

Probabilistic Capacity Planning Framework for Electric Vehicle Charging Stations with Overstay

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Abstract—Public charging stations provide charging services as well as parking for the growing population of electric vehicles (EVs). Effective management of these facilities is becoming crucial, with a significant proportion of drivers remaining parked even after the services are completed. This phenomenon, known as *overstay*, results in an underutilization of station resources and becomes a barrier to other electric vehicle drivers seeking charging services. To that end, this article presents (i) stochastic modeling of charging stations with overstaying customers, and (ii) a methodology to calculate station capacities with respect to a performance metric probability of loss of load that represents the percentage of unsatisfied demand. The station model is constructed using a two-dimensional Markov chain reflecting interactions among the “idle”, “charging”, and “overstaying” customers. Initially, the generalized small-scale charging station model is studied to investigate the behavior of the station parameters. Then, the general model is extended using the statistical large deviation theory to cover the case of large-scale charging stations. Effective demand, a deterministic quantity, for each charger is calculated, and station capacity is calculated in terms of the above-mentioned performance metric. The case studies demonstrate that calculating effective demand-based capacity leads to substantial savings when provisioning station resources. However, a significant proportion of these savings diminish with increasing rate of overstay customers and durations.

Index Terms—electric vehicles, charging needs, stochastic process, overstay

I. INTRODUCTION

The pressure for emission reductions in the transportation sector requires the phaseout of gasoline- and diesel-fueled internal-combustion vehicles (ICEVs) which are powered with gasoline or diesel [1]. technological options for providing a lower-carbon transport alternative to ICEVs [2]. Despite significant improvements in the battery technology [3], the adoption rates are hindered by lack of sufficient public charging infrastructures needed to support drivers who do not have access to a dedicated charger. In fact, recent measurement studies show that the EV driving experience is very risk-adverse due to range anxiety and limited charger coverage [4].

Public charging stations can consist of faster chargers (50+kW) and/or slower chargers (7.2-22kW) to accommodate different customer needs [5]. At public charging facilities, EV drivers often encounter congestion [6] when all chargers are occupied or some EVs *overstay* [7], [8]. The former occurs during peak traffic hours, while the latter could happen more frequently when EV owners keep their vehicles parked even if the charging service has been completed [9]. EV overstay

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Fig. 1: Charging and overstay statistics [10].

is a serious issue as it reduces charging station utilization, increases station congestion, and decreases access to chargers at public charging locations.

There are only a few published literature on managing EV overstay problem. In [9] interviews were conducted with electric vehicle owners to learn their acceptance of different management styles for overstay charging, such as authoritative (enforce rules), collective (employee-driven rules), and non-managed (no rules). In [7], charging station planning problem with overstay is studied. EVs who overstay are interchanged with waiting customers to increase charger utilization. The model is applied to real-world statistics collected from parking lots in California. It is shown that average charging duration is 2 hours, however, 90% of the EVs tend to overstay for 75% of their charging duration. In our previous work [10], we examined the charging behavior of EVs located in a major North American University Campus. In Fig. 1, charging and overstay statistics are presented. It can be seen that a sizable portion of the EVs stay extended periods of time due to lack of control schemes.

In [8], the overstay problem is addressed by devising pricing schemes to induce human behavior and reduce station congestion. In real-world applications, overstay is tackled by offering myopic prices such that an overstay fee is charged after a short grace period when an EV is connected and not charging. For instance, in Tesla Superchargers, overstaying EVs are charged 0.5 USD/min and 1 USD/min when the station is 100% occupied [11].

Markov chain-based modelling of EV charging stations has

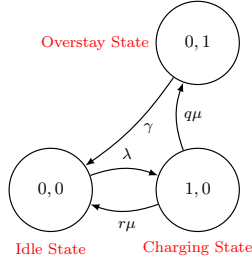


Fig. 2: Markov chain model for single state charger.

been presented in the literature. In our previous work [12], we proposed a Markov Modulated Poisson Process to compute optimal size of on-site storage units for EV charging stations. In [13], a continuous time Markov chain is used to model the behavior of a single-charger multiple-socket charging stations. Similar to our work, blocking probability is used as the main performance metric to size station capacity.

