta_0, \theta_1, \theta_2) \end{aligned} \quad (1)$$

where x_0 and y_0 are the coordinates of the center of mass of the space vehicle, θ_0 is the orientation of the vehicle, and θ_1 and θ_2 are the joint variables. The motion of the center of mass of the space vehicle is governed by the holonomic constraint due to linear momentum conservation. For zero initial linear momentum, this can be reduced to the form

$$x_0 = f_3(\theta_0, \theta_1, \theta_2), \qquad y_0 = f_4(\theta_0, \theta_1, \theta_2) \quad (2)$$

Since we are looking into the repeatability problem of a planar space robot, we consider closed trajectories of the joint variables. If the orientation of the space vehicle trace a closed curve as the joints move along a closed trajectory, it is clear from Eqs.(1) and (2) that all the configuration variables including x_0, y_0, x_E, and y_E will move along closed trajectories. This is not true in the general case.

When the joints move along closed trajectories, and the system maintains zero angular momentum, the change in the orientation of the space vehicle is expressed as a line integration along the closed path in the joint space. This line integral may be conviniently expressed as a surface integral using the generalized Stokes' theorem on a manifold. If D is an oriented manifold of dimension k, and if ω is a $(k-1)$-form on D, then from Stokes' theorem [2] we have

$$\int_{\partial D} \omega = \int_D d\omega \quad (3)$$

where, ∂D is the path of the line integration and is the boundary of the domain D, and $d\omega$ is a differential k-form obtained by exterior differentation of ω. In the case of a space robot with two links, the domain D is a two dimensional manifold, and the differential 1-form on D is given as

$$\begin{aligned} d\theta_0 &= -\left(\frac{B}{A}\right) d\theta_1 - \left(\frac{C}{A}\right) d\theta_2 \\ &= g_1(\theta_1, \theta_2)\, d\theta_1 + g_2(\theta_1, \theta_2)\, d\theta_2 \end{aligned} \quad (4)$$

where, θ_0 is the orientation of the space vehicle, θ_1 and θ_2 are the joint variables of the manipulator, and A, B, and C are functions of θ_1 and θ_2. The expressions for A, B, and C are defined in the Appendix.
Using Stokes' theorem, the line integration of $d\theta_0$ along a path ∂D on the two-dimensional manifold D of θ_1 and θ_2 is expressed as

$$\begin{aligned} \int_{\partial D} d\theta_0 &= \int_{D^\alpha} \left[\frac{\partial g_2}{\partial \theta_1} - \frac{\partial g_1}{\partial \theta_2}\right] d\theta_1 \wedge d\theta_2 \\ &= \int_D \left[\frac{\partial g_2}{\partial \theta_1} - \frac{\partial g_1}{\partial \theta_2}\right] (d\theta_1 \wedge d\theta_2) \cdot \alpha \\ &= \pm \int_D \left[\frac{\partial g_2}{\partial \theta_1} - \frac{\partial g_1}{\partial \theta_2}\right] d\theta_1 d\theta_2 \end{aligned} \quad (5)$$

where "$\wedge$" denotes the exterior product, and α, the orientation of D has the same orientation as $dx_1 \wedge dx_2$ when the direction along the path is counterclockwise, otherwise α has the same orientation as $dx_2 \wedge dx_1$. The symbol "$\cdot$" in Eq.(5) denotes the inner product operation between $dx_1 \wedge dx_2$ and α.

If the constraint given by Eq.(4) were a holonomic constraint, then we would have

$$\frac{\partial g_2}{\partial \theta_1} = \frac{\partial g_1}{\partial \theta_2} \quad (6)$$

Then the change in the variable θ_0 would be zero for all closed paths in the domain D because of the integrable nature of the constraints. This would ensure repeatability. Our contention is that integrability is a sufficient condition for repeatability but is not a necessary condition. For a space robot where the condition given by Eq.(6) does not hold good, repeatability can be still achieved for specific closed paths in the domain D. Let us define

$$\left(\frac{\partial g_2}{\partial \theta_1} - \frac{\partial g_1}{\partial \theta_2}\right) \equiv F(\theta_1, \theta_2) \quad (7)$$

