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Research Article

Fault Diagnosis and Fault-Tolerant Control for Leader-Follower Multi-Agent Systems with Time Delay

Chang Zhang, Yawei Wu, and Lina Yao

School of Electrical Engineering, Zhengzhou University, Zhengzhou 450001, China

Correspondence should be addressed to Lina Yao; michelle_lnxq@126.com

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Under the guidance of the directed graph, this paper studies the fault diagnosis (FD) and fault-tolerant control (FTC) of the leader-follower multi-agent systems (MASs) with time delay. Actuator faults and disturbances are considered in this study, which make the work of this paper more challenging. Firstly, a robust distributed observer on the basis of the relative estimation error is put forward to obtain the fault estimation information. Then, after getting the fault information obtained by the observer, a dynamic compensation FTC protocol on the basis of the relative output tracking error is developed to make the output of followers can track the leader's output. Finally, the proposed algorithm is verified by an example and satisfactory results are obtained.

1. Introduction

In recent years, with the large-scale application of MAS such as UAV (unmanned aerial vehicle) formation flying, multi-unmanned vehicles, and so on, the control of MAS has gradually become a research hotspot. For instance, in [1], a pinning control strategy is proposed for multi-agent systems with exogenous disturbances. In [2], heterogeneous multi-agent system with input constraints composed by second-order linear and nonlinear agents is discussed. A point-to-point tracking control based on iterative learning is proposed in [3] for a class of nonlinear multi-agent systems. A new collaborative output regulation control method is proposed in [4] for the problem of resilient practical co-operative output regulation of heterogeneous multi-agent systems subject to denial-of-service (DoS) attacks. The problem of event-triggered consistent control of nonlinear uncertain systems in the presence of unknown parameters and external disturbances is studied in [5]. Also, there are many other research results on multi-agent control methods such as [6–10]. Compared with a single agent, MAS can obviously accomplish some more challenging tasks, but at the same time, the topology of the multi-agent system becomes more complex. With the topology of MASs becoming more complex, how to ensure the safety and reliability of MASs has also attracted much attention. In order to improve the reliability of MAS, it is necessary to study FD and FTC of MAS.

In the design of FTC algorithm, the prior knowledge of fault estimation information is usually required. So, FD is essential to obtain the information. Nowadays, the problem of FD in MAS has received widespread attention. Adaptive observer, sliding mode observer (SMO), and unknown input observer (UIO) are commonly used in FD of MAS. In [11], for a linear MAS which has actuator faults, an adaptive distributed fault estimation observer on the basis of the relative output estimation error is proposed. In literature [12], a robust distributed fault estimation algorithm on the basis of SMO is developed for linear MAS. In [13], an UIO is constructed for linear MAS with actuator fault only according to the relative output information. In addition, some other algorithms have been presented in [14–20] for the problem of fault diagnosis in multi-agent systems.

It is worth noting that most of the aforementioned studies do not consider the occurrence of time delay in MAS. In practical systems, communication delay will inevitably occur, so it is of great practical importance to consider the possible time delay when studying multi-agent systems. FD and FTC problems for stochastic distribution control systems with time delay have been studied in [21, 22], but it does not take into account the perturbation of the system. The FD and FTC method for systems with fast time-varying delay was proposed in the literature [23]. But it is based on the classical observer error to construct the fault diagnosis algorithm. The FD and FTC problems for nonlinear systems with time delay and linear time-delaying systems containing different parameters have been studied in the literature [24, 25]. In [26], a distributed fault estimation algorithm on the basis of relative output estimation error is developed for interconnected SDC system with time delay. However, most of the aforementioned studies dealing with the time delay problem are for single systems. Few studies have been conducted for fault diagnosis of multi-agent systems with time delay, which is one of the motivations for the research in this paper.

After obtaining the fault estimation information, the FTC strategy can be designed based on the obtained information so that the output of the follower system tracks that of the leader's system even after a failure occurs. Fault-tolerant control of multi-agent systems has attracted a lot of attention from researchers in recent years in order to reduce the damage caused by the system fault. An adaptive backstepping sliding mode FTC strategy is studied for a second-order MAS with actuator fault in [27]. The problem of consensus tracking for MAS with actuator fault types of interruption and partial fault is studied in [28]. But time delay is not considered in this literature. In literature [29], a distributed adaptive FTC protocol on the basis of the network local information is introduced for a kind of MAS, and it also does not contain time delay of the system. In addition, a fault-tolerant feedback controller according to the relative output of neighboring agents is often used to control MAS [16, 30–33].

In this research, a FD and FTC strategy is devised for MAS with time delay. The major contributions of this research cover the following. (1) Time delay and disturbance are considered in MAS, which increases the complexity of FD and FTC. (2) A novel FD observer according to the relative estimation error is developed and the parameters may be gained by addressing linear matrix inequality (LMI). (3) A dynamic compensation FTC protocol on the basis of the relative output tracking error is studied, and the corresponding parameters can be gained according to LMI.

The structure of this paper is arranged as follows. In Section 2, the system model of leader-follower MAS is introduced and some necessary assumptions are established. In Section 3, this paper proposes a robust observer on the basis of relative estimate error to get the fault estimation information. In Section 4, a dynamic compensation controller according to the relative output tracking error is designed to control the leader-follower MAS. In Section 5, a MAS with five agents is taken as an example under a directed graph to prove the validity of the introduced scheme. At last, the conclusions are drawn in Section 6.

Notation. The matrix of $n \times n$ is expressed as $R^{n\times n}$. The inverse and transpose of matrix A are expressed as A^{-1}, A^T, respectively. A^+ is the pseudo-inverse of matrix A. The $n \times n$ identity matrix can be expressed as I_n. Kronecker product is represented by the symbol $\otimes$. $\|\cdot\|_\infty$ and $\|\cdot\|$ represent the infinite norm and 2-norm of the vector or matrix, respectively.