In this paper, we propose a probabilistic capacity planning framework for EV charging stations with Overstay customers. First, we develop a two-dimensional continuous time Markov chain model to capture station state. Then, we present how to calculate station capacity in respect to a quality of service target represented by probability of loss of load that shows the percentage of unsatisfied demand. Second, the proposed model is extended for large-scale parking lots. In this case, the proposed model exploits the stochastic behavior of chargers represented by idle, charging, and overstay states. Then, the effective demand, a deterministic quantity, of each charger is computed using large deviations theory and station capacity is computed at the cost of denying service to a small fraction of customers. Third, we develop case studies to demonstrate how the length and percentage of overstay EVs impact the overall station capacity and discuss ways of reducing this pricing policies.

II. PROBLEM FORMULATION

A. System Description

We consider an EV charging station with large number of homogenous chargers, with index denoted by $i = \{1, 2, \dots, N\}$. Since most chargers have a power factor close to 1 [14], the capacity of the station (or the size of the supporting transformer) is denoted by P and has a unit in kW. The EV arrival process is assumed to follow a Poisson process with parameter λ (for similar assumptions, see [15] and [16]). An arriving customer could have two types of demand: (i) energy demand for filling up the EV's battery and (ii) parking space demand. It is assumed that if there is an idle parking space, an arriving customer immediately starts charging and both demand types are satisfied. During the charging, charger i 's charging power is assumed to be constant at C_i .

Upon service completion, one of the two distinct events could occur: (i) the EV could continue to stay parked (*overstay* event) and continue to occupy a charger without drawing power or (ii) the EV could leave the system. Similar to the previous assumption, both event types are assumed to

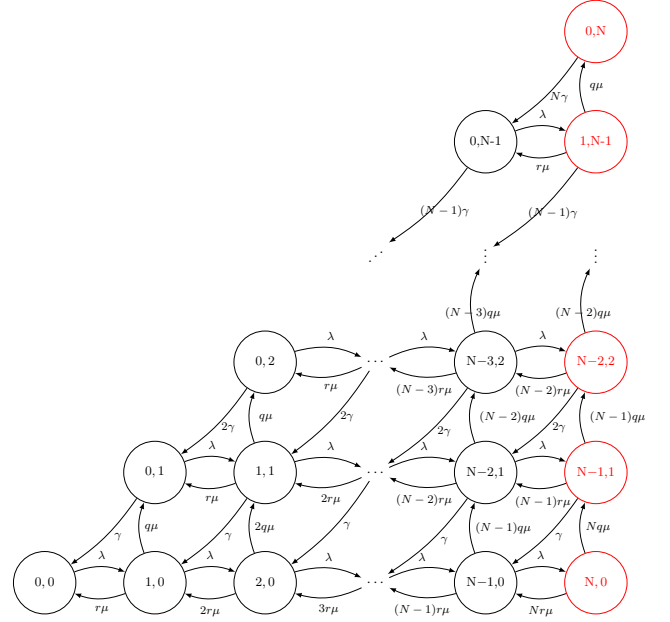


Fig. 3: Generalized Markov-chain model for the charging station with overstay customers.

follow a Poisson process with rates μ and γ , respectively. It is further assumed that an EV in *overstay* state cannot go back to charging state and leaves the station after an overstay period. Similarly, an EV cannot use the station as a parking lot without requesting charging service. To that end, a single charger is modelled with a three state two-dimensional continuous time Markov chain as depicted in Fig. 2. A state is described by a tuple (j, k) , where j denotes the number of EVs getting charged, hence j could take values between 0 and N (for single charger case $N = 1$). Moreover, k denotes the number of EVs that are in *overstay* state which, similar to the previous case, k takes values between 0 and N . For instance, in state $(1, 0)$ there is an EV getting charged and upon service completion, the system state can move to state $(0, 0)$ with rate $r\mu$ or move to overstay state $(0, 1)$ with rate $q\mu$. Note that coefficients r and q represent the percentage of EVs leaving the station and overstaying, respectively. They take values between 0 and 1 and their sum adds to 1, i.e. $r + q = 1$. For instance, if $r = 1$, there is no overstay in the system, all EVs leave immediately after service completion. Similarly, if $r = 0.5$, then half of the customer leave and the remaining half moves to overstay. In this paper, it is assumed that r and q are constant for all states, and we will explore their impacts on the system performance. However, in our future work they will be used as pricing-based control parameters which will determine the amount of overstay values.

