# NONLINEAR ADAPTIVE CONTROL OF A 6 DOF PARALLEL MANIPULATOR

**Marcel Honegger**
Institute of Robotics, ETH Zurich, Switzerland, honegger@ifr.mavt.ethz.ch

**ABSTRACT**
This paper presents the kinematic and dynamic modeling and experimental results with nonlinear adaptive control of the Hexaglide, a new 6 dof parallel manipulator that is used as a high speed milling machine.
The dynamic equations in a linear form for use in a nonlinear adaptive control scheme are first derived using a method based on the virtual work principle. These equations are then implemented on a powerful controller for real-time compensation of dynamical forces and on-line dynamic calibration.
Experimental results show that the proposed nonlinear adaptive controller is capable of identifying the dynamical parameters and that it outperforms conventional linear axis controllers by far.

## INTRODUCTION
The Hexaglide milling machine is a fully parallel structure with 6 dof (see figure 1). Its kinematics is similar to the well-known Stewart platform, but instead of variable length bars, the tool attached to the platform is guided in the workspace with constant length bars linked to 6 linear direct drive motors, that are distributed on 3 linear rails [1].
The most important advantage of such parallel manipulators over serial chain robots is certainly the possibility to keep all motors fixed to a base, with the consequence that the moved mass is much smaller and fast movements can be performed. Apart from this, parallel manipulators can also achieve a higher stiffness and are thus well suited for the application as high speed milling machine.
Fast and accurate movements require however a good control of the actuators. The closed mechanical chains make the dynamics of parallel manipulators coupled and highly nonlinear. To minimize the tracking errors these dynamical forces need to be compensated by the controller.
In order to perform a good compensation, the parameters of the dynamic model of the manipulator must be known precisely. Some of these parameters, such as masses, can be calculated from CAD-drawings or measured when the robot is disassembled. Other parameters, especially the friction coefficients, are more difficult to obtain. A simple way to identify these parameters is however to use a nonlinear adaptive control algorithm. This leads to accurate parameters and can be performed on-line, so that varying parameters, such as friction, can continuously be updated.

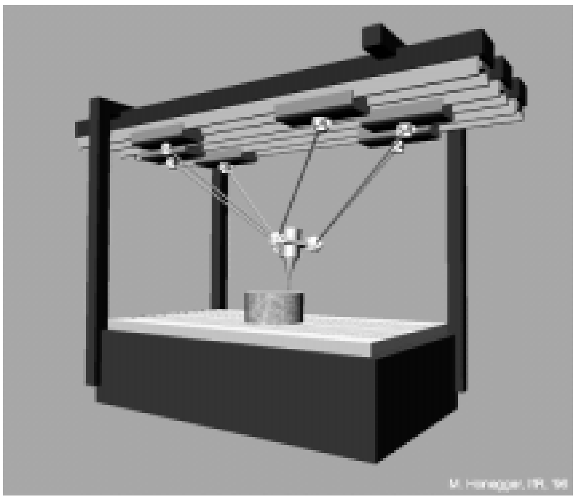


**Figure 1:** Sketch of the Hexaglide

This paper is organized as follows. In the next section the kinematic and dynamic model of the Hexaglide machine is developed. The following section describes the scheme of the nonlinear adaptive controller. Its implementation on a powerful controller hardware and experimental results are finally presented in the last section.

## KINEMATIC AND DYNAMIC MODEL

As there is no closed form solution for the direct kinematics of general parallel manipulators, the following modeling is only based on the inverse kinematics.

