

# Probabilistic localization with a blind robot

Lawrence H. Erickson<sup>1</sup>, Joseph Knuth<sup>1</sup>, Jason M. O’Kane<sup>2</sup>, Steven M. LaValle<sup>1</sup>  
Department of Computer Science  
University of Illinois at Urbana-Champaign<sup>1</sup>  
Urbana, IL 61801 USA  
{lericks4, knuth, lavalle}@uiuc.edu

Department of Computer Science and Engineering  
University of South Carolina<sup>2</sup>  
Columbia, SC 29208 USA  
{jokane}@cse.sc.edu

**Abstract**—Researchers have addressed the localization problem for mobile robots using many different kinds of sensors, including rangefinders, cameras, and odometers. In this paper, we consider localization using a robot that is virtually “blind”, having only a clock and contact sensor at its disposal. This represents a drastic reduction in sensing requirements, even in light of existing work that considers localization with limited sensing. We present probabilistic techniques that represent and update the robot’s position uncertainty and algorithms to reduce this uncertainty. We demonstrate the experimental effectiveness of these methods using a Roomba autonomous vacuum cleaner robot in laboratory environments.

## I. INTRODUCTION

Localization is one of the best-studied problems in mobile robotics. Accurate knowledge of the robot’s position within its environment is widely considered to be essential for mobile robots to be useful. Although robots often obtain such knowledge by some combination of sensor measurements, motion estimates, and pre-supplied initial conditions, information from sensors is usually the primary means of eliminating position uncertainty. Understanding the role of sensing in the localization process is therefore essential to a complete understanding of localization problems.

In the mobile robotics literature, localization problems take many forms. In this paper, we consider active global localization, in which a robot has access to a complete map of its environment but is totally ignorant of its position. The robot must purposefully direct its motions to eliminate that uncertainty. Using probabilistic techniques, we demonstrate that a certain global active localization problem can be solved with very limited sensing capabilities. This work is distinguished from prior probabilistic localization techniques by the limitations on the robot’s sensing. We consider a differential drive robot equipped with a contact sensor and a clock, but no other sensors.

This work is motivated by a desire to understand the *information requirements* of important robotic tasks. By finding very simple robots that are able to complete certain tasks, we begin to identify necessary conditions on the sensing and motion capabilities for completing that task. In a more directly practical sense, studying robots with very simple sensing schemes is profitable because such robots are better suited (for reasons of cost and complexity) for deployment in large cooperative teams.

Prior theoretical work showed that active global localization problems can be solved under severe sensor lim-

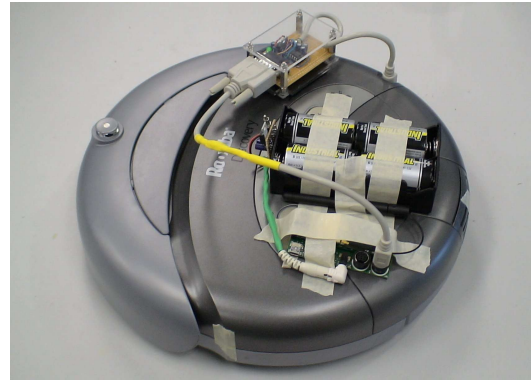


Fig. 1. A Roomba autonomous vacuum cleaner robot.

itations [22]. However, that work relies in crucial ways on assumptions that the robot’s sensing and control are perfect. For example, certain portions of [22] depend on the robot’s ability to slide along the environment boundary without triggering its contact sensor. If the robot’s control admits even the slightest directional errors, such motions are impossible. In practice, sensing is rarely fully accurate and control is rarely perfect. In this paper we present new algorithms that are robust to these errors and demonstrate experimentally that these algorithms are effective.

The most successful published approaches to localization are generally based on either particle filters [8], [11], [12], [17], extended Kalman filters [13], [18], or grid-based discretizations [3], [5], [6], [14]. The novelty of our work is that we use a robot model in which the sensing and motion capabilities are severely limited. These limitations introduce geometric issues, requiring new algorithms for both pose tracking and active localization. Our work also draws inspiration from theoretical results on localization that use geometrical reasoning and set-based representations of uncertainty [10], [22], [27].

The minimalist approach we take has a long history in robotics. Researchers have studied the implications of sensing limitations for navigation [19], [24], exploration [1], [7], and manipulation [2], [20] tasks. The general problem of determining the information requirements of robotic tasks is taken up in [9], [21]. Our work contributes to this line of research by demonstrating that, with appropriate adaptations, such minimalist models are applicable in experimental contexts.

We conducted experiments using Roomba autonomous vacuum cleaner robots, shown in Figure 1. The Roomba is attractive as a research platform because it is inexpensive and readily available, and because its sensors closely match the abstract models we use.

Like many approaches to the localization task, our basic method is probabilistic. In fact, our work can be considered as exploring a kind of probabilistic information space [16], which arises in partially observable Markov decision processes (POMDPs) [15], [25], [28]. As our algorithm progresses, the robot maintains an approximate, discrete probability distribution over positions along the environment boundary. This distribution is updated in response to motions by the robot. For active localization, we use an entropy-based heuristic to choose uncertainty-reducing motions. The algorithm constructs a localization plan consisting of several subplans, each of which results in a monotonic decrease in entropy, even if the entropy temporarily increases within a subplan. This willingness to tolerate temporary increases in uncertainty is crucial to effective handling of the multimodal distributions that arise in our problems.

The remainder of this paper is organized as follows. We present formal definitions for our robot model and for passive and active localization problems in Section II. Algorithms to solve the passive and active problems appear in Sections IV and III respectively. Details about our experiments are in Section V, followed by discussion and conclusion in Section VI.

## II. PROBLEM STATEMENT

In this section, we introduce our robot model and define the passive and active localization problems we solve.

### A. Robot model

A point robot with orientation moves in an environment  $W \subset \mathbb{R}^2$  that is planar, closed, bounded, and polygonal. The environment need not be simply connected. Let  $\partial W \subset W$  represent the boundary of  $W$  and let  $n$  denote the number of vertices of  $\partial W$ . The robot knows its initial orientation, but not its initial position within  $W$ .

The robot is equipped with a contact sensor and a clock, but no other sensors. We consider two types of motions that these sensors enable.

- *Rotations* – Using its clock, the robot can rotate in place by dead reckoning. Motion by dead reckoning is notoriously noisy, so we model the error in the amount of rotation by a zero-mean density  $p$ . We assume that  $p$  is strictly increasing below its mean and strictly decreasing above its mean, and that  $p$  has a continuous cumulative distribution function. Figure 2 shows an example, in which a robot starts in the center of a rectangular environment and moves upward.
- *Translations* – The robot can move forward, but since it lacks odometry, the only reliable translation it can make is to move forward until it reaches the environment boundary. We assume that the robot travels in a straight line, but because we explicitly model orientation errors,

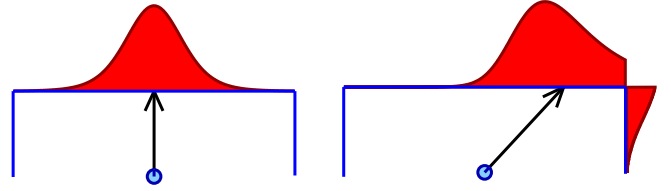


Fig. 2. A robot with Gaussian rotational error starts in the center of a rectangular environment and moves forward to the environment boundary. The probability density over points along the boundary is shown. For illustration purposes, the variance of the distribution is impractically large. [left] The robot moves upward. Although the resulting distribution appears superficially to be Gaussian, it is not. [right] The robot moves at an angle. Observe that, because of geometric effects, the mean of the resulting distribution need not be endpoint of the robot's nominal trajectory.

our algorithms are robust to small deviations from this assumption. Since we are interested in solving localization problems with a little sensor information as possible, we ignore the robot's clock during translations.

The robot's motions can be described as a sequence of discrete stages, in each of which the robot makes a single rotation, then moves forward until its contact sensor is triggered. We number these stages with consecutive integers  $k = 1, 2, \dots$ . Since the robot can only move between points along  $\partial W$ , we need not consider the points in the interior of  $W$  as possible locations for the robot.

Note that rotational errors accumulate over time, and the robot's true heading will become more uncertain as more stages go by. Since the error in the robot's orientation at stage  $k$  will be the sum of the error at stage  $k$  plus all preceding error, we can say that the random variable  $R_k$  representing the orientation error at step  $k$  is  $R_k = \sum_{i=1}^k r_i$ , in which  $r_i$  is a random variable describing the error resulting from step  $i$ . Note that the  $r_i$ 's are independent and identically distributed according to  $p$ . Let  $p_k$  denote the distribution of  $R_k$ . The result of this sum will depend on the single-stage error distribution  $p$ . In the special case where  $p$  is Gaussian with variance  $\sigma^2$ ,  $p_k$  is Gaussian with variance  $k\sigma^2$ .

### B. Localization problems

We consider two related localization problems:

- *Passive localization* – The robot's motions are controlled by an external decision maker. The problem is to efficiently maintain and update the probability distribution of possible states of the robot.
- *Active localization* – The robot's primary task is to eliminate uncertainty in its position. The problem is to choose motions so that the robot will be certain it is in a disk of radius  $\epsilon$  with a probability at least  $1 - \delta$ .

Observe that to solve the active localization problem typically requires as a "subroutine" a solution to the passive localization problem.

Unfortunately, as the robot moves, the distribution of possible states along  $\partial W$  becomes increasingly difficult to represent analytically, even if both the prior and error distributions are well-behaved. Moreover, geometric features within the environment will cause discontinuities that complicate the analytical representation of the distribution even further. To

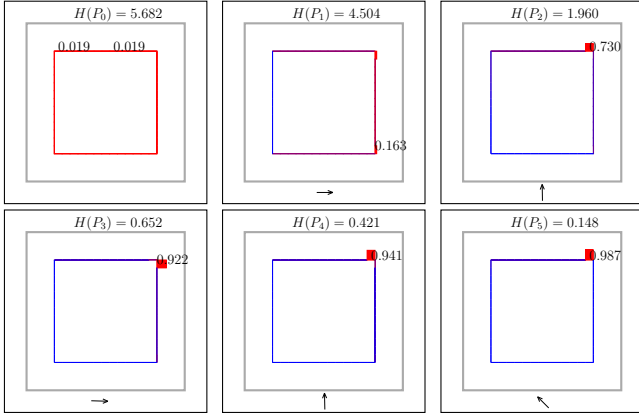


Fig. 3. A simple square environment, along with a 5-step plan that solves the active localization problem in that environment. The initial state is shown in the top left. The final state, which concentrates nearly all of the probability mass in one corner of the square, is shown in the lower right. The small arrows in each picture show the direction of movement at that step.

combat this complexity, we approximate the true distribution by discretizing the boundary of the environment into small cells of size at most  $2\epsilon$  and recording the amount of probability mass in each of these cells. Such a nonparametric representation is well-suited for representing the complex, multimodal distributions that arise in global localization. This discretization can be viewed as a piecewise-constant approximation to the underlying density function. We divide each edge  $e$  of  $\partial W$  into  $\lceil \text{length}(e)/(2\epsilon) \rceil$  equally-spaced cells. Let  $S = \{s_1, \dots, s_m\}$ , in which each  $s_i \in S$  is line segment in  $\partial W$ , denote the set of discrete cells generated in this way.

Under this discretization, we can represent the robot's uncertainty as an  $m$ -dimensional column vector

$$P_k = \begin{bmatrix} P_{k,1} & \cdots & P_{k,m} \end{bmatrix}^T, \quad (1)$$

in which  $P_{k,i}$  is the probability of the robot being in cell  $s_i$  at stage  $k$ . We assume a uniform prior, so that

$$P_{0,i} = \frac{\text{length}(s_i)}{\text{perimeter}(\partial W)}. \quad (2)$$

If additional information about the robot's starting position is available (for example, a known starting position), this initial condition can be changed accordingly.

In this context, the input to the passive localization problem is the environment  $W$ , a discretization  $S$ , a motion direction  $u$ , and a probability vector  $P_k$ ; the output is a probability vector  $P_{k+1}$  the updated to reflect this motion. Similarly, note that if at least  $1 - \delta$  of the probability mass is concentrated in a single cell, we can be certain (modulo any errors introduced by the discretization) that the active localization problem has been solved. Figure 3 shows a starting distribution and possible solution for a very simple environment.

### III. PASSIVE LOCALIZATION

In this section, we solve the passive localization problem, in which the task is to observe the robot's execution

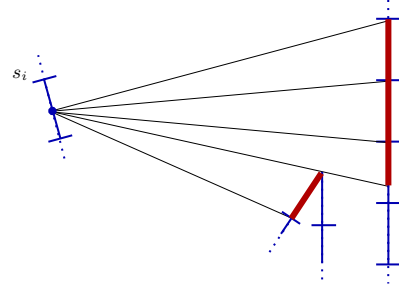


Fig. 4. Computing the set  $V_i$  of cells visible from a cell  $s_i$ . The diagram depicts three cells fully visible from the midpoint of  $s_i$  and one cell that, because of an obstruction, is only partially visible. All four cells are included in  $V_i$ .

and maintain a probability distribution of possible positions within the environment.

