Proceedings of the 2006 IEEE International Conference on Robotics and Automation
Orlando, Florida - May 2006

Model-Based Estimation of Off-Highway Road Geometry using Single-Axis LADAR and Inertial Sensing

Lars B. Cremean†, Richard M. Murray

Division of Engineering and Applied Science
California Institute of Technology
†lars@caltech.edu

***Abstract*— This paper applies some previously studied extended Kalman filter techniques for planar road geometry estimation to the domain of autonomous navigation of off-highway vehicles. In this work, a clothoid model of the road geometry is constructed and estimated recursively based on road features extracted from single-axis LADAR range measurements. We present a method for feature extraction of the road centerline in the image plane, and describe its application to recursive estimation of the road geometry. We analyze the performance of our method against simulated motion of varied road geometries and against closed-loop detection, tracking and following of desert roads. Our method accomodates full 6 DOF motion of the vehicle as it navigates, constructs consistent estimates of the road geometry with respect to a fixed global reference frame, and requires an estimate of the sensor pose for each range measurement.**

***Index Terms*— Navigation, sensing, LADAR, sensor fusion, road estimation.**

I. INTRODUCTION

Estimation of road geometry is an important task for a variety of automotive applications in intelligent transportation systems, because it enables prediction and evaluation of the future path of the vehicle. This information is particularly pertinent to driver assistance technologies that involve detection and response to other vehicles or hazards on this path, including adaptive cruise control and collision warning systems.

Since road curvature parameters change as function of distance along the road, they can be viewed as the state and output of a time-varying process as the vehicle moves along the road. Recursive estimation of these parameters using Kalman filtering techniques has become a standard for road (and many other types of) estimation, and this approach has been applied in recent years for human navigation on well-structured paved roads with relatively clear boundary markings (see for example [1], [2], [3]). Common assumptions for these environments include planar road geometry, negligible sensor pitch and roll, and that the vehicle is traveling along the center of the road or lane. In rural or underdeveloped areas, however, many of these assumptions can break down, and novel feature extraction algorithms are needed. Reference [4] outlines a feature extraction method that finds the straight line segments in the Cartesian ground plane which can be applied to such situations; this work presents a complementary method for feature filtering and extraction.

This work applies recursive estimation schemes for road estimation to “off-highway” environments, where roads are typically not painted with boundary markings and in many cases are unpaved. Off-highway navigation presents a special challenge for road estimation due to the rough motion of the sensor and the lack of visual structure found in highways and improved roads. We therefore make no assumption that the pitch and roll of the vehicle are negligible, but rather require a full 6 DOF estimate of the sensor pose. In this way, we are able to associate inertial and range information to do road feature extraction in a global coordinate system. The only assumption that we make about the vehicle pose relative to the road is that features of the road lie within the sensor field of view. As in other work, we do assume planar geometry for the road, but this assumption could be lifted with extension of the ideas presented here.

While off-highway scenes have been studied recently using image processing and computer vision techniques, as in [5], we have chosen to begin this work using a single axis laser detection and ranging (LADAR) measurement system. Advantages of using LADAR include operability in unfavorable lighting conditions and the ability to use direct range measurements to represent road features in an inertially fixed reference system. Extensions within the framework we provide here will be able to accomodate LADAR and image-based sensing together, but these are outside the scope of this paper.

Our work is motivated by the problem of reliable fully autonomous navigation of ground vehicles in unstructured environments. Solutions to this problem will see application in places where human operation of vehicles is typically too costly and/or too dangerous. This will most ostensibly be seen in military transport operations within the next ten to twenty years; while we anticipate economically and technologically feasible further application to construction, agriculture, manufacturing and mining activities in twenty to fifty years time. After several generations of advancement in the reliability of autonomous navigation, the level of autonomy could be sufficient to provide blind and disabled individuals personal transportation solutions (with assistance from the individual for high-level navigation instruction).

While road feature identification in static camera images has been studied for on- and off-highway environments, and recursive road estimation has been studied using both camera and LADAR sensing techniques in urban environments, this paper represents some of the first work of the authors' knowledge that applies recursive estimation techniques to road estimation explicitly for navigation of off-highway roads. The main contributions of this paper are the application of recursive road estimation techniques to off-highway environments, estimation of complete road geometry in the global coordinate system, development of techniques to accommodate pitching and rolling of the vehicle during navigation, and performance evaluation of these techniques.

These contributions are presented as follows. Basic assumptions and the road model are given in Sections II and III. The measurement model and recursive estimation framework are presented in Section IV. We present road feature extraction techniques in Section V and the results from simulation and autonomous operation in Sections VI and VII. A summary and brief look toward future work are presented in Section VIII.

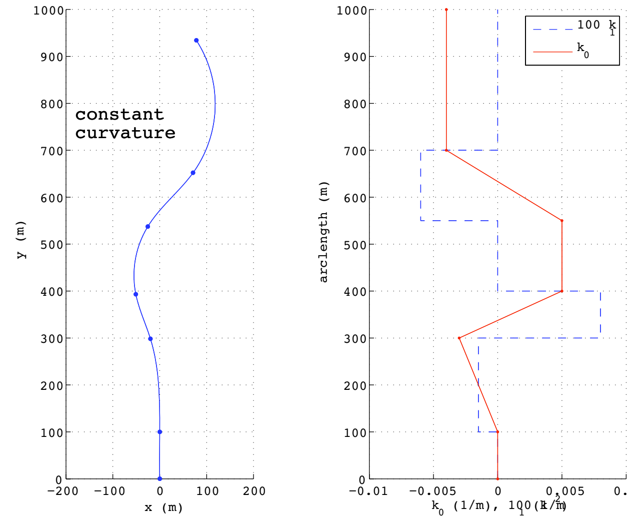

Fig. 1. Simulated road geometry (left) and curvature as a function of arclength (right). In our coordinate conventions, negative curvature corresponds to a left-hand turn.

II. Assumptions

This work presents a solution for estimating the road geometry as the vehicle travels along the road. We maintain the following fundamental assumptions throughout this paper. We assume

1) that a forward-looking sensor is mounted on the vehicle. We assign to this sensor both a Cartesian coordinate system $\mathcal{S}$ and an image coordinate system $\mathcal{I}$,
2) that we are able to extract estimates of road features in either of the sensor-assigned coordinate systems,
3) that we have, at any given time, an estimate of the full 6 DOF pose of the sensor with respect to some fixed inertial (global) coordinate system, which we will call $\mathcal{G}$,
4) that the roughness of the road is small relative to the roughness outside the boundaries of the road, and
5) that there are small heading changes in the road at the scale of the sensing horizon.

Assumption 3) can be satisfied with some combination of GPS, inertial sensing and odometry. We have done this through related work that is outside of the scope of this paper by using GPS and IMU data as inputs to a Kalman filter, following the principles of [6]. With assumptions 2) and 3) and a range sensor, the road feature estimates can all be represented in the fixed coordinate system $\mathcal{G}$. The primary use of the inertial sensing is to provide a way to estimate the road geometry with respect to a fixed coordinate system.

We will further assume that the vehicle is driven such that the road geometry is maintained in the sensor's field of view, but no further assumption is made regarding the relative position or orientation between the vehicle and the road. The road is assumed planar, and the width of the road is assumed fixed.

The estimation framework used here is an implementation of the extended Kalman filter (EKF). An EKF is used because the speed of the vehicle is not assumed constant and because the measurement model is linearized about the small heading changes assumed in 5. The state variables of the Kalman filter are the local curvature κ_0 and arc length rate of change of curvature κ_1. Local here indicates the position along the road centerline that is closest to the vehicle coordinate system (assumed coincident with the sensor coordinate system $\mathcal{S}$).

We enlist another coordinate system, $\mathcal{R}$, that is attached to the road with its x-axis pointing tangent to the road, but which is able to be repositioned as the road estimate progresses. One can think of the road estimate construction as analogous to laying down model railroad tracks one after the other, with $\mathcal{R}$ positioned at the end of the last laid track segment. Forward road geometry estimates extend from this coordinate system origin, and represent the current curvature estimates.

III. Road Model

We have chosen a piecewise clothoid to model the road centerline geometry. The centerline is represented as a planar twice differentiable curve as a function of arc-length, $\mathbf{r}(s) \in \mathbb{R}^2$, and is parameterized by curvature $\kappa(s)$. In the clothoid model, the curvature profile is assumed to be piecewise linear, i.e. each segment of the road in this model corresponds to a constant arc-length rate of change of curvature. For segment i which covers the arc-length interval $s \in [s_i, s_{i+1})$, the curvature is given as $\kappa_i(s - s_i) = \kappa_{0,i} + (s - s_i)\kappa_{1,i}$. The twice differentiable assumption on the curve guarantees curvature continuity.

