Non-linear control of a Narrow Tilting Vehicle

Fabien Claveau, Philippe Chevrel, Lama Mourad

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F. Claveau, Ph. Chevrel, and L. Mourad
IRCCyN, IMT - Ecole des Mines de Nantes
La Chantrerie, 4 Rue Alfred Kastler, BP 20722
44307 NANTES, FRANCE
fabien.claveau@mines-nantes.fr,
philippe.chevrel@mines-nantes.fr

***Abstract*— Narrow Tilting Vehicles (NTVs) are the convergence of a car and a motorcycle. They are expected to be the new generation of city cars considering their practical dimensions and lower energy consumption. But considering their height to breadth ratio, in order to maintain lateral stability, NTVs should tilt when cornering. Unlike the motorcycle's case, where the driver tilts the vehicle himself, the tilting of an NTV should be automatic. Two tilting systems are available; Direct and Steering Tilt Control, the combined action of these two systems being certainly the key to improve considerably NTVs dynamic performances. Focusing on the lateral dynamic of NTVs, multivariable control strategies based on linear robust control theory, were already proposed in the literature, assuming decoupling with the longitudinal dynamic. In this paper a 4 DoF model of the main longitudinal and lateral dynamics is considered, and its differential flatness is demonstrated. The three flat outputs have furthermore a particular physical meaning, making possible the design of a simple external control loop complying with the driver demands.**

Keywords—Narrow Tilting Vehicle* (NTV)*, Vehicle Dynamics, Flat Systems, Non-Linear State Feedback

I. Introduction

A new generation of cars is currently being studied which will be more practical and efficient in relation to traffic congestion and parking problems in urban areas. These cars are small narrow commuter vehicles, hence saving energy, and are approximately half as wide as a conventional car (less than 1 m). Considering their geometry (approximately 2.5 m long, 1 m wide and 1.5 m high), these cars are characterized by a high centre of gravity, which makes roll stability an issue. To reduce this risk, they may have to lean into corners like two-wheeled vehicles. Some three- and four-wheels NTV projects have already been proposed by several companies. The Ford Gyron is one of the earliest prototypes while General Motors developed the Lean Machine, with a manual lean system controlled by the driver. More recently, Brink Dynamics developed the Carver, a three-wheeled car with a rotating body but a non-tilting rear engine, while the manufacturer Lumeneo proposed the Smera. Two mechanical systems are available to tilt the vehicle [1]-[4]: Direct Tilt Control (DTC) and Steering Tilt Control (STC), see Fig. 1:

- the DTC system is based on a dedicated actuator mounted on the longitudinal axis of the NTV, providing a torque (M_t) to tilt the vehicle.
- the STC actuator requires a Steer-by-Wire system: the steering angle (δ_{driv}) applied by the driver is modulated by the STC system (δ_c) to control the tilt angle using counter-steering. The tilting strategy is therefore directly inspired by the action of a bicycle or motorcycle rider.

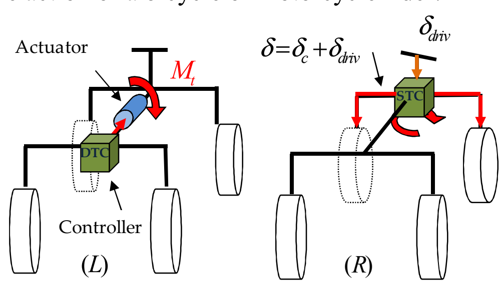

Fig. 1. Tilting actuators: DTC (left) and STC (right) systems

STC systems are not well suited for low longitudinal speeds (*e.g.* less than 8 $m.s^{-1}$ [4]), demanding a large counter-steering to tilt the vehicle, which deviates it significantly from its trajectory. In contrast, the STC system may be more efficient than the DTC one at high speed, as a large torque is required by the DTC when entering a bend if the tilting torque occurs a little late. To benefit from the complementary advantages of both systems, several projects have involved the STC and DTC systems working together [4]-[12].

