# Task Allocation and Mobile Base Station Deployment in Wireless Powered Spatial Crowdsourcing

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***Abstract*—Wireless power transfer (WPT) is a promising technology to prolong the lifetime of sensor and communication devices, i.e., workers, in completing crowdsourcing tasks by providing continuous and cost-effective energy supplies. In this paper, we propose a wireless powered spatial crowdsourcing (SC) framework which consists of two mutual dependent phases: task allocation phase and data crowdsourcing phase. In the task allocation phase, we propose a Stackelberg game based mechanism for the SC platform to efficiently allocate spatial tasks and wireless charging power to each worker. In the data crowdsourcing phase, the workers may have an incentive to misreport its real working location to improve its own utility, which manipulates the SC platform. To address this issue, we present a strategyproof deployment mechanism for the SC platform to deploy its mobile base station. We apply the Moulin's generalized median mechanism and analyze the worst-case performance in maximizing the SC platform's utility. Finally, numerical experiments reveal the effectiveness of the proposed framework in allocating tasks and charging power to workers while avoiding the dishonest worker's manipulation.**

***Index Terms*—Spatial crowdsourcing, facility location, mechanism design, wireless power transfer**

## I. INTRODUCTION

Integrated with advanced sensing and communication techniques, mobile devices can help complete diverse location-aware tasks, such as large-scale data acquisition and analysis of real-time traffic monitoring or weather measurements at different places. By focusing on the geospatial data, a new paradigm called *spatial crowdsourcing (SC)* [1] has received increasing attention in the last few years [2], [3]. Typically, there are three entities in the SC system, including an online SC platform, requesters and workers. As a core component of the SC system, the SC platform serves as a broker which allows requesters to post tasks and recruits workers to complete them. Each employed worker then stays at or travels to its target task area to collect and transmit the requested data back. We hence study the interactions between the SC platform and the workers.

Most existing work assumes that there is always a reliable communication infrastructure and enough energy available for workers to complete the data transmission. However, this may not be realistic especially when workers have to perform tasks in remote areas without a wireless base station deployed. Moreover, the workers can be battery-powered wireless mobile devices. Fortunately, studies [4], [5] in wireless powered sensor networks have illustrated the feasibility of using wireless power transfer (WPT) [6] in sensing data collection to prolong the lifetime of sensors. In view of this, we consider a paradigm called *wireless powered spatial crowdsourcing* where the SC platform deploys a mobile base station (BS), e.g., robotics, drones or vehicles to assist the data collection. Moreover, the mobile BS serves as the infrastructure for communication and wireless power transfer.

To ensure a successful and stable operation of the crowdsourcing system, designing an incentive mechanism that stimulates workers' participation and efficiently allocates tasks is essential [7]. A number of studies have proposed mechanisms satisfying various requirements, such as profitability, strategyproofness, i.e., truthfulness, and individual rationality [8]. Nevertheless, in wireless powered crowdsourcing networks, the reward offered by the SC platform to workers is the wireless power supply which is the major difference with those existing mechanisms, the incentive of which is based on monetary reward[1]. The difference introduces a few major issues for incentive mechanism design in wireless powered crowdsourcing networks, and the following questions have to be answered. First, what is the optimal total charging power supply from the SC platform for maximizing its utility? The SC platform can encourage workers to transmit sensed data at a higher transmission rate, i.e., more collected data per unit time, but it is at the cost of a higher power supply. Second, how to allocate the tasks and charging power to workers which are spatially distributed in the task area? The allocation is based on not only each worker's sensing cost, but also the working location which clearly affects the communication cost and transferred power. Note that the workers' sensing cost and working location can be private information and unknown to the SC platform. Lastly, how to deploy the mobile BS



[1]The monetary reward can be tokens, virtual money, reputation, etc.

taking the worker's strategic behavior into account? Since the worker's working location is private, workers need to report their own locations before the mobile BS chooses the best location to deploy. Under the assumption of rationality, a worker may misreport its location to increase its own utility while reducing the SC platform's utility. Figure 1 shows a simple example. In the task area, there are one dishonest worker at location $L_{\rm A}$ and two honest workers respectively at $L_{\rm B}$ and $L_{\rm C}$. The SC platform should place the mobile BS at $L_{\rm M}$ for optimal utility if all workers report true locations $L_{\rm A}, L_{\rm B}$ and $L_C$. However, the dishonest worker has an incentive to report a fake location $L'_{\rm A}$, so that according to the reported locations $L'_{\rm A}, L_{\rm B}$ and $L_{\rm C}$, the mobile BS will be deployed at $L'_{\rm M}$. In this case, the dishonest worker at $L_{\rm A}$ can be closer to the mobile BS and then enjoy more transferred power from the mobile BS while consuming less power to transmit its sensed data. Meanwhile, it inevitably increases other workers' and SC platform's energy consumption and damages their utilities finally. Most current studies on incentive mechanisms for the crowdsourcing system have not addressed such issue yet. In this paper, we propose a strategyproof and energy-efficient SC framework which jointly solves the problems of task and wireless charging power allocation as well as the truthful working location reporting. In the framework, there are two phases: task allocation phase and data crowdsourcing phase. In the task allocation phase, the SC platform determines and announces a fixed total charging power supply. All workers interested in participating have to choose and submit their preferred transmission rate for the corresponding portion of the supplied power. We use the Stackelberg game to model the interactions between workers and the SC platform, in which the SC platform determine each worker's transmission rate and allocated power. Next, in the data crowdsourcing phase, the mobile BS requests for workers' working locations. Based on the Moulin's generalization median rule [9], the mobile BS applies the median mechanism in determining its service location to deploy. Then, the workers start to perform sensing tasks. We analyze the worst-case performance of the median mechanism for maximizing the SC platform's utility.

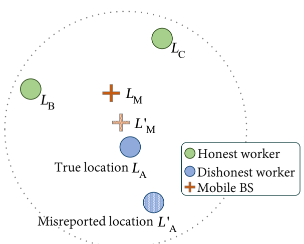


Fig. 1. An example where a dishonest worker misreports its true location.