Then we realize that the change in the orientation of the space robot for a positive direction of travel is equivalent to

$$\begin{aligned} \int_{\partial D} d\theta_0 &= \int_D F(\theta_1, \theta_2)\, d\theta_1 d\theta_2 \\ &= F(\theta_1^*, \theta_2^*) \int_D d\theta_1 d\theta_2 = F(\theta_1^*, \theta_2^*)\, \pi(D) \\ (\theta_1^*, \theta_2^*) &\in D \end{aligned} \quad (8)$$

where, Eq.(8) was obtained by the application of the mean value theorem of integral calculus. The function F can be shown to be continuous in the entire domain D and hence the mean value theorem applies. θ_1^* and θ_2^* denote some point within the domain D, and $\pi(D)$ is the measure of the domain D; in this case it is simply equal to the area enclosed within the closed curve ∂D. $F(\theta_1^*, \theta_2^*)$ can also be interpreted as the mean value of the function F, defined in Eq.(7), taken over the domain D. If this mean value happens to be zero, then we would have a zero net change in the orientation of the space vehicle when the joints move along closed paths. This would ensure repeatability in the motion of the end-effector in the workspace. We are now ready to state the necessary condition for the repeatable motion of the space vehicle.

Proposition: A necessary condition for the repeatable motion of the space vehicle is that the closed path ∂D which is the boundary of the domain D in the joint space should enclose at least one point where the function F defined by Eq.(7) is equal to zero.

The proof of the proposition stated above is quite straightforward and is left to the reader.

Before we go on to the next section we would just like to point out that all nonholonomic systems do not exhibit pseudo-holonomic behavior. In the classical example of the rolling disk [4], the two nonholonomic constraints are

$$\begin{aligned} dx - r\,\sin\alpha\, d\theta &= 0 \\ dy - r\,\cos\alpha\, d\theta &= 0 \end{aligned} \tag{9}$$

It can be shown that the change in the dependent variables for closed loop motion of the independent variables θ and α are given by

$$\begin{aligned} \int_{\partial D} \begin{pmatrix} dx \\ dy \end{pmatrix} &= \pm \int_D rF(\theta,\alpha)d\theta d\alpha \\ F(\theta,\alpha) &\equiv \begin{pmatrix} -\cos\alpha & \sin\alpha \end{pmatrix}^T \end{aligned} \tag{10}$$

Since the function $F(\theta,\alpha)$ is not equal to zero at any point in the space of θ and α, the rolling disk does not satisfy the necessary condition for repeatability. Consequently, it does not admit repeatable motions.

3 Planning Repeatable Paths for Planar Space Robots

In this section we present a simple method to plan repeatable paths for the space robot, shown in Fig.1. All paths that will ensure repeatability will have to satisy the necessary condition for repeatability, developed in the last section. Therefore, we first take a look at all points in the θ_1-θ_2 space where the function $F(\theta_1,\theta_2)$ in Eq.(7) is identically zero. The set of all such points constitute a smooth curve, as seen in Fig.2.

We assume our closed path to have an elliptical shape. This path, as seen in Fig.3, can be parameterized as follows:

$$\begin{aligned} \theta_1 &= \theta_{10} + a\cos\phi\cos 2\pi t - b\sin\phi\sin 2\pi t \\ \theta_2 &= \theta_{20} + a\sin\phi\cos 2\pi t + b\cos\phi\sin 2\pi t \\ t &\in [0,1] \end{aligned} \tag{11}$$

where, a, and b are the major and minor axes of the ellipse, ϕ is the angle of inclination of the ellipse with the θ_1 axis, and θ_{10} and θ_{20} are the coordinates of the center of the ellipse. The velocities of the joints of the manipulator can be easily obtained from the above equation as a function of time. Consequently, the rate of change of the orientation of the space vehicle can be obtained from Eq.(4) as a function of time.

We start with an initial elliptical path which is characterized by the parameters θ_{10}, θ_{20}, a, b, and ϕ. The initial choices of these parameters are quite arbitrary. We only make sure that (1) the ellipse encompasses at least one point where the function F defined by Eq.(7) is equal to zero, and (2) the elliptical path lies in the workspace of the robot. Condition 1 can be easily satisfied by considering Fig.2 which provides the set of all points where the function F vanishes. Condition 2 can be taken care of by applying methods discussed in [9].