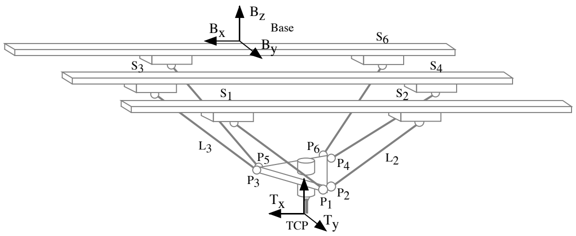


**Figure 2:** Definition of frames and parameters

### Kinematics of the Hexaglide

For modeling the Hexaglide structure, a base reference frame $B$ is defined as shown in figure 2. A second frame $T$ is attached to the tool center point (TCP) of the robot. The points linking the legs to the platform are noted $P_i$, $i = 1 \ldots 6$, and each leg is attached to the linear motor at the point $S_i$, $i = 1 \ldots 6$. The pose of the TCP is represented by the vector

$$x = \begin{bmatrix} x & y & z & \alpha & \beta & \gamma \end{bmatrix}^T, \tag{1}$$

where $x$, $y$, $z$ are the cartesian positions of the TCP and $\alpha$, $\beta$, $\gamma$ the fixed angles ZYX representation of its rotation [2]. The rotation matrix between the frames $T$ and $B$ is thus given by:

$${}^B_T R = R_x(\alpha) \cdot R_y(\beta) \cdot R_z(\gamma). \tag{2}$$

Using the rotation matrix (2), the platform joint positions $P_i$ can be transformed to the base reference frame $B$. The joint coordinates $q_i$ can then be calculated using the following equation:

$$q_i = {}^B P_{i,x} + (-1)^i \cdot \sqrt{l_i^2 - \Delta y_i^2 - \Delta z_i^2} \tag{3}$$

where

$$\Delta y_i = S_{i,y} - {}^B P_{i,y} \text{ and } \Delta z_i = S_{i,z} - {}^B P_{i,z}.$$

The joint angles finally lead to the joint vector

$$q = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 & q_5 & q_6 \end{bmatrix}^T. \tag{4}$$

The Jacobian matrix $J$ that gives the relation between joint velocities and the cartesian velocity of the TCP can be derived using the partial differentiation of the inverse geometric model of the machine. $J$ is however representation dependent. In order to find the instantaneous linear and angular velocities of the TCP, it has to be multiplied by the representation dependent matrix $E^+$:

$$\dot{x}_{0T} = E^+ J \dot{q} = J_{0T} \dot{q} \tag{5}$$

with

$$E^+ = \begin{bmatrix} I_{3\times 3} & 0_{3\times 3} & & \\ & 1 & 0 & \sin\beta \\ 0_{3\times 3} & 0 & \cos\alpha & -\cos\beta\sin\alpha \\ & 0 & \sin\alpha & \cos\beta\cos\alpha \end{bmatrix}$$

The joint acceleration $\ddot{q}$ and the linear as well as the angular acceleration of the TCP can finally be obtained by numerical differentiation of $\dot{q}$ and $\dot{x}_{0T}$ respectively.

### Dynamic equations

Using the inverse kinematic model, the dynamic equations can be obtained in the cartesian space in the following form [2,3]:

$$\tau = M(x)\ddot{x} + V(x, \dot{x})\dot{x} + G(x) + F(\dot{x}) \tag{6}$$

$M(x)$ denotes the mass matrix of the system, $V(x, \dot{x})$ a matrix with velocity dependent forces, $G(x)$ the gravity force vector and $F(\dot{x})$ finally a vector containing friction forces. The equations in this form can be obtained using well known energy based methods such as the Lagrangian technique.
To apply an adaptation law to the control algorithm it is however necessary to rewrite the dynamic equations in a linear form in the dynamical parameters.

$$\tau = \Psi(x, \dot{x}, \ddot{x})p \tag{7}$$

In this form, $\Psi$ is a matrix containing the nonlinear equations and the states of the robot and $p$ a vector containing the dynamical parameters.
To simplify the computation, the dynamics of the six links has been neglected [4,5]. The model of the Hexaglide consists therefore of seven bodies, the six motors and the platform. A further simplification can be made by calculating the dynamics of the six motors in the joint space. The dynamic equations can then be rewritten in the following way:

$$\tau = \begin{bmatrix}\Psi_{1..6} & \Psi_7\end{bmatrix}\begin{bmatrix}p_{1..6}\\ p_7\end{bmatrix}, \tag{8}$$