The robot receives as input a description of  $W$ , represented as a doubly-connected edge list. As a preprocessing step, we compute for each cell  $s_i$  in the environment discretization  $S$  a list  $V_i \subseteq S$  of cells that are visible, either fully or partially, from the midpoint of  $s_i$ . See Figure 4. This step can be accomplished in time  $O(mn \log n)$  by computing the visibility polygon in  $W$  of each such midpoint [23].

Given the environment  $W$ , a commanded motion direction  $u$ , and a belief distribution  $P_k$ , the problem of passive localization is to compute an updated distribution  $P_{k+1}$ . To accomplish this, we compute a transition matrix  $R_{u,k}$  such that

$$P_{k+1} = R_{u,k} P_k \quad (3)$$

The interpretation of  $R_{u,k}$  is that the entry at row  $i$ , column  $j$  contains the fraction of probability mass that moves from cell  $s_i$  to cell  $s_j$ , under a motion in direction  $u$ . We use the subscript  $k$  on the transition matrix  $R$  to emphasize the dependence on time-varying the orientation error distribution  $p_k$ .

It remains to describe how to compute  $R_{u,k}$ . Column  $i$  of this matrix describes how probability mass moves from  $s_i$  to each other cell of the discretization. Since the robot moves in a straight line, this fraction is nonzero only for cells visible from  $s_i$ , that is, the cells in  $V_i$ . For each of these, we compute two angles  $\theta_1$  and  $\theta_2$  that bound the interval<sup>1</sup> of orientation errors that, given commanded motion direction  $u$ , leads the robot from the midpoint of  $s_i$  into a visible portion of  $s_j$ . By integrating  $p_k$  over the interval between  $\theta_1$  and  $\theta_2$ , we obtain the transition probability from cell  $s_i$  to cell  $s_j$ . Since  $p_k$  has a continuous cumulative distribution function, the integral is guaranteed to exist. After all of the visible cells in  $V_i$  are accounted for, the remaining probability remains concentrated in  $s_i$ . This procedure is pictured Figure 5 and summarized in Algorithm 1. The algorithm runs in

<sup>1</sup>Note that, if the visibility of  $s_j$  by the midpoint of  $s_i$  is obstructed by an obstacle small relative to  $\epsilon$ , then visible portion of  $s_j$  need not be a segment. Although, in general, this indicates the  $\epsilon$  is too large for localization in  $W$ , this case can be handled in Algorithm 1 by integrating over each connected component of the visible portion of  $s_j$  and summing the results.

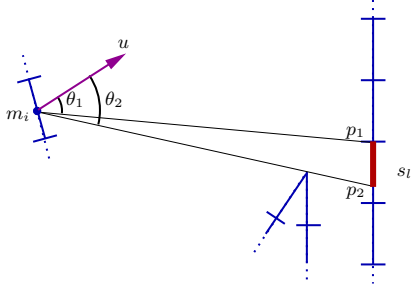


Fig. 5. Computing a single entry in  $R_{u,k}$ . Orientation errors between  $\theta_1$  and  $\theta_2$  will lead the robot from  $s_i$  to  $s_l$ .

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**Algorithm 1** *PassiveLocalization*( $W, S, P_k, u$ )

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1:  $R_{u,k} \leftarrow m \times m$  matrix of zeros
2: for  $i \in \{1, \dots, m\}$  do
3:    $m_i \leftarrow$  midpoint of  $s_i$ 
4:   for  $s_l \in V_j$  do
5:     if  $s_l \neq s_i$  then
6:        $\overline{p_1 p_2} \leftarrow$  maximal subset of  $s_l$  visible from  $m_i$ 
7:        $\theta_1 \leftarrow u - \text{ANGLE}(p_1 - m_i)$ 
8:        $\theta_2 \leftarrow u - \text{ANGLE}(p_2 - m_i)$ 
9:        $R_{u,k,l,i} \leftarrow \left| \int_{\theta_1}^{\theta_2} p_k(\phi) d\phi \right|$ 
10:    end if
11:  end for
12:   $R_{u,k,j,i} \leftarrow 1 - \sum_{1 \leq j \leq m, j \neq i} R_{u,k,j,i}$ 
13: end for
14:  $P_{k+1} \leftarrow R_{u,k} P_k$ 
15: return  $P_{k+1}$ 

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time  $O(m^2)$ , which (holding  $\text{perimeter}(\partial W)$  constant), is  $O(\epsilon^{-2})$ .

