Spectral Analysis of Suspension System of a Commercial City Bus

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***Abstract*—There is an ever increasing demand for intelligent and efficient urban vehicle systems that fulfill several requirements, e.g., low cost maintainability and high passenger comfort. Concerning these goals reliable methods are needed to model and to evaluate the imposed performances. In this paper a spectral analysis of the suspension system of a commercial city bus is presented. Based on experimental data taken on a city bus, the vibrations emerging on the wheels and the body are analyzed in the frequency domain. The goal of the analysis is to characterize the main eigenfrequencies of the suspension system and its damping in amplitude and also to evaluate both the road and the suspension system in terms of passenger comfort according to ISO standards.**

I. Introduction

Vertical acceleration of vehicles generated by variations in road surface induces discomfort of passengers, may cause harm in freight and further impairs the road surface. The role of vehicle suspension system is to isolate vehicle body from vertical accelerations in order to provide a more comfortable ride and a reduced effect on the road. Assessing technical conditions of the road network and of fleets of public vehicles in a city provides an important support to operators in making decisions on the scheduling and harmonization of maintenance works.

Both goals can be achieved by means of collecting measurements of the suspension systems of the fleet of vehicles during normal every-day operation, and performing a vibration analysis that helps to objectively evaluate both the road conditions and the technical conditions of the vehicles. Such an analysis may consists of the following main steps.

1) Constructing a mathematical model for describing nominal vibration mechanism serves as references for evaluating both the technical condition of the suspension system of a specific vehicle, and conditions of road surface. The reference model should be constructed based on experiments of specific circumstances where the road surface is undamaged, the road is straight, the vehicle speed is constant, the vertical dynamics of the vehicle is excited by well specified test obstacles placed on the road. With repeated experiments a set of nominal models are obtained from which a so called *nominal uncertainty model set* can be constructed, [3], [5].
2) Divide the road network into segments and repeat the modeling for every segment during normal operation of the vehicles. Compare the models with the nominal uncertainty model set. The differences have multiple sources: road surface variations, unmodeled dynamics of the suspension system excited by the lateral and longitudinal accelerations and roll and pitch motions of the vehicle and variations in the mass of load. In order to separate the effects by the different sources, the following two tasks can be performed.
    a) the effects of lateral/longitudinal/roll and pitch dynamics should be filtered out from the vertical measurements with the help of measurements and estimation of these disturbing motions
    b) measurements taken on the same segments with multiple vehicles are compared in order to separate the effects of road and the effect of change in the technical conditions of the vehicles.

To begin with, a spectral analysis of the suspension system is presented in this paper. Based on experimental data taken on a city bus, the vibrations emerging on the wheels and the body are analyzed in the frequency domain. One goal of the analysis is to characterize the main eigenfrequencies of the suspension system and its damping in amplitude. The second goal of the analysis is to evaluate both the road and the suspension system in terms of passenger comfort according to ISO standards.

The paper is organized as follows. In order to fix the ideas and to introduce the main notations, the applied spectral analysis tools are presented in Section II. The measurement setting and the experimental conditions are described in Section III. Preparation of the raw measurement data (filtering, resampling) is discussed in Section IV and the spectral analysis of the suspension systems is performed in Section V.

II. Preliminaries on spectral analysis

The goal of spectral estimation is to describe the frequency-distribution of the power contained in a signal, based on a finite set of data. The power spectral density (PSD) of a *wide-sense stationary random process* x_t is mathematically related to the autocorrelation sequence by the discrete Fourier transformation (DFT). In terms of normalized frequency, this is given by [4], [6]

$$P_{xx}(f) = T_s \sum_{m=-\infty}^{\infty} R_{xx}(m) e^{-i2\pi m f T_s}, \tag{1}$$

where the autocorrelation sequence is defined by

$$R_{xx}(m) = \frac{E[(x_t - \mu)(x_{t+m} - \mu)]}{\sigma^2} \tag{2}$$

where μ denotes the mean of the sequence,

$$\sigma^2 = \int_{-f_N}^{f_N} P_{xx}(f) df$$

denotes variance with Nyquist frequency $f_N = \frac{1}{2T_s}$, and $E(.)$ denotes expectation.

