Analysis of Phase-Locked Loops using the Best Linear Approximation

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Analysis of Phase-Locked Loops using the Best Linear Approximation

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***Abstract*—During the early design stage of Phase-Locked Loops, linear models are thoroughly used to analyse the steady-state behaviour. In reality, the envisioned linear performance is degraded due to nonlinearities present in the actual implementation. Lately, a nonlinear modelling technique based on the Best Linear Approximation has been developed which allows to verify the validity of this linear model and, in addition, permits to characterise the nonlinear distortions present in the system. Incorporating this Best Linear Approximation in the design stage allows to intuitively analyse the nonlinear behaviour of the Phase-Locked Loop.**

***Index Terms*—Phase-Locked Loop, Nonlinear Distortion, Least Squares Approximation**

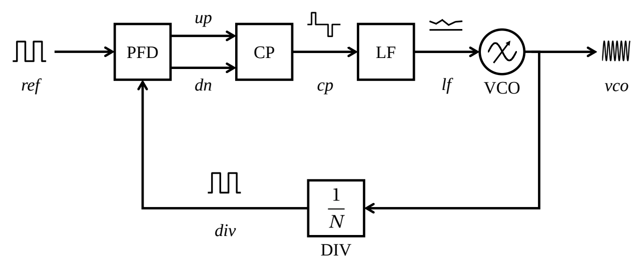


Fig. 1. Considered CP-PLL architecture. The PLL consists of five major building blocks: the Phase-Frequency Detector (PFD), Charge Pump (CP), Loop Filter (LF), Voltage-Controlled Oscillator (VCO) and the frequency DIVider (DIV). The italic scripted text represents the signal names used throughout this paper.

I. Introduction

The Phase-Locked Loop (PLL) is the beating heart of every modern wireless communication system by providing the local oscillator function in transceivers [1]. Although, the PLL is omnipresent in the current information age, the design and realisation of such a loop remain challenging tasks.

Typically, a linear model in the phase domain is used to describe the behaviour of the PLL [2]. Although this linearised model is a powerful tool for the design of PLLs, the nonlinear distortion that is present in the system is completely neglected. Modelling the full nonlinear behaviour of a system can be accomplished with Volterra-based approaches [3], [4], but they lack simplicity and ease of use. The Best Linear Approximation (BLA) strikes a balance between the linear design framework and the Volterra theory. The BLA approximates the behaviour of a nonlinear system in least square sense by a linear system and gives an indication about the nonlinear distortions as well [5], [6], [7]. By doing so, designers will already be familiar with the notions introduced by the proposed method and can extend their existing skillset with a minimum of effort. In the end, the BLA methodology enables:

- the extraction of the 'best' linearised behaviour
- the separation of steady-state and transient response
- the separation of even and odd nonlinear distortions
- the characterisation of the additive noise contribution

In this paper, a method is developed which allows to study PLLs with existing BLA-based modelling and analysis tools. The proposed methodology requires an extension of the excitation signal, i.e. random phase multisines, to the phase domain. Additionally, the advantages and disadvantages of the proposed extension on the simulation and extraction techniques are discussed.

Section II discusses the methodology, including the PLL architecture, the BLA, and the extended multisine excitation in the phase domain. Section III develops a system-level simulation framework of the PLL using Simulink/Matlab. Finally, Section IV examines the closed loop architecture and the nonlinear distortions generated by non-idealities in the charge pump.

II. Methodology

This Section elaborates on the underlying theoretical concepts for the nonlinear analysis. Furthermore, a particular PLL architecture was chosen to exemplify the proposed methdology. First, the considered PLL architecture and its working principle are explained. Next, the theory of the BLA, the excitation signal used, and the extraction technique are introduced.

A. Phase-Locked Loop

The prime purpose of a phase-locked loop is to synthesize a periodic signal whose frequency can be accurately controlled. A PLL achieves this by synchronising the phase and frequency of its internal oscillator to the reference input signal. Essentially, its operation is based on a feedback system which is sensitive to the phase difference between the input and feedback signal. In this paper, the so-called Charge Pump Phase-Locked Loop (CP-PLL) is extensively studied (Fig. 1) [2], [8].

