# FULLY CONVOLUTIONAL FRACTIONAL SCALING

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## ABSTRACT

We introduce a fully convolutional fractional scaling component, FCFS. Fully convolutional networks can be applied to any size input and previously did not support non-integer scaling. Our architecture is simple with an efficient single layer implementation. Examples and code implementations of three common scaling methods are published.

*Index Terms*— FCN, Fully convolutional network, scaling, fully convolutional layer, pixelshuffle, fractional scaling, fully convolutional scaling

# 1. INTRODUCTION

Image scaling is a ubiquitous image processing operation. Neural networks based only on convolutions have the nice property that they can be applied to any size object. This family of architectures is named FCN, Fully Convolutional Networks. Many FCN models consist of up/down-sampling layers, albeit with integer factors. Here we present a fully convolutional fractional scaling component for CNNs, FCFS.

Various tasks, such as **instance**semantic segmentation [1] and [2], style transfer [3], super-resolution [4], image compression [5], satellite image segmentation [6], general object detection [7], etc. have state-of-the-art solutions based on integer scale up/down-sampling layers embedded in FCN architectures.

Up/down scaling architectures are extensive in computer vision fields. Zhang et al. [8] used the well known Laplacian Pyramids, together with a deep neural network to train a super-resolution model. Luo et al. [9] the authors used an up-sampling sequence of layers to find the optical-flow of an image. Various super-resolution models as [10], [11] and [12] tried to find the proper HR-counterpart of an LR-image when it's acquisition isn't predetermined. Maeda et al. [13] implemented an unpaired super resolution model based on cycleconsistency and up/down scaling layers. Saeedan et al.[14] succeeded to preserve important image details during down scaling with average-pooling layers. In addition [15] reported success on using using the *pixelshuf fle* component to up-scale an input image by any integer scaling integer factor.

Audio processing also uses scaling methods, [16] studied the appearance of artifacts on audio signals after applying upsampling methods.

Works like [17] and [18] used scaling layers and FCN to detect anomalies in medical images and brain 3D reconstruction respectively.

In this paper, we suggest a fully convolutional fractional, generalizing the integer, scaling component. Our architecture has an elegant and simple single layer implementation that allows easy integration in any FCN. Implementations of three common scaling methods: "nearest-neighbour", "bilinear" and "bicubic" interpolation can be found in Project Page

# 2. PREVIOUS WORK

The fractional convolutional scaling was proposed, first time, in [19], and later have been exceeded by [20]. It has stochastic implementations. Their works randomly pools the input with overlapping patches to achieve the desired fraction. Afterwards [21] suggested bilinear average pooling, their work is applicable only for scaling factors (denoted by f) in range  $1 \le f \le 2$ . All the former mentioned work aimed only for down-scaling tasks.

Another work is [22], their motivation was compressing video and they suggested an architecture with fixed input and output shapes.

Our solution is not restricted to fixed sizes and doesn't use stochastic sampling, providing a clean fully convolutional component to perform fractional down/up scaling.

## 3. APPROACH

For simplicity, we develop the theory for 1D tensor convolutions. Then we present the generalization for the higher dimensional cases.

**1D discrete tensor convolution:** Given an array x and a convolution kernel h of size 2K + 1, the convolution of x and h is:

$$(x * h)[i] = \sum_{k=-K}^{K} x[i-k] \cdot h[k]$$

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#### 3.0.1. Stride, Padding & Pixelshuffle

Our algorithm is based on three operations: **stride**, **padding**, and **pixelshuffle**.

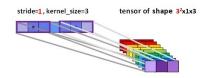
**Padding:** Given an array x. Padding by 2p is the concatenation  $c^p |x| c^p$ . There are other padding methods including reflection, zeros, and repetition of edge pixels.



Fig. 1. Padding an array of length 5 by 2.

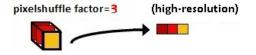
**Stride:** Given an array x and a convolution kernel h, the convolution of x and h with stride s is

$$(x *_{s} h)[i] = (x * h)[s \cdot i]$$



**Fig. 2**. 1D Tensor convolution with padding=3, stride=4 out\_channels=3 on a length 5 array.

**Pixel Shuffling:** Let the pixelshuffle be r, given a tensor of shape  $r \times N$  pixelshuffle [23] reshapes and rearranges it's elements in a tensor of shape rN.



**Fig. 3**. Pixelshuffle of  $3 \times 1$  tensor returning a tensor of shape 3.

## 3.1. Fully Convolutional Fractional Scaling

The architecture we purpose, FCFS, carries out fractional scaling. **Fully Convolutional Fractional Scaling**: Given a real array x and a scaling factor  $\frac{r}{s}$ , we define the following algorithm.