B. Generic Station Model

The Markov chain-based model described above could be generalized for a station with N chargers, as shown in Fig. 3. In this case, the transition rates are updated according to the number of EVs in the system. The bottom row shows

the state when there is no overstay, while the remaining rows shows the states with overstay. The state $(0, N)$ represent the case where all EVs have completed charging service, and they are all in overstay state. The rightmost states (shown in red) are “unsatisfied demand” states or “loss of load” (LoLP) states because all station resources are completely utilized and an arriving customer will not be accepted to the system. Therefore, the ratio of unsatisfied demand or *loss-of-load probability* (LoLP) naturally serves as a performance metric to compute station capacity for given system parameters such as arrival, departure, and overstay rates.

It can be seen from Fig. 3 that the number of states grows with the number of chargers and can be written as

$$\tau = \frac{(N+1) \times (N+2)}{2}. \quad (1)$$

The steady state probabilities for each state π_i , $i \in N$ can be computed numerically by solving

$$\pi \mathbf{Q} = \mathbf{0} \quad \text{and} \quad \sum_{i=1}^{\tau} \pi_i = 1, \quad (2)$$

where π is the vector of steady state probabilities and \mathbf{Q} is $\tau \times \tau$ an infinitesimal generator matrix whose elements contain the transition rate from one state to another. The elements of the matrix \mathbf{Q} denoted by row index l and column index r has the following structure

$$q_{kl} \geq 0, \forall k \neq l \quad \text{and} \quad q_{kk} = -\sum_{k \neq l} q_{kl}, \forall k. \quad (3)$$

For small-scale charging stations, the minimum station capacity can be found by computing station capacity in respect to varying station parameters. However, in (2), there are τ system states which grows exponentially as the station size grows, and numerical evaluation becomes computationally prohibitive. Therefore, for large scale public stations, we propose a statistical sizing approach described in the next section.

III. STATISTICAL CAPACITY PLANNING

Our primary goal is to find minimum station capacity such that the LoLP does not exceed a small value denoted by δ . We start our analysis by investigating the individual charger states. As described in the previous section, a charger can have three distinct states namely, “idle”, “charging”, or “overstay”. From electrical energy demand, a charger can have two states:

- 1) Charger is at “Off” state when no electricity is drawn. The probability of being at the “Off” state is the sum of probabilities of being at “idle” and “overstay” states, that is $P(\text{Off}) = P(\text{idle}) + P(\text{overstay})$.
- 2) Charger is at “On” state when electrical power is drawn to charge EVs. This time, the probability is written as $P(\text{On}) = P(\text{charging})$

To that end, $P(\text{Off})$ and $P(\text{On})$ can be computed by constructing \mathbf{Q} matrix from (3) and solving the balance equations given

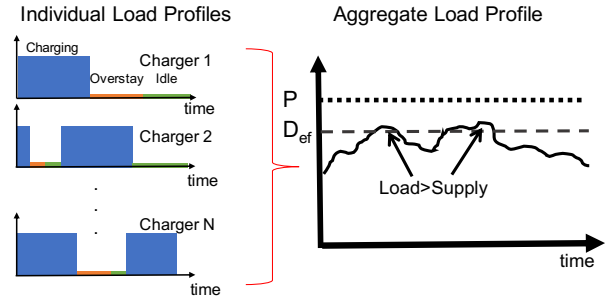


Fig. 4: Overview of station load profile and effective demand.

in (2). For a single charger case, the infinitesimal generator matrix becomes

$$\mathbf{Q} = \begin{bmatrix} -\lambda & \lambda & 0 \\ r\mu & -\mu & q\mu \\ \gamma & 0 & -\gamma \end{bmatrix} \quad (4)$$

By inserting (4) in (2), the probability of being “Off” becomes

$$P(\text{Off}) = \frac{q\mu(1+\lambda)}{\mu\gamma + \lambda\gamma + q\mu\lambda}. \quad (5)$$

Similarly, probability of being “On” becomes

$$P(\text{On}) = \frac{\lambda\gamma}{\mu\gamma + \lambda\gamma + q\mu\lambda}. \quad (6)$$

Let $D_i(s)$ denote the amount of electrical power drawn by the charger at time s , then the probability distribution of $D_i(s)$ is given by

$$D_i = \begin{cases} C_i, & \text{with probability } P(\text{On}) \\ 0, & \text{with probability } P(\text{Off}) \end{cases} \quad (7)$$

From statistical capacity planning standpoint, the worst approach would be to allocate peak rate (C_i) for each charger, that is $P = \sum_{i=1}^N C_i$. Due to statistical distribution of D_i as shown in (7), actual station capacity could be significantly lower than peak rate. To that end, we are interested in computing minimum station capacity P such that the probability of LoLP does not exceed a small probability δ , that is

$$P(\delta) = \begin{cases} \min & P \\ \text{s.t.} & P\left(\sum_{i=1}^N C_i\right) \leq \delta \end{cases} \quad (8)$$

From (8), the “expected” demand can be re-written,

$$\sum_i E(D_i) = \sum_{i=1}^N C_i \times P_i(\text{On}) < P, \forall i, \quad (9)$$

given $0 < P_i(\text{On}) < 1$. Even though the inequality given in (9) is valid, it does not provide statistical guarantees to provide charging service with maximum LoLP performance (δ) as given in (8). To that end, we introduce the concept of “effective demand” which represents effective utilization of a charger for a given LoLP target.

A. Computation of Effective Demand

We are interested in computed effective demand D_{ef} for each charger such that aggregate demand exceeds total station capacity P with a small probability denoted by δ . An overview of the system is presented in Fig. 4. In other words, we aim to compute the fraction of time that total station demand is higher than supporting transformer capacity (P) in the long run. Let us define an indicator function as $1_{(\cdot)}$. Since charging demand at each slot $D_i(s)$ are independently distributed, the aggregate profile forms a stationary and ergodic process as below

$$\lim_{t \rightarrow \infty} \frac{1}{t} \sum_{s=0}^{t-1} 1_{\left\{ \sum_{i=1}^N D_i(s) > D_{ef} \right\}} = \mathbb{P} \left\{ \sum_{i=1}^N D_i > D_{ef} \right\}. \quad (10)$$

Since all chargers are identical, effective demand can be normalized as $D_{ef} = Nd$, where d is the effective usage of a single charger which can be computed by solving

$$\mathbb{P} \left\{ \sum_{i=1}^N D_i > Nd \right\} \leq e^{-\delta}. \quad (11)$$

Applying Chernoff's bound (see [17]) to (11) yields to

$$\mathbb{P} \left\{ \sum_{i=1}^N D_i > Nr \right\} \leq e^{-NA(d)}, \quad (12)$$

where $A(d)$ is the rate function and written as

$$A(d) = \sup_{\theta > 0} \{ \theta d - \log(\mathbb{E}\{e^{\theta D}\}) \} \quad (13)$$

The rate function $A(d)$ is key to compute effective demand d and has the following four properties. First, the rate function $A(d)$ is concave in θ and convex in d . Proving concavity requires examining the properties of the logarithmic expression $\log(\mathbb{E}(\cdot))$. To confirm this property, the definition of convexity can be applied and Hölder's Inequality (see [18]) can be used to complete the proof. The second part is trivial as the effective demand d has a linear relation with the rate function, therefore, $A(d)$ is convex in d . The second property is a corollary to the first one. Due to the convexity assumption, the supremum over θ is guaranteed and can be found through a search process. Third, without loss of generality, the effective demand d is greater than or equal to the expected usage as given in (9) and less than or equal to normalized state capacity P/N . Fourth, the rate function takes positive values for $d > \mathbb{E}(D)$ and $A(\mathbb{E}(D)) = 0$. This property can be proven by taking the derivative of $A(d)$ at the origin $\theta = 0$, that is $\frac{\partial A(\theta, d)}{\partial \theta} \big|_{\theta=0} = d - \mathbb{E}\{D\}$. The result will be positive since $d > \mathbb{E}(D)$, hence $A(d) > 0$. For the second case, the rate function will occur at the origin, since it is concave in θ .