Our goal is now to change the five parameters of the ellipse so that the value of the surface integral given by Eq.(5) is equal to zero. Of the five different parameters a and b are not allowed to change independent of one another. This is because we want to eliminate the trivial solution where the surface integral is zero because the area of the closed path is equal to zero. One simple way to avoid this situation is to impose the restriction that the area of the ellipse is a constant. This is equivalent to the constraint

$$a\,db + b\,da = 0 \tag{12}$$

We define a function V as follows

$$V = \zeta^2, \qquad \zeta \equiv \int_D F(\theta_1,\theta_2)\; d\theta_1 d\theta_2 \tag{13}$$

and solve the unconstrained minimization problem by implicitly assuming that a and b are dependent. While there are many methods for unconstrained minimization, we choose the simplest method of steepest descent [14]. Other alternative methods that can be used are the conjugate direction method by Fletcher and Reeves [3], and the variable metric method [14] that offer improvement over the method of steepest descent. In our case the method of steepest descent works well and therefore we adopted it only for its simplicity.

The correct choice of the independent parameters θ_{10}, θ_{20}, ϕ, and a that provied us with the steepest direction of descent of the function V are computed as

$$\begin{aligned} d\theta_{10} &= -\zeta\,\frac{\partial\zeta}{\partial\theta_{10}}, & d\theta_{20} &= -\zeta\,\frac{\partial\zeta}{\partial\theta_{20}}, \\ d\phi &= -\zeta\,\frac{\partial\zeta}{\partial\phi}, & da &= -\zeta\,\frac{\partial\zeta}{\partial a} \end{aligned} \tag{14}$$

In Eq.(14), the quantities $(\partial\zeta/\partial\theta_{10})$, $(\partial\zeta/\partial\theta_{20})$, $(\partial\zeta/\partial\phi)$, and $(\partial\zeta/\partial a)$ are computed by numerical partial differentiation. While computing the term $(\partial\zeta/\partial a)$ it has to be remembered that a change in a is accompanied by a change in b given by the constraint in Eq.(12).

The optimization technique discussed above provides us with a systematic way to reach the local minimum value of the function V. If this minimum value is zero, then we have converged upon the desired path around which the space robot will exhibit holonomic behavior. In the general case, the method of steepest descent does not guarantee the convergence of a function to its global minimum value. However, in our case the method always converged to the global minimum value of $V = 0$, because of the particular nature of the function F in Eq.(7).

4 Simulations

We carried out several computer simulations. Here we present results of one particular case. The kinematic and dynamic parameters of the planar space robot were chosen to be

Table of Kinematic and Dynamic Parameters

	Mass (kg)	Inertia (kg-m^2)	Length (m)
Vehicle	27.440	1.520	$r = 0.20$
Link-1	5.380	0.115	$l_1 = 0.50$
Link-2	2.640	0.028	$l_2 = 0.35$

The initial parameters of the elliptical path were arbitrarily chosen as

$$\begin{aligned} a &= 1.50000, \quad b = 1.00000, \quad \phi = 0.75000 \\ \theta_{10} &= 0.50000 \quad \theta_{20} = 0.50000 \end{aligned} \tag{15}$$

where the units are in radians. For these set of values, the numerical value of the surface integral ζ was found to be $\zeta = -0.162775$. The convergence criterion was set at $|\,\zeta\,| \leq 1.0 \times 10^{-8}$. The values of the path parameters after convergence were

$$\begin{aligned} a &= 1.31117, \quad b = 1.14381, \quad \phi = 0.79302 \\ \theta_{10} &= 0.34094 \quad \theta_{20} = -0.07054 \end{aligned} \tag{16}$$

The two elliptical paths are shown in Fig.4. Ellipse I corresponds to the initial choice of the path parameters given by Eq.(15) for which the value of $\zeta = -0.162775$. Ellipse II corresponds to the optimized values of the path parameters given by Eq.(16) and the value of ζ for this path was $\zeta = -9.9636 \times 10^{-9}$. The sinusoidal curve in Fig.2 is inset in Fig.4. This curve passes through both paths I and II and therefore these paths satisfy the necessary condition discussed in section 2.

Figures 5 and 6 depict the motion of the end-effector of the space robot for 20 cycles for the elliptical paths I and II respectively. The end-effector configuration is seen to drift in Fig.5 but has negligible drift for the closed path in Fig.6. The magnitude of the drift was computed to be approximately 76.96 mm/cycle in the case of path I whereas it was only 0.87 mm/cycle for path II.