Fundamentally, the operation of the PLL can be considered as strongly nonlinear, especially during the lock-in process. Although nonlinear behaviour is inherently present in the system, it is possible to come up with a linear, behavioural

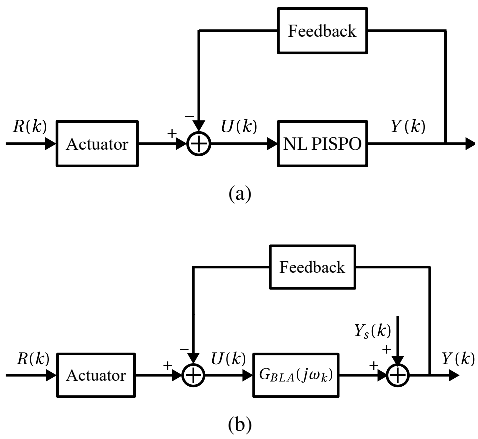


Fig. 2. Closed loop configurations of the original NonLinear Period In Same Period Out (NL PISPO) system (a) and the corresponding BLA (b).

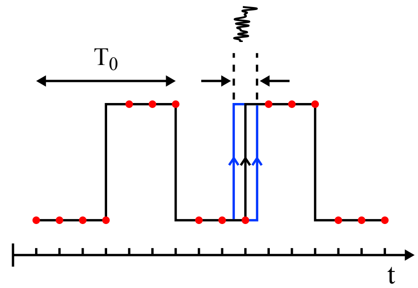


Fig. 3. Jitter applied to a sampled (•) digital clock with period T_0.

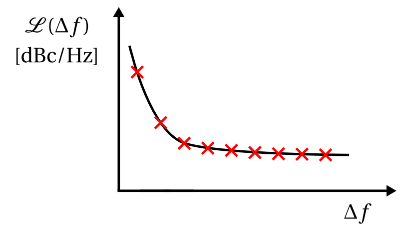


Fig. 4. Approximation (×) of the phase noise profile $\mathcal{L}(\Delta f)$ (–).

model by considering the phase of the signals [9]. With this linear model, classical control theory concepts can be used during the design of the PLL.

This linearisation involves the notion of phase noise in non-ideal oscillators [10]. To introduce this concept, consider the output $y(t)$ generated by a non-ideal oscillator which is subjected to random phase fluctuations $\varphi(t)$ caused by noise and intrinsic nonlinear behaviour

$$y(t) = A_c \cos(\omega_c t + \varphi(t)) \tag{1}$$

where ω_c is the carrier angular frequency and A_c the output amplitude which is assumed constant. Phase noise is the frequency domain representation of these random phase fluctuations $\varphi(t)$. It is usually expressed in dBc/Hz and is represented by the one-sided noise power $\mathcal{L}(\Delta f)$ relative to the carrier in a 1 Hz bandwidth centered around a particular frequency offset Δf from the carrier frequency f_c.

B. Best Linear Approximation

The nonlinear distortion generated by a system can be significantly large and hence needs to be taken into account when modelling the system. The BLA was introduced for exactly this purpose. It provides a linear approximation G_{BLA} in least squares sense of the nonlinear system for the class of Gaussian-like excitation signals (fixed power spectral density and probability density function) [11]. This approximation can be described in the frequency domain with the succeeding input-output relationship (Fig. 2)

$$Y(k) = G_{BLA}(j\omega_k)U(k) + Y_s(k) \tag{2}$$

where $U(k)$ and $Y(k)$ are the Discrete Fourier Transform (DFT) of, respectively, the input $u(t)$ and output $y(t)$. The index k represents the k^{th} multiple of the frequency resolution f_0, used when performing the DFT. The contributions present in the output spectrum which cannot be described by the linear model G_{BLA} are described by Y_s. These so-called stochastic nonlinear contributions have noise-like properties and are uncorrelated with, but not independent of, the excitation signal [12].

Identification of a system operating in closed loop requires to involve the known reference signal $r(t)$ such that an unbiased estimate can be obtained (Fig. 2) [12]

$$G_{BLA}(j\omega_k) = \frac{S_{yr}(j\omega_k)}{S_{ur}(j\omega_k)} = \frac{\mathcal{F}\{\mathbb{E}\{y(t)r(t-\tau)\}\}}{\mathcal{F}\{\mathbb{E}\{u(t)r(t-\tau)\}\}} \tag{3}$$

where S_{yr} and S_{ur} are, respectively, the cross-power spectra of the output and input towards the applied reference. The expected value in S is taken with respect to the random phase realisation of the reference.