For standard automobiles, this parameterization corresponds to continuous nominal motion of the steering wheel as the road is traveled; this is a common consideration in the design of roads and highways. Figure 1 depicts an example simulated road geometry, determined completely by the curvature rate profile and the initial position, initial orientation, and initial curvature of the road.

The orientation and position of the road centerline in $SE(2)$ can be easily recovered from the curvature profile, and are given as a function of arc-length by

$$\begin{aligned} \theta(s) &= \theta_0 + \int_0^s \kappa(\tau)\ d\tau \\ x(s) &= x_0 + \int_0^s \cos\theta(\tau)\ d\tau \\ y(s) &= y_0 + \int_0^s \sin\theta(\tau)\ d\tau \end{aligned} \tag{1}$$

Using the small angle approximation for the angle $\theta(\tau)$ (justified by choosing the road coordinate aligned with the initial θ_0), Eqns. 1 can be transformed into a spatio-temporal representation using the chain rule and assuming some time dependent speed $v(t)$ along the curve. The resulting differential equations are

$$\begin{aligned} \dot{y}(s) &= v(t)\ \left(\theta_0 + \kappa_0 s + \frac{1}{2}\kappa_1 s^2\right) \\ \dot{\theta}(s) &= v(t)\ (\kappa_0 + \kappa_1 s) \\ \dot{\kappa}(s) &= v(t)\ \kappa_1 \end{aligned} \tag{2}$$

We use the representation of Eqns. (2) to provide both the propagation and the measurement model for our Kalman filter design. A time integral of that equation at a fixed lookahead distance s provides the measurement model with y (the lateral offset in the road coordinates) as the measurement variable, and propagating the road coordinate system forward with the vehicle provides the dynamic equations for tracking of road parameters.

The evolution of the curvature variables as the vehicle moves forward at speed $v(t)$ is modeled by

$$\begin{bmatrix} \dot{\theta}_0 \\ \dot{\kappa}_0 \end{bmatrix} = \begin{bmatrix} 0 & v(t) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_0 \\ \kappa_0 \end{bmatrix} + \begin{bmatrix} 0 \\ w(t) \end{bmatrix}, \tag{3}$$

where $\dot{(\ )} \triangleq \frac{d}{dt}(\ )$ and $w(t)$ represents a noise term that drives the evolution of κ_0. The noise signal $w(t)$ is assumed to be Gaussian and white with $p(w) = \mathcal{N}(0, Q)$. Eqn. (3) forms the process dynamics for our estimator, to be used in the propagation step.

Note that there is no input in this process model, but any a priori information about the road could be incorporated into the model here. Note also that we have chosen to use a state formulation with heading and curvature, rather than curvature and curvature rate, as this resulted in better performance in closed-loop testing. A comparative analysis of the different approaches is not within the scope of this paper, but warrants attention.

In compact form, we re-express Eqn. (3) as

$$\dot{\mathbf{x}} = A(t)\mathbf{x} + \mathbf{w}(t). \tag{4}$$

We approximate Eqn. (4) with the first order difference equation

$$\mathbf{x}_k = A_k \mathbf{x}_{k-1} + \mathbf{w}_k \tag{5}$$

Fig. 2. Our test platform is a 2005 Ford E-350 Econoline van modified by Sportsmobile of Fresno, California. A roof-mounted LADAR and INS provide the raw data used for navigation of desert roads.

where $A_k \triangleq (A\Delta t + I)$.