The rest of the paper is organized as follows. In Section II, we describe the system model of wireless powered spatial crowdsourcing. Section III proposes the task and charging power allocation mechanism. In Section IV, we present the strategyproof mechanism for mobile BS deployment in data crowdsourcing phase. In Section V, we provide the experimental results. Finally, we conclude the paper in Section VI.

## II. System Model: Wireless Powered Spatial Crowdsourcing Market

The wireless powered spatial crowdsourcing system includes the requesters, the SC platform residing in the cloud and the workers with mobile sensing devices[2]. Initially, the requesters publish spatial tasks with requirements, such as the target task area, the task period and the sensed data type. Then, the SC platform advertises the task information to *registered workers* on behalf of the requesters and collects the crowdsourced data. As shown in Fig. 2, we denote by $\mathcal{N} = \{1, \ldots, N\}$ the set of workers and denote by $\boldsymbol{A}_t$ the task area on a Cartesian coordinate plane. The worker $i$'s working location $L_i$ is described by a 2-tuple, i.e., $L_{i\in\mathcal{N}} = (x_i, y_i)$. We use $L_{\rm M} = (x_{\rm M}, y_{\rm M}) \in \boldsymbol{A}_t \subseteq \mathbb{R}^2$ to represent the deployed mobile BS's service location projected on the XY-plane and use $h$ to denote its height. In the task area, worker $i$ has its own working area $\boldsymbol{A}_i$ and its working location $L_i$ falls in this area, i.e., $L_i \in \boldsymbol{A}_i \subseteq \boldsymbol{A}_t \subseteq \mathbb{R}^2$.

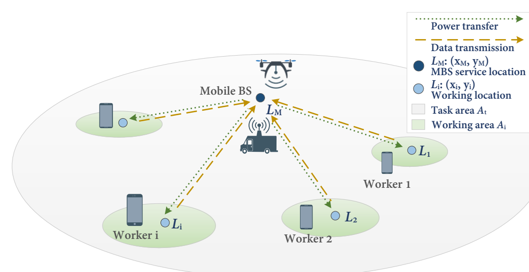


Fig. 2. Data transmission and power transfer in the data crowdsourcing phase.

### A. Power cost model

*1) Worker's power cost:* We consider an FDD system where sufficient channels are available to ensure interference-free transmission. Moreover, we assume that the communication channels are dominated by line-of-sight (LoS) links. Given the mobile BS's service point $L_{\rm M}$, we can write the worker $i$'s transmission rate according to Shannon's formula as follows:

$$r_i = B\log_2\left(1 + \frac{P_i^{\rm t}\delta}{\sigma^2 d_i^\alpha}\right) = B\log_2\left(1 + \frac{P_i^{\rm t} g}{d_i^\alpha}\right) \quad (1)$$

where $g = \frac{\delta}{\sigma^2}$ is the channel gain to noise ratio (CNR), $\delta$ represents the corresponding channel power gain at the reference distance of 1 meter, $\sigma^2$ is the noise power at the receiver mobile BS, $B$ is the channel bandwidth in hertz, $P_i^{\rm t}$ is worker $i$'s transmission power, and $\alpha \geq 2$ is the path-loss exponent. In addition, we define

$$\begin{aligned} d_i &= d_i(L_{\rm M}) = d((x_i, y_i), (x_{\rm M}, y_{\rm M})) \\ &= \sqrt{(x_i - x_{\rm M})^2 + (y_i - y_{\rm M})^2 + h^2} \end{aligned} \quad (2)$$

[2]The workers can be human, unmanned vehicles or robots.

as the euclidean distance between the worker $i$ and the mobile BS. Note that $h$ is the height of the mobile BS. Hereby, we can derive the worker $i$'s transmission power as

$$P_i^{\mathrm{t}} = \frac{(2^{\frac{r_i}{B}} - 1)}{g} d_i^{\alpha}. \quad (3)$$

Besides the power used to transmit data, for the worker $i$, we have the power cost function of data sensing $P_i^{\mathrm{s}} = b_i r_i$ where $b_i$ is the energy cost per bit. Here, the power cost of data sensing is linear to the sampling rate [10], i.e., the transmission rate. Therefore, the worker $i$'s total power cost $P_i$ can be expressed as follows:

$$P_i = P_i^{\mathrm{t}} + P_i^{\mathrm{s}} = \frac{(2^{\frac{r_i}{B}} - 1)}{g} d_i^{\alpha} + b_i r_i. \quad (4)$$

*2) Power cost of the mobile base station:* The mobile BS consumes energy mainly for WPT to workers. If the charging power transferred to the worker $i$ is $P_i^{\mathrm{c}}$, the mobile BS at the service location has to consume power $P_i^{\mathrm{c}'}$ derived as follows [11]:

$$P_i^{\mathrm{c}'} = \frac{P_i^{\mathrm{c}} d_i^{\alpha}}{\eta \Gamma} = P_i^{\mathrm{c}} d_i^{\alpha} \kappa, \quad (5)$$

where $\kappa = \frac{1}{\eta\Gamma}$, $0 < \eta < 1$ denotes the overall receiver energy conversion efficiency, and $\Gamma$ denotes the combined antenna gain of the transmit and receive antennas at the reference distance of 1 meter.