The Basics of Algebraic Graph Theory. In algebraic graph theory, each agent can be considered as a node in the graph. The information exchange of agents can be represented by $G = (V, E, S)$, where $V = \{V_1, V_2, \ldots, V_n\}$ represents the set containing all the nodes in the graph, $E \subset V \times V$ represents the set containing all the edges, and $E_{ji} = (V_j, V)_i$ represents that agent i can transmit the information to agent j. The adjacency matrix of algebraic graph theory can be expressed as $S = [S_{ij}] \in R^{n\times n}$, where $s_{ii} = 0$ and if $E_{ij} \in E$ holds, $s_{ij} = 1$; otherwise, $s_{ij} = 0$. Furthermore, for a directed graph, the in degree and out degree of node i are defined as $\sum_{j=1}^N a_{ij} \sum_{j=1}^N a_{ji}$, respectively. In this study, undirected graphs are considered, where $\sum_{j=1}^N a_{ij} \sum_{j=1}^N a_{ji}$. The Laplace matrix of graph G can be expressed as $L = [L_{ij}] \in R^{n\times n} = D - A$, where $D \triangleq \mathrm{diag}\{\sum_{j\in V_1} a_{1j} \sum_{j\in V_2} a_{2j}, \ldots, \sum_{j\in V_n} a_{nj}\}$. The matrix $G = \mathrm{diag}\{g_i\} \in R^{N\times N}$ can represent the communication between the leader system and the follower system, where the i-th agent can gain the information of the leader, $g_i = 1$, or else, $g_i = 0$.

2. Model Description

Considering the MAS with N followers and 1 leader, the state equation of the i-th follower system is shown as follows:

$$\begin{aligned} \dot{x}_i(t) &= Ax_i(t) + A_d x_i(t-d) + Bu_i(t) + Ef_i(t) + Fd_i(t), \\ y_i(t) &= Cx_i(t) \quad i = 1, \ldots, N, \end{aligned} \tag{1}$$

where $x_i \in R^n$ denotes the state of the follower system, $y_i \in R^m$ denotes the output, d represents a constant delay, $u_i \in R^p$ is the control input of the system, $f_i(t) \in R^s$ stands for actuator fault and it is bounded, and $d_i(t) \in R^l$ represents the external disturbance or system uncertainty. In this article, A, $A_d B$, E, F, C are constant real matrices of appropriate dimensions. It is supposed that matrices C and E are of full rank and the pair (A, C) is observable.

The leader's system state space equation is shown below:

$$\begin{aligned} \dot{x}_l(t) &= Ax_l(t) + A_d x_l(t-d) + Br(t), \\ y_l(t) &= Cx_l(t), \end{aligned} \tag{2}$$

where $x_l \in R^n$ represents the state vector of the leader system, $y_l \in R^m$ is the output vector, and $r \in R^p$ is the known given reference input trajectory.

Assumption 1. It is assumed that system parameters B, E satisfy the following condition: $\mathrm{rank}(B, E) = \mathrm{rank}(B)$.

It is worth noting that when Assumption 1 is satisfied, there is a matrix B^* that satisfies the following condition:

$$(I - BB^*)E = 0. \tag{3}$$

3. Fault Diagnosis

FD is required to get an estimation of the fault. Design the following robust observer as

$$\begin{aligned} \dot{\hat{x}}_i &= A\hat{x}_i + A_d\hat{x}_i(t-d) + Bu_i(t) + E\hat{f}_i, \\ \hat{y}_i &= C\hat{x}_i\,(i = 1,\cdots N), \end{aligned} \tag{4}$$

where $\hat{x}_i \in R^n$ denotes the state estimation vector, $\hat{y}_i \in R^p$ denotes the output estimation vector, and $\hat{f}_i(t)$ indicates the estimation vector of the fault, which can be obtained by the dynamic compensator $g(x)$.

Different from the traditional residual form, relative estimation errors are often used in MAS, and it can be shown as follows:

$$\begin{aligned} \varepsilon_i &= \sum_{j\in N_i} a_{ij}\left\{\left[y_i(t) - \hat{y}_i\right] - \left[y_j(t) - \hat{y}_j(t)\right]\right\} \\ &\quad + g_i\left[y_i(t) - \hat{y}_i(t)\right]. \end{aligned} \tag{5}$$

The dynamic compensator $g(x)$ can be indicated as

$$\begin{cases} \dot{x}_{ki}(t) = A_k x_{ki}(t) + B_k\varepsilon_i(t), \\ \hat{f}_i(t) = C_k x_{ki}(t) + D_k\varepsilon_i(t), \end{cases} \tag{6}$$

where $x_{ki} \in R^{ni}$ is the state vector of the dynamic compensator.

Denote

$$\begin{aligned} e_{x_i} &= x_i - \hat{x}_i, \\ e_{f_i} &= f_i - \hat{f}_i. \end{aligned} \tag{7}$$

Then, the observation error system is formulated as follows:

$$\begin{aligned} \dot{e}_{x_i}(t) &= Ae_{x_i}(t) + A_d e_{x_i}(t-d) \\ &\quad + E\left(f_i(t) - \hat{f}_i(t)\right) + Fd_i(t). \end{aligned} \tag{8}$$

Denote $z_i = \begin{bmatrix} e_{x_i}^T & e_{f_i}^T \end{bmatrix}^T, w_i = \begin{bmatrix} f_i^T & d_i^T \end{bmatrix}^T$, and the following formula can be further gained as

$$\begin{aligned} \dot{e}_{x_i}(t) &= Ae_{x_i}(t) + A_d e_{x_i}(t-d) + C_1 w_i - E\hat{f}_i, \\ z_i &= A_1 e_{x_i} + C_2 w_i - D_1\hat{f}_i, \end{aligned} \tag{9}$$

where

$$\begin{aligned} C_1 &= \begin{bmatrix} E & F \end{bmatrix}, \\ A_1 &= \begin{bmatrix} I \\ 0 \end{bmatrix}, \\ C_2 &= \begin{bmatrix} 0 & 0 \\ I & 0 \end{bmatrix}, \\ D_1 &= \begin{bmatrix} 0 \\ I \end{bmatrix}. \end{aligned} \tag{10}$$