C. Multisine excitations

The random phase multisine (MS) is characterised by two important properties; it is random and periodic. Its random behaviour permits to mimic real-life signals while the periodicness allows to perform measurements/simulations with a high signal-to-noise ratio [7]. A random phase multisine consisting of N components is defined as:

$$s(t) = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} A_k \cos(2\pi k f_0 t + \varphi_k) \tag{4}$$

where A_k and φ_k are, respectively, the amplitude and phase of the k^{th} spectral line of the MS excitation and f_0 the frequency resolution. The phases φ_k are uniformly distributed over the interval $[0, 2\pi)$ and the amplitudes A_k can be freely chosen, depending on the application.

Introducing the notion of random phase MS signals in the voltage and current domain is a common and straightforward practice in the field of frequency domain system identification [13]. However, when the phase of the signal is considered, the conventional methods have to be extended such that the MS and the previously introduced concept of oscillator phase noise can be combined.

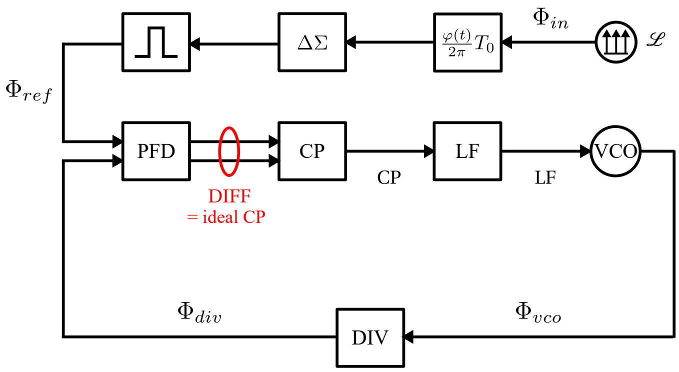


Fig. 5. Frequency domain scheme of the PLL. The capitalised signals represent the Fourier transform of the signals in their respective voltage or current domain. Φ_x is the Fourier transform of the signal x in the phase domain. The additional signal *DIFF* has been defined as the difference between the UP and DN output of the PFD.

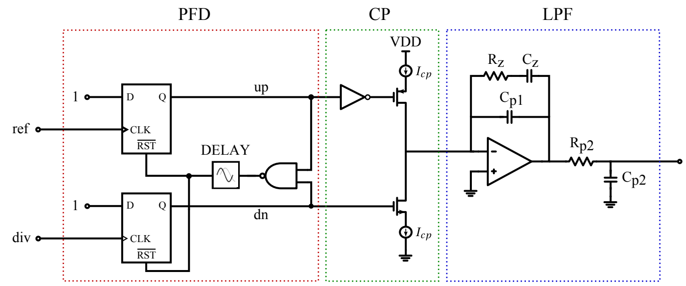


Fig. 6. System-level implementation of the PFD, CP and LPF used in Simulink.

D. Multisine excitations in the phase domain

The general principle is to apply the MS excitation as the jitter signal for the digital reference clock of the PLL, as is showcased in Fig. 3. In digital communication systems, jitter describes the relative (uncertain) time deviation of the clock from the true presumed periodic signal.

Starting from a desired phase noise profile $\mathcal{L}(\Delta f)$, a discrete approximation of this profile can be constructed with the MS (Fig.4). This discrete MS profile can afterwards be converted to the phase noise $\varphi(t)$, expressed in radians. The relative time deviation $\delta(t)$, to be applied to the reference clock, can then be retrieved as follows:

$$\delta(t) = \frac{\varphi(t)}{2\pi}T_0 \tag{5}$$

Concerning the practical implementation of this MS jitter excitation in transient-like simulation software (e.g. Simulink), a quantisation of the time needs to be performed. This quantisation in time is unavoidable, since the rising/falling edge cannot be manually shifted with a non-integer multiple of the fixed discrete time step. The drawback of this quantisation is that inherent distortion is introduced in the excitation signal and has to be accounted for with the BLA-extraction method.

Alternative techniques using timed simulations (VHDL/Verilog, variable-step solvers) exist which make it possible to circumvent this quantisation by timing the precise moment on which the transition of the edge occurs. However, these techniques have difficulties to properly handle analog signals (VCO and loop filter) and make frequency domain identification more difficult.