### B. Utility function in the wireless powered spatial crowdsourcing system

We evaluate the utility of the crowdsourced data based on the transmission rate, which combines two common metrics, i.e., the data size and timeliness. For example, the requesters may perform the data analysis and prediction based on the real-time crowdsourced data. Higher data transmission rate means that the requesters can process more data during a unit time and yield more accurate prediction results. The quality of the crowdsourced data is equivalent to the quality of the SC task completion. We use a logarithmic function of the total data rate $R$ to characterize the quality $q$ of the SC task completion as follows:

$$q(R) = a_1 \log(1 + a_2 R) = a_1 \log(1 + a_2 \sum_{i \in \mathcal{N}} r_i), \quad (6)$$

where $R = \sum_{i \in \mathcal{N}} r_i$, $a_1$ and $a_2$ are parameters. Taking the power cost of WPT (5) into consideration, the SC platform's utility function can be expressed as

$$\begin{aligned} u_m &= q(\tilde{R}) - \sum_{i \in \mathcal{N}} P_i^{\mathrm{c}'} \\ &= a_1 \log(1 + a_2 \sum_{i \in \mathcal{N}} r_i) - \sum_{i \in \mathcal{N}} P_i^{\mathrm{c}} d_i^{\alpha} \kappa. \end{aligned} \quad (7)$$

Similarly, we obtain the worker $i$'s utility function as

$$u_i = P_i^{\mathrm{c}} - P_i = P_i^{\mathrm{c}} - \frac{(2^{\frac{r_i}{B}} - 1)}{g} d_i^{\alpha} - b_i r_i. \quad (8)$$

### C. The procedure of wireless powered spatial crowdsourcing

*1) Task allocation phase:* First, the SC platform announces the task area $\boldsymbol{A}_t$ and a total charging power supply $P_{\mathrm{c}}$ ($P_{\mathrm{c}} = \sum_{j \in \mathcal{N}} P_j^{\mathrm{c}}$) to assist workers in data crowdsourcing. The charging power $P_i^{\mathrm{c}}$ transferred to worker $i$ is proportional to the worker $i$'s contribution (the data transmission rate) to the SC system, i.e., $P_i^{\mathrm{c}} = \frac{r_i}{R} P_{\mathrm{c}} = \frac{r_i}{\sum_{j \in \mathcal{N}} r_j} P_{\mathrm{c}}$. Based on the sensing tasks and the other workers' responses, each worker reports the preferred data rate $r_i$ to maximize its own utility. We assume that the workers are *risk-averse*, which means that they choose to minimize the uncertainty and the possible loss in the future. This concept can be found in the well-known prospect theory [12]. A common example is that many people prefer to deposit money at the bank for safe keeping and low return instead of buying financial products with high risk of loss. As workers have not set out to find the suitable working place and perform the allocated task, they are exposed to the uncertainty of working location $L_i$ and the mobile BS's service location $L_{\mathrm{M}}$ which are only known in the next data crowdsourcing phase. Each worker plans for the worst case where the mobile BS is deployed at the farthest location from it. Since worker $i$ knows its own working area $\boldsymbol{A}_i$ and the task area $\boldsymbol{A}_t$, it can calculate the maximum distance $D_i$ between $L_i$ and $L_M$, i.e., $D_i = \max_{L_M \in \boldsymbol{A}_t, L_i \in \boldsymbol{A}_i} d_i$. Note that the worker $i$'s utility function in (8) is monotonically decreasing with $d_i$. When making the decision on the transmission rate $r_i$, the worker $i$ sets $D_i$ as the distance away from the mobile BS. Then, the worker cannot suffer a loss due to the uncertain distance with the mobile BS in the data crowdsourcing phase. We denote by $\mathbf{r} = (r_1, r_2, \ldots, r_N)$ the workers' reported transmission rate vector. In addition, we use $\mathbf{r}_{-i} = (r_1, \ldots, r_{i-1}, r_{i+1}, \ldots, r_N)$ to denote the reported transmission rate vector for all workers except the worker $i$. Hereby, the worker $i$'s utility function in the task allocation phase can be expressed as

$$\bar{u}_i(r_i, \mathbf{r}_{-i}, P_{\mathrm{c}}) = \frac{r_i}{\sum_{j \in \mathcal{N}} r_j} P_{\mathrm{c}} - \frac{(2^{\frac{r_i}{B}} - 1)}{g} D_i^{\alpha} - b_i r_i. \quad (9)$$

The SC platform's utility in (7) is rewritten as

$$\bar{u}_m(P_{\mathrm{c}}, \mathbf{r}) = a_1 \log(1 + a_2 \sum_{i \in \mathcal{N}} r_i) - \sum_{i \in \mathcal{N}} \frac{r_i}{\sum_{j \in \mathcal{N}} r_j} P_{\mathrm{c}} D_i^{\alpha} \kappa. \quad (10)$$

*2) Data crowdsourcing phase:* Once the total charging power supply $P_c$, each worker's allocated charging power $P_i^c$ and transmission rate $r_i$ have been determined in the task allocation phase, workers travel to the working location and the SC platform sends out the mobile BS to serve the workers. However, the mobile BS has to know each worker's working location. Then, it can decide the service location $L_{\mathrm{M}}$ for maximizing the SC platform's utility. To make workers reveal their private working location $L_i$, the mobile BS organizes the following voting process on the spot.

1) The mobile BS first broadcasts its deployment mechanism, i.e, the mechanism or method to place the mobile BS according to the locations reported by workers, to the task area.

2) Once receiving the notification, each worker sends its working location $L_i$ to the mobile BS.
3) Based on the collected locations and the deployment mechanism, the service point location $L_{\mathrm{M}}$ is calculated for the mobile BS to deploy.

Let $\mathsf{M}$ denote the applied deployment mechanism which takes the workers' reported working location vector $\mathbf{L} = (L_1, \ldots, L_i, \ldots, L_{\mathrm{M}})$ as input and outputs the mobile BS's service location $L_{\mathrm{M}}$, i.e., $L_{\mathrm{M}} = \mathsf{M}(\mathbf{L})$. During the above voting process, a worker $i$ may have the incentive to improve its own utility in (8) by misreporting its true working location $L_i$. To make the location voting process robust and implementable, our designed mobile BS deployment mechanism should have the strategyproofness (truthfulness) property, which is defined as follows:

**Definition 1.** (Strategyproofness) Regardless of other workers' reported locations, a worker $i$ cannot increase the utility by misreporting its working location $L_i$. Formally, given a deployment mechanism $\mathsf{M}$ and a misreported location $L_i'$, we should have

$$\hat{u}_i(\mathsf{M}((L_i, \mathbf{L}_{-i}))) \geq \hat{u}_i(\mathsf{M}((L_i', \mathbf{L}_{-i}))) \, \forall L_i' \neq L_i \tag{11}$$

where $\mathbf{L}_{-i}$ is the vector containing all workers' working locations except the worker $i$'s.