We are further interested in the behavior of the resultant probability bound. According to Cramer's Theorem (used in large deviation asymptotics, see [19]), the probabilistic bound is quite tight for systems with large N (e.g. large parking lots), while for stations with small number of chargers, the devised

method could lead to over-utilization of station resources. Recall that the real world application of this study is to reduce the cost of investments for supporting network elements (e.g. transformers). Therefore, the infrastructure saving rate would be the ratio of effective demand to the peak charger rate, that is $\frac{d}{C}$.

IV. RESULTS

A. Small-scale Fast Charging Station

In the first case study, we consider two small scale fast charging stations equipped with $N = 5$ and $N = 6$ chargers. We use the generic Markov chain model discussed in Section II-B and explore how system parameters affect the station performance metric, $\mathbb{P}(\text{LoLP})$ or the percentage of unsatisfied demand. The numerical evaluation is carried out with the following parameter setting. The unit time slot is assumed to be 1 hour. The charger rate is set as $\mu = 2$, meaning that on average it takes 30 minutes to charge an EV. Three different arrival rate parameters are chosen ($\lambda = 3, 4$, and 5) EVs/hour. The rate of leaving the overstay state (γ) is varied from $\gamma = 1$ to $\gamma = 12$ to represent average overstay lengths between 5 minutes to 60 minutes. Similarly, the percentage of EVs who prefer to overstay are denoted by parameter, q which is varied between 10% and 90%. Note that since $r + q = 1$, the remaining population inherently represents the portion of EVs who immediately leave the station after getting charged.

In Fig. 5, the results are presented by solving (2). It can be observed that the duration of overstay when most EVs prefer to park extra time in the charging station significantly impacts the station performance. For instance, for an average overstay length of 30 minutes, that is $\gamma = 2$, the percentage of unsatisfied demand for $q = 0.9$ is fourfold higher than for the case with $q = 0.1$ (or 10% of Overstay EVs). Moreover, as the overstay length gets shorter, the differences in performance metric diminishes. This is mainly because the station stays in the bottom row of Fig. 3 and limited vertical transitions occur.

In Fig. 6, the same case study is carried out for a station with $N = 6$ chargers. It can be seen that, increasing the charger number significantly contributes to improving the station performance. For instance, for traffic intensity of $\lambda = 4$, the LoLP performance goes below 5% for all overstay rates when the average overstay length is 20 minutes. To that end, these results could be used to support capacity planning of such stations. Another key observation can be made by comparing the amount of LoLP reduction caused by overstay % of EVs determined by parameter q and the number of chargers N . For instance, assume that the station operates under medium traffic load (figures (b)) and average overstay duration is 30 minutes. If the ratio of overstay EVs could be reduced from 90% to 30% via pricing policies for the charging station with 5 chargers, then, similar station performance (e.g. 4%) would be achieved with a station with 6 chargers. Therefore, admission control policies could be instrumental in reducing infrastructure cost, and we leave this as a future study. Overall, the presented results could be used to determine the number of chargers,

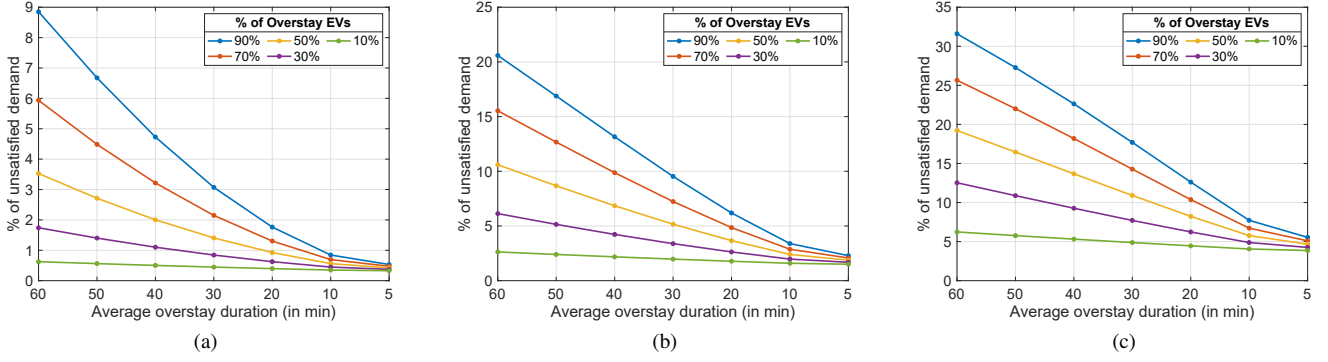


Fig. 5: Performance evaluation of a small scale charging station with $N=5$ chargers with different traffic intensity (EVs/hour) (a) $\lambda = 2$; (b) $\lambda = 3$; (c) $\lambda = 4$. Y-axis represents the station performance (% of unsatisfied demand or $P(\text{LoLP})$).