E. Extraction technique

Odd random phase multisines, exciting only odd harmonics of the base frequency, are used to extract the BLA and acquire a measure of the nonlinear behaviour [7]. At the excited frequencies, multiple phase realisations of φ_k (Eq. 4) are used to retrieve an estimate of the BLA and the odd nonlinear distortions [7]. Next, a linear interpolation of G_{BLA} from the excited to the in-band, non-excited even frequencies is performed. Afterwards, the following correction on the output spectrum can be carried out for all the in-band frequencies

$$Y_c(k) = Y(k) - G_{BLA}(j\omega_k)U(k) \tag{6}$$

where $Y_c(k)$ represents the corrected output spectrum.

This correction is required to compensate for the spectral impurities present in the input spectrum [12]. Energy present in the corrected output spectrum at the even detection lines provides a measure for the even nonlinearities. The proposed method allows to fully characterise the nonlinear behaviour of a system operating in both open and closed loop.

III. Simulation Framework

All the components present in the PLL can be examined by determining the BLA from the reference to their respective input and output (see Section II-B). The frequeny domain scheme in Fig. 5 can be used for this purpose.

The phase noise profile $\mathcal{L}$ of the reference signal is considered as the input. Next, the equivalent time domain jitter signal is calculated (Eq. 5) and quantised by a 2^{nd}-order $\Delta\Sigma$-modulator [14]. Finally, the quantised jitter signal is applied to a reference digital clock such that the high frequency reference oscillator can be constructed (Φ_{ref}).

For the study of the PFD and CP an additional signal, *DIFF*, has been defined which is the difference between the *UP* and *DN* output of the PFD. The use of this *DIFF* signal is allowed since the valuable information coming from the PFD is contained in this derived quantity. In order to verify the methodology discussed in the previous section, a 4^{th}-order type-II CP-PLL [15] is analysed using Simulink. A system-level implementation of the different building blocks of the PLL (see Fig. 6 for the PFD, CP and LPF) is used during these simulations.

Concerning the VCO, an ideal behavioural model is adopted which relates the angular frequency of the sinusoidal oscillator (ω) with the input voltage (v_{in}) as:

$$\omega(v_{in}) = \omega_c + K_{vco}v_{in} \tag{7}$$

where ω_c and K_{vco} represent, respectively, the free-running angular frequency and the gain of the voltage-controlled oscillator. Furthermore, a generic integer-N frequency divider is used. The properties which had to be defined for the different components incorporated in the PLL are all listed in Table I.

Component	Property	Value	Units	Description
PFD	f_{ref}	100	MHz	Reference frequency
	f_{div}	100	MHz	Divider frequency
	ΔT	1	ns	Delay time
CP	I_{cp}	1	μA	Current gain
	D	1	%	Current mismatch
	$\Delta\tau$	9.26	ps	Asymmetric delay
LPF	f_{co}	1	MHz	Crossover frequency
	M_{gain}	-21.4	dB	Gain margin
	M_{phase}	57.6	$^\circ$	Phase margin
VCO	f_c	1.8	GHz	Free-running frequency
	K_{vco}	2π 400	Mrad/(Vs)	Gain
DIV	N	18	-	Division ratio
MS	f_{res}	24.4	kHz	Frequency resolution
	f_{min}	24.4	kHz	Minimal frequency
	f_{max}	1	MHz	Maximal frequency
	$\mathcal{L}(\Delta f)$	-85	dBc/Hz	Uniform phase noise profile
Simulink	f_s	108	GHz	Sampling frequency
	M	5	-	Number of phase realisations
	P	2	-	Number of periods
	T	$20 f_{co}$	s	Transient time

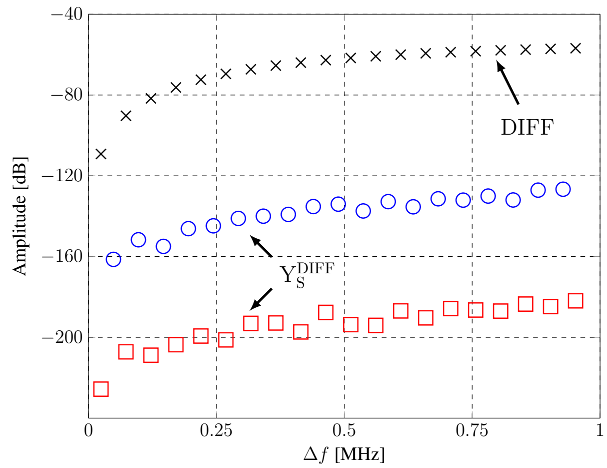
TABLE I
PROPERTIES USED FOR THE SIMULATION OF THE PLL.