#### IV. ACTIVE LOCALIZATION

Now we turn to the problem of active localization. We present an algorithm that chooses motions for the robot in order to eliminate uncertainty in its position. The intuition is to chain together a sequence of subplans, each of which “merges” the probability mass from two cells into a single destination. This basic structure is inspired by the algorithm of [22], but because we admit errors in control, the algorithm requires significant modifications. Our algorithm is greedy in the sense that it selects, from a group of candidate subplans, the subplan that makes the most “progress” toward localization.

We propose a certain form of entropy as a progress measure in Section IV-A, describe how we generate subplans in Section IV-B, and combine these two elements to form a complete active localization plan in Section IV-C.

##### A. Progress measure

We follow [5] and others in using entropy as a heuristic for measuring the progress of the algorithm. Recall that  $P_k$  approximates the density of a continuous random variable representing the robot’s true position. Let  $f_{P_k} : \partial W \rightarrow [0, 1]$  denote the (piecewise constant) approximation to the true

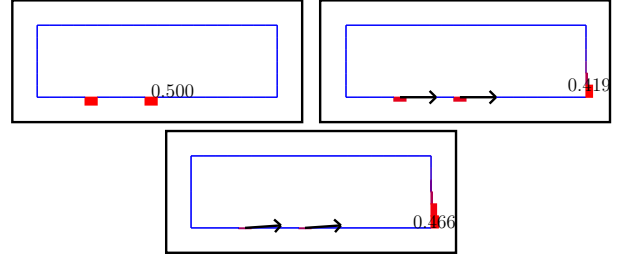


Fig. 6. Adjusting robot’s motion angle. [top left] An initial, bimodal distribution. [top right] Because of orientation errors, a motion parallel to the wall leaves a large fraction of the probability mass unaffected. [bottom] A small adjustment to the motion direction, computed by Algorithm 2 with  $\alpha = 0.1$ , corrects the problem.

density function induced by  $P_k$ . This density has differential entropy

$$h(f_{P_k}) = - \int_{\partial W} f_{P_k}(x) \log f_{P_k}(x) dx \quad (4)$$

$$= - \sum_{i=1}^m \int_{s_i} f_{P_k}(x) \log f_{P_k}(x) dx \quad (5)$$

$$= - \sum_{i=1}^m \text{length}(s_i) P_{k,i} \log P_{k,i}. \quad (6)$$

Note in particular that this formulation differs from the discrete entropy of  $P_k$ , because the contribution of each cell to  $h(f_{P_k})$  is weighted by the size of that cell. Our algorithm selects a series of motions  $u_1, \dots, u_k$  intended to minimize  $h(f_{P_k})$ .

##### B. Candidate subplans

Suppose two cells  $s_i$  and  $s_j$  each have nonzero probability in  $P_k$ . What actions by the robot will transfer (most of) the probability mass in these cells into a single common destination? A solution to a similar problem for a robot without errors appears in [22]. The approach is based on pursuit-evasion, in which one point (representing a possible position of the robot) chases another, repeatedly moving in the direction of the first step of the shortest path in  $W$  between the two points, until they finally merge.

Unfortunately, this solution is not directly applicable, because it often generates motions that require the robot to move very close to  $\partial W$  without triggering its contact sensor. In extreme cases, the robot makes a “collapsing transition” by sliding along an edge of  $\partial W$ . Such motions are not reliable for the robot model in this work, because so much probability mass lies between angles occupied by the wall the robot is currently touching. This probability mass corresponds to the possibility that the robot may collide with the wall immediately. Figure 6 illustrates the phenomenon.

This behavior can be minimized by adjusting the robot’s motion direction away from the boundary, thereby increasing the probability that the robot will move. Select an algorithm parameter  $\alpha \in (0, 1/2]$ , representing the maximum allowable chance of the robot failing to make a move. Then, given a desired motion direction  $u$  and a cell  $s_i$ , we can compute a

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**Algorithm 2** AdjustAction( $u, s_i, k$ )