where $\Psi_{1..6}$ and $p_{1..6}$ describe the dynamics of the six motor bodies and $\Psi_7$ and $p_7$ that of the platform. Determining $\Psi_{1..6}$ and $p_{1..6}$ is straight forward. The dynamic forces of the motors consist of inertia and Coulomb as well as viscous friction:

$$\Psi_{1..6} = \begin{bmatrix}\ddot{q}_1 & sign(\dot{q}_1) & \dots & 0 & \dot{q}_1 & \dots & 0\\ \dots & \dots & \dots & \dots & \dots & \dots & \dots\\ \ddot{q}_6 & 0 & \dots & sign(\dot{q}_6) & 0 & \dots & \dot{q}_6\end{bmatrix} \tag{9}$$

and

$$p_{1..6} = \begin{bmatrix}m_{1..6} & f_{c1} & f_{c2} & \dots & f_{c6} & f_{v1} & f_{v2} & \dots & f_{v6}\end{bmatrix}^T$$

For simplicity, the masses of the six motors, that should all be equal, are collected together to just one parameter $m_{1..6}$. The Coulomb and viscous friction coefficients are however considered independently.

The determination of $\Psi_7$ and $p_7$ is more complicated. A body-oriented method based on the virtual work principle is used to derive the dynamic equations of the platform in a linear form [6].

The vector of dynamical parameters is given by:

$$p_7 = \begin{bmatrix}m_7 & m_7 r_x & m_7 r_y & m_7 r_z & I_{xx} & I_{yy} & I_{zz}\end{bmatrix}^T \tag{10}$$

It consists of the mass of the platform, the first moments (location of the mass center point relative to the TCP) and the inertia moments $I_{xx}$, $I_{yy}$ and $I_{zz}$. The frame attached to the TCP is supposed to be oriented such that the inertia moments $I_{xy}$, $I_{xz}$ and $I_{yz}$ can be neglected. The matrix $\Psi_7$ is given by

$$\Psi_7 = J_{0T}^T \begin{bmatrix}{}^B a_7 & {}^B_T R \cdot {}^T\Lambda_7 & 0_{3\times 3}\\ 0_{3\times 1} & -{}^B\hat{a}_7 \cdot {}^B_T R & {}^B_T R \cdot {}^T\psi_{\omega 7}\end{bmatrix}, \tag{11}$$

where the Jacobian matrix $J_{0T}$, defined in (5), projects the forces from the cartesian into the joint space. The acceleration $a_7$ and the skew symmetric matrix $\hat{a}_7$ corresponding to the cross product are defined as follows:

$${}^B a_7 = \begin{bmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}+g\end{bmatrix} \text{ and} \tag{12}$$

$$-{}^B\hat{a}_7 = \begin{bmatrix}0 & -\ddot{z}-g & \ddot{y}\\ \ddot{z}+g & 0 & -\ddot{x}\\ -\ddot{y} & \ddot{x} & 0\end{bmatrix}. \tag{13}$$

The matrices $\Lambda_7$ and $\psi_7$ are finally defined as

$${}^T\Lambda_7 = \begin{bmatrix}-\omega_y^2-\omega_z^2 & -\dot{\omega}_z+\omega_x\omega_y & \dot{\omega}_y+\omega_x\omega_z\\ \dot{\omega}_z+\omega_x\omega_y & -\omega_x^2-\omega_z^2 & -\dot{\omega}_x+\omega_y\omega_z\\ -\dot{\omega}_y+\omega_x\omega_z & \dot{\omega}_x+\omega_y\omega_z & -\omega_x^2-\omega_y^2\end{bmatrix}, \tag{14}$$

$${}^T\psi_{\omega 7} = \begin{bmatrix}\dot{\omega}_x & -\omega_y\omega_z & \omega_y\omega_z\\ \omega_x\omega_z & \dot{\omega}_y & -\omega_x\omega_z\\ -\omega_x\omega_y & \omega_x\omega_y & \dot{\omega}_z\end{bmatrix}, \tag{15}$$

where the angular velocity $\omega$ of the platform in the tool coordinate system is given by

$${}^T\omega_7 = {}^B_T R^T E_r^+ \begin{bmatrix}\dot{\alpha}\\ \dot{\beta}\\ \dot{\gamma}\end{bmatrix} \tag{16}$$

with

$$E_r^+ = \begin{bmatrix}1 & 0 & \sin\beta\\ 0 & \cos\alpha & -\cos\beta\sin\alpha\\ 0 & \sin\alpha & \cos\beta\cos\alpha\end{bmatrix}. \tag{17}$$

The dynamic equations are now given in a compact linear form, that can be used for real-time computation in a nonlinear adaptive control scheme.

## NONLINEAR ADAPTIVE CONTROL

In the last years, many nonlinear adaptive control schemes have been investigated [7,8,9,10]. These approaches require however that the dynamic model can be calculated in the joint space from actual joint coordinates. As the dynamic model of fully parallel manipulators can only be calculated in the cartesian space, and because calculating the actual cartesian position using a numerical algorithm for the direct kinematics is very time consuming, a control scheme is required that computes the dynamics from desired values.

A further assumption with most control schemes is that the velocity of the actuators $\dot{q}$ can be measured. With the Hexaglide and in many other robotic applications,

this is however not the case. Some researchers were dealing with this problem and proposed to use nonlinear velocity observers or linear observers in combination with nonlinear controllers [11,12,13].
For controlling the Hexaglide, a scheme is proposed that is similar to the so called computed torque controller [8], with the modification that the dynamic model is computed from desired values instead of actual joint coordinates. The control scheme is shown in figure 3 below.

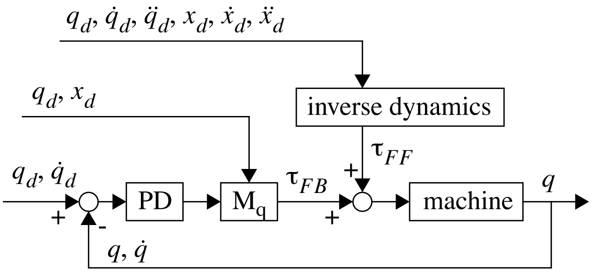


**Figure 3:** Scheme of model based controller

It consists mainly of a feedforward part of the inverse dynamics and a nonlinear feedback loop. The control law is as follows:

$$\tau = \Psi p + M_q \ddot{q}_{FB} \tag{18}$$

where $\Psi p$ is the inverse dynamic model in a linear form (equation 8), calculated from desired values.

$$\Psi p = \begin{bmatrix} \Psi_{1..6}(\dot{q}_d, \ddot{q}_d) & \Psi_7(x_d, \dot{x}_d, \ddot{x}_d) \end{bmatrix} \begin{bmatrix} p_{1..6} \\ p_7 \end{bmatrix} \tag{19}$$

The feedback part is calculated as follows. The mass matrix that relates the joint accelerations with the actuator forces can be determined as

$$M_q = M_{1..6}(q_d) + J^T M_7(x_d) J \tag{20}$$

and the feedback of position and velocity errors as

$$\ddot{q}_{FB} = K_p(q_d - q) + K_d(\dot{q}_d - \dot{\hat{q}}). \tag{21}$$

The controller gains $K_p$ and $K_d$ can be determined by pole placement to obtain a critically damped system as follows:

$$K_p = \omega^2 I \quad \text{and} \quad K_d = 2\omega I \tag{22}$$

As the velocities $\dot{q}$ of the six motors can not be measured directly, they have to be observed from measured motor positions. The following linear 2nd order velocity observer is used:

$$\dot{\hat{q}} = z + H_1(q - \hat{q}) \tag{23}$$

$$\dot{z} = \ddot{q}_d + \ddot{q}_{FB} + H_2(q - \hat{q})$$

with $H_1 = 2\omega_0 I$ and $H_2 = \omega_0^2 I$.

The poles of the observer are set by pole-placement also. They should be chosen so that the observer has a higher bandwidth than the controller.

For the adaptation of the dynamical parameters $p$ the following algorithm that is based on the minimization of tracking errors is used [8]:

$$\dot{p} = \Gamma \Psi^T M_q^{-1} \ddot{q}_{FB} \tag{24}$$

$\Gamma$ is a positive definite matrix composed of learning factors, to tune the speed of adaptation of the different parameters. $p$ is then adapted by integration of (24).

## IMPLEMENTATION AND RESULTS

The implementation of the proposed control algorithm requires a fast controller hardware. Therefore the controller is based on a 300MHz PowerPC 604e RISC-Processor board in a VME chassis. To control the motors in torque mode, measure the motor positions etc., the VME system is further equipped with fast D/A converters, encoder counters and digital IO boards.

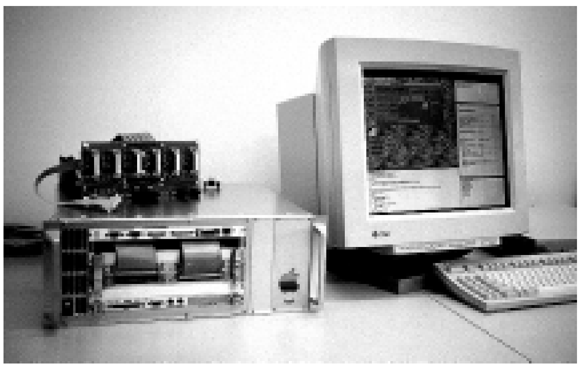

**Figure 4:** Controller hardware and XOberon development environment

All processes such as the control algorithm, the dynamics computation or the path planner are implemented as time critical tasks running on top of XOberon [14], an object-oriented hard real-time operating system with a deadline driven scheduler.
The closed loop controller as well as the path planning process together with the coordinate transformation have a sampling time of 300$\mu$s, while the dynamic model is calculated every 900$\mu$s. The adaptation of dynamical

parameters and other tasks like NC-kernel and security processes are however only called every few ms.

The following figure shows the prototype of the Hexaglide milling machine, developed by the Institute of Machine Tools at the ETH, on which the experiments have been performed.

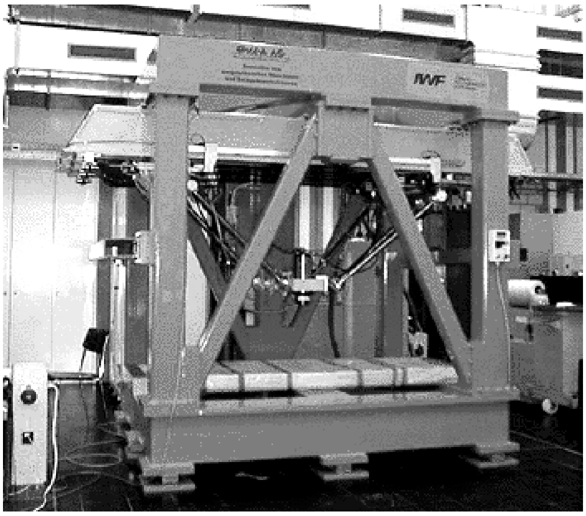

**Figure 5:** Hexaglide milling machine

With a length of the linear guideways at the upper base of 3 m and leg-lengths of 1 m this machine can achieve a workspace of about 500x500x300 mm. The linear direct drives that move the upper joints on the guideways have a maximum force of 2500 N each. The maximum velocity of the TCP is 1 m/s and the acceleration about 1 g in every direction within the workspace.

In order to test the nonlinear controller, a set of dynamical parameters for the model has to be identified first.

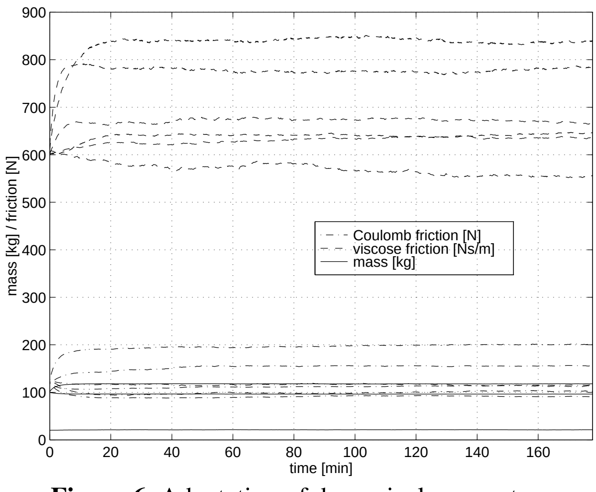


**Figure 6:** Adaptation of dynamical parameters

This identification is performed by the adaptation algorithm. Figure 6 shows the evolution of these parameters, starting from first estimates, while the machine performs a simple translational movement.
The actual parameters are learned within 5 to 10 minutes. While the mass parameters remain constant after a few minutes, the varying friction parameters, especially the viscous frictions, are continuously updated.

| | |
|---|---|
| motor mass $m_{1..6}$ | 97.6 kg |
| motor friction $f_c$ | 91 - 201 N |
| motor friction $f_v$ | 555 - 839 Ns/m |
| platform mass $m_7$ | 118.7 kg |
| 1st moment $m_7r_z$ | 21.3 kgm |
| 2nd moment $I_{xx}$ | 7.4 kgm$^2$ |
| 2nd moment $I_{yy}$ | 7.7 kgm$^2$ |
| 2nd moment $I_{zz}$ | 2.9 kgm$^2$ |

**Table 1:** “Learned” dynamical parameters

The following figure shows results of circle trajectories with different velocities, when the actuators are controlled with conventional linear PD controllers with a static stiffness of 20 N/$\mu$m. For a better illustration the errors in the plot are 40x amplified. It can easily be seen that increasing the velocity of the TCP leads to increased tracking errors.

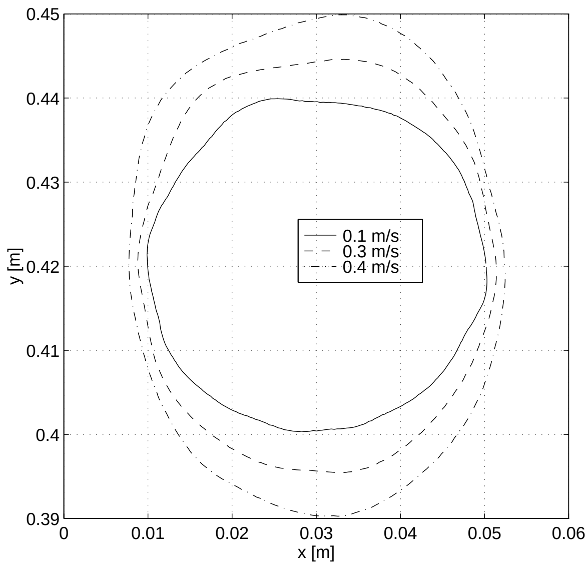


**Figure 7:** Circle test with linear axis controllers

The next plot shows a comparison of results obtained with linear axis controllers and with the nonlinear adaptive controller. It is the same circle trajectory, with a velocity of 0.4 m/s.

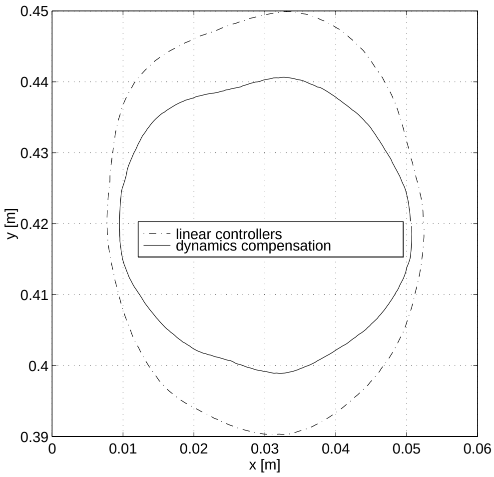


**Figure 8:** Linear vs. nonlinear control

| values in [$\mu$m] | min | max | Ø | σ |
|---|---|---|---|---|
| PD, 0.1 m/s | -17 | 19 | -2 | 9 |
| PD, 0.3 m/s | 24 | 127 | 73 | 34 |
| PD, 0.4 m/s | 56 | 253 | 143 | 66 |
| CTC, 0.4 m/s | -11 | 34 | 13 | 10 |