A. Periodogram

The PSD can be estimated directly from the sampled data (nonparametric methods). The simplest method is the *periodogram* which is the Fourier transform of the biased estimate of the autocorrelation sequence

$$\hat{P}_{xx}(f) = \frac{T_s}{N} \left| \sum_{t=1}^{N} x_t e^{-i2\pi f(t-1)} \right|^2, \quad -f_N < f < f_N \tag{3}$$

For a one-sided periodogram, the values at all frequencies except 0 and f_N are multiplied by 2 so that the total power is conserved.

Periodograms can be computed by using DFT. The DFT and the inverse DFT of a sequence of sampled data x_t, $t = 1, 2, ..., N$, are

$$X_k = \sum_{t=1}^{N} x_j \omega_N^{(t-1)k}, \qquad k = 0, 1, ..., N-1 \tag{4}$$

$$x_t = \frac{1}{N} \sum_{k=0}^{N-1} X_k \omega_N^{-(t-1)k} \tag{5}$$

where, with the imaginary unit $i = \sqrt{-1}$,

$$\omega_N = e^{-\frac{2\pi i}{N}}. \tag{6}$$

Then, the one sided periodogram is computed by

$$P_{xx}(f_k) = \begin{cases} \frac{T_s}{N}|X_k|^2, & k = 0 \text{ or } k = \frac{N}{2}, \\ 2\frac{T_s}{N}|X_k|^2, & k = 1, 2, ..., \frac{N}{2} - 1 \end{cases} \tag{7}$$

$$f_k = \frac{k}{NT_s}, \qquad k = 0, 1, 2, ..., \frac{N}{2} \tag{8}$$

The periodogram is asymptotically unbiased, however, in some cases the periodogram is a poor estimator of the PSD even when the data record is long. This is due to the variance of the periodogram,

$$\text{Var}(P_{xx}(f_k)) = \begin{cases} 2P_{xx}^2(f_k), & k = 0 \text{ or } k = \frac{N}{2}, \\ P_{xx}^2(f_k), & k = 1, 2, ..., \frac{N}{2} - 1 \end{cases} \tag{9}$$

which does not tend to zero as the data length N tends to infinity. In statistical terms, the periodogram is not a consistent estimator of the true power spectral density of a wide-sense stationary process.

Fig. 1. The test vehicle

B. Welch's method

Welch's technique to reduce the variance of the periodogram breaks the time series into segments, usually overlapping [7]. Welch's method computes a modified periodogram for each segment and then averages these estimates to produce the estimate of the power spectral density. Because the process is wide-sense stationary and Welch's method uses PSD estimates of different segments of the time series, the modified periodograms represent approximately uncorrelated estimates of the true PSD and averaging reduces the variability. The segments are typically multiplied by a window function, such as a Hamming window, so that Welch's method amounts to averaging modified periodograms.

Welch's method yields a biased estimator of the PSD. The expected value of the PSD estimate is:

$$E(P_{Welch}(F)) = \frac{T_s}{N} \int_{-f_N}^{f_N} |W(f - \xi)|^2 P_{xx}(\xi) d\xi, \tag{10}$$

where $W(f)$ id the Fourier transformation of the window function. Welch's estimator is asymptotically unbiased. For a fixed length data record, the bias of Welch's estimate is larger than that of the periodogram because the length of the segments is less than the length of the entire data sample. The variance of Welch's estimator is difficult to compute. Basically, the variance is inversely proportional to the number of segments whose modified periodograms are being averaged.

III. Experimental conditions

A. Route-ways

In order to test the behavior of a commercial city bus (test vehicle, see Figure 1), the bus was driven on two specific route in real traffic conditions, see Figure 2. The bus passed the dedicated route two times. From these experiments, the suspension system and the passenger comfort will be evaluated according to ISO standards.

B. Sensor allocation

On the test bus a 10-channel accelerometer and a specific computerized data acquisition system has been installed. Near the four spindle on each of the Z-axis direction a PCB ICP-type accelerometer was mounted to measure the unsprung mass vertical acceleration. On the floor plate, over these sensors, three differential DC MEMS accelerometers were mounted along the Z-axis direction to measure the vertical acceleration of the sprung mass at front right and the rear on both sides, see Figure 3. The sampling time of the data acquisition system was set to 20 kHz.