IV. RESULTS

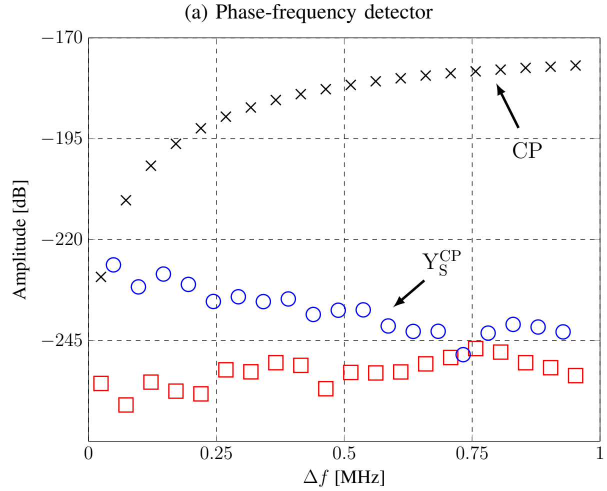
The identification of the PFD and CP both considers $\mathcal{L}(\Delta f)$ as the reference signal. Three more signals are considered meaningful during the analysis: $\Phi_{ref} - \Phi_{div}$ as input of the PFD; *DIFF* as the output and input of, respectively, the PFD and the CP; *CP* as output of the charge pump. In order to take into account the effect of process variations in semiconductors, the influence of non-idealities (mismatches in the current sources and asymmetric arrival of the *up* and *dn* output) on the generated nonlinear distortion by the CP is verified.
To showcase the capabilities of the earlier introduced methodology (Section II), the nonlinear distortions generated by the PFD and the CP are examined by inspection of the corrected output spectra (Fig. 7). These corrected output spectra show the nonlinear distortions which are intrinsically generated by the system itself.

The following observations can be made from Fig. 7(a)-7(c):

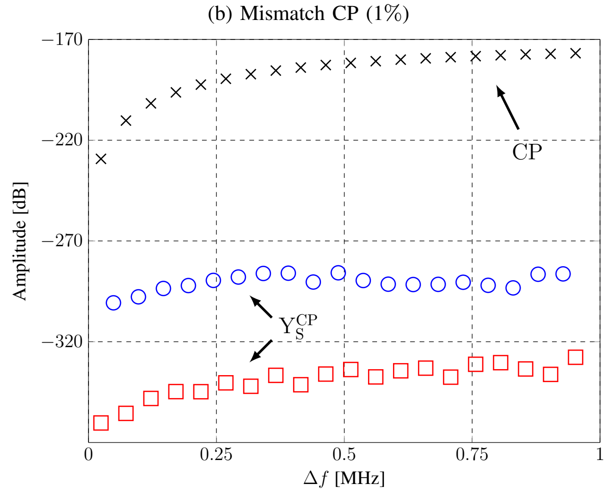
- The PFD deviates from the anticipated, completely linear behaviour (no distortions). Simulations show that a dominant even nonlinearity is present in the PFD which is independent of the delay time used.
- The introduced non-idealities in the CP both exhibit a dominant even nonlinear behaviour due to asymmetry.
- Distortions caused by mismatch in the CP are much more pronounced than those generated by an asymmetric delay. Additional effort should thus be spent during the design stage to minimise the current mismatch as much as possible.



(a) Phase-frequency detector



(b) Mismatch CP (1%)



(c) Asymmetric arrival time CP (9.26 ps)

Fig. 7. Output (Y) and corrected output spectra (Y_S) of the PFD and CP with included non-idealities (mismatch in current sources and asymmetric arrival time of the inputs). $\times$: excited odd harmonics, $\circ$: RMS of even distortions, $\square$: RMS of odd distortions.

V. Conclusion

A methodology has been developed which permits to study the linearised behaviour of the different subsystems of a Phase-Locked Loop, and their contributions to the nonlinear distortion. An extension of the multisine excitation signal to the phase domain has been proposed such that existing BLA-based analysis methods could be used. This extension with the multisine phase domain signal is especially important when the user does not have full control over the (fixed or variable) time-step used in their time-domain solver. The proposed methodology has been illustrated on a non-ideal CP-PLL and shows that the suggested method is able to characterise the nonlinear distortion levels without using the proposed simulation techniques or models.

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