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1:  $(x_1, x_2) \leftarrow$  endpoints of  $s_i$ , ordered so that the interior
   of  $W$  is on the clockwise side.
2:  $m \leftarrow (x_1 + x_2)/2$ 
3:  $\psi_1 \leftarrow \text{ANGLE}(x_1 - m)$ 
4:  $\psi_2 \leftarrow \text{ANGLE}(x_2 - m)$ 
5:  $u_{min} \leftarrow \min(u, \text{ANGLE}((x_2 - x_1)^\perp))$ 
6:  $u_{max} \leftarrow \max(u, \text{ANGLE}((x_2 - x_1)^\perp))$ 
7:  $u \leftarrow (u_{max} + u_{min})/2$ 
8:  $b \leftarrow \int_{[\psi_2, \psi_1]} p_k(\theta) d\theta$ 
9: if  $b \leq \alpha$  then
10:   return  $u$ 
11: end if
12: while  $|b - \alpha| > \zeta$  do
13:   if  $b < \alpha$  then
14:      $u_{min} \leftarrow u$ 
15:   else
16:      $u_{max} \leftarrow u$ 
17:   end if
18:    $u \leftarrow (u_{max} + u_{min})/2$ 
19:    $b \leftarrow \int_{[\psi_2, \psi_1]} p_k(\theta) d\theta$ 
20: end while
21: return  $u$ 
```

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new motion direction  $u'$ , defined as the direction closest to  $u$  that leaves at most  $\alpha$  probability of the robot not moving from  $s_i$ . The resulting angle  $u'$  will always be between  $u$  and the angle normal to the  $s_i$  in the interior direction. This  $u'$  can be approximated to within a tolerance  $\zeta$  using binary search. Algorithm 2 summarizes this method, which runs in time  $O(\log(1/\zeta))$ .

This adjustment technique allows us to construct a subplan that attempts to unify the probability mass in a given pair of cells  $s_i$  and  $s_j$ . We use the midpoints of the cells as representatives, and apply the pursuit-evasion technique introduced in [22] to unify those representatives. At each step, however, we apply Algorithm 2 to adjust those motions to allow for orientation errors. This process continues until the two representatives are merged. Since, in contrast to [22], some motions may actually increase the robot's uncertainty, we truncate the subplan at the stage at which entropy is lowest. The complete algorithm to generate subplans appears in Algorithm 3.

### C. A complete localization plan

Now we can state the complete active localization algorithm, which appears in Algorithm 4. At each step, it considers a set of candidate subplans generated by Algorithm 3 and appends to its master plan the candidate that improves the entropy the most. This process continues until no entropy-reducing candidate can be found.

Which candidate subplans should be considered? Ideally, all  $m^2 - m$  possibilities should each be evaluated, but because generating each subplan is computationally expensive, this approach is impractical. Instead, we choose an algorithm parameter  $N$  and we assign a score  $P_{k,i}P_{k,j}$  to each pair

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**Algorithm 3** CandidateSubplan( $s_i, s_j, P_k$ )

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```
1:  $q_1 \leftarrow$  midpoint of  $s_i$ 
2:  $q_2 \leftarrow$  midpoint of  $s_j$ 
3:  $P_{min} \leftarrow P_k$ 
4:  $\pi \leftarrow$  empty list of actions
5: while  $|q_1 - q_2| > \epsilon/2$  do
6:    $u \leftarrow$  first step of the shortest path from  $q_1$  to  $q_2$ 
7:    $s \leftarrow$  cell in  $S$  containing  $q_1$ 
8:    $u \leftarrow \text{ADJUSTACTION}(u, s, k)$ 
9:   append  $u$  to  $\pi$ 
10:   $R_{u,k} \leftarrow \text{PASSIVELOCALIZATION}(W, S, P_k, u)$ 
11:   $P_{k+1} \leftarrow R_{u,k}P_k$ 
12:   $k \leftarrow k + 1$ 
13:  if  $h(f_{P_k}) < h(f_{P_{min}})$  then
14:     $\pi_{min} \leftarrow \pi$ 
15:     $P_{min} \leftarrow P_k$ 
16:  end if
17:   $q_1 \leftarrow \text{SHOOTRAY}(W, q_1, u)$ 
18:   $q_2 \leftarrow \text{SHOOTRAY}(W, q_2, u)$ 
19: end while
20: return  $(\pi_{min}, P_{min})$ 
```

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$(s_i, s_j)$  of cells. The algorithm considers only the  $N$  pairs with the highest scores. This had the effect of excluding low-probability unions that would more likely scatter the probability mass around the environment rather than lower the entropy. Note that the pairs  $(s_i, s_j)$  and  $(s_j, s_i)$  are distinct (and therefore considered separately) but always have identical scores. In practice, we obtained acceptable results setting  $N$  as low as 10.