## III. Task and Wireless Transferred Power Allocation Mechanism

We utilize the Stackelberg game [13] to analyze the model introduced in the task allocation phase (Section II-C1). Typically, there are two levels in the Stackelberg game. In the first (upper) level, the SC platform acts as a leader which strategizes and announces the total charging power supply $P_c$. In the second (lower) level, each worker is a follower which determines the strategy, i.e., the preferred transmission rate $r$, to maximize its own utility. Mathematically, the SC platform chooses the strategy $P_c$ by solving the following optimization problem:

$$(\mathrm{P1}) \ \max_{P_c \geq 0} \bar{u}_m(P_c, \mathbf{r}).$$

Meanwhile, the worker $i$ makes the decision on its reported $r_i$ to solve the following problem:

$$(\mathrm{P2}) \ \max_{r_i \geq 0} \bar{u}_i(r_i, \mathbf{r}_{-i}, P_c).$$

The objective of the Stackelberg game is to find the *Stackelberg Equilibrium (SE)* points. We first introduce the concept of the SE in our proposed model.

**Definition 2.** (Stackelberg Equilibrium) Let $\tilde{P}_c$ be a solution for Problem P1 and $\tilde{\mathbf{r}}$ be a solution for Problem P2 of the worker $i$. Then, the point $(\tilde{P}_c, \tilde{\mathbf{r}})$ is a SE for the proposed Stackelberg game if it satisfies the following conditions:

$$\bar{u}_m(\tilde{P}_c, \tilde{\mathbf{r}}) \geq \bar{u}_m(P_c, \tilde{\mathbf{r}}), \tag{12}$$

$$\bar{u}_i(\tilde{r}_i, \tilde{\mathbf{r}}_{-i}, \tilde{P}_c) \geq \bar{u}_i(r_i, \tilde{\mathbf{r}}_{-i}, \tilde{P}_c), \tag{13}$$

for any $(P_c, \mathbf{r})$ with $P_c \geq 0$ and $\mathbf{r} \succeq 0$.

In general, the first step to obtain the SE is to find the perfect *Nash Equilibrium (NE) [13]* for the non-cooperative transmission Rate Determination Game (RDG) in the second level. Then, we can optimize the strategy of the SC platform in the first level. Given a fixed $P_c$, the NE is defined as a set of strategies $\mathbf{r}^{\mathrm{ne}} = (r_1^{\mathrm{ne}}, \ldots, r_N^{\mathrm{ne}})$ that no worker can improve its utility by unilaterally changing its own strategy while other workers' strategies are kept unchanged. To analyze the NE, we introduce the concept of *concave game* and the theorem about the *existence* and *uniqueness* of NE in a concave game.

**Definition 3.** (Concave game [14]) A game is called concave if each worker $i$ chooses a strategy $r_i$ to maximize utility $\bar{u}_i$, where $\bar{u}_i$ is concave in $r_i$.

**Theorem 1.** *([14]) Concave games have (possibly multiple) Nash Equilibrium. Define $N \times N$ matrix function $\mathbf{H}$ in which $\mathbf{H}_{ij} = \frac{\partial^2 \bar{u}_i}{\partial r_i \partial r_j}, i, j \in \mathcal{N}$. Let* $\mathbf{H}^{\mathrm{T}}$ denote the transpose of $\mathbf{H}$. *If $\mathbf{H} + \mathbf{H}^{\mathrm{T}}$ is strictly negative definite, then the Nash equilibrium is unique.*

Hereby, we calculate the first-order and second-order derivatives of the worker $i$'s utility function $\bar{u}_i(r_i, \mathbf{r}_{-i}, P_c)$ with respect to $r_i$ as follows:

$$\frac{\partial \bar{u}_i}{\partial r_i} = \frac{P_c \sum_{k \in \mathcal{N}_{-i}} r_k}{(\sum_{j \in \mathcal{N}} r_j)^2} - \frac{D_i^{\alpha} \ln 2}{B} 2^{\frac{r_i}{B}} - b_i, \tag{14}$$

$$\frac{\partial^2 \bar{u}_i}{\partial r_i^2} = -\frac{2 P_c \sum_{k \in \mathcal{N}_{-i}} r_k}{(\sum_{k \in \mathcal{N}} r_k)^3} - \frac{D_i^{\alpha} \ln^2 2}{B^2} 2^{\frac{r_i}{B}}. \tag{15}$$

Since $\frac{\partial^2 \bar{u}_i}{\partial r_i^2} < 0$, $\bar{u}_i(r_i, \mathbf{r}_{-i}, P_c)$ is a strictly concave function with respect to $r_i$. Then, the non-cooperative RDG is a concave game and the NE exists. Given any $P_c > 0$ and any strategy profile $\mathbf{r}_{-i}$ ($\sum_{j \in \mathcal{N}_{-i}} r_j > 0$), the worker $i$'s best response strategy $\gamma_i$ exists and is unique. To prove the uniqueness of the NE, we also calculate the second-order mixed partial derivative of $\bar{u}_i$ for $i \in \mathcal{N}$ with respect to $r_{j \in \mathcal{N}_{-i}}$ as follows:

$$\frac{\partial^2 \bar{u}_i}{\partial r_j^2} = \frac{2 r_i}{(\sum_{j \in \mathcal{N}} r_j)^3} P_{\mathrm{c}}, \ \frac{\partial^2 \bar{u}_i}{\partial r_i \partial r_j} = \frac{r_i - \sum_{k \in \mathcal{N}_{-i}} r_k}{(\sum_{k \in \mathcal{N}} r_k)^3} P_{\mathrm{c}},$$