Denote

$$\begin{aligned} e_x &= \begin{bmatrix} e_{x_1}^T & e_{x_2}^T & \cdots & e_{x_N}^T \end{bmatrix}^T, \\ u &= \begin{bmatrix} u_1^T & u_2^T & \cdots & u_N^T \end{bmatrix}^T, \\ d &= \begin{bmatrix} d_1^T & d_2^T & \cdots & d_N^T \end{bmatrix}^T, \\ \varepsilon &= \begin{bmatrix} \varepsilon_1^T & \varepsilon_2^T & \cdots & \varepsilon_N^T \end{bmatrix}^T, \\ f &= \begin{bmatrix} f_1^T & f_2^T & \cdots & f_N^T \end{bmatrix}^T, \\ \hat{f} &= \begin{bmatrix} \hat{f}_1^T & \hat{f}_2^T & \cdots & \hat{f}_N^T \end{bmatrix}^T, \\ x_k &= \begin{bmatrix} x_{k1}^T & x_{k2}^T & \cdots & x_{kN}^T \end{bmatrix}^T, \\ w &= \begin{bmatrix} w_{k1}^T & w_{k2}^T & \cdots & w_{kN}^T \end{bmatrix}^T. \end{aligned} \tag{11}$$

Combining (5) and (9), it can be deduced that

$$\begin{cases} \dot{e}_x(t) = (I_N\otimes A)e_x(t) + (I_N\otimes A_d)e_x(t-d) + (I_N\otimes C_1) \\ w(t) - (I_N\otimes E)\hat{f}(t), \\ z(t) = (I_N\otimes A_1)e_x(t) + (I_N\otimes C_2)w(t) - (I_N\otimes D_1)\hat{f}(t), \\ \varepsilon(t) = ((L+G)\otimes C)e_x(t). \end{cases} \tag{12}$$

The global system of the dynamic compensator $g(x)$ can be expressed as follows:

$$\begin{cases} \dot{x}_g(t) = \left(I_N\otimes A_g\right)x_g(t) + \left(I_N\otimes B_g\right)\varepsilon(t), \\ \hat{f}(t) = \left(I_N\otimes C_g\right)x_g(t) + \left(I_N\otimes D_g\right)\varepsilon(t). \end{cases} \tag{13}$$

Denote $\zeta(t) = [e_x^T(t)x_g^{T(t)}]^T$, and combine equations (12) and (13), and it can be gained that

$$\begin{cases} \dot{\zeta}(t) = \overline{A}\zeta(t) + \overline{A_d}\xi(t-d) + \overline{B}w(t), \\ z = H\xi(t) + \overline{C}w(t), \end{cases} \tag{14}$$

where

$$\begin{aligned} \overline{A} &= \begin{bmatrix} I_N\otimes A - \left((L+G)\otimes ED_gC\right) & -\left(I_N\otimes EC_g\right) \\ (L+G)\otimes B_gC & I_N\otimes A_g \end{bmatrix}, \\ \overline{A_d} &= \begin{bmatrix} I_N\otimes A_d & 0 \\ 0 & 0 \end{bmatrix}, \\ \overline{B} &= \begin{bmatrix} I_N\otimes C_1 \\ 0 \end{bmatrix}, \\ H &= \begin{bmatrix} I_N\otimes A_1 - \left((L+G)\otimes D_1D_gC\right) & -I_N\otimes D_1C_g \end{bmatrix}, \\ \overline{C} &= \left(I_N\otimes C_2\right). \end{aligned} \tag{15}$$

It can be further found that

$$\begin{aligned} \overline{A} &= \overline{A_1} + \overline{LKC_1}, \\ H &= H_1 + \overline{DKC_1}, \end{aligned} \tag{16}$$

where

$$\begin{aligned}
\overline{A_1} &= \begin{bmatrix} I\otimes A & 0 \\ 0 & 0 \end{bmatrix}, \\
\overline{L} &= \begin{bmatrix} -I\otimes E & 0 \\ 0 & I\otimes I \end{bmatrix}, \\
\overline{K} &= \begin{bmatrix} I\otimes D_g & I\otimes C_g \\ I\otimes B_g & I\otimes A_g \end{bmatrix}, \\
\overline{C_1} &= \begin{bmatrix} ((L+G)\otimes C) & 0 \\ 0 & I_N\otimes I \end{bmatrix}, \\
\overline{D} &= \begin{bmatrix} -I\otimes D_1 & 0 \end{bmatrix}, \\
H_1 &= \begin{bmatrix} I\otimes A_1 & 0 \end{bmatrix}.
\end{aligned} \tag{17}$$

Lemma 1. *Given a symmetric matrix* $M = \begin{bmatrix} M_{11} & M_{12} \\ * & M_{22} \end{bmatrix}$, *where* M_{11} *and* M_{22} *are symmetric matrices, the following inequality condition can be obtained:* $(1) M<0$, $(2) M_{22}<0$, $M_{11}-M_{12}M_{22}^{-1}M_{12}^T<0$, $(3) M_{11}<0, M_{22}-M_{12}^T M_{11}^{-1}M_{12}<0$.

Lemma 2. *For the parameter* $\beta>0$, *if there are positive definite symmetric matrices (PDSM)* P, Q *meeting the LMI*

$$\Psi = \begin{bmatrix} A^TP+PA+Q & PA_d & PB & C^T \\ * & -Q & 0 & 0 \\ * & * & -\beta I & D^T \\ * & * & * & -\beta I \end{bmatrix} < 0, \tag{18}$$

then the following linear system:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t-d) + Bv(t), \\ y(t) &= Cx(t) + Dv(t), \end{aligned} \tag{19}$$

should meet the H_∞ *performance index* $\|y(t)\|_2 \le \beta^2 \|v(t)\|_2$.