IV. Measurement Model

The underlying measurement model used follows as in [1], and relies on small heading changes between the road coordinate x-axis and the heading of the road at the so-called lookahead location at which the road measurement is taken. With this small angle approximation, $\cos\theta_r \approx 1$ and $\sin\theta_r \approx \theta_r$, and the lateral location of the centerline in the road coordinate at lookahead distance x_m is recovered from Eqns. (1) as

$$y \approx \frac{1}{2}\theta_0 x_m + \kappa_0 x_m^2 + \frac{1}{6}\kappa_1 x_m^3 \tag{6}$$

Formulated with heading and curvature only, we can eliminate the κ_1 term, and therefore find the following as the measurement equation associated with our process model:

$$y(t) = \begin{bmatrix} x_m & \frac{1}{2}x_m^2 \end{bmatrix} \begin{bmatrix} \theta_0 \\ \kappa_0 \end{bmatrix} + \nu(t) \tag{7}$$

We use a discrete version of Eqn. (7) for use with the discrete EKF described below,

$$y_k = C_k \mathbf{x}_k + \nu_k. \tag{8}$$

The signal ν_k represents our measurement noise, which we assume to be white, Gaussian, with $p(\nu) = \mathcal{N}(0, R)$, where R is the measurement noise (co)variance. This measurement noise can be estimated from a statistical analysis of a sequence of recorded measurements.

The discrete process and measurement Eqns. (5) and (8) were used in the design of the extended Kalman filter. Letting $\hat{x}_k$ denote the state estimate, and given an initial estimate of the process covariance P_0, the propagation equations are given by

$$\hat{x}_k^- = A_k \hat{x}_{k-1} \tag{9}$$
$$P_k^- = A_k P_{k-1} A_k^T + Q \tag{10}$$

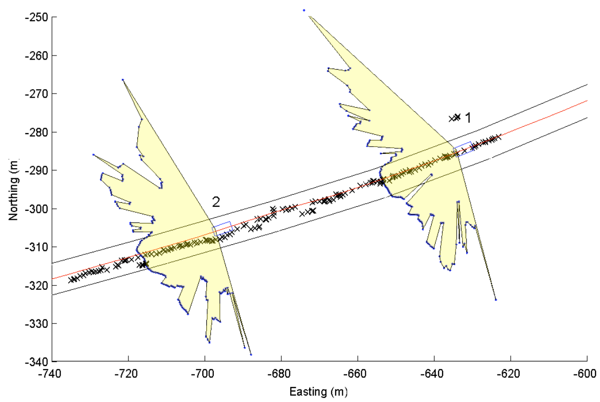

Fig. 3. A pair of scans taken by our test vehicle several seconds apart during autonomous operations on a slight right hand curve. The sensor is mounted over the cab of the vehicle and is mounted with approximately -7 degrees pitch. Pitching and rolling of the vehicle and sensor cause the intersection of the scan with flat ground to move relative to the vehicle. Estimated vehicle (roll, pitch) in position 1 are $(1.21, \ -1.74)$ degrees and $(0.94, \ -1.58)$ degrees for position 2. Approximate road boundaries are shown as parallel lines. The $\times$s are the result of the road feature extraction techniques of Section V. The thin line is the trace of the vehicle position as it followed the road estimate.

Fig. 4. Visual forward image corresponding to position 2 of Fig. 3. LADAR-based road feature estimation has the advantage of insensitivity to lighting conditions such as long shadows, but is limited to a single lookahead range. Computation times are significantly faster with LADAR feature extraction.

and the update equations are

$$\begin{aligned} K_k &= P_k^- C_k^T (C_k P_k^- C_k^T + R)^{-1} && (11)\\ \hat{x}_k &= \hat{x}_k^- + K_k(y_k - C_k \hat{x}_k^-) && (12)\\ P_k &= (I - K_k C_k) P_k^-. && (13) \end{aligned}$$

The matrix K_k represents the Kalman gain here; see [7] for a good background reference. Note that since the matrices A and C depend on the current speed and current lookahead distance, they are not constant but rather are dependent on the timestep k.

The inputs to this extended Kalman filter implementation are the sequence of estimates y_k for the road centerline lateral coordinate (in the road coordinate system $\mathcal{R}$) at the lookahead corresponding to a given scan. The calculation of these estimates given the range images and sensor pose estimate is the subject of the next Section.