**FCFS(input:** Tensor)  $\rightarrow$  Tensor: x = pad(x, padding=2K)  $x = conv(x, out\_channels=r, stride=s, kernel\_shape=2K+1, kernel\_weights=W)$ **return** pixelshuffle(x, factors=r)

### 3.1.1. Description

The scaling is relative to the padded tensor of shape N. The architecture contains only a single hidden layer, whose shape is  $r \times \frac{N}{s}$ . Given a scale factor  $\frac{r}{s}$  we apply a convolution with stride = s. Each of the r convolution kernels produces an interpolation for offset  $i + \frac{j-1}{r}$  s.t.  $j \in [1, ..., r]$ . Thus the hidden layer is of shape  $r \times \frac{N}{s}$ . Applying pixelshuffle results in an array with the desired shape of  $\frac{r}{s}N$ 

The parameters K, kernel\_shape, and kernel\_weights depend on the interpolation method see examples section 5. The hidden layer's shape is bilinearly dependent on output shape and r, which is the space and time complexity of the component.

## 4. 2D & ND EXTENSIONS

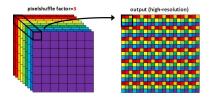
The adaptations needed for 2D and ND are straightforward. Special attention needs to be paid to *Pixelshuffle*.

## 4.1. ND Pixelshuffle

Here we propose a slight generalization of Pixelshuffle. **PixelShuffle:** For  $n \ge 2$  and a tensor of shape  $(r_1 \cdot \ldots \cdot r_i \cdot \ldots \cdot r_n) \times N_1 \times \ldots \times N_i \times \ldots \times N_n$ , Pixelshuffle rearranges the elements to a new tensor of shape  $r_1N_1 \times \ldots \times r_iN_i \times \ldots \times r_nN_n$ 

$$Out[i_1, ..., i_i, ..., i_n] = x[r, \lfloor \frac{i_1}{r_1} \rfloor, ..., \lfloor \frac{i_i}{r_i} \rfloor, ..., \lfloor \frac{i_n}{r_n} \rfloor]$$
$$r = \sum_{t=0}^{n-1} (\prod_{j=1}^{n-t-1} r_j)((i_{n-t} - 1) \mod r_{n-t})$$

Figure 4 illustrates the formula.



**Fig. 4**. Pixelshuffle of a  $3^2 \times 7 \times 7$  tensor to a  $3 \cdot 7 \times 3 \cdot 7$  matrix.

### 4.2. ND Fully Convolutional Fractional Scaling

To scale an ND input signal by scaling factors:  $\frac{r_1}{s_1}, ..., \frac{r_n}{s_n}$  for the different dimensions.

**FCFS(input:** Tensor)  $\rightarrow$  Tensor:  $\mathbf{x} = \text{pad}(\mathbf{x}, \text{padding}=2K_1, \cdots, 2K_n)$   $\mathbf{x} = \text{conv2d}(\mathbf{x}, \text{out\_channels}=\prod_i r_i,$   $\text{stride}=[s_1, ..., s_n],$  $\text{kernel\_shape}=[2K_1 + 1, ..., 2K_n + 1],$  kernel\_weights=W return pixelshuffle(x, factors= $[r_1, ..., r_n]$ )

## 5. EXAMPLES

#### 5.1. Illustration of FCFS

Figure 5 illustrates FCFS on a  $5 \times 5$  image with a  $\frac{3}{2}$  up-sacle factor. The output image is a  $9 \times 9$  image,

$$9 \times 9 = \frac{3}{2} \cdot (5+1) \times (5+1) \Longrightarrow output = \frac{3}{2} \cdot input$$

as expected.

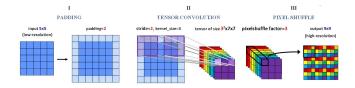


Fig. 5. Illustration of 2D  $\frac{3}{2}$  Fully Convolutional Fractional Scaling.

## 5.2. Convolution's kernel weights

FCFS supports various scaling-methods through the parameters. In this section, we present kernel weights for various image scaling-methods.

Consider  $f = \frac{3}{2} = \frac{r}{s}$  scaling. According to the offsets described in 3.1.1, we have  $3^2 = 9(=r^2)$  different kernels. We present the kernel of offsets (1, 1) and (1, 3) denoted by  $W_{1,1}$  and  $W_{1,3}$ .