where $\frac{\partial^2 \bar{u}_i}{\partial r_j^2} \geq 0$ and $\frac{\partial^2 \bar{u}_i}{\partial r_i \partial r_j} \leq 0$ if $r_i \leq \sum_{k \in \mathcal{N}_{-i}} r_k$, $\forall i \in \mathcal{N}$. Then, we have the specific expression of the matrix function $\mathbf{H}$ defined in Theorem 1. Furthermore, the matrix function $\mathbf{H} + \mathbf{H}^{\mathrm{T}}$ can be decomposed into a sum of several $N \times N$ matrix functions: $\mathbf{H} + \mathbf{H}^{\mathrm{T}} = \mathbf{U} + \mathbf{V} + \sum_{k \in \mathcal{N}} \mathbf{C}^k$, where $\mathbf{U}_{ij} = \begin{cases} 0 & i \neq j \\ \frac{\partial^2 \bar{u}_i}{\partial R_i^2} & i = j \end{cases}$, $\mathbf{V}_{ij} = \sum_{k \in \mathcal{N}} \frac{\partial^2 \bar{u}_k}{\partial r_i \partial r_j}$ and $\mathbf{C}_{ij}^k = \begin{cases} 0 & i = k \,\mathrm{or}\, j = k \\ -\frac{\partial^2 \bar{u}_k}{\partial r_i \partial r_j} & \mathit{otherwise}. \end{cases}$ Since $\frac{\partial^2 \bar{u}_i}{\partial r_i^2} < 0$ and $\frac{\partial^2 \bar{u}_i}{\partial r_i \partial r_j} \leq 0$ if $r_i \leq \sum_{k \in \mathcal{N}_{-i}} r_k$, $\forall i \in \mathcal{N}$, we can find that $\mathbf{U}$ is strictly negative definite and $\mathbf{V}$, $\sum_{k \in \mathcal{N}} \mathbf{C}^k$ are negative semi-definite. Thus, $\mathbf{H} + \mathbf{H}^{\mathrm{T}}$ is proved to be strictly negative definite which indicates the NE in the RDG is unique. In other words, once the SC platform decides a strategy $P_{\mathrm{c}}$, the workers' strategies, i.e., the transmission rates, will be uniquely determined. We then can use the iterative best response [15] to find the SE point $\tilde{P}_{\rm c}$ in the first level, i.e., the optimal strategy of $P_{\rm c}$.

## IV. Mobile BS Deployment Mechanism in Data Crowdsourcing Phase

Given the calculated SE points $(\tilde{P}_c, \tilde{\mathbf{r}})$ from the task allocation phase, the specific problems for the SC platform and workers in the data crowdsourcing phase are as follows,

$$\max_{L_M \in \boldsymbol{A}_t} \hat{u}_m(L_{\rm M}) = a_1 \log(1 + a_2 \sum_{i \in \mathcal{N}} \tilde{r}_i) - \sum_{i \in \mathcal{N}} \frac{\tilde{r}_i}{\sum_{j \in \mathcal{N}} \tilde{r}_j} \tilde{P}_c d_i^{\alpha} \kappa, \quad (16)$$

$$\max_{L_{\rm M} \in \boldsymbol{A}_t} \hat{u}_i(L_{\rm M}) = \frac{\tilde{r}_i}{\sum_{j \in \mathcal{N}} \tilde{r}_j} \tilde{P}_c - \frac{(2^{\frac{\tilde{r}_i}{B}} - 1)}{g} d_i^{\alpha} - b_i \tilde{r}_i, \quad (17)$$

where $d_i = d_i(L_{\rm M})$ is defined in (2). To address the mobile BS's location problem introduced in Section II-C2, we first introduce an important concept of 2-*dimensional single-peaked preference* for the discussed problem.

**Definition 4.** *(2-dimensional single-peaked preference [16])* Let $\mathbf{L}_{\rm M}$ be the set of possible mobile BS's service locations output by the deployment mechanism $\mathsf{M}$ on the XY-plane where $X$ and $Y$ are respectively a one-dimensional axis. A worker $i$'s preference for the mobile BS's location is 2-dimensional single-peaked with respect to $(X, Y)$ if 1) there is a single most-preferred location outcome $\tilde{L}_i^{\rm M} \in \mathbf{L}_{\rm M}$, and 2) for any two outcomes $L'_{\rm M}, L''_{\rm M} \in \mathbf{L}_{\rm M}$, $L'_{\rm M} \succeq_i L''_{\rm M}$ whenever $L''_{\rm M} <_\rho L'_{\rm M} <_\rho \tilde{L}_i^{\rm M}$ or $\tilde{L}_i^{\rm M} <_\rho L'_{\rm M} <_\rho L''_{\rm M}$ for $\forall \rho \in \{X, Y\}$, i.e., both $X$ and $Y$ axes.

In the above definition, $L'_{\rm M} \succeq_i L''_{\rm M}$ straightforwardly means $L'_{\rm M}$ is preferred by worker $i$ to $L''_{\rm M}$. "$<_\rho$" is a strict ordering by worker $i$ on the dimension $\rho$. An intuitive explanation of this condition is that $L'_{\rm M}$ is preferred by worker $i$ to $L''_{\rm M}$ as long as $L'_{\rm M}$ is nearer to its most-preferred location $\tilde{L}_i^{\rm M}$ on each dimension.

**Proposition 1.** *In the data crowdsourcing phase, the worker's preference for the mobile BS's service location is 2-dimensional single-peaked.*

*Proof:* We first expand the worker $i$'s utility function given in (17) as $\hat{u}_i(L_{\rm M}) = \hat{u}_i(x_{\rm M}, y_{\rm M}) = -\frac{(2^{\frac{\tilde{r}_i}{B}}-1)}{g} \left((x_i - x_{\rm M})^2 + (y_i - y_{\rm M})^2 + h^2\right)^{\frac{\alpha}{2}} + \frac{\tilde{r}_i}{\sum_{j \in \mathcal{N}} \tilde{r}_j} \tilde{P}_c - b_i \tilde{r}_i$. It is clear that $\hat{u}_i$ is concave with respect to $(x_{\rm M}, y_{\rm M})$ and there is a unique optimal solution $\tilde{L}_i^{\rm M} = (x_i, y_i)$ to maximizing the utility. In other words, the worker $i$'s most preferred mobile BS's service location is its working location, i.e., $\tilde{L}_i^{\rm M} = L_i = (x_i, y_i)$, which satisfies the first condition in Definition 4. In the task area $\boldsymbol{A}_t$, we randomly choose two locations $L'_{\rm M} = (x'_{\rm M}, y'_{\rm M})$ and $L''_{\rm M} = (x''_{\rm M}, y''_{\rm M}) \in \boldsymbol{A}_t$. Note that the concavity of $\hat{u}_i$ guarantees the concavity on one dimension if fixing the variable on the other dimension. $L''_{\rm M} <_X L'{}_{\rm M} <_X \tilde{L}_i^{\rm M}$ implies that $\hat{u}_i((x''_{\rm M}, y)) < \hat{u}_i((x'_{\rm M}, y)) < \hat{u}_i((x_i, y))$ for any $y$ on $Y$ axis and then$|x'_{\rm M} - x_i| < |x''_{\rm M} - x_i|$. We can have the similar implication from $L''_{\rm M} <_Y L'{}_{\rm M} <_Y \tilde{L}_i^M$. If $L''_{\rm M} <_X L'{}_{\rm M} <_X \tilde{L}_i^{\rm M}$ and $L''_{\rm M} <_Y L'_{\rm M} <_Y \tilde{L}_i^M$ are both satisfied, we can have $(x_i - x'_{\rm M})^2 + (y_i - y'_{\rm M})^2 < (x_i - x''_{\rm M})^2 + (y_i - y''_{\rm M})^2$ and thus $\hat{u}_i(L''_{\rm M}) = \hat{u}_i((x''_{\rm M}, y''_{\rm M})) < \hat{u}_i(L'_{\rm M}) = \hat{u}_i((x'_{\rm M}, y'_{\rm M}))$. Therefore, the worker $i$ prefers $L'_{\rm M}$ to $L''_{\rm M}$, i.e., $L'_{\rm M} \succeq_i L''_{\rm M}$, which proves the second condition in Definition 4 and completes the proof. ■

**Theorem 2.** *(Moulin's one-dimensional generalized median mechanism [9]) A mechanism* $\mathsf{M}$ *for single-peaked preferences in a one-dimensional space is strategyproof and anonymous if and only if there exist* $N+1$ *constants* $\tau_1, \ldots, \tau_{N+1} \in \mathbb{R} \cup (-\infty, +\infty)$ *such that:*

$$\mathsf{M}(\mathbf{L}^{\rm M}) = \text{median}(\tilde{L}_1^{\rm M}, \ldots, \tilde{L}_N^{\rm M}, \tau_1, \ldots, \tau_{N+1}) \quad (18)$$

*where* $\mathbf{L}^{\rm M} = \{\tilde{L}_1^{\rm M}, \ldots, \tilde{L}_N^{\rm M}\}$ *is the set of workers' most-preferred mobile BS's locations and* median *is the median function. An outcome rule* $\mathsf{M}$ *is anonymous, if for any permutation* $\boldsymbol{T}'$ *of* $\boldsymbol{T}$*, we have* $\mathsf{M}(\boldsymbol{T}') = \mathsf{M}(\boldsymbol{T})$ *for all* $\boldsymbol{T}$.

**Theorem 3.** *(Multi-dimensional generalized median mechanism [16]) A mechanism for multi-dimensional single-peaked preferences in a multi-dimensional space is strategyproof and anonymous if and only if it is an* $m$*-dimensional generalized median mechanism, which straightforwardly applies the one-dimensional generalized median mechanism on each of the* $m$ *dimensions.*

**Algorithm 1** MED mechanism

**Input:** Workers' reported locations $\mathbf{L} = (L_1, \ldots, L_i, \ldots, L_N)$.
**Output:** Mobile BS's service point location $L_M = (x_M, y_M)$.
1: **begin**
2: Respectively sort the workers' locations on x-axis $\mathbf{x} = (x_1, \ldots, x_N)$ and y-axis $\mathbf{y} = (y_1, \ldots, y_N)$ in ascending order.
3: **if** $N$ is odd **then**
4: $x_M \leftarrow x_{\frac{N+1}{2}}, y_M \leftarrow y_{\frac{N+1}{2}}$
5: **else**
6: $x_M \leftarrow \frac{x_{\frac{N}{2}} + x_{\frac{N}{2}+1}}{2}, y_M \leftarrow \frac{y_{\frac{N}{2}} + y_{\frac{N}{2}+1}}{2}$
7: **end if**
8: **end**

A straightforward mechanism is the median mechanism [9], [16], as shown in Algorithm 1. We simply name it as MED mechanism $\mathsf{M}_{\rm MED}$. This algorithm directly computes the median of workers' reported locations as the mobile BS's service point location. Apparently, it is a special case of the multi-dimensional generalized median mechanism, so it is strategyproof. We next analyze its performance by comparing it with the optimal mechanism $\mathsf{M}_{\rm OPT}$ which achieves the maximum utility of the SC platform without considering incentive constraints. Let $\tilde{r}_{\max}, \tilde{r}_{\min}$ respectively denote the maximum and the minimum transmission rate among workers, i.e., $\tilde{r}_{\max} = \max(\tilde{\mathbf{r}}), \tilde{r}_{\min} = \min(\tilde{\mathbf{r}})$ .

**Proposition 2.** *The worst-case performance of the MED mechanism* $\mathsf{M}_{\rm MED}$ *for maximizing the SC platform's utility can guarantee* $\hat{u}_m(\mathsf{M}_{\rm MED}(\boldsymbol{L})) \geq \varphi - 2^{\frac{\alpha}{2}} N^{\frac{\alpha}{2}-1} \frac{\tilde{r}_{\max}}{\tilde{r}_{\min}} (\varphi - \hat{u}_m(\mathsf{M}_{\rm OPT}(\boldsymbol{L})))$, where $\varphi = a_1 \log(1 + a_2 \sum_{i \in \mathcal{N}} \tilde{r}_i)$.