Proof. Select the Lyapunov function shown below:

$$V(t) = x^T(t)Px(t) + \int_{t-d}^{t} x^T(t)Qx(t). \tag{20}$$

It can be further obtained that

$$\begin{aligned} \dot{V}(t) = {} & x^T(t)\left(PA + A^TP + Q\right)x(t) + +2x^T(t)PA_d x(t-d) \\ & + 2x^T(t)PBv(t) - x^T(t-d)Qx(t-d). \end{aligned} \tag{21}$$

When linear system (19) meets the H_∞ performance index $J = (1/\beta)y^T(t)y(t) - \beta v^T(t)v(t) < 0$, the inequality $\Xi = \dot{V} + J < 0$ holds. It can be found that

$$\Xi = \begin{bmatrix} x(t) \\ x(t-d) \\ v(t) \end{bmatrix}^T \left(\begin{bmatrix} PA+A^TP+Q & PA_d & PB \\ * & -Q & 0 \\ * & * & -\beta I \end{bmatrix} + \frac{1}{\beta} \begin{bmatrix} C^T \\ 0 \\ D^T \end{bmatrix} \begin{bmatrix} C & 0 & D \end{bmatrix} \right) \begin{bmatrix} x(t) \\ x(t-d) \\ v(t) \end{bmatrix} < 0. \tag{22}$$

Combined with Lemma 1, it can be obtained that $\Xi<0$ is equivalent to $\Psi<0$. The proof is completed.

Theorem 1. *For scalars* $\delta_1>0, \delta_2>0$, *if there is a PDSM* $\overline{P} = \begin{bmatrix} I_N\otimes P_{11} & 0 \\ 0 & I_N\otimes P_{22} \end{bmatrix}$ *and matrix* $Y = \begin{bmatrix} (L+G)\otimes Y_{11} & I_N\otimes Y_{12} \\ (L+G)\otimes Y_{21} & I_N\otimes Y_{22} \end{bmatrix}$ *satisfying the following LMI:*

$$\pi = \begin{bmatrix} \pi_{11} & \pi_{12} & I\otimes A_d & 0 & I\otimes C_1 & \pi_{16} & I\otimes P_{11} & 0 \\ * & \pi_{22} & 0 & 0 & 0 & \pi_{26} & 0 & I\otimes P_{22} \\ * & * & -\delta_2 I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\delta_2 I & 0 & 0 & 0 & 0 \\ * & * & * & * & -\delta_1 I & (I\otimes C_2)^T & 0 & 0 \\ * & * & * & * & * & -\delta_1 I & 0 & 0 \\ * & * & * & * & * & * & -\delta_2^{-1} I & 0 \\ * & * & * & * & * & * & * & -\delta_2^{-1} I \end{bmatrix} < 0, \tag{23}$$

where $Y = \overline{KC_1P}$, $\pi_{11} = I_N \otimes P_{11}A + I_N \otimes A^T P_{11} - (L+G)\otimes EY_{11} - (L+G)^T \otimes Y_{11}^T E^T$, $\pi_{12} = -I_N \otimes EY_{12} - (L+G)^T \otimes Y_{21}^T$, $\pi_{22} = I_N \otimes Y_{22} + I_N \otimes Y_{22}^T$, $\pi_{16} = I_N \otimes P_{11}A_1^T - (L+G)^T \otimes Y_{11}^T D_1^T$, $\pi_{26} = -I_N \otimes Y_{12}^T D_1^T$ *then system (14) is stable and satisfies the* H_∞ *performance index* $\|z(t)\|_2 \le \delta_1^2 \|v(t)\|_2$.

Proof. From Lemma 2, if the H_∞ performance index $\|z(t)\|_2 \le \delta_1^2 \|v(t)\|_2$ is satisfied in system (14), then there are P, Q to make the following inequality hold:

$$\begin{bmatrix} \overline{A}^T P + P\overline{A} + Q & P\overline{A}_d & P\overline{B} & H^T \\ * & -Q & 0 & 0 \\ * & * & -\delta_1 I & \overline{C}^T \\ * & * & * & -\delta_1 I \end{bmatrix} < 0. \tag{24}$$

By combining equation (16), it can be further found that

$$\begin{bmatrix} \overline{A}_1^T P + P\overline{A}_1 + P\overline{LKC}_1 + \overline{C}_1^T \overline{K}^T \overline{A}^T P^T + Q & P\overline{A}_d & P\overline{B} & H_1^T + \overline{C}_1^T \overline{K}^T \overline{D}^T \\ * & -Q & 0 & 0 \\ * & * & -\delta_1 I & \overline{C}^T \\ * & * & * & -\delta_1 I \end{bmatrix} < 0. \tag{25}$$

Multiplying the left side and the right side of inequality (25) by $\operatorname{diag}\{P^{-1} \ \ I \ \ I \ \ I\}$, it can be obtained that

$$\begin{bmatrix} \Phi & \overline{A}_d & \overline{B} & P^{-1}\overline{C}_1^T \overline{K}^T \overline{D}^T + P^{-1} H_1^T \\ * & -Q & 0 & 0 \\ * & * & -\delta_1 I & \overline{C}^T \\ * & * & * & -\delta_1 I \end{bmatrix} < 0, \tag{26}$$

where $\Phi = P^{-1}\overline{A}_1^T + \overline{A}_1 P^{-1} + \overline{LKC}_1 P^{-1} + P^{-1} \ \ \overline{C_1^T}\overline{K}^T \overline{L}^T + P^{-1} Q (P^{-1})^T$.

Combining Lemma 1 and defining $Y = \overline{KC}_1 P^{-1}, P^{-1} = \overline{P}$, it can be found that

$$\begin{bmatrix} \overline{PA}_1^T + \overline{A}_1 \overline{P} + \overline{LY} + \overline{YL}^T & \overline{A}_d & \overline{B} & Y^T \overline{D}^T + XH_1^T & \overline{P} \\ * & -Q & 0 & 0 & 0 \\ * & * & -\delta_1 I & \overline{C}^T & 0 \\ * & * & * & -\delta_1 I & 0 \\ * & * & * & * & -Q^{-1} \end{bmatrix} < 0. \tag{27}$$

For convenience, choose $Q = \delta_2 I$ and bring $\overline{A}_1, \overline{L}, \overline{A}_d, \overline{D}, H_1, \overline{C}, \overline{P}, Y$ into the inequality (27), and it can found that $\pi < 0$. The proof is completed. After $\overline{P}, Y$ is gained by solving LMI ($\pi < 0$), the parameter matrix $\overline{K}$ of the dynamic compensator $g(x)$ can be obtained by $\overline{K} = Y(\overline{C}_1 \overline{P})^{+}$.

4. Fault-Tolerant Control

After getting the fault estimation information, the FTC strategy based on the relative output estimation is devised to make the output of the follower system still track the leader's output after a failure occurs.

The state tracking error between the i-th follower and leader is indicated as follows:

$$e_i(t) = x_i(t) - x_0(t). \tag{28}$$

Then, it can be further found that

$$\begin{aligned} \dot{e}_i(t) &= Ae_i(t) + A_d e_i(t-d) + Bu_i(t) \\ &\quad + Ef_i(t) + Fd_i(t) - Br(t). \end{aligned} \tag{29}$$

Denote

$$e(t) = \left[e_1^T(t) \ \ e_2^T(t) \ \ \cdots \ \ e_N^T(t) \right]^T. \tag{30}$$

Then, the global system may be indicated that

$$\begin{aligned} \dot{e}_i(t) &= (I_N \otimes A)e(t) + (I_N \otimes A_d)e(t-d) + (I_N \otimes B)u(t) \\ &\quad + (I_N \otimes E)f(t) - (I_N \otimes B)r(t) + (I_N \otimes F)d(t). \end{aligned} \tag{31}$$

Define the relative output tracking error of the i-th agent as shown below:

$$\begin{aligned} \mu_i(t) &= \sum_{j \in N_I} a_{ij}\big(y_i(t) - y_j(t)\big) + g_i\big(y_i(t) - y_0(t)\big) \\ &= ((L+G) \otimes C)e(t). \end{aligned} \tag{32}$$

A dynamic compensation FTC strategy according to the relative output tracking error is developed as follows:

$$\begin{aligned} \dot{\psi}_i(t) &= A_{k1}\psi_i(t) + B_{k1}\mu_i(t), \\ u_i(t) &= C_{k1}\psi_i(t) + D_{k1}\mu_i(t) - B^* E\hat{f}_i + r, \end{aligned} \tag{33}$$

where $\psi_i(t) \in R^q$ is the state vector of the dynamic compensation fault-tolerant controller.

Denote

$$\begin{aligned} \psi(t) &= \left[\psi_1^T(t) \ \ \psi_2^T(t) \ \ \cdots \ \ \psi_N^T(t) \right]^T, \\ \mu(t) &= \left[\mu_1^T(t) \ \ \mu_2^T(t) \ \ \cdots \ \ \mu_N^T(t) \right]^T. \end{aligned} \tag{34}$$

Then, the global system of the dynamic compensation FTC strategy (33) can be indicated as follows:

$$\begin{aligned}\dot{\psi}(t) &= (I_N \otimes A_{k1})\psi(t) + (I_N \otimes B_{k1})\mu(t),\\ u(t) &= (I_N \otimes C_{k1})\psi(t) + (I_N \otimes D_{k1})\mu(t) - (I_N \otimes B^* E)\hat{f}\\ &\quad + (I_N \otimes I)r.\end{aligned} \tag{35}$$

Define $S(t) = \left[e^T(t) \;\; \psi^T(t) \right]^T$, $\overline{d} = \left[e_f^T \;\; d^T \right]^T$ and substitute equations (35) and (32) into (31), and it can be obtained that

$$\begin{aligned}\dot{S}(t) &= A_c S(t) + A_{dc} S(t-d) + \overline{E}\overline{d},\\ e(t) &= C_c S(t),\end{aligned} \tag{36}$$

where

$$\begin{aligned}A_c &= \begin{bmatrix} I_N \otimes A + (L+G)\otimes BD_{k1}C & I \otimes BC_{k1}\\ (L+G)\otimes B_{k1}C & I \otimes A_{k1}\end{bmatrix},\\ A_{dc} &= \begin{bmatrix} I\otimes A_d & 0\\ 0 & 0\end{bmatrix},\\ \overline{E} &= \begin{bmatrix} I\otimes E & I\otimes F\\ 0 & 0\end{bmatrix},\\ C_c &= \begin{bmatrix} I & 0\end{bmatrix}.\end{aligned} \tag{37}$$

Then, it can be further found that

$$A_c = A_{c1} + L_{c1}\overline{K_1 C_c}, \tag{38}$$

where

$$\begin{aligned}A_{c1} &= \begin{bmatrix} I_N\otimes A & 0\\ 0 & 0\end{bmatrix},\\ L_{c1} &= \begin{bmatrix} I_N\otimes B & 0\\ 0 & I_N\otimes I\end{bmatrix},\\ \overline{C_c} &= \begin{bmatrix} (L+G)\otimes C & 0\\ 0 & I_N\otimes I\end{bmatrix},\\ \overline{K_1} &= \begin{bmatrix} I_N\otimes D_{k1} & I_N\otimes C_{k1}\\ I_N\otimes B_{k1} & I_N\otimes A_{k1}\end{bmatrix}.\end{aligned} \tag{39}$$

Theorem 2. *For scalars $\gamma_1 > 0, \gamma_2 > 0$, if there is a PDSM $P_1 = \begin{bmatrix} I_N\otimes P_{111} & I_N\otimes P_{112}\\ I_N\otimes P_{112}^T & I_N\otimes P_{122}\end{bmatrix}$ and a matrix $Y_c = \begin{bmatrix} I_N\otimes Y_{c11} & I_N\otimes Y_{c12}\\ I_N\otimes Y_{c21} & I_N\otimes Y_{c22}\end{bmatrix}$ satisfying the following LMI:*

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & I\otimes P_{111}A_d & 0 & I\otimes P_{111}E & I\otimes P_{111}F & I & 0 & 0\\ * & \Pi_{22} & I\otimes P_{112}^T A_d & 0 & I\otimes P_{112}^T E & I\otimes P_{112}^T F & 0 & I & 0\\ * & * & -\gamma_2 I & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & -\gamma_2 I & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & -\gamma_1 I & 0 & 0 & 0 & 0\\ * & * & * & * & * & -\gamma_1 I & 0 & 0 & 0\\ * & * & * & * & * & * & -\gamma_1 I & 0 & 0\\ * & * & * & * & * & * & * & -\gamma_1 I & 0\\ * & * & * & * & * & * & * & * & -\gamma_1 I\end{bmatrix} < 0, \tag{40}$$

where $Y_c = P_1 L_{c1}\overline{K_1}$, $\Pi_{11} = I\otimes A^T P_{111} + I\otimes P_{111}A + (L+G)\otimes Y_{c11}C + (L+G)^T\otimes C^T Y_{c11}^T + \gamma_2 I$, $\Pi_{12} = I\otimes A^T P_{112} + I\otimes Y_{c12} + (L+G)^T\otimes C^T Y_{c21}^T$, $\Pi_{22} = I\otimes Y_{c22} + I\otimes Y_{c22}^T + \gamma_2 I$ then system (36) is stable and meets H_∞ performance index $\|e(t)\|_2 \le \gamma_1^2\|\overline{d}\|_2$.

Proof. It is known from Lemma 2 that when system (36) meets the H_∞ performance index $\|e(t)\|_2 \le \gamma_1^2\|\overline{d}\|_2$, there are positive definite matrices P_1, Q_1 satisfying the following inequality:

$$\begin{bmatrix} A_c^T P_1 + P_1 A_c + Q_1 & P_1 A_{dc} & P\overline{E} & C_c\\ * & -Q_1 & 0 & 0\\ * & * & -\gamma_1 I & 0\\ * & * & * & -\gamma_1 I\end{bmatrix} < 0. \tag{41}$$

Substituting equation (38) into (41), it can be found that

$$\begin{bmatrix} A_{c1}^T P_1 + P_1 A_{c1} + P_1 L_{c1}\overline{K_1 C_c} + \overline{C_c^T K_1^T} L_{c1}^T P_1 + Q_1 & P_1 A_{dc} & P\overline{E} & C_c\\ * & -Q_1 & 0 & 0\\ * & * & -\gamma_1 I & 0\\ * & * & * & -\gamma_1 I\end{bmatrix} < 0. \tag{42}$$

Define $Y_c = P_1 L_{c1}\overline{K_1}$, and it can be further obtained that

$$\begin{bmatrix} A_{c1}^T P_1 + P_1 A_{c1} + Y_c\overline{C_c} + \overline{C_c^T} Y_c + Q_1 & P_1 A_{dc} & P\overline{E} & C_c \\ * & -Q_1 & 0 & 0 \\ * & * & -\gamma_1 I & 0 \\ * & * & * & -\gamma_1 I \end{bmatrix} < 0. \tag{43}$$

For convenience, choose $Q_1 = \gamma_2 I$ and substitute $P_1, Y_c, A_c, A_{dc}, \overline{E}, C_c$ into (43), and it can obtained that $\Pi < 0$. The proof is completed. After P_1, Y_c is gained by solving LMI $\Pi < 0$, the parameter matrix $\overline{K_1}$ of the dynamic compensation FTC strategy can be gained by $\overline{K_1} = (P_1 L_{c1})^{+} Y_c$.

5. Simulation Example

In the simulation section, we choose a MAS consisting of five kinds of aircraft to illustrate the effectiveness of the proposed algorithm in the paper; in addition, the state delay caused by wireless transmission is considered in the aircraft model. The given parameters of the aircraft are shown as follows:

$$\begin{gathered} A = \begin{bmatrix} -0.0227 & 1 & -0.0002 \\ -17.1 & -0.178 & -12.2 \\ 0 & 0 & -6.67 \end{bmatrix}, \\ A_d = \begin{bmatrix} -0.5 & 0 & 0 \\ 0.5 & -0.5 & 0.5 \\ 0 & 0 & -3.35 \end{bmatrix}, \\ B = \begin{bmatrix} 0 \\ 0 \\ 6.67 \end{bmatrix}, \\ E = \begin{bmatrix} 0 \\ 0 \\ 6.67 \end{bmatrix}, \\ F = \begin{bmatrix} 0.2 \\ 0.2 \\ 0.2 \end{bmatrix}, \\ C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \\ d = 2, d_i(t) \end{gathered} \tag{44}$$

The topology structure of the system is given in Figure 1. The code of 0 acts as the leader, and other agents represent the followers. The Laplacian matrix L and the leader adjacency matrix G are shown below:

$$\begin{gathered} L = \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}, \\ G = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \end{gathered} \tag{45}$$

The parameters of the dynamic compensator $g(x)$ are designed as follows:

$$\begin{gathered} \delta_1 = 2.5, \\ \delta_2 = 1, \\ A_k = \begin{bmatrix} -98.4895 & -0.0112 & -0.0977 \\ -0.0109 & -98.0100 & 0.2348 \\ -0.0976 & 0.2383 & -98.2363 \end{bmatrix}, \\ B_k = \begin{bmatrix} -2.3121 & 2.2697 \\ 28.5611 & -25.7432 \\ -69.8578 & 97.7782 \end{bmatrix}, \\ C_k = \begin{bmatrix} 53 & -74.5 & 154.7 \end{bmatrix}, \\ D_k = \begin{bmatrix} -86.8803 & -114.7595 \end{bmatrix}. \end{gathered} \tag{46}$$

The parameters of the dynamic compensation FTC strategy based on the relative output tracking error are shown as follows:

$$\begin{gathered} \gamma_1 = 2, \\ \gamma_2 = 2, \\ B^* E = 1, \\ A_{k1} = \begin{bmatrix} -0.3139 & 0.0294 & 0.0008 \\ 0.0297 & -0.5474 & -0.0015 \\ 0.0027 & 0.000 & -0.5770 \end{bmatrix}, \\ B_{k1} = \begin{bmatrix} 0.0032 & -0.0054 \\ -0.0440 & 0.0707 \\ 0.2329 & 0.3992 \end{bmatrix}, \\ C_{k1} = \begin{bmatrix} -0.2882 & 0.2756 & 2.5922 \end{bmatrix}, \\ D_{k1} = \begin{bmatrix} -33.2715 & -56.1697 \end{bmatrix}. \end{gathered} \tag{47}$$

In order to verify the effectiveness of the proposed FD algorithm, two different fault forms, time-varying fault and constant fault, are selected here for the experiment. The specific forms of the two faults are described as follows.

Case 1. Only agents 1 and 4 have constant faults, while agents 2 and 3 have no faults.

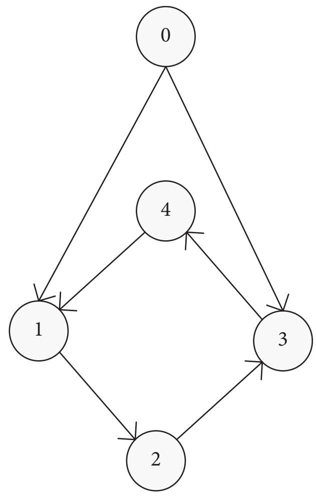

Figure 1: The topology of the MAS.

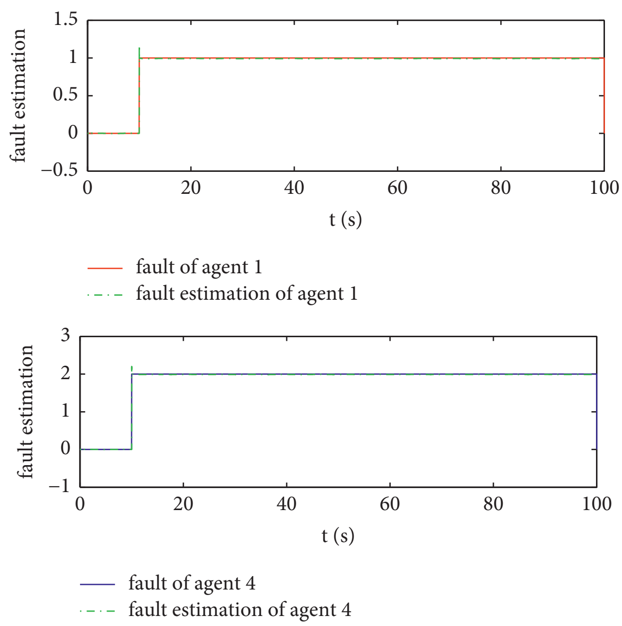

Figure 2: Constant fault and fault estimation.

$$f_1 = \begin{cases} 0 & t < 10 \\ 1 & t \geq 10 \end{cases}, \quad f_4 = \begin{cases} 0 & t < 10 \\ 2 & t \geq 10 \end{cases}, \quad f_2 = f_3 = 0. \tag{48}$$

Case 2. Only agents 1 and 4 have time-varying faults, while agents 2 and 3 have no faults.

$$\begin{aligned} f_1 &= \begin{cases} 0 & t < 10 \\ \sin(0.5(t-10)) & t \geq 10 \end{cases}, \\ f_4 &= \begin{cases} 0 & t < 10 \\ 2\cos(0.5(t-10)) & t \geq 10 \end{cases}, \quad f_2 = f_3 = 0. \end{aligned} \tag{49}$$

Based on the proposed fault diagnosis algorithm, the observer is designed to estimate the constant and time-varying fault, respectively, and the results are shown in Figures 2 and 3. From the simulation results, it can be seen

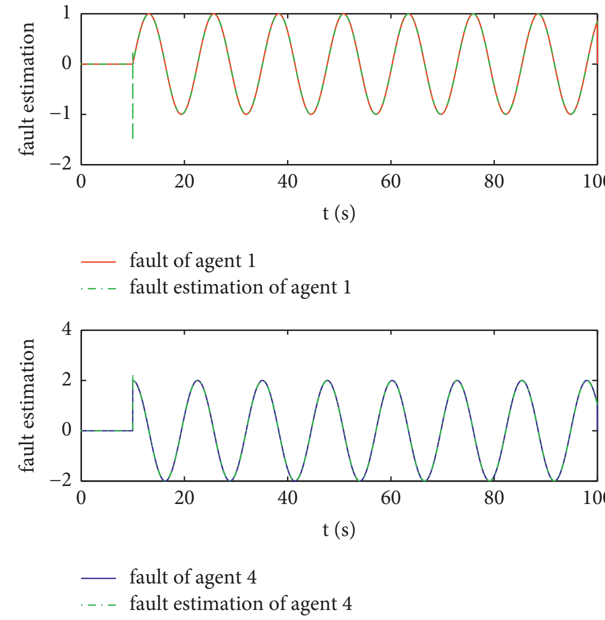

Figure 3: Time-varying fault and fault estimation.