#### 5.2.1. Nearest neighbour interpolation

From [24] nearest neighbour:

$$W_{1,1} := \begin{bmatrix} 1. & 0.0 \\ 0.0 & 0.0 \end{bmatrix} \quad W_{1,3} := \begin{bmatrix} 0.0 & 1.0 \\ 0.0 & 0.0 \end{bmatrix}$$

### 5.2.2. Bilinear interpolation

From [24], bilinear interpolation which is based on the 4 nearest pixels around the point of interpolation:

$$W_{1,1} := \begin{bmatrix} 0.44 & 0.22 \\ 0.22 & 0.11 \end{bmatrix} \quad W_{1,3} := \begin{bmatrix} 0.22 & 0.44 \\ 0.11 & 0.22 \end{bmatrix}$$

#### 5.2.3. Biqubic interpolation

From [24] bicubic interpolation as derived from the formula, published in [24]:

$$W(\Delta) = \begin{cases} 1.5|\Delta|^3 - 2.5|\Delta|^2 + 1 & \text{for } |\Delta| \le 1, \\ -0.5|\Delta|^3 + 2.5|\Delta|^2 - 4|\Delta| - 4a & \text{for } 1 < |\Delta| < 2, \\ 0 & \text{otherwise,} \end{cases}$$

 $\Delta = x - i, y - j$ . Where the x, y the subpixel point of interpolation and i, j are the integer coordinates of the input image.

	0.16	0.16	0.07		0.13	0.13	0.13
$W_{1,1} :=$	0.16	0.16	0.07	$W_{1,3} :=$	0.13	0.13	0.13
	0.07	0.07	0.03	$W_{1,3} :=$	0.05	0.05	0.05

### 6. EXPERIMENTS

To test the time complexity and quality of FCFS we ran the following experiments:

- 1. We compared running times of FCFS to *torch.resize* [25] for various scaling factors.
- 2. We computed two commonly used metrics, PSNR [26] and SSIM [26]. again comparing FCFS to *torch.resize* [25], for various scaling factors.

### 6.1. Empirical methods

### 6.1.1. Scaling methods

Each experiment was repeated 100 times. We tested for the three different scaling - methods: "nearest neighbours", "bilinear-interpolation" and "bicubic-interpolation" The weights were implemented as described in section 5. For each method, six up-scaling factors and six down-scaling factors were tested

#### 6.1.2. Hardware & Datasets

The experiments were carried out on NVIDIA RTX2070 GPU.

The dataset used was Celeb\_A [27] with more than 200K celebrity images with 1024x1024 pixel resolution.

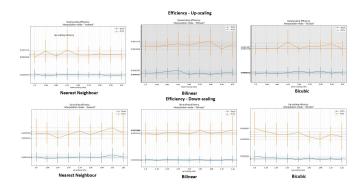
#### 6.2. Results

#### 6.2.1. Experiment results

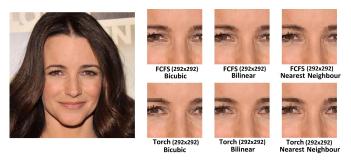
The first experiment's running time results are presented in figure 6. No significant difference was found between *torch.resize* and FCFS, neither in up-scaling nor in downscaling. The FCFS was 0.0003 seconds slower on average.

The second experiment's visual sameness results are presented in figures 7 and 8. Zooming in shows the visual artifacts that slightly differ between *FCFS* and *torch.resize*.

Figures 9 and 10 show distances between the output images for different *scaling* – *methods*. PSNR and SSIM values above 20 and close to 1.00 respectively support visual sameness [26]. The experiment shows the consistency of FCFS with the *torch.resize* implementation.



**Fig. 6.** Efficiency graphs for up/down-scaling tasks  $\times$  different *scaling* – *methods* 

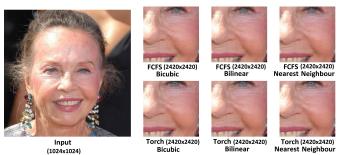


**Fig. 7**. Down-scaling by factor  $=\frac{2}{11}$ 

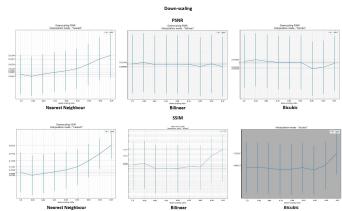
# 7. SUMMARY & FUTURE WORK

We introduced a fully convolutional fractional scaling component-FCFS that is as efficient as the fixed shape scaling component (torch.resize).

The benefit from a convolution based approach is the ability to learn weights. FCFS allow to train the *kerenel\_weights* and adjust them both in **shape** and **values** to the particular task. We aim to invest more effort in this direction in future work.



**Fig. 8**. Up-scaling by factor  $=\frac{27}{11}$ .



**Fig. 9.** PSNR & SSIM graphs for down-scaling tasks  $\times$  different *scaling* – *methods*.

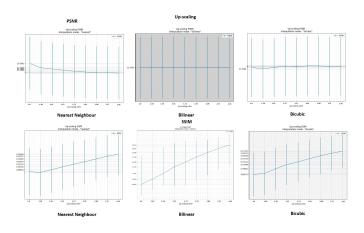


Fig. 10. PSNR & SSIM graphs for up-scaling tasks  $\times$  different *scaling* – *methods*.

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