*Proof:* We expand the SC platform's utility function in (16) as follows:

$$\hat{u}_m((x_{\rm M}, y_{\rm M})) = \varphi - \frac{\tilde{P}_c\kappa}{\sum_{j\in\mathcal{N}}\tilde{r}_j}\sum_{i\in\mathcal{N}}\left(\tilde{r}_i^{\frac{2}{\alpha}}(x_i-x_M)^2+(y_i-y_M)^2+h^2\right)^{\frac{\alpha}{2}}, \quad (19)$$

where $\varphi = a_1\log(1+a_2\sum_{i\in\mathcal{N}}\tilde{r}_i)$. Let $x_{\rm med}, \overline{x}$ and $y_{\rm med}, \overline{y}$ respectively denote the median and mean of $\mathbf{x} = (x_1, \ldots, x_N)$ and $\mathbf{y} = (y_1, \ldots, y_N)$. Also, we use $(x_{\rm opt}, y_{\rm opt})$ to denote the optimal solution to maximizing the expression in (19), i.e., $\mathsf{M}_{\rm OPT}(\boldsymbol{L}) = (x_{\rm opt}, y_{\rm opt})$. We also note that the optimal solution to minimizing the $\sum_{i\in\mathcal{N}}\tilde{r}_i^{\frac{2}{\alpha}}\left((x_i-x_{\rm M})^2+(y_i-y_{\rm M})^2+h^2\right)$ is $(x^*, y^*)$ where $x^* = \frac{\sum_{i\in\mathcal{N}}\tilde{r}_i^{\frac{2}{\alpha}}x_i}{\sum_{i\in\mathcal{N}}\tilde{r}_i^{\frac{2}{\alpha}}}$ and $y^* = \frac{\sum_{i\in\mathcal{N}}\tilde{r}_i^{\frac{2}{\alpha}}y_i}{\sum_{i\in\mathcal{N}}\tilde{r}_i^{\frac{2}{\alpha}}}$. As $\tilde{r}_{\min} \le \tilde{r}_i$, we have

$$\begin{aligned}&\tilde{r}_{\min}^{\frac{2}{\alpha}}\sum_{i\in\mathcal{N}}\left((x_i-\overline{x})^2+(y_i-\overline{y})^2+h^2\right)\\ &\le \sum_{i\in\mathcal{N}}\tilde{r}_i^{\frac{2}{\alpha}}\left((x_i-x^*)^2+(y_i-y^*)^2+h^2\right). \quad (20)\end{aligned}$$

According to [17, Theorem 4.3], we have $\sum_{i\in\mathcal{N}}(x_i - x_{\rm med})^2 \le 2\sum_{i\in\mathcal{N}}(x_i-\overline{x})^2$ and $\sum_{i\in\mathcal{N}}(y_i-y_{\rm med})^2 \le 2\sum_{i\in\mathcal{N}}(y_i-\overline{y})^2$. Then, it is not hard to verify that

$$\begin{aligned}&\tilde{r}_{\min}^{\frac{2}{\alpha}}\sum_{i\in\mathcal{N}}\left((x_i-x_{\rm med})^2+(y_i-y_{\rm med})^2+h^2\right)\\ &\le 2\tilde{r}_{\min}^{\frac{2}{\alpha}}\sum_{i\in\mathcal{N}}\left((x_i-\overline{x})^2+(y_i-\overline{y})^2+h^2\right), \quad (21)\end{aligned}$$

$$\begin{aligned}&\tilde{r}_{\min}\left(\sum_{i\in\mathcal{N}}\left((x_i-x_{\rm med})^2+(y_i-y_{\rm med})^2+h^2\right)\right)^{\frac{\alpha}{2}}\\ &\le 2^{\frac{\alpha}{2}}\tilde{r}_{\min}\left(\sum_{i\in\mathcal{N}}\left((x_i-\overline{x})^2+(y_i-\overline{y})^2+h^2\right)\right)^{\frac{\alpha}{2}}\\ &\le 2^{\frac{\alpha}{2}}\tilde{r}_{\min}\left(\sum_{i\in\mathcal{N}}\left((x_i-x^*)^2+(y_i-y^*)^2+h^2\right)\right)^{\frac{\alpha}{2}}\\ &\le 2^{\frac{\alpha}{2}}\left(\sum_{i\in\mathcal{N}}\tilde{r}_i^{\frac{2}{\alpha}}\left((x_i-x^*)^2+(y_i-y^*)^2+h^2\right)\right)^{\frac{\alpha}{2}}. \quad (22)\end{aligned}$$

Since $\alpha \ge 2$, we can prove that

$$\begin{aligned}&\tilde{r}_{\min}\sum_{i\in\mathcal{N}}\left((x_i-x_{\rm med})^2+(y_i-y_{\rm med})^2+h^2\right)^{\frac{\alpha}{2}}\\ &\le \tilde{r}_{\min}\left(\sum_{i\in\mathcal{N}}\left((x_i-x_{\rm med})^2+(y_i-y_{\rm med})^2+h^2\right)\right)^{\frac{\alpha}{2}}. \quad (23)\end{aligned}$$

Hence, based on Theorem 1 in [18] and the facts that $\tilde{r}_i \le \tilde{r}_{\max}$ and $\frac{\alpha}{2} \ge 1$, we can obtain

$$\begin{aligned}&2^{\frac{\alpha}{2}}\left(\sum_{i\in\mathcal{N}}\tilde{r}_i^{\frac{2}{\alpha}}\left((x_i-x^*)^2+(y_i-y^*)^2+h^2\right)\right)^{\frac{\alpha}{2}}\\ &\le 2^{\frac{\alpha}{2}}\left(\sum_{i\in\mathcal{N}}\tilde{r}_i^{\frac{2}{\alpha}}\left((x_i-x_{\rm opt})^2+(y_i-y_{\rm opt})^2+h^2\right)\right)^{\frac{\alpha}{2}}\\ &\le 2^{\frac{\alpha}{2}}\tilde{r}_{\max}\left(\sum_{i\in\mathcal{N}}\left((x_i-x_{\rm opt})^2+(y_i-y_{\rm opt})^2+h^2\right)\right)^{\frac{\alpha}{2}}\\ &\le 2^{\frac{\alpha}{2}}N^{\frac{\alpha}{2}-1}\tilde{r}_{\max}\sum_{i\in\mathcal{N}}\left((x_i-x_{\rm opt})^2+(y_i-y_{\rm opt})^2+h^2\right)^{\frac{\alpha}{2}}.\end{aligned} \quad (24)$$