In addition to the acceleration transducer system a GPS based track recorder was also installed.

C. Segmentation of the route-way

The data collected from the route-way experiments was segmented into smaller records. Eight pieces, each of length 10.24 sec, were cut. The measurements are affected by both the road surface and the vehicle dynamics. In order to avoid the disturbing effects caused by the lateral and longitudinal acceleration of the bus, the criteria in selecting the segments was the possible constant speed and the straight track. A sample data record can be observed in Figure 4.

IV. Characterization of Measurements

The row data is sampled by 20 kHz resulting in a large data set. The amplitudes of the DFT coefficients (4) of a typical measurement is shown in Figure 5. A significant amount of energy is present at hight frequencies in the signal, while the useful information about the suspension system and for evaluating passenger comfort is up to about 15Hz. The high frequency components of the signal is typically due to electromagnetic noise of the environment, the power network and its harmonics, and also the mechanical vibrations of the engine and gear system. In order to reduce the amount of data, which is important mainly in the model identification phase of future work, the data must be resampled. The new sampling frequency is chosen to be $f_s = 1/T_s = 200Hz$ which is sufficiently high to capture the frequency content of the suspension system. However, resampling must be taken with care, due to aliasing effects. Without anti-aliasing filtering, high frequency components of the signal (noise) appear at lower frequencies after resampling and alter the low frequency components of the signal. For more details on aliasing effects, see [2].

An ideal filter was used as antialiasing filter. First, the DFT components of the 20 kHz signal which correspond to frequencies higher than 25Hz were zeroed out. Then the filtered 20 kHz signal was reconstructed by using inverse DFT (5). In Figure 6 the original signal in time-domain is plotted with green line and the filtered and reconstructed signal with blue line. (Only a 01 sec period is plotted.) This filtered signal can be resampled with $f_s = 200Hz$. For illustration, the original signal is resampled also without filtering. The result is plotted by a red line in Figure 6. The difference between the two resampled signals can be observed in the frequency domain in Figure 7. (The spectrum of the filtered and resampled signal coincides with the spectrum of the original 20 kHz signal.)

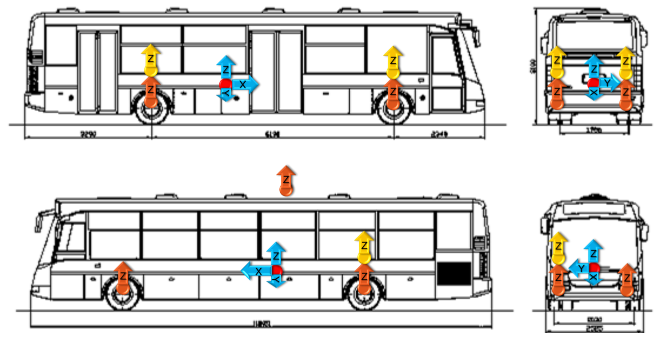

Fig. 3. Sensor allocation

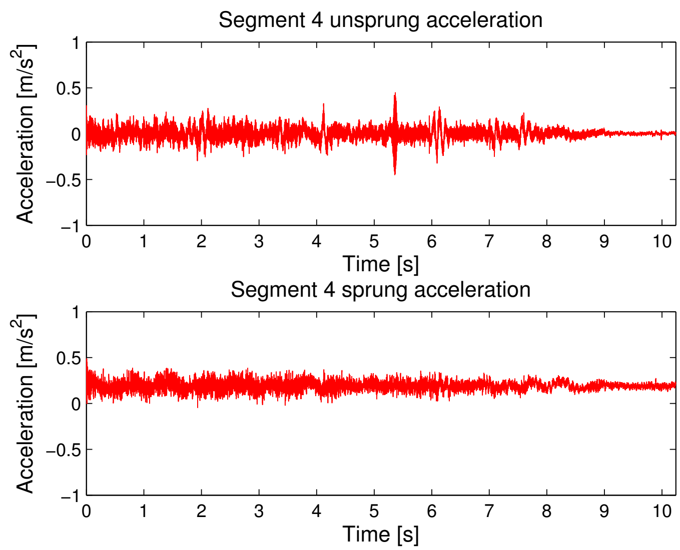

Fig. 4. Measured time-domain acceleration data at the front right. Unsprung mass (top) and sprung mass (down).