## V. EXPERIMENTAL RESULTS

We have implemented this algorithm and evaluated its effectiveness in localizing a Roomba autonomous vacuum cleaner robot. In this section, we describe those experiments.

Since the Roomba is a disk rather than a point, we perform computations using the configuration space of the robot in  $W$ , rather than  $W$  itself. Where the boundary of the configuration space is a circular arc, we use a piecewise-linear approximation by segments of length less than  $\epsilon$ .

Note that even as simple a robot as the Roomba is equipped with several sensors that we ignore in our models. In particular, the robot has an infrared wall sensor with range approximately 5cm, a more powerful infrared sensor for receiving remote control commands, encoders for each wheel, and several sensors that report on the internal conditions of the robot (battery voltage, battery current, etc.). Although some of these sensors provide information that might be helpful for localization, we ignore them in this work because our intention is to find minimal sensor configurations that enable localization solutions. Additionally, because of scalability problems that may eventually arise due to accumulating error in dead reckoning, this algorithm was created with the intention of eventually using a compass instead of a clock, calibration data, and initial heading. Therefore, we do not use the clock for anything other than rotating.



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**Algorithm 4** ActiveLocalization( $W$ )

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1:  $S \leftarrow$  discretization of  $\partial W$  into cells no larger than  $2\epsilon$ 
2:  $\pi \leftarrow$  empty list of actions
3:  $P_0 \leftarrow \left[ \frac{\text{length}(s_1)}{\text{perimeter}(\partial W)} \quad \dots \quad \frac{\text{length}(s_m)}{\text{perimeter}(\partial W)} \right]^T$ 
4:  $k \leftarrow 0$ 
5: loop
6:    $\mathcal{S} \leftarrow S \times S - \{(s, s) \mid s \in S\}$ 
7:   sort  $\mathcal{S}$  by decreasing values of  $P_{k,i}P_{k,j}$ 
8:   delete all but the first  $N$  elements from  $\mathcal{S}$ 
9:    $P_{min} \leftarrow P_k$ 
10:   $\pi_{min} \leftarrow$  empty list of actions
11:  for  $(s_i, s_j)$  in  $\mathcal{S}$  do
12:     $(\pi_{can}, P_{can}) \leftarrow \text{CANDIDATESUBPLAN}(s_i, s_j, P_k)$ 
13:    if  $h(f_{P_{can}}) < h(f_{P_{min}})$  then
14:       $\pi_{min} \leftarrow \pi_{can}$ 
15:       $P_{min} \leftarrow P_{can}$ 
16:    end if
17:  end for
18:  if  $\pi_{min}$  is empty then
19:    return  $\pi$ 
20:  else
21:    append  $\pi_{min}$  to  $\pi$ 
22:     $k \leftarrow k + \text{length}(\pi_{min})$ 
23:     $P_k \leftarrow P_{min}$ 
24:  end if
25: end loop
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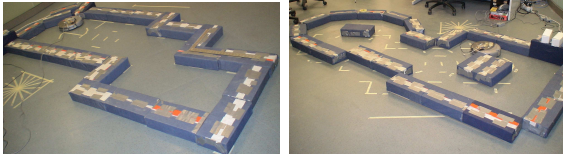


Fig. 7. Two laboratory environments we used to test Algorithm 4. The floor is a pitted vinyl surface, and the walls are covered cinderblocks.

#### A. Error modeling

Since the robot uses dead reckoning to rotate, calibration was required to minimize the rotational error. The robot was commanded to rotate a certain amount of time in an effort to get it to rotate a certain number of radians. Our results in this calibration indicate that the manufacturer-specified rotational speeds were off by small amounts, though large enough that they had to be considered. We also confirmed that the same calibration works well for two other Roombas not used for collecting the calibration data. The calibrations were made on a floor with pitted vinyl tiles, and it is possible that different flooring materials would require different calibrations.

At each stage, the rotation error is relatively small. In our experiments, we modelled this error by using for  $p$  a Gaussian distribution with variance  $\sigma^2 = 0.0001$ . Recall, however, that at stage  $k$ , the accumulated orientation error  $p_k$  has variance  $\sigma_k^2 = k\sigma^2$ .