5 Conclusion

One of the objectives of this paper was to show that integrability is merely a sufficient condition for repeatability; it is by no means a necessary condition. We elucidated our point through the example of a planar space robot. Though a space robot does not admit repeatable motion in the general case due to its nonholonomy, it is possible to find exceptions to the rule. We showed in this paper that there exist "holonomic loops" on which the space robot exhibit holonomic behavior globally. We presented a simple method based on optimization techniques to plan these trajectories that result in repeatable motion of the planar space robot.

References

[1] Alexander, H. L. and Cannon, R. H., 1987, "Experiments on the Control of a Satellite Mounted Manipulator", Proc. American Control Conference.

[2] Arnold, V. I., 1989, "Mathematical Methods of Classical Mechanics", Springer Verlag.

[3] Fletcher, R., and Reeves, C. M., 1964, "Function Minimization by Conjugate Gradients", "Computer Journal", Vol. 7, No. 2, pp. 149-154.

[4] Greenwood, D. T., 1988, "Principles of Dynamics", Prentice Hall, Englewood Cliffs, NJ.

[5] Longman, R. W., Lindberg, R. E., and Zedd, M. F., "Satellite Mounted Robot Manipulators: New Kinematics and Reaction Moment Compensation", International Journal of Robotics Research, Vol. 6, No. 3, pp. 87-103.

[6] Miyazaki, F., Masutani, Y., and Arimoto, S., 1988, "Sensor Feedback using approximate Jacobian", USA-Japan Symposium on Flexible Automation, pp. 139-145.

[7] Mukherjee, R., and Zurowski, M., 1993, "Reorientation of a structure in space using a three link rigid manipulator", AIAA Conference on Guidance, Control and Navigation, Monterey, CA.

[8] Nakamura, Y., and Mukherjee, R., 1991, "Nonholonomic Motion Planning of Space Robots via a Bi-Directional Approach", IEEE Transactions on Robotics and Automation, Vol. 7, No.4, pp. 500-514.

[9] Papadopoulos, E., and Dubowsky, S., 1989, "On the Dynamic Singularities in the Control of Free-Floating Space Manipulators", ASME Winter Annual Meeting: Dynamics and Control of Multibody/Robotic Systems with Space Applications, DSC-Vol. 15, pp. 45-51.

[10] Reyhanoglu, M., and McClamroch, N. H., 1992, "Planar Reorientation Maneuvers of Space Multibody Systems using Internal Controls", AIAA Journal of Guidance, Control, and Dynamics, Vol. 15, No. 6, pp. 1475-1480.

[11] Sreenath, N., 1992, "Nonlinear Control of Planar Multibody Systems in Shape Space", Mathematics of Control, Signals and Systems, Vol. 5, pp. 343-363.

[12] Umetani, Y., and Yoshida, K., 1987, "Continuous Path Control of Space Manipulators Mounted on OMV", Acta Astronautica, Vol. 15, No. 12, pp. 981-986.

[13] Vafa, Z., and Dubowsky, S., 1987, "On the Dynamics of Manipulators in Space Using the Virtual

Manipulator Apoproach", IEEE International Conference on Robotics and Automation, (Raleigh, NC), pp. 579-585.

[14] Vanderplaats, G. N., "Numerical Optimization Technologies for Engineering Design", McGraw Hill Book Company, San Francisco.

[15] Walsh, G., and Sastry, S., 1991, "On Reorienting Linked Rigid Bodies using Internal Motion", Proc. IEEE Conference on Decision and Control, (Brighton, England), pp. 1190-1195.

[16] Yamada, K., and Yoshikawa, S., 1992, "Arm Trajectory Design of a Space Robot", Proc. of the 18th International Symp. on Space Technology and Science, Kagoshima, Japan.