V. Road Feature Extraction

A LADAR image array can be considered as a 2D Cartesian pixel map where each pixel (u, v) represents an azimuth, elevation pair $(\theta, \ \phi)$. The values in this pixel map for a given scan represent the range measured in that direction. Let the sensor x-axis be aligned with the measurement at $(\theta, \phi) = (0, 0)$. If we consider a Cartesian coordinate system with its x-axis pointed along the range measurement vector, a ZYX Euler rotation with (roll, pitch, yaw) = $(0, -\phi, -\theta)$ will transform the point $p = (\rho, 0, 0)$ into the sensor frame. For our single axis LADAR, the elevation is zero and the azimuth lies in the interval $[-90, \ 90]$ degrees. The range image becomes one dimensional and the Cartesian coordinates in the sensor frame are reduced to

$$\begin{bmatrix} x_s \\ y_s \\ z_s \end{bmatrix} = \rho \begin{bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{bmatrix} \qquad (14)$$

Since we are using a range sensor for feature extraction, we are able to transform any feature between any of the coordinate systems we have defined so far – namely the image coordinates $\mathcal{I}$, sensor coordinates $\mathcal{S}$, road coordinates $\mathcal{R}$ or global coordinates $\mathcal{G}$ – with ease. We have a choice, therefore, of performing feature extraction in any of these coordinates.

Fig. 3 depicts two full range scans from a bird's-eye perspective, where the LADAR sensor is rigidly mounted on the front of the vehicle at a height of 2.4 meters and pitched downward by approximately 6.9 degrees with respect to flat ground. On flat ground in this configuration, the scan plane intersects the ground on a line perpendicular to the vehicle orientation at approximately 20 meters from the sensor position. Positive pitching of the vehicle causes this line to move further away, and positive roll causes the line to rotate clockwise. Feature extraction in this space can be based on finding and filtering the straight segments that correspond to the road surface and road boundaries. This has been achieved with considerable success as shown in [3] for navigation at low speeds (up to 4 m/s) in urban environments.

Fig. 5 shows the range image corresponding to the scan shown in position 2 of Fig. 3. Features in the scan due to vegetation, road berms and flat road are all apparent to the trained human eye, if correlated with the camera image of Fig. 4.

The method used in this paper to extract road features is to perform a search in the image plane for best candidate section for flat ground. With the assumption of flat ground, the ideal range image that would result is a function of the height of the sensor above the ground z_s, the pitch of the sensor ϕ_s, the roll of the sensor ψ_s, and the sensor scan angle

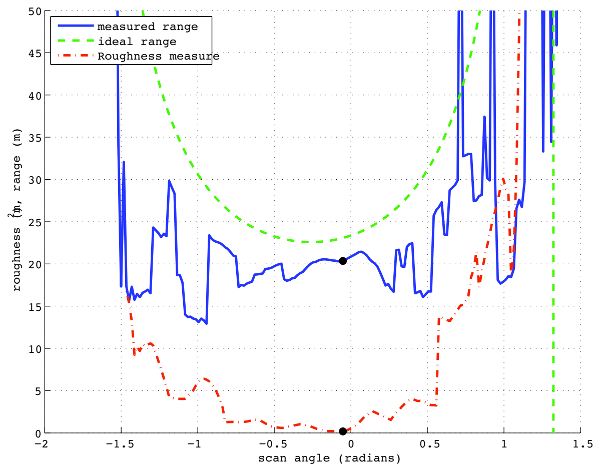

Fig. 5. The range image corresponding to position 2 in Fig. 3, and the image that would result in a scan of flat ground. The difference between these two is used to extract features that correspond to the road centerline.

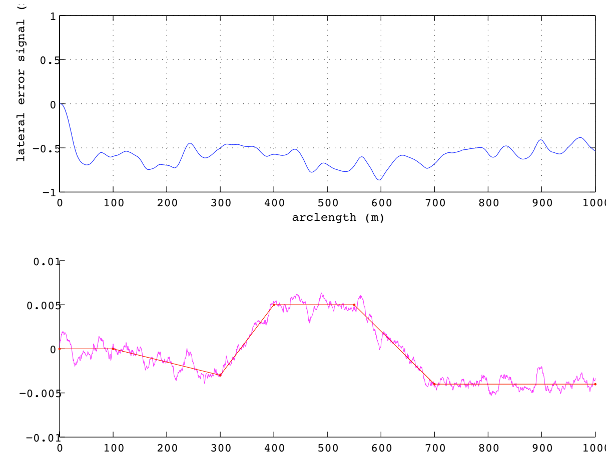

Fig. 6. Typical simulation performance for the simulated road geometry given in Fig. 1. Noisy data was simulated to generate lateral offset estimates from the current road geometry estimate. Simulation results show good tracking of the curvature of the road and small lateral errors even at curvature transition segments. Curvature rate estimates are not shown.