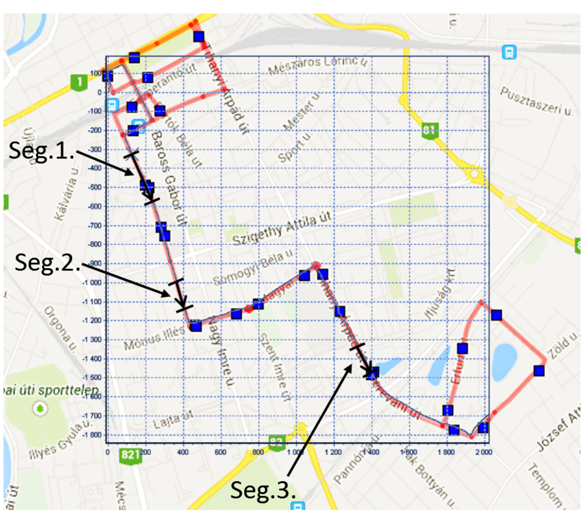

Fig. 2. Dedicated route for the identification experiment

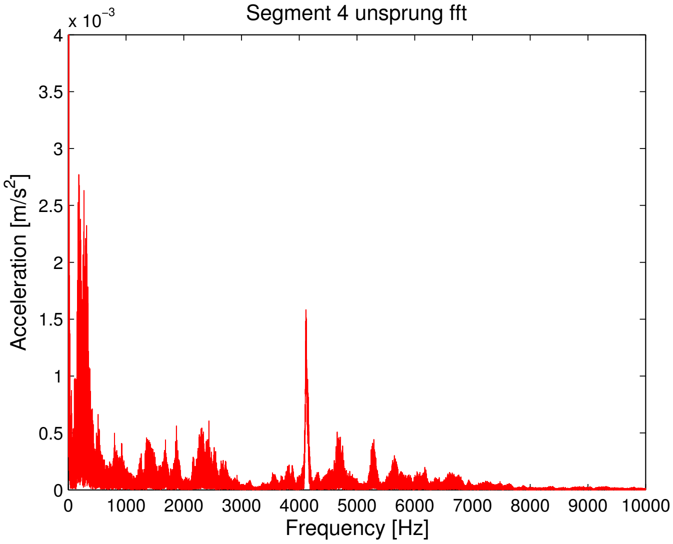

Fig. 5. Measured acceleration data in the frequency-domain - front right unsprung mass

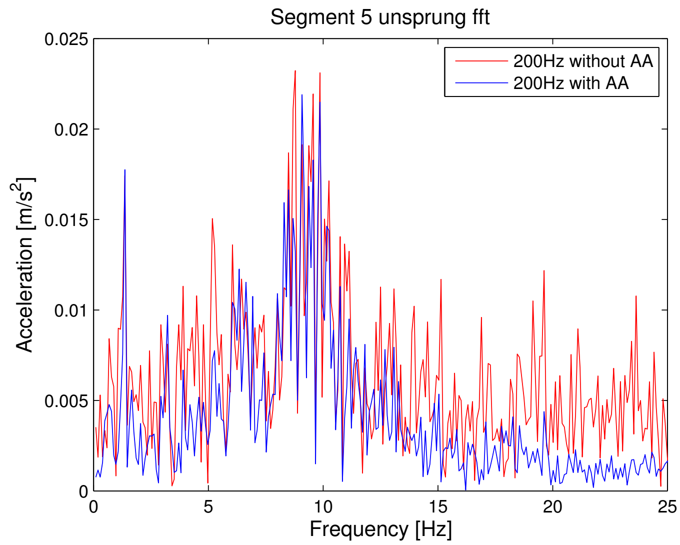

Fig. 7. Effects of anti-aliasing filtering on the low frequency components of the signal.

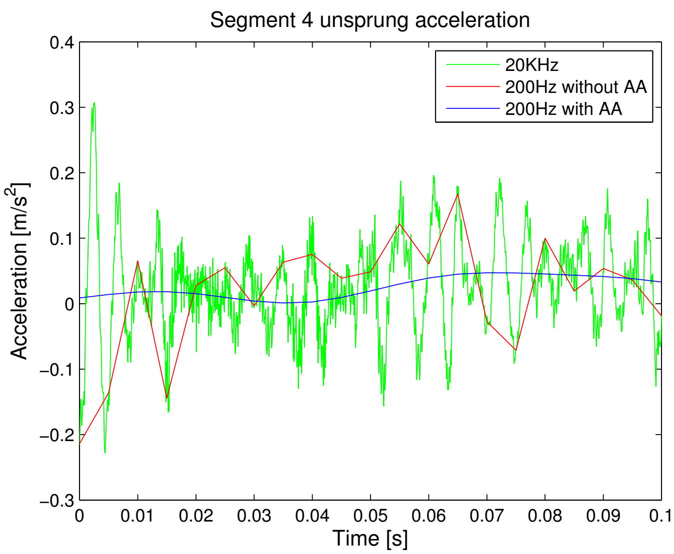

Fig. 6. Anti-aliasing filtering

V. Spectral analysis of the suspension system

Since the vibration load on the passengers and the driver is mainly influenced by the quality of the road and the dynamical parameters of the chassis, it is worth evaluating the road sections and the suspension system in terms of the ride comfort.

A. Indicators of ride comfort

According to the VDI 2057 recommendation and the ISO 2631 standard the ride comfort depend on the following indicators.

- The driver and the travelers may suffer vertical excitations only about $1\ Hz$, more precisely in the range of frequencies $0.75...1.45\ Hz$.
- A frequency weighted RMS in the form

$$RMS = \left[\frac{1}{T} \int_0^T a_w(t)^2 dt \right]^{1/2} \tag{11}$$

  is computed where vertical accelerations measured on the chassis (sprung mass) and various human sensitivities are also taken into consideration in a_w which denotes the frequency weighted sprung mass acceleration. According to the ISO 2631 standard the RMS values are classified as in Table I. (In this paper the RMS of a_w is computed without frequency weighting.)
- A *damping factor* of the suspension system is defined as the amplitude ratio of the sprung and unsprung mass accelerations. The damping factor is a function of frequency and is computed by

$$D(f) = \frac{A_s(f)}{A_u(f)}, \tag{12}$$

  where $A_s(f)$ and $A_u(f)$ are the power spectral density estimation of the sprung mass and the unsprung mass accelerations, respectively, computed at frequency f.

B. Analysis results

The vertical vibration levels (RMS) for all segments are computed according to (11) from the sprung mass acceleration time-domain data and are presented in Figure 8. The values are between 0.19 and 0.25 showing an acceptable level of passenger comfort (compare with Table I.) on the eight selected road segments.

TABLE I
PERCEPTION OF RIDE COMFORT ACCORDING TO ISO-2631-1

vibration level (RMS)	Perception
< 0.315	Not uncomfortable
$0.315 - 0.63$	A little uncomfortable
$0.5 - 1$	Fairly uncomfortable
$0.8 - 1.6$	Uncomfortable
$1.25 - 2.5$	Very uncomfortable
> 2	Extremely uncomfortable