Considering only the DTC actuator, the STC one, or both, several control strategies can be found in the literature; most of them are based on SISO control strategies such as PD / PID controllers as in [3]-[6],[10],[13], tuned potentially thanks to a LQ criterion [3], but assuming a natural decoupling between the roll dynamic of the NTV and the other ones (yaw, longitudinal velocity…). Recent results proposed by Mourad *et al.* [8], [9] provide multivariable controllers to control the lateral dynamics of SDTC (STC+DTC) vehicles, design thanks to the H_2 control theory. A gain-scheduling strategy is also implemented to make the control law robust to longitudinal velocity variations. In fact, few papers take into consideration the coupling between the longitudinal and the lateral dynamics of these vehicles, and more generally the intrinsic non-linear behavior of NTVs. Considering only the roll dynamic of a DTC vehicle, Piyabongkaran *et al.* in [3] provide a first non-linear controller (feedback linearization), and most recent results provided by Roquiero *et al.* (see *e.g.* [12]) lead to an interesting non-linear control strategy based on sliding mode, dealing both with the roll and the longitudinal dynamic of a STC narrow vehicle.

Motivated by the fact that non-linear control strategies can potentially reach better solution than the linear ones, it is demonstrated in this paper that a 4 Degrees of Freedom, DoF, non-linear model of a SDTC narrow vehicle has flatness properties, and more precisely it can be linearized thanks to a static state feedback [14]-[16]. Such non-linear systems are quite interesting, as once linearized, one can implement linear control strategy on the new input / output mapping. Furthermore, flat outputs can be found, with interesting physical meanings (Huygens oscillation center [16],[17]), making easier the design of the external tracking loop. This work can be seen as a generalization of results proposed by Fuchshumer *et al.* [18] demonstrating that the non-linear longitudinal – lateral so-called bicycle model is flat.

The paper is organized as follows: Section 2 presents the SDTC NTV 4 DoF model and the associated assumptions. Section 3 makes some reminders on the definition and the properties of flat systems, used in Section 4 to demonstrate the flatness property of the NTV model. The linearizing state feedback is defined in Section 5, as well as the external loop based on a simple trajectory generator and PI/PD controllers. Results obtained in simulation are shown in Section 6. The conclusion and perspectives are presented in Section 7.

II. 4 DoF non-linear NTV model

A. 4 DoF Model of the Longitudinal-Lateral Dynamics

Several NTV models were proposed in the literature; see *e.g.* [4],[10]-[13],[19] or [8],[9] for a short survey of the different models. The University of Minnesota has proposed several non-linear and linear models [1],[3],[5],[6], in particular a 3 DoF non-linear model to study the lateral dynamics of NTVs.

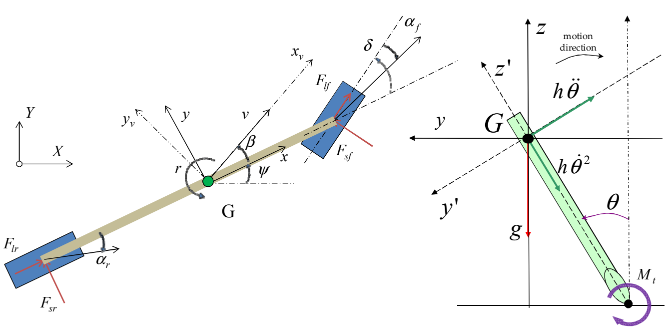

Fig. 2. Four DoF of the tilting vehicle: top view (left) and rear view (right)

In this paper a 4 DoF model is proposed, based on the non-linear 3 DoF model of the lateral dynamics of a NTV [1], considering the longitudinal speed of the vehicle as a parameter of the model, and the so-called 3 DoF "bicycle" model modeling the lateral and longitudinal dynamics of the vehicle. As depicted in Fig. 2, the 4 DoF are the longitudinal and lateral position (x,y) of the vehicle, the tilt angle θ, and the yaw angle ψ. In the absolute reference (XYZ), the reference (xyz) is attached to the centre of gravity G of the vehicle, with (xy) the horizontal plane, (x) being parallel with the longitudinal axis of the vehicle. The reference $(x'y'z')$ is also attached to the centre of gravity, but leans with the chassis, *i.e.* (x) and (x') are the same. The reference $(x_v y_v z_v)$ is also linked to G, but the axis (x_v) is collinear to the longitudinal speed of the vehicