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Motion generation for formations of robots: a geometric approach

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Abstract

This paper develops a method for generating smooth trajectories for mobile robots in formation. The problem of trajectory generation is cast in terms of designing optimal curves on the Euclidean group, $SE(3)$. Specifically, the method generates the trajectory that minimizes the total energy associated with the translations and rotations of the robots, while maintaining a rigid formation. When the mobile robots are nonholonomic, trajectories that allow rigid formations to be maintained must satisfy appropriate constraints. An efficient non-iterative algorithm to obtain near-optimal trajectories is described. Finally, the approach is illustrated with examples involving formations of aircrafts.

1 Introduction

Multi-robotic systems are versatile and efficient in exploration missions, military surveillance, and co-operative manipulation tasks. Recent research on such systems include work on cooperative manipulation [7], multi-robot motion planning [11], mapping and exploration [6], behavior-based formation control [2], and software architectures for multi-robotic systems [8]. In all these paradigms, the motion planning and control of the *team* of robots in formation can be modeled as a triple $(\mathbf{g}, \mathbf{r}, \mathbf{H})$, where $\mathbf{g} \in SE(3)$ represents the gross position and orientation of the team (for example, the pose of the leader), $\mathbf{r}$ is a set of shape variables that describes the relative positions of the robots in the team, and $\mathbf{H}$ is a control graph which describes the control strategy used by each robot [5]. In this paper, we are primarily interested in trajectory generation, *i.e.*, the trajectory $\mathbf{g}(t)$.

The basic idea is to consider the formation of robots as a rigid body, and investigate its motion. Virtual structures have been proposed in [10] and used for motion planning and coordination and control of space-crafts in [3]. Our definition of a rigid formation requires the distances between robots, or reference points on robots, to remain fixed. Such a rigid formation is geometrically defined as a polyhedron formed by the reference points of each robot. The relative orientations of each robot are not restricted in such a rigid formation.

In this paper we build on the results from [4, 12] to generate trajectories that satisfy the rigid formation constraint and the overall energy of motion is minimized. The geometry of the formation induces a metric on $SE(3)$, which is the sum of the individual kinetic energies of the robots. In [4] we proved that this metric is naturally inherited from the ambient manifold $GA(3)$. Our method involves three steps: (1) the generation of optimal trajectories for the virtual body in $GA(3)$; (2) the projection of the trajectories from $GA(3)$ to $SE(3)$, and (3) the translation of the motion to position trajectory for each individual robot. The procedure is invariant with respect to both the parameterization of the motion and the choice of the inertial frame.

It should be noted that there are several papers that address the controllability of holonomic and nonholonomic mobile robots that are relevant to this work. For our purposes, we will cite the work of d'Andrea-Novel *et.al.* [1] who derive controllers for three-dimensional nonholonomic mobile robots for following trajectories in $\mathbb{R}^3$.

2 Background and problem formulation

2.1 The Lie groups $SO(3)$ and $SE(3)$

Let $GL(n)$ denote the general linear group of dimension n, which is a smooth manifold and a Lie group. The rotation group on $\mathbb{R}^n$ is a subgroup of the general linear group, defined as

$$SO(n) = \{R \,|\, R \in GL(n, \mathbb{R}),\, RR^T = I,\, \det R = 1\}$$

$GA(n) = GL(n) \times \mathbb{R}^n$ is the affine group. $SE(n) = SO(n) \times \mathbb{R}^n$ is the special Euclidean group, and is the set of all rigid displacements in $\mathbb{R}^n$. Special consideration will be given to $SO(3)$ and $SE(3)$.

The Lie algebras of $SO(3)$ and $SE(3)$, denoted by

$so(3)$ and $se(3)$ respectively, are given by:

$$so(3) = \{\hat{\omega} \in \mathbb{R}^{3\times 3},\ \hat{\omega}^T = -\hat{\omega}\},$$

$$se(3) = \left\{ \begin{bmatrix} \hat{\omega} & v \\ 0 & 0 \end{bmatrix} \mid \hat{\omega} \in so(3), v \in \mathbb{R}^3 \right\}$$

where $\hat{\omega}$ is the skew-symmetric matrix form of the vector $\omega \in \mathbb{R}^3$. Given a curve

$$A(t) : [-a, a] \to SE(3),\ A(t) = \begin{bmatrix} R(t) & d(t) \\ 0 & 1 \end{bmatrix}$$

an element $\zeta(t)$ of the Lie algebra $se(3)$ can be associated to the tangent vector $\dot{A}(t)$ at an arbitrary point t by:

$$\zeta(t) = A^{-1}(t)\dot{A}(t) = \begin{bmatrix} \hat{\omega}(t) & R^T\dot{d} \\ 0 & 0 \end{bmatrix} \qquad (1)$$

where $\hat{\omega}(t) = R^T\dot{R}$ is the corresponding element from $so(3)$.

Consider a rigid body moving in free space. Assume any inertial reference frame $\{F\}$ fixed in space and a frame $\{M\}$ fixed to the body at point O' as shown in Figure 1. A curve on $SE(3)$ physically represents a

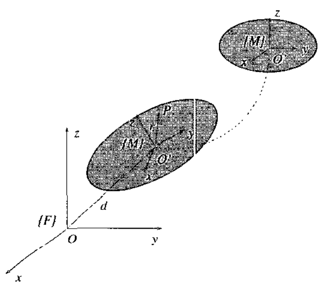

Figure 1: The inertial frame and the moving frame

motion of the rigid body. If $\{\omega(t), v(t)\}$ is the vector pair corresponding to $\zeta(t)$, then ω corresponds to the angular velocity of the rigid body while v is the linear velocity of O', both expressed in the frame $\{M\}$. In kinematics, elements of this form are called twists and $se(3)$ thus corresponds to the space of twists. The twist $\zeta(t)$ computed from Equation (1) does not depend on the choice of the inertial frame $\{F\}$.

If P is an arbitrary point on the rigid body with position vector r in frame $\{M\}$ (Figure 1), then the velocity of P in frame $\{M\}$ is given by

$$v_P = \Gamma(r)\zeta,\ \ \Gamma(r) = [\ -\hat{r} \quad I_3\ ] \qquad (2)$$

where ζ is the twist of the rigid body.

2.2 Problem formulation

Consider N robots moving (rotating and translating) in space. We choose a reference point on each robot. It is convenient (but not necessary) to fix it at the center of mass O_i of each robot. We say N robots move in *rigid formation* if the motion preserves the distances between any two reference points, or, equivalently, the reference points form a rigid polyhedron. A moving frame $\{M_i\}$ is attached to each robot at O_i. Robot i has mass m_i and matrix of inertia H_i with respect to frame $\{M_i\}$. Given the positions d_i of the centers of mass with respect to a reference frame $\{F\}$, the center of mass O' of the system of robots has coordinates $d = \sum_{i=1}^N (m_i d_i)/\sum_{i=1}^N m_i$ with respect to the same inertial frame. The formation is interpreted as a virtual rigid body with centroid O', to which we attach a moving frame $\{M\}$ (Figure 2). Let R, $R_i \in SO(3)$

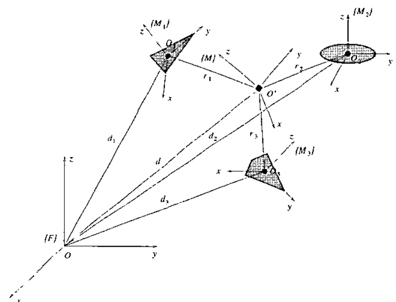

Figure 2: A formation of $N = 3$ robots of various shapes

denote the rotations of $\{M\}$ and $\{M_i\}$ in $\{F\}$. If r_i is the position vector of O_i in $\{M\}$, then moving in *rigid formation* preserves the r_i's, or, equivalently, the trajectory of each O_i in $\{F\}$ has the form:

$$d_i(t) = d(t) + R(t)r_i,\ i = 1, \dots, N \qquad (3)$$

If v_i is the velocity of O_i in $\{M_i\}$, condition (3) is the same as

$$v_i(t) = R_i(t)^T R(t)\Gamma(r_i)\zeta(t) \qquad (4)$$

provided that r_i satisfies equation (3) at some instant (say $t = 0$). $\zeta \in se(3)$ is the twist of the virtual rigid body and $\Gamma(r_i)$ is as in equation (2).

Given two poses of the system of robots d^0, R^0 at $t = 0$ and d^1, R^1 at $t = 1$ such that they have the same rigid formation, (*i.e.*, have the same r_i's) and given the individual orientations R_i^0 at $t = 0$ and R_i^1 at $t = 1$, we want to generate interpolating motion for each robot so that: (i) the rigid formation is preserved, (ii) the total kinetic energy is minimized, and (iii) the nonholonomic constraints (if any) are satisfied. To mathematically formalize the energy requirement, we need to define a meaningful metric on $SE(3)$.

2.3 Riemannian metrics

We define a left or right invariant metric in $SO(n)$ ($SE(n)$) by introducing an appropriate metric in $GL(n)$ ($GA(n)$).

For $SO(3)$, let W be a symmetric positive definite 3×3 matrix. For any $M \in GL(3)$ and any $X, Y \in T_M GL(3)$, define

$$< X, Y >_{GL} = Tr(X^T Y W) \tag{5}$$

The quadratic term in (5) is symmetric and positive definite, therefore a proper metric in $GL(3)$. Let R be an arbitrary element from $SO(3)$ and X, Y be two vectors from $T_R SO(3)$. The metric on $SO(3)$ at R inherited from $GL(3)$ is of the form

$$< X, Y >_{SO} = \omega_x^T G \omega_y \tag{6}$$

where

$$G = Tr(W) I_3 - W, \; W = \frac{1}{2} Tr(G) I_3 - G \tag{7}$$

and ω_x, $\omega_y \in \mathbb{R}^3$ are the vector forms of the corresponding twists from $so(3)$.
For $SE(3)$, similar to $SO(3)$, let

$$\tilde{W} = \left[\begin{array}{cc} W & 0 \\ 0 & w \end{array} \right] \tag{8}$$

be a symmetric positive definite 4×4 matrix, where W is the matrix of the metric given by (5) and $w \in \mathbb{R}$. If X and Y are two vectors in the tangent space at an arbitrary point $B \in GA(3)$, then the following form defines a metric:

$$< X, Y >_{GA} = Tr(X^T Y \tilde{W}) \tag{9}$$

If A is an arbitrary element from $SE(3)$ and $X, Y \in T_A SE(3)$, then the restriction of (9) to $SE(3)$ gives:

$$< X, Y >_{SE} = \left[\begin{array}{cc} \omega_x^T & v_x^T \end{array} \right] \tilde{G} \left[\begin{array}{c} \omega_y \\ v_y \end{array} \right], \tag{10}$$

$$\tilde{G} = \left[\begin{array}{cc} G & 0 \\ 0 & w I_3 \end{array} \right]$$

where G is given by (7) and $\{\omega_x, v_x\}$ is the vector representation of the twist corresponding to X.
The norm of metric (10) gives the kinetic energy of a moving body, with mass $m = 2w$ and matrix of inertia about frame $\{M\}$ placed at the centroid $H = 2G$. If $\{M\}$ is aligned with the principal axes of the body, then G is diagonal.

2.4 Optimal trajectories on $SE(3)$

We can use the norm induced by metric (9) to define the distance between elements in $GA(3)$. Using this distance, for a given $B \in GA(3)$, we define *the projection* of B on $SE(3)$ as being the closest $A \in SE(3)$ with respect to metric (9). The following result is stated and proved in [4]:

Proposition 2.1 *Let $B \in GA(3)$ with the following block partition*

$$B = \left[\begin{array}{cc} B_1 & B_2 \\ 0 & 1 \end{array} \right], \; B_1 \in GL(3), \; B_2 \in \mathbb{R}^3$$

and U, Σ, V the singular value decomposition of $B_1 W$:

$$B_1 W = U \Sigma V^T \tag{11}$$

Then the projection of B on $SE(3)$ is given by

$$A = \left[\begin{array}{cc} U V^T & B_2 \\ 0 & 1 \end{array} \right] \in SE(3) \tag{12}$$

Based on Proposition 2.1, a procedure for generating near optimal curves on $SE(3)$ follows: generate the curves in $GA(3)$ and project them on $SE(3)$. In [4], we prove that the overall procedure is left invariant (*i.e.*, the generated trajectories are independent of the choice of the inertial frame $\{F\}$). The projection method can be used to generate near optimal interpolating motion between end poses (geodesics) or poses and velocities (minimum acceleration curves). In what follows, the given boundary conditions will be denoted by $R^0, d^0, \dot{R}^0, \dot{d}^0$ at $t = 0$ and $R^1, d^1, \dot{R}^1, \dot{d}^1$ at $t = 1$. The differential equations to be satisfied by geodesics and minimum acceleration curves on $SE(3)$ equipped with metric (10) are derived in [12]. The translational part is easily integrable: $d(t) = d^0 + (d^1 - d^0)t$, $t \in [0, 1]$ for geodesics and

$$\begin{aligned} d(t) &= d^0 + \dot{d}^0 t + (-3d^0 + 3d^1 - 2\dot{d}^0 - \dot{d}^1)t^2 \\ &\quad + (2d^0 - 2d^1 + \dot{d}^0 + \dot{d}^1)t^3 \end{aligned} \tag{13}$$

for minimum acceleration curves. If the projection method is used, the rotation is given by $R(t) = U(t)V^T(t)$, where $M(t)W = U \Sigma V^T$ with

$$M(t) = [R^0 + (R^1 - R^0)t]W$$

for geodesics and

$$\begin{aligned} M(t) &= R^0 + \dot{R}^0 t + (-3R^0 + 3R^1 - 2\dot{R}^0 - \dot{R}^1)t^2 \\ &\quad + (2R^0 - 2R^1 + \dot{R}^0 + \dot{R}^1)t^3 \end{aligned} \tag{14}$$

for minimum acceleration curves.

3 Generation of smooth motion

3.1 Fully-actuated robots

In this section we will assume that each robot can actuate each of its 6 degrees of freedom. Assume the initial ($t = 0$) and final ($t = 1$) poses of each robot are given in frame $\{F\}$ and the problem is to generate interpolating motion while keeping formation and minimize the total kinetic energy of the system.

Obviously, the initial and final poses should have the same formation, *i.e.* have the same r_i's.

The total instantaneous kinetic energy is given by $T = \sum_{i=1}^N \zeta_i^T \tilde{G}_i \zeta_i$ where $\zeta_i = [\omega_i^T \; v_i^T]^T \in se(3)$, the twist corresponding to the i-th robot is assumed in the vector form and $\tilde{G}_i$ is of the form described in equation (10) with $G_i = (1/2)H_i$ and $w_i = (1/2)m_i$. Using equation (4), $\sum_{i=1}^N m_i r_i = 0$, and orthogonality of rotation matrices, the total kinetic energy becomes $T = \sum_{i=1}^N \omega_i^T G_i \omega_i + \zeta^T \tilde{G} \zeta$ where $\tilde{G}$ is dependent on the mass and the geometry of the formation by

$$\tilde{G} = \frac{1}{2}\begin{bmatrix} \sum_{i=1}^N(-m_i\hat{r}_i^2) & 0 \\ 0 & \sum_{i=1}^N m_i I_3 \end{bmatrix} \tag{15}$$

Therefore, using the formation constraint, the problem reduces from dimension $6N$ to $3N+6$ and is separable into $N+1$ independent subproblems:

$$(i) \min_{\omega_i} \int_0^1 \omega_i^T G_i \omega_i dt, \; i = 1, \dots, N,$$

$$(ii) \min_{\zeta} \int_0^1 \zeta^T \tilde{G} \zeta dt$$

The solutions to problems (i) are geodesics on $SO(3)$ for metric G_i. The solution to problem (ii) is the geodesic on $SE(3)$ for metric $\tilde{G}$. Both these problems can be solved as described in Section 2.4.

We are now in the position to outline a procedure for generating smooth motion for each robot to interpolate between two given positions while preserving the formation and minimizing the kinetic energy. First, we locate the centroids O_i, attach the frames $\{M_i\}$ and, from the geometric properties of each robot, calculate the inertia matrices H_i. If $\{M_i\}$ is aligned with the principal axes of robot i, then H_i is diagonal, but this is not necessary. If the initial and final rotations R_i^0, R_i^1 of robot i are given, then the rotation of the robot is the interpolating geodesic on $SO(3)$ equipped with metric $G_i = (1/2)H_i$. Given $d_1^0, d_2^0, \dots, d_N^0$ at $t=0$ and $d_1^1, d_2^1, \dots, d_N^1$ at $t=1$, the initial and final positions of the centroid of the fictitious rigid body are given by $d^j = (\sum_{i=1}^N m_i d_i^j)/(\sum_{i=1}^N m_i)$, $j = 0, 1$. A frame $\{M\}$ is fixed to the virtual rigid body at its center of mass, which will give the initial and final orientations of the formation, R^0 and R^1. The r_i's are then determined by $r_i = {R^0}^T(d_i^0 - d^0)$, which will induce a metric $\tilde{G}$ on the $SE(3)$ of the formation, according to (15). The geodesic $(d(t), R(t))$ on the $SE(3)$ of the formation with boundary conditions (d^0, R^0), (d^1, R^1) can be found using the relaxation method or the projection method. The trajectory of the centroid of each robot is finally determined in form (3).

3.2 Under-actuated robots

In this section we assume that a subset of M from the N robots are under-actuated. We consider an airplane-like model, for illustration. Suppose robot i, $i = 1, \dots, M$ can only translate along the x-axis of $\{M_i\}$. Besides the formation restriction (4), in this case we have an additional constraint of the form $C\zeta = 0$. In this airplane-like model, C is a $2M \times 6$ matrix obtained by stacking 2×6 matrices of the form

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} R_i^T R \Gamma(r_i), \; i = 1, \dots, M \tag{16}$$

Therefore, the system of N robots moves while keeping formation $\{r_1, \dots, r_N\}$ and observing the nonholonomic constraints if and only if ζ, the twist of the fictitious rigid body, is in the null space of matrix C given by (16). In other words, the formation of N robots of which M are under-actuated can move (*i.e.*, $\zeta \neq 0$) if and only if the null space of matrix C is nonempty, or, equivalently, if rank$(C) \leq 5$.

As in Section 3.1, assume the initial $(d_1^0, R_1^0), \dots, (d_N^0, R_N^0)$ and final $(d_1^1, R_1^1), \dots, (d_N^1, R_N^1)$ poses of each robot are so that the rigid formation is preserved. Let C^0 and C^1 denote the initial and final C matrices and assume that rank$(C^0) \leq 5$ and rank$(C^1) \leq 5$ (*i.e.*, motion of formation is possible both at $t=0$ and $t=1$). Let $\{a_1, \dots, a_n\}$ and $\{b_1, \dots, b_m\}$ $(n, m \leq 5)$ be the coordinates of ζ^0 and ζ^1 in some basis of the null spaces of C^0 and C^1. If we write $\zeta^0 = [{\omega^0}^T \; {v^0}^T]^T$ and $\zeta^1 = [{\omega^1}^T \; {v^1}^T]^T$, then $\dot{R}^0 = R^0\hat{\omega}^0$, $\dot{R}^1 = R^1\hat{\omega}^1$, $\dot{d}^0 = R^0 v^0$, $\dot{d}^1 = R^1 v^1$ are all linear in a_i's and b_i's. The metric on the $SE(3)$ of the virtual rigid body is determined by the mass properties of the robots and the geometry of the formation in the form given by (15). Minimum acceleration curves $(M(t), d(t))$ are generated in $GA(3)$ with the corresponding metric $\tilde{W}$ in the form (14), (13).

The energy of the curve in $GA(3)$

$$E(a_i; b_i) = \int_0^1 \left[Tr(\dot{M}^T \dot{M} W) + w\dot{d}^T \dot{d} \right] dt$$

is a positive definite quadratic in a_i's and b_i's. Let a_i^* and b_i^* be the arguments minimizing E. The corresponding curve is then projected onto $SE(3)$, as described in Section 2.4. The obtained $(R(t), d(t))$ will determine trajectories for each O_i using equation (3). The generated $d_i(t)$, $i = 1, \dots, N$ have the property that the corresponding $v_i(0)$ and $v_i(1)$ are in accordance with the nonholonomic constraints. A position tracking algorithm [1] can be used for the M nonholonomic robots. The resulting tracking error will be zero. For the remaining $N - M$ holonomic robots, rotational motion can be generated separately as described in Section 3.1.

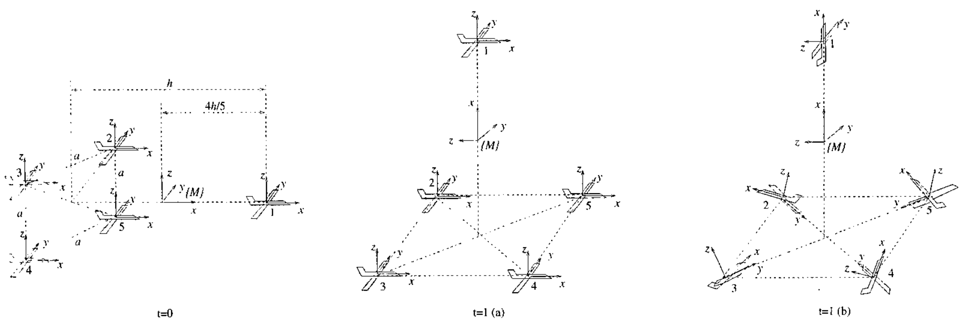

Figure 3: A formation of 5 identical airplanes.

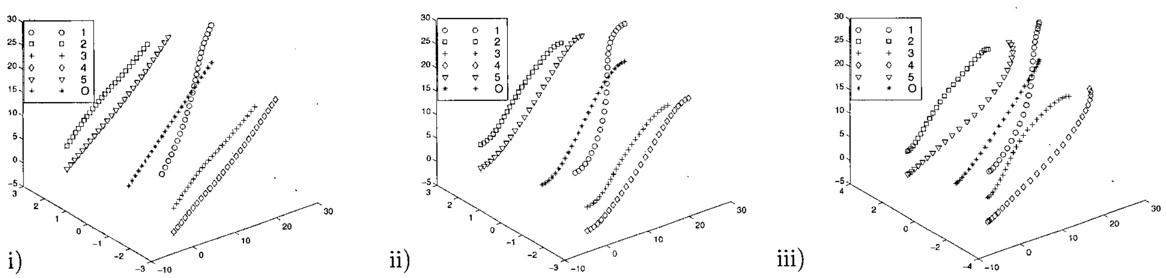

Figure 4: The generated position trajectory for airplane i, $i = 1, \ldots, 5$ and the curve described by the centroid of the virtual rigid body (O) for the case $m = 1$, $a = 5$, $h = 10$, $X = 20$, $Y = 20$: i) The airplanes are assumed holonomic; ii) End orientations as in case (a); and iii) End orientations as in case (b).

4 Examples

A spatial formation of $N = M = 5$ identical airplanes is described in Figure 3. With the formation frame $\{M\}$ fixed at the centroid and oriented as shown in the figure, the geometry of the formation is described by:

$$r_1 = \begin{bmatrix} \frac{4h}{5} \\ 0 \\ 0 \end{bmatrix}, \; r_2 = \begin{bmatrix} -\frac{h}{5} \\ \frac{a}{2} \\ \frac{a}{2} \end{bmatrix}, \; r_3 = \begin{bmatrix} -\frac{h}{5} \\ -\frac{a}{2} \\ \frac{a}{2} \end{bmatrix},$$

$$r_4 = \begin{bmatrix} -\frac{h}{5} \\ -\frac{a}{2} \\ -\frac{a}{2} \end{bmatrix}, \; r_5 = \begin{bmatrix} -\frac{h}{5} \\ \frac{a}{2} \\ -\frac{a}{2} \end{bmatrix}$$

The geometry will induce metrics in $SE(3)$ and $GA(3)$ as follows: If m is the mass of each airplane, then the matrix $\tilde{G}$ of the kinetic energy metric as defined in (15) is given by 10 with

$$G = \frac{m}{2}\mathrm{diag}\{2a^2,\, a^2 + \frac{4h^2}{3},\, a^2 + \frac{4h^2}{5}\},\; w = \frac{3m}{2}$$

The matrix of the metric $\tilde{W}$ in the ambient manifold $GA(3)$ is determined using (7) and (8). Let the formation at $t = 0$ be given by

$$R_1^0 = R_2^0 = R_3^0 = R_4^0 = R_5^0 = R^0 = I_3,\; d^0 = [0\; 0\; 0]^T$$

Some elementary but tedious algebra gives the null space of the matrix C^0 in the form:

$$\mathrm{Null}(C^0) = \mathrm{span}([0\; 0\; 0\; 1\; 0\; 0]^T)$$

Therefore, as expected, the formation can move at $t = 0$ while observing the nonholonomic constraints only by translating along the x axis of $\{M\}$. We consider two possible end configurations at $t = 1$, both described by

$$d^1 = [X\; 0\; Y]^T,\; R^1 = \mathrm{Rot}(y, -90)$$

The cases (a) and (b) differ by the individual orientations of each airplane. In case (a),

$$R_1^1 = R_2^1 = R_3^1 = R_4^1 = R_5^1 = I_3$$

while in case (b)

$$R_1^1 = \mathrm{Rot}(y, -90),\; R_2^1 = \mathrm{Rot}(z, -135)\mathrm{Rot}(y, -45),$$

$$R_3^1 = \mathrm{Rot}(z, -45)\mathrm{Rot}(y, -45),$$

$$R_4^1 = \mathrm{Rot}(z, 45)\mathrm{Rot}(y, -45),$$

$$R_5^1 = \mathrm{Rot}(z, 135)\mathrm{Rot}(y, -45).$$

The corresponding null spaces are again 1-dimensional:

$$(a)\ \mathrm{Null}(C^1) = \mathrm{span}([0\ 0\ 0\ 0\ 0\ 1]^T),$$

$$(b)\ \mathrm{Null}(C^1) = \mathrm{span}([\frac{\sqrt{2}}{a}\ 0\ 0\ 1\ 0\ 0]^T)$$

The interpolating minimum acceleration curves on the ambient manifold $(M(t), d(t))$, which depend on a_1 and b_1 as described in Section 3.2, are given by

$$(a)\ M(t) = \begin{bmatrix} 1+f(t) & 0 & f(t) \\ 0 & 1 & 0 \\ -f(t) & 0 & 1+f(t) \end{bmatrix}$$

$$d(t) = \begin{bmatrix} a_1 t - Xf(t) + (b_1 - 2a_1)t^2 + (a_1 - b_1)t^3 \\ 0 \\ -Zf(t) \end{bmatrix}$$

$$(b)\ M(t) = \begin{bmatrix} 1+f(t) & g(t) & f(t) \\ 0 & 1 & g(t) \\ -f(t) & 0 & 1+f(t) \end{bmatrix}$$

$$d(t) = \begin{bmatrix} a_1 t + 3Xt^2 + (-2X + a_1)t^3 \\ 0 \\ (3Z - b_1)t^2 + (-2Z + b_1)t^3 \end{bmatrix}$$

where

$$f(t) = -3t^2 + 2t^3,\ g(t) = \frac{\sqrt{2}}{a} b_1 t^2 (1-t)$$

The energy is minimized along the above curves in metric $\tilde{W}$ if $a_1 = D, b_1 = -D$ in case (a) and $a_1 = -19X/48, b_1 = 3Z/4(1+2m)$ in case (b). The corresponding curve is then projected onto $SE(3)$ as described in Section 2.4. The resulting position trajectories for each airplane and for the centroid are shown in Figure 4 for the holonomic case in i) and for the two nonholonomic cases (a) and (b) in ii) and iii) respectively. Note that in cases ii) and iii), the end orientations are consistent with the nonholonomic restrictions. Also, in the holonomic case i), the trajectory of the centroid is a uniformly parameterized line, as expected. We should emphasize that we have ignored the problem of controllability of the aircrafts along the given position trajectory. We instead refer the reader to [1].

5 Conclusion and future work

We consider the trajectory generation problem for a formation of N arbitrarily shaped robots of which M are under-actuated. Our method yields trajectories that (nearly) minimize a measure of total energy while satisfying the nonholonomic constraints and maintaining a rigid formation (the errors in our approximation method are quantified in [4]). The resulting curves are invariant with respect to the choice of the inertial frame and parameterization. Our ongoing research is devoted to including changes in the shape of the formation to accommodate velocity constraints and to generating strategies and controllers for switching among pre-defined formations.

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