θ, and is given by

$$\rho_{flat}(\theta) = \frac{z_s}{\cos\theta \sin\phi_s - \cos\phi_s \sin\psi_s \sin\theta}. \tag{15}$$

The feature extraction of a point on the road centerline is done by considering the difference $\rho_e(\theta)$ between the LADAR range image $\rho(\theta)$ and the flat ground range image $\rho_{flat}(\theta)$ as calculated above. An optimization problem is posed to find the most likely road center feature in the scan, and can be expressed as

$$\min_i \sigma(\rho_e(W_i)) \tag{16}$$

where $\sigma(\cdot)$ represents a variance and W_i is the scan angle interval that corresponds to the road width (which is here assumed fixed). Fig. 5 also shows the solution of this optimization (performed by brute force but still quite fast) for the scan shown.

We perform the search through the image plane by considering several overlapping discrete window positions in the scan as candidates for a road cross-section. The smoothness of each road-window candidate is calculated by taking the variance of $\rho_e(\theta)$ for the range of θ that corresponds to the window. The range measurement at the center of the minimum variance window is used to compute the measurement for each update step of the extended Kalman Filter.

Note that this algorithm will work only in the situation where the roughness of the road is small relative to the roughness outside the boundaries of the road. If the terrain on either side of the road is also smooth, false outlier road features are likely to occur. If road berms are geometrically significant, restricting the search to a region around the current estimate of the road can improve results in this situation.

VI. Simulation Results

The extended Kalman filter estimation scheme as presented above, but with the curvature and curvature rate formulation, was simulated over the road geometry depicted in Fig. 1. The vehicle was simulated to move along the road at 5 m/s and scans were simulated to provide noisy measurements of the road centerline. The process noise covariance was set to Q = diag(5.0e-5, 5.0e-2) and the measurement noise variance was set to R =3.0e-4. The value of the process noise covariance was chosen to be consistent with results from previously published literature (see [8]). The initial covariance estimate was set as P_0 = diag(5.0e-5, 5.0e+1). The resulting estimated versus actual curvature parameters for one of these simulated runs is presented in Fig. 6, along with a trace of lateral error as a function of arc-length from the beginning of the simulated road.

The noise provided in this simulation is significant, designed to be comparable with the noise observed from analysis of collected LADAR scans using the feature extraction methods of Section IV. Even with this amount of noise, these simulation results are comparable to previous results, see for example [8] for results from a two-clothoid model.

A few practical comments on the estimator design. For the lookahead distances used here (approx. 20 m), no estimator parameters were found that give good estimation performance of $\kappa_1(s)$. Poor κ_1 estimates sometimes caused oscillations in the κ_0 estimates, which provided a large part of the incentive to investigate the heading and curvature formulation in real-world runs.

VII. Autonomous Operation Results

The EKF as designed above was also implemented to determine the performance of the road estimation scheme with real data, processed using the methods in Sec. V to provide road feature estimates in the road coordinate system to the extended Kalman filter presented in section IV. The process and measurement noise covariance were set to be Q = diag(5.0e-4, 5.0e-5), R =5.0e2. These were tuned heuristically to find a good balance between matching the

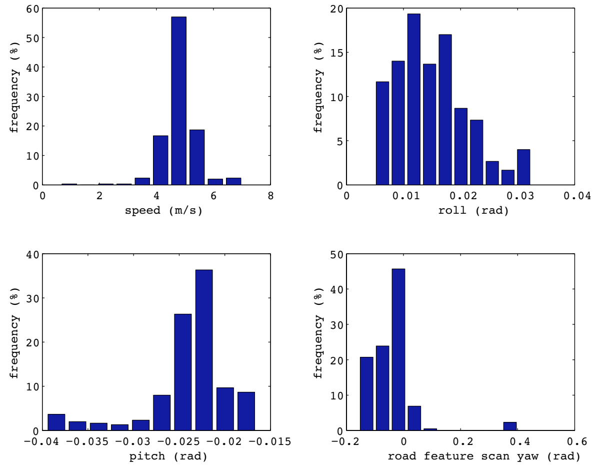

Fig. 7. Speed, pitch and roll histograms for a sample segment of the data on which the techniques presented in this paper were tested. This data represents a sampling of 1000 full LADAR scans with 181 scanpoints each, which corresponds to about 43 seconds of data at 75 Hz.

local curvature and reducing the lateral offset error in the estimate.

The data presented was collected while running the road feature detection and road geometry estimation as described above, in closed loop, while our test vehicle controlled the throttle, brake and steering in order to track the road estimate. Successful tracking of the road was consistently achieved while driving a section of desert road at speeds between 4 and 6 m/s.

LADAR scans and synchronized state data were collected at a rate of approximately 75 Hz. Figure 7 shows some sample statistics from the collected data, including traveling speed, roll, pitch and relative yaw to the estimated road centerline features. These are indications of the degree to which the flat ground and the constant speed assumptions are violated, which require us to use the extended Kalman filter implementation to account for changing lookahead distances and non-constant process matrix A_k.

Example performance of the algorithms on collected data is depicted in Fig. 3. Approximate road boundaries are shown, as well as extracted road centerline feature estimates ($\times$s), and the resultant vehicle trace. These results indicate quality performance of the road feature extraction algorithms of Section IV, although outliers are present, and they also indicate the ability of the filtering algorithm to handle outlier as well as noisy data. In particular, the road estimate conforms well to the series of measurements, while not being affected terribly by the outliers.

VIII. Summary and Future Work

We have developed an extended Kalman filter framework for estimation of road geometry that has been applied in simulation and to closed-loop autonomous operation in off-highway environments. Results from simulation and from real data indicate that reliable estimation and tracking of off-highway road geometry is possible for use in autonomous systems and future intelligent vehicles. We have presented feature extraction methods for the road centerline that provide good tracking in moderate off-road environments.

Several improvements and extensions are the subject of future research. In general, careful testing in a wider variety of road types will provide insight for the improvement of these algorithms and likely enable us to eliminate several of the assumptions presented in the beginning of this paper.

The extent to which the filter is able to handle outlier measurements is limited. Exploration of other solutions to the optimization problem posed in Section IV for extraction of road feature estimates will be necessary to improve the conditions under which the application of these techniques will be successful. Initial investigation suggests that restricting the search for the road features in a scan to a local region around the current estimate may prove to be an intelligent way to reduce the number of outlier measurements.

In addition, there is clear benefit to extending the estimation framework to include online estimation of road width. Supervised learning, matched filter and other techniques could enable us to extract estimates of left and right road boundaries as well as road center.

Finally, there are indications that significant performance increases might be had from combining road feature extraction techniques from LADAR sensing and monocular vision to generate a unified road model for hetereogeneous sensor suites. These would take advantage of the long range sensing properties of the camera (enabling vanishing point detection) with short range geometric information from the LADARs to better estimate the global road geometry.

References

[1] Ernst D. Dickmanns and Birger D. Mysliwetz, "Recursive 3-d road and relative ego-state recognition," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 14, no. 2, pp. 199–213, 1992.

[2] A. Kirchner and Th. Heinrich, "Model based detection of road boundaries with a laser scanner," in *Proc. of IEEE Internatonal Conference on Intelligent Vehicles*, 1998, pp. 93–98.

[3] W.S. Wijesoma, K.R.S. Kodagoda, and A.P. Balasuriya, "Road-boundary detection and tracking using ladar sensing," *IEEE Transactions on Robotics and Automation*, vol. 20, no. 3, pp. 456–464, 2004.

[4] Heiko Cramer and Gerd Wanielik, "Road border detection and tracking in non cooperative areas with a laser radar system," in *Proceedings of German Radar Symposium*, 2002.

[5] Christopher Rasmussen, "Grouping dominant orientations for ill-structured road following.," in *Computer Vision and Pattern Recognition (1)*, 2004, pp. 470–477.

[6] Mohinder S. Grewal, Lawrence R. Weill, and Angus P. Andrews, *Global Positioning Systems, Inertial Navigation, and Integration*, Wiley-Interscience, 2000.

[7] Greg Welch and Gary Bishop, "An introduction to the Kalman filter," Tech. Rep. TR 95-041, University of North Carolina, 1995.

[8] Deepak Khosla, "Accurate estimation of forward path geometry using two-clothoid road model," in *Proc. of IEEE Intelligent Vehicles Symposium*, 2002, pp. 154–159.