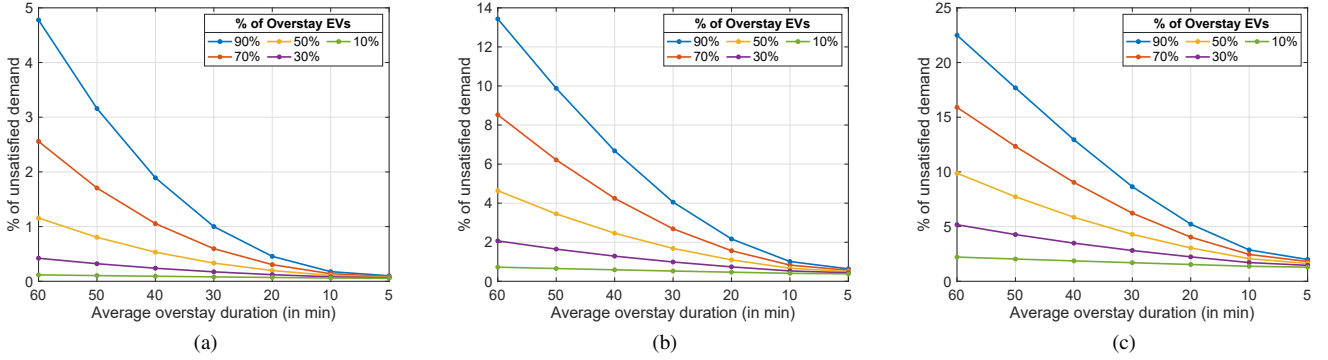


Fig. 6: Performance evaluation of a small scale charging station with $N=6$ chargers with different traffic intensity (EVs/hour) (a) $\lambda = 2$; (b) $\lambda = 3$; (c) $\lambda = 4$. Y-axis represents the station performance (% of unsatisfied demand or $P(\text{LoLP})$).

hence, the capacity of supporting network equipments for a given LoLP target, peak traffic demand, and overstay statistics.

B. Capacity Planning at Large-scale Charging Stations

Next, we present a case study to compute the effective demand and capacity for large-scale charging stations. Let us assume that there are $N = 100$ identical level-2 chargers ($C_i = 7.2$ kW for $i = \{1, \dots, 100\}$) deployed at a parking lot. It is assumed that, on average, 20 cars arrive to the station in one hour. Using the superposition property of (or thinning) Poisson process, arrival rate per charger becomes $\lambda = 0.2$. Similarly, on average EVs stay 4 hours, that is $\mu = 0.25$ and average overstay length is 1 hour, that is $\gamma = 1$. The number of EVs who prefer to overstay is 50%, hence q is set to 0.5. Using equations (6) and (5), the probability of being “On” and “Off” are found as $P(\text{On}) = 0.4211$ and $P(\text{Off}) = 0.5789$. We aim to compute effective demand for $P(\text{LoLP})=0.005$ (or 0.5% unsatisfied demand), therefore, from (12), δ is found as 5.29. Next, using the concavity of the rate function, the optimal θ that maximizes the rate function is calculated as

$$\frac{\partial f(\theta, r)}{\partial \theta} = 0 \rightarrow \theta^* = \frac{1}{C} \text{Log} \left(\frac{d \times P(\text{Off})}{P(\text{On})(C-d)} \right). \quad (14)$$

Then, the rate function becomes

$$A(d) = \frac{d}{C} \text{Log} \left(\frac{d \times P(\text{Off})}{P(\text{On})(C-d)} \right) - \text{Log} \left(P(\text{Off}) + \frac{d \times P(\text{Off})}{C-d} \right) \quad (15)$$

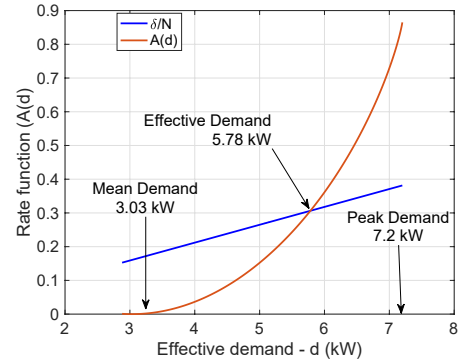


Fig. 7: Effective demand calculation.

Now, using the equation $\delta = N \times A(d)$, we can compute the effective demand d as 5.78 kW. Therefore, for the given parameter setting, instead of allocating $100 \times 7.2 = 720$ kW infrastructure, $5.78 \times 100 = 578$ kW capacity will suffice and nearly 20% of the station capacity would be saved. The computation is illustrated in Fig. 7.

Next, we consider a charging lot with $N = 200$ chargers and investigate the impacts of the percentage of overstay EVs (determined by parameter r) and overstay length (determined by parameter γ). Using the arguments above, the station parameters are set as $\mu = 0.25$, $\lambda = 0.6$, and average overstay duration is one hour ($\gamma = 1$). Then, the parameter

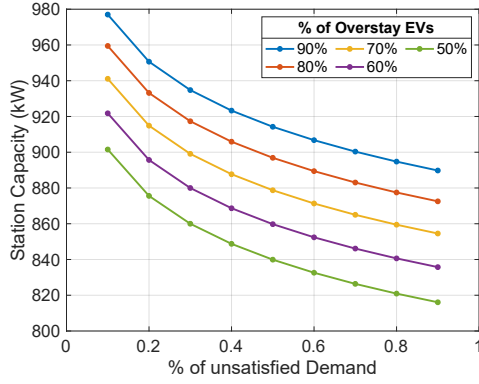


Fig. 8: Effective demand calculation for fixed overstay length and varying r .

δ is varied to provide % of unsatisfied demand from 0.1% to 0.9%. For five different $q = 1 - r$ values (from 0.9 to 0.5), the station capacity is computed and presented in Fig. 8. From the results, two important observations can be made. First, as more EVs stay parked after charging service, more capacity is needed to provide quality of service because the utilization of chargers serving non-overstay EVs becomes higher. Second, as the LoLP target becomes tighter (e.g. 0.01%), more station resources are needed.

As a second evaluation, parameter q is fixed to 0.5 and average overstay length is varied from 1 hour to 3 hours. As shown in Fig. 9, station capacity increases as the average charging duration increases due to the same reason discussed above. It is noteworthy that if the peak demand is allocated for each charger, then, $200 \times 7.2 = 1440$ kW would be required. With the proposed effective demand approach, the station capacity is reduced by more than 30% for the cases discussed above by sacrificing to decline a small percentage of customers.

V. CONCLUSION

In this paper, we presented a Markov chain model for EV charging stations with overstay customers. We discussed how to solve the steady state probability distributions for small scale charging stations. For large scale charging stations, we expanded the initial model and presented a stochastic capacity planning model that computes station capacity with respect to probability of loss of load (main quality of service metric). The model assigns a deterministic capacity to each charger that depends on overstay lengths and rates. The case studies showed that as the overstay periods increase, the station operators need to increase the station capacity to provide charging service within quality of service targets. As a future work, we will develop pricing mechanisms to induce human behavior and reduce overstay lengths and examine the trade-off between charging tariffs and charging operational profit.

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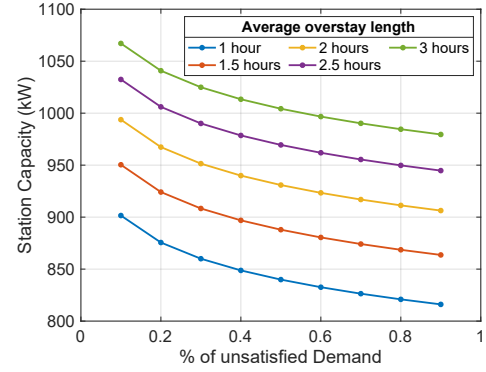


Fig. 9: Effective demand calculation for fixed r and varying overstay length γ .

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