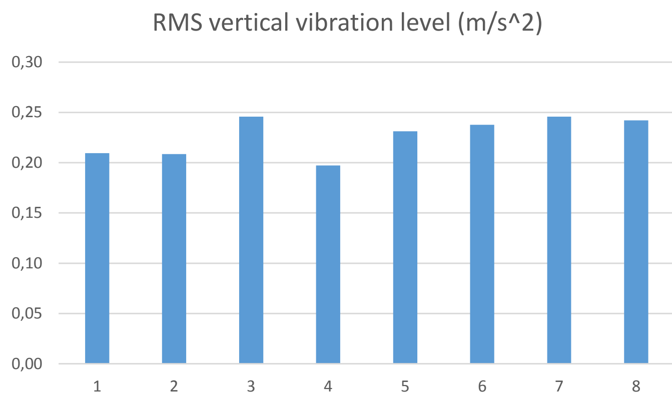

Fig. 8. Vertical vibration level (RMS) computed for the 8 road segments

For the evaluation of ride comfort around 1Hz and to determine the damping factor the power spectral density is estimated for all segments and for both acceleration sensors in the front right side of the bus. A typical PSD computation is shown in Figure 9, where the periodogram (7)-(8) and the Welch's method (Section II-B) are plotted. The periodogram provides an asymptotically unbiased estimate of the PSD, but its variance is quite large. Welch's method provides also an asymptotically unbiased estimate of the PSD. Its variance depends on the chosen window length and the overlapping rate.

A simplified model of a suspension system, the so called quarter car model, consists of two rigid masses, one for the sprung mass and one for the unsprung mass, which are connected by a spring and a damper (possibly of nonlinear characteristics) and the unsprung mass is connected to the road with a spring and a small negligible damper (tire). The linearized system with realistic parametrization has two lightly damped modes, each with a complex pair of poles. It is expected from this physical interpretation that the PSDs of the measured vertical accelerations of the two masses will show two peaks according to the two modes of the transfer function. The two peaks can be observed in Figure 9 at about 1 Hz and about 8 Hz. These frequencies are approximately the same for all segments. The corresponding amplitudes of the PSDs are obtained based on the Welch method and summarized in Figures10-12.

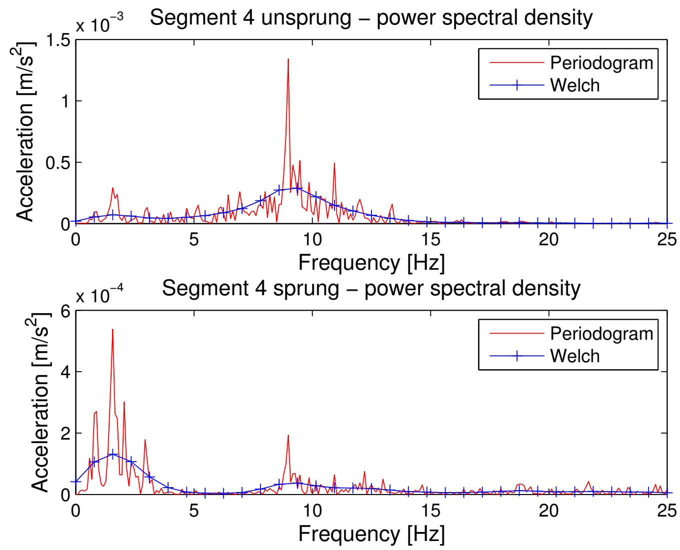

Fig. 9. Power spectral density of the measurements of Segment 4

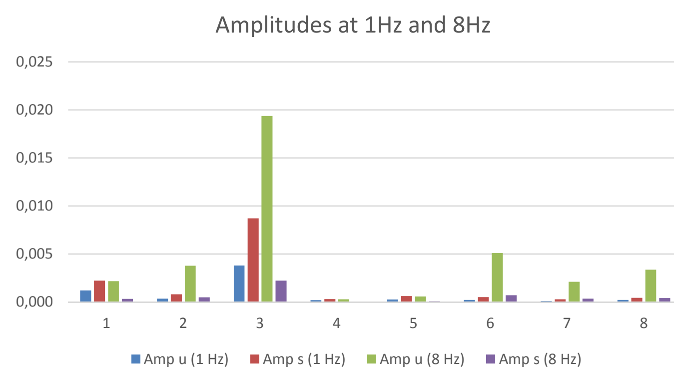

Fig. 10. Power spectral density of sprung and unsprung mass acceleration measurements at the two peaks. *Amp u* and *Amp s* denotes unsprung and sprung, respectively.