Combining the above inequalities, we have

$$\begin{aligned}&\varphi - \frac{\tilde{P}_c\kappa}{\sum_{j\in\mathcal{N}}\tilde{r}_j}\sum_{i\in\mathcal{N}}\left((x_i-x_{\rm med})^2+(y_i-y_{\rm med})^2+h^2\right)^{\frac{\alpha}{2}}\\ &\ge \varphi - \left(\frac{\tilde{P}_c\kappa}{\sum_{j\in\mathcal{N}}\tilde{r}_j}2^{\frac{\alpha}{2}}N^{\frac{\alpha}{2}-1}\frac{\tilde{r}_{\max}}{\tilde{r}_{\min}}\right.\\ &\qquad\left.\sum_{i\in\mathcal{N}}\left((x_i-x_{\rm opt})^2+(y_i-y_{\rm opt})^2+h^2\right)^{\frac{\alpha}{2}}\right). \quad (25)\end{aligned}$$

Finally, we can conclude that $\hat{u}_m(\mathsf{M}_{\rm MED}(\boldsymbol{L})) \ge \varphi - 2^{\frac{\alpha}{2}}N^{\frac{\alpha}{2}-1}\frac{\tilde{r}_{\max}}{\tilde{r}_{\min}}(\varphi - \hat{u}_m(\mathsf{M}_{\rm OPT}(\boldsymbol{L})))$. ■

## V. Simulation Results and Discussions

In this section, we conduct simulations based on realistic data to evaluate the performance of our proposed framework and strategyproof deployment mechanisms. Unless otherwise stated, the simulation configuration is set as follows. We consider a $[0, 50]\times[0, 50]$ square-meter area as the SC task area $\boldsymbol{A}_t$. The number of registered workers is set as $N = 50$. We generate worker $i$'s working location from the bivariate uniform distribution, i.e., $\mathcal{P}^{\rm u} = \begin{cases}1 & (x,y)\in\boldsymbol{A} = [25-\beta_i, 25+\beta_i]^2\\ 0 & otherwise.\end{cases}$, where $\beta_i \in \mathbb{R}$ is uniformly distributed in $[0, 25]$. Hereby, the maximum distance $D_i$ can naturally be calculated. We set the height of the mobile BS $h = 5\,\text{m}$, the channel gain to noise ratio $g = 90\,\text{dB}$, the bandwidth of each subchannel $B = 60\,\text{MHz}$, the data utility parameters $a_1 = 1000$ and $a_2 = 200$, the energy conversion efficiency $\eta = 0.5$, the antenna gain $\Gamma = -30\,\text{dB}$, and the path-loss exponent $\alpha = 2$. The sensing energy cost per bit $b_i$ is generated from the uniform distribution on $[10^{-4}, 1.5\times10^{-4}]$. Each measurement is averaged over 100 instances. Figure 3 demonstrates the impact of the number of workers on the SC platform's utility, the average worker's utility and the number of employed workers in the task allocation phase. When the number of registered workers increases, the SC platform's utility and the number of employed workers gradually increase but with a diminishing return. These results reflect that when more workers are employed, the SC platform has to consume more charging power for the same marginal utility. By contrast, the

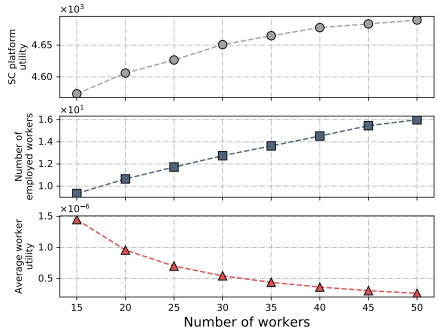


Fig. 3. Impact of the number of workers on the SC platform's utility (top), the number of employed workers (middle) and the average worker's utility (bottom) in the task allocation phase.

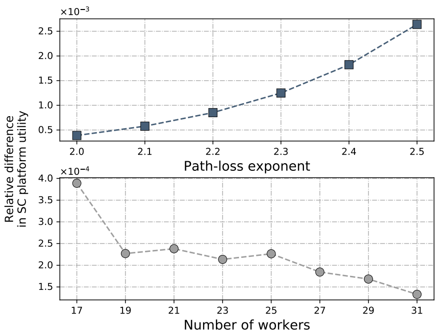


Fig. 4. The SC platform utility achieved per different mechanism with varied number of employed workers and varied path-loss exponent in the data crowdsourcing phase.

average worker's utility decreases with the increase of registered workers because of the more fierce competition among workers. Figure 4 illustrates the performance gap between the MED mechanism and the optimal mechanism in the average case. We use the *relative difference* metric which is defined as the ratio of the SC platform's utility difference achieved from the OPT mechanism and the MED mechanism over the SC platform's utility achieved from the OPT mechanism, i.e., $\frac{\hat{u}_m(\mathsf{M}_{\mathrm{OPT}}(\boldsymbol{L}))-\hat{u}_m(\mathsf{M}_{\mathrm{MED}}(\boldsymbol{L}))}{\hat{u}_m(\mathsf{M}_{\mathrm{OPT}}(\boldsymbol{L}))}$. When the radio environment gets worse (a larger $\alpha$), we can see the decreasing performance of the MED mechanism compared to the optimal mechanism. However, if more workers participate, the performance of the MED mechanism is closer to the optimal results. This is mainly due to the fact that the generated workers' locations follows a similar symmetric uniform distribution.

## VI. Conclusion

In this paper, we have proposed a wireless powered spatial crowdsourcing framework composed of two phases. We have proven that the proposed Stackelberg game based incentive mechanism can help the SC platform efficiently allocate the tasks and the wireless charging power in the task allocation phase. For the deployment of the mobile BS in the data crowdsourcing phase, we have adopted the median mechanism as the strategyproof mobile BS deployment mechanism. Besides avoiding the dishonest worker's manipulation, the proposed SC framework can efficiently allocate tasks and charging power which is shown by experimental results.


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