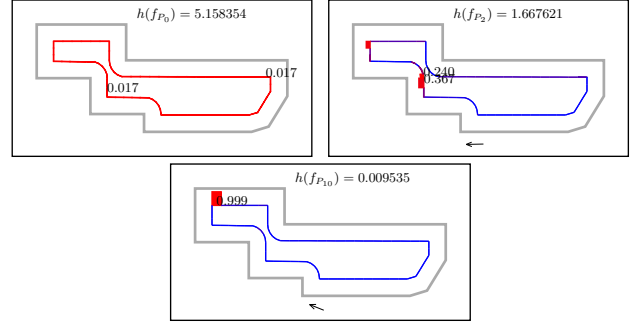


Fig. 8. Localization in a simple environment. [top left] The initial probability distribution around the environment boundary. [top right] An intermediate step resulting from the first two steps of the plan. [bottom] The resulting distribution after the localization plan is run.

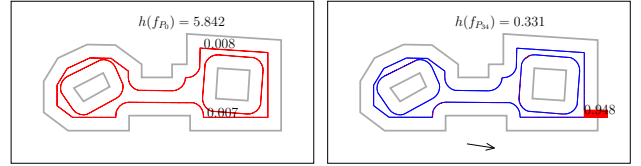


Fig. 9. Localization in a non-simple environment. [left] The initial probability distribution. [right] The final probability distribution.

#### B. Execution examples

We tested Algorithm 4 on two synthetic environments, depicted in Figure 7. Figure 8 shows a localization plan for the environment in the top portion of Figure 7. Our implementation computed a 10-stage plan that concentrated essentially all of the probability mass in a single cell. The Roomba was able to repeatedly localize itself in this environment using this plan, without any failures. Our algorithm computes a similar plan even if the variance in the rotational error is increased by a factor of five. This occurs largely because the environment is relatively simple, having few concavities and no holes.

Figure 9 shows a significantly more difficult environment. There are two holes, and there is a narrow corridor in the center of the environment around which it is difficult for a Roomba to navigate. It also contains a segment in the upper left corner where the environment width is only larger than the robot by a few centimeters. Our implementation computed a 34-stage plan that concentrated almost 95% of the probability mass in a single cell. The Roomba was also able to consistently localize itself in this environment.

We also tested the algorithm in simulation (but not with physical experiments) on about 12 other environments. We were able to solve most of them without any parameter tuning using a standard set of parameters. The left portion of Figure 10 shows one of these, along with the final state after executing a 52-step localization plan. The environments requiring special tuning of  $\alpha$ ,  $N$ , and  $\epsilon$  shared the feature of having portions between which it is difficult to move without stopping in the interior and turning, an action that the robot is incapable of performing. The right portion of Figure 10 shows an extreme example of this feature, where movements by our robot between arms of the environment are difficult

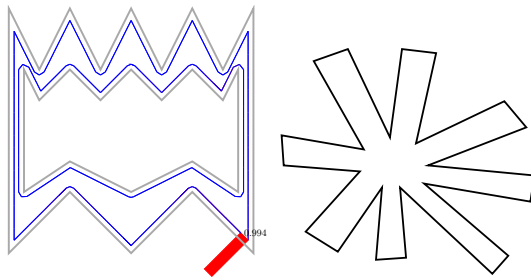


Fig. 10. [left] The final probability distribution after the algorithm was run on a complicated, jagged environment. [right] A problematic environment for the algorithm. The algorithm could not solve this environment with reasonable parameters.

to execute reliably.

## VI. DISCUSSION AND CONCLUSIONS

In this paper, we presented algorithms for both passive and active localization problems a robot equipped with only a contact sensor and a clock, and demonstrated its usefulness experimentally. We have left several important areas unexplored.

First, the active localization algorithm depends on several constants that must be hand-tuned. The discretization resolution  $\epsilon$  can be eliminated using dynamic discretization methods [4]. We found that the performance of Algorithm 4 depends on only weakly on  $N$ , the number of candidate subplans considered. It remains a challenging problem to choose  $\alpha$  in an automated way that balances overly long localization plans (if  $\alpha$  is too small) against the inability to deal with multimodal distributions (if  $\alpha$  is too large).

Second, the environments we used for our experiments are relatively small and artificial. We are actively working to implement our techniques in a much larger, more realistic office environment. We expect larger environments to significantly increase the computation requirements and magnify the noise of the robot's control.

Finally, we are also interested in solving navigation problems with similar sensor limitations. This problem is related to both path planning and active localization, because the robot must carefully plan paths that keep uncertainty at manageable levels throughout the robot's motion. Prior work on so-called "coastal navigation" with longer-range sensors [26] is also relevant. More detailed tasks, such as mapping or delivery, might require even more informative sensors.

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