**Table 2:** Statistics of errors along the circle trajectory

The nonlinear controller is able to keep the tracking errors much smaller than the linear axis controllers. Even at high speed motion, a good accuracy is achieved, that conventional linear controllers are only able to achieve when moving slowly.

## CONCLUSIONS

Due to their potential advantages for high-speed motion and accuracy, parallel mechanisms have currently caught the interest of the machine-tool community. This paper showed that these mechanisms can attain a high accuracy only when a nonlinear control is used, and provided methods for implementing such a controller.

A nonlinear adaptive controller with velocity observer was designed and implemented on a 6 dof parallel manipulator with direct drive actuators. The experiments showed that the nonlinear controller reduced the tracking errors by a factor of about 10 relative to linear joint controllers, when fast movements were performed.

## REFERENCES

1. Wiegand, A., Hebsacker, M., Honegger, M.,
   Parallele Kinematik und Linearmotoren: Hexaglide - ein neues, hochdynamisches Werkzeugmaschinenkonzept, Technische Rundschau, Nr. 25, 1996.
2. Craig, J.J.,
   Introduction to Robotics, Mechanics and Control, 2nd Edition, Addison-Wesley Publishing Company, 1989.
3. Lebret, G., Liu, K., Lewis, F.L.,
   Dynamic Analysis and Control of a Stewart Platform Manipulator, Journal of Robotic Systems, Vol. 10, No. 5, 1993.
4. Ji, Z.,
   Study of the effect of Leg Inertia in Stewart Platforms, Proc. IEEE ICRA, Atlanta GA, USA, 1993.
5. Honegger, M.,
   Steuerung und Regelung einer Stewart-Plattform als Werkzeugmaschine, Internal Report, Institute of Robotics, ETHZ, 1996.
6. Codourey, A., Burdet, E.,
   A Body-oriented Method for Finding a Linear Form of the Dynamic Equation of Fully Parallel Robots, Proc. IEEE ICRA, Albuquerque NM, USA, 1997.
7. Slotine, J.-J. E., Li, W.,
   On the Adaptive Control of Robot Manipulators, Int. Journal of Robotics Research, Vol. 6, No. 3, 1987.
8. Craig, J.J.,
   Adaptive Control of Mechanical Manipulators, Addison-Wesley Publishing Company, 1988.
9. Sadegh, N., Horowitz, R.,
   Stability and Robustness Analysis of a Class of Adaptive Controllers for Robotic Manipulators, Int. Journal of Robotics Research, Vol. 9, No. 3, 1990.
10. Burdet, E., Sprenger, B., Codourey, A.,
    Experiments in Nonlinear Adaptive Control: A Step Towards Applications, Proc. IEEE ICRA, Albuquerque NM, USA, 1997.
11. Canudas de Wit, C., Fixot, N.,
    Adaptive Control of Robot Manipulators via Velocity Estimated Feedback, IEEE Transactions on Automatic Control, Vol. 37, No. 8, 1992.
12. Berghuis, H., Nijmeijer, H.,
    A Passivity Approach to Controller-Observer Design for Robots, IEEE Transactions on Robotics and Automation, Vol. 9, No. 6, 1993.
13. Kaneko, K., Horowitz, R.,
    Repetitive and Adaptive Control of Robot Manipulators with Velocity Estimation, IEEE Transactions on Robotics and Automation, Vol. 13, No. 2, 1997.
14. Brega, R.,
    A Real-Time Operating System Designed for Predictability and Run-Time Safety, Proc. MOVIC'98, Zurich, Switzerland, 1998.