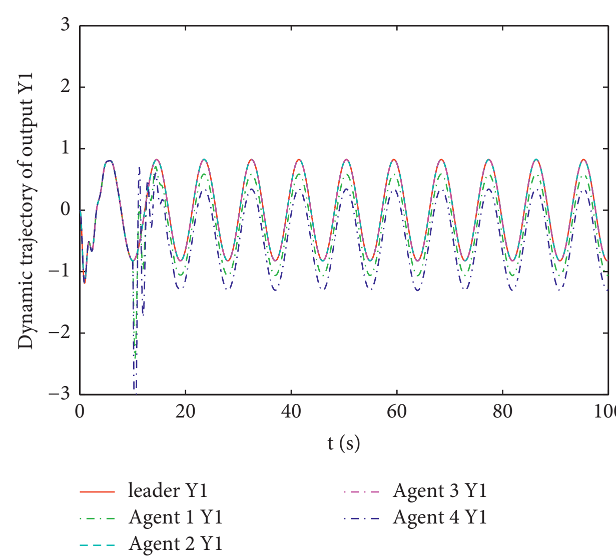

Figure 4: Profiles of output trajectories y_{i1} without fault-tolerant controller.

that the designed observer has good estimation results for both constant and time-varying faults. The output trajectory of the multi-agent system in which the fault occurred is shown in Figures 4 and 5. It can be seen from the figures that the output of the followers no longer tracks the leader's trajectory due to the presence of the fault. After fault-tolerant control is carried out on the system, its output trajectory when failure occurs is shown in Figures 6 and 7. By comparison, it can be seen that the fault-tolerant control algorithm proposed in this paper can make the MAS have good tracking performance even after the fault occurs.

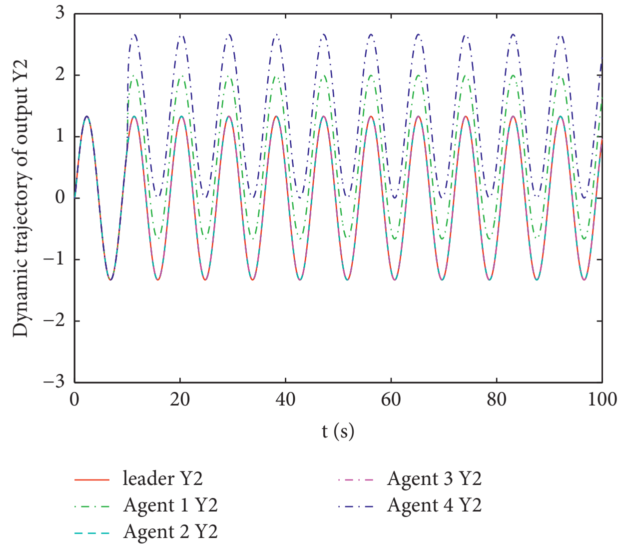

Figure 5: Profiles of output trajectories y_{i2} without fault-tolerant controller.

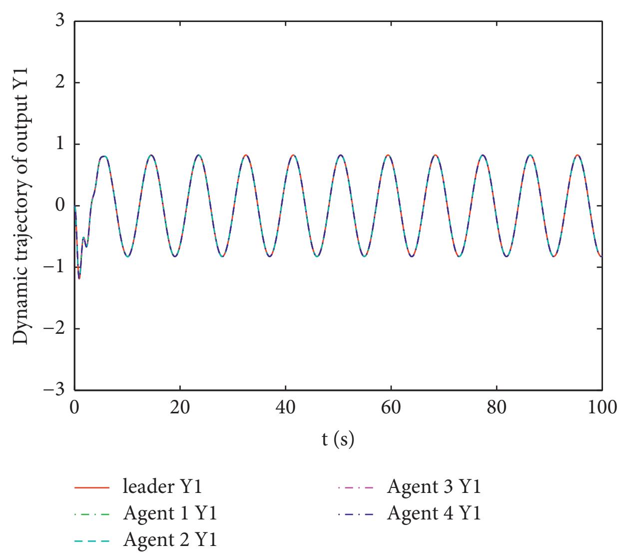

Figure 6: Profiles of output trajectories y_{i1} with fault-tolerant controller.

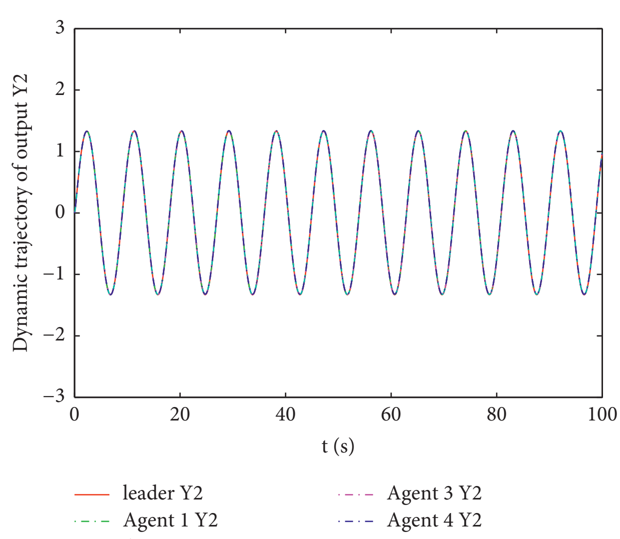

Figure 7: Profiles of output trajectories y_{i2} with fault-tolerant controller.

6. Conclusions

A novel fault diagnosis and fault-tolerant control (FTC) protocol is proposed for the leader-follower multi-agent systems (MASs) with time delay in this paper. At first, design an observer on the basis of the relative estimation error information to estimate the fault of the original MAS. It is worth noting that the fault estimation information is obtained by the dynamic compensator proposed in this paper. Then, after obtaining fault information, a novel dynamic compensation FTC protocol based on relative output tracking error is used in the system to allow the follower's output to still track the leader's output when a fault occurs. In the end, an example of five-aircraft MAS is given to prove the validity of the developed algorithm. However, there are some limitations in this article. The time delay of the system is time invariant and the leader and follower have the same system structure. So, the research on heterogeneous agent systems is interesting which will be focused in our future study.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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