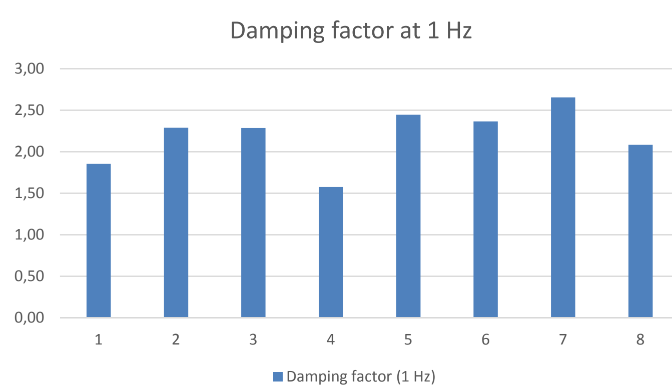

Fig. 11. Damping factor at 1Hz computed according to (12) (Values greater than 1 represent magnification.)

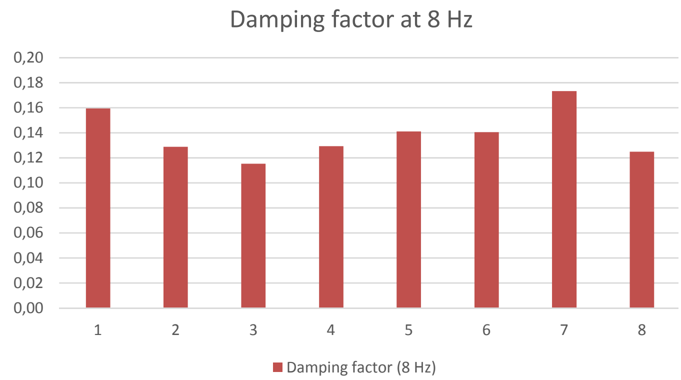

Fig. 12. Damping factor at 8Hz computed according to (12)

From the values shown in Figure 10 the damping factor of the suspension system can be computed. Figure 12 shows that the energy of vertical vibration at the front right wheel is suppressed by the suspension system. The energy measured at the chassis is reduced to 11...18% as compared to the energy measured at the wheel. On the other hand, the energy is amplified at 1Hz according to Figure 11, but this does not spoil the feel of comfort of the passengers.

VI. Conclusion

The suspension system of a city bus has been analyzed based on the spectral analysis of real acceleration measurements. Damping of the road excitation by the suspension system and several passenger comfort indicators have been computed. The applied analysis method, as a nonparametric tool, is the first step in system modeling and identification.

Future work includes the identification of both physical and black box models of the system, followed by uncertainty modeling for road surface classification and suspension systems diagnosis. Further goal is to construct mathematical models, as in reference [1], based on experimental data for an adaptive intelligent active suspension control system.

References

[1] J. Bokor, L. Palkovics, P. Michelberger and P. Várlaki, *Design of active wheel suspension system using RLQR and $\mathcal{H}_\infty$ control*, VEHICLE SYSTEM DYNAMICS, 1993

[2] W.A. Gardner, *Statistical spectral analysis. A nonprobabilistic theory*, Prentice Hall, Englewood Cliffs, New Jersey

[3] P. Gáspár, Z. Szabó and J. Bokor, *Parameter identification of a suspension system and road disturbance estimation.* INTERNATIONAL JOURNAL OF VEHICLE SYSTEMS MODELLING AND TESTING 2:(2) pp. 128-137, 2007

[4] L. Ljung, *System Identification (2nd Ed.): Theory for the User*, Prentice Hall PTR, Upper Saddle River, NJ, USA, 1999

[5] P. Michelberger, J. Bokor, A. Keresztes and P. Várlaki, *Identification of a Multivariable Linear-model for Road Vehicle (bus) Dynamics From Test Data*, INTERNATIONAL JOURNAL OF VEHICLE DESIGN 8:(1) pp. 96-114, 1987

[6] *Signal Processing Toolbox User's Guide*, The MathWorks, Inc., 2001

[7] P.D. Welch, *The Use of Fast Fourier Transform for the Estimation of Power Spectra: A Method Based on Time Averaging Over Short, Modified Periodograms*, IEEE Transactions on Audio and Electroacoustics, Vol. AU-15. Pgs. 70-73. 1967