Appendix

The terms A, B, and C in Eq.(4) are defined as follows

$$
\begin{aligned}
A \equiv I_t \quad &+ \quad \frac{1}{M} r^2 m_0 (m_1 + m_2) \\
&+ \quad \frac{l_1^2}{4M}(m_0 m_1 + m_1 m_2 + 4 m_0 m_2) \\
&+ \quad \frac{l_2^2}{4M} m_2 (m_0 + m_1) \\
&+ \quad \frac{1}{M} m_0 (m_1 + 2m_2) r l_1 cos\, \theta_1 \\
&+ \quad \frac{1}{M} m_2 (m_0 + 0.5 m_1) l_1 l_2 cos\, \theta_2 \\
&+ \quad \frac{1}{M} m_0 m_2 r l_2 cos(\theta_1 + \theta_2)
\end{aligned}
$$

$$
\begin{aligned}
B \equiv I_1 \quad &+ \quad I_2 + \frac{l_1^2}{4M}(m_0 m_1 + m_1 m_2 + m_0 m_2) \\
&+ \quad \frac{l_2^2}{4M} m_2 (m_0 + m_1) \\
&+ \quad \frac{2}{M} m_0 (m_1 + 2m_2) r l_1 cos\, \theta_1 \\
&+ \quad \frac{1}{M} m_2 (m_0 + 0.5 m_1) l_1 l_2 cos\, \theta_2 \\
&+ \quad \frac{1}{2M} m_0 m_2 r l_2 cos(\theta_1 + \theta_2)
\end{aligned}
$$

$$
\begin{aligned}
C \quad &\equiv \quad I_2 + \frac{l_2^2}{4M} m_2 (m_0 + m_1) + \frac{1}{2M} m_0 m_2 l_1 l_2 cos\, \theta_2 \\
+ \quad &\frac{1}{2M} m_0 m_2 r l_2 cos(\theta_1 + \theta_2)
\end{aligned}
$$

where, m_0, m_1, and m_2 are the masses of the space vehicle and the two links of the manipulator, I_0, I_1, and I_2 are the moment of inertias of the space vehicle and the two links about their center of masses, r is the distance of the first joint from the center of mass of the vehicle, l_1 and l_2 are the lengths of the two links, $M \equiv m_0 + m_1 + m_2$, and $I_t \equiv I_0 + I_1 + I_2$.

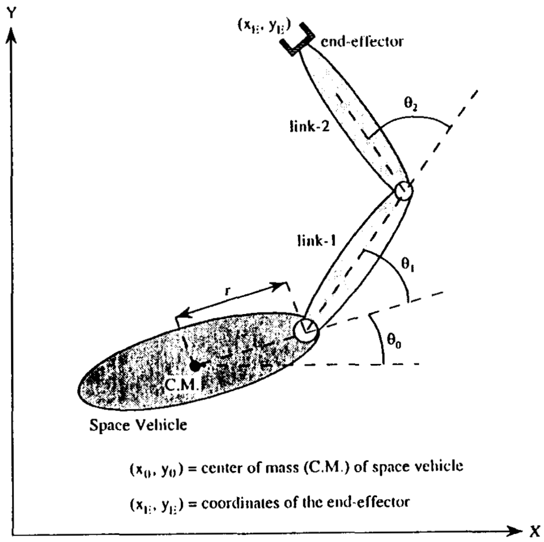

Figure 1. A planar space robot with two links is capable of exhibiting pseudo-holonomic behavior.

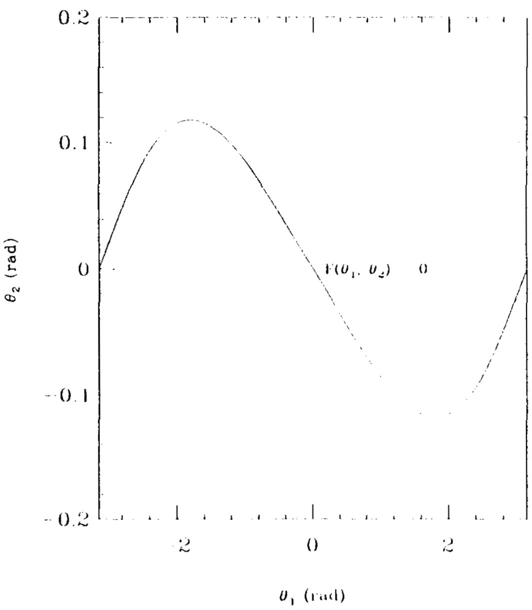

Figure 2. All the points in the θ-θ_2 space where $F(\theta_1, \theta_2) = 0$.

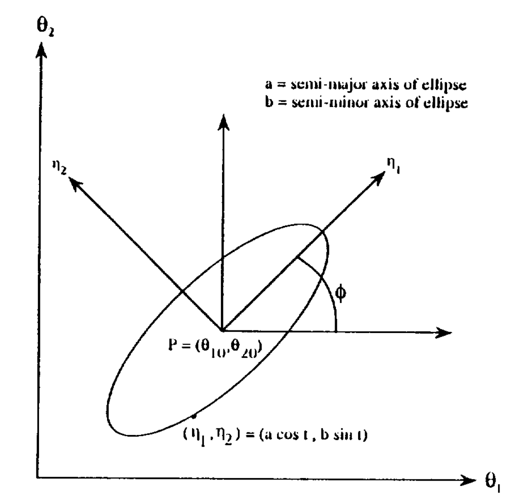

Figure 3. Parametric representation of the elliptical path in the θ_1-θ_2 space.

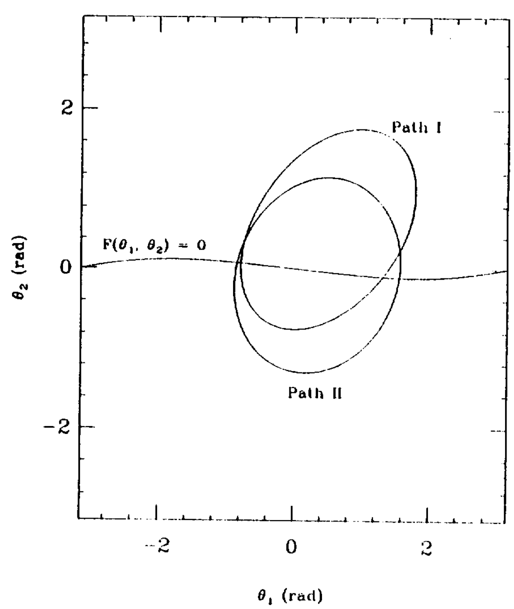

Figure 4. Elliptical paths in the joint space of the planar space robot.

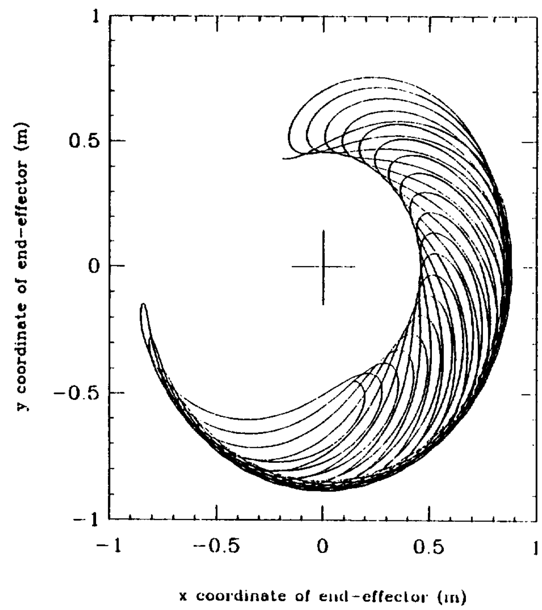

Figure 5. End-effector drift in 20 cycles for path I.

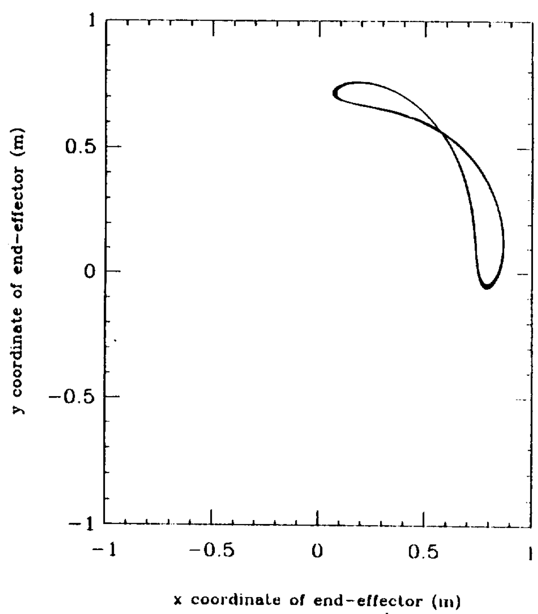

Figure 6. Repeatable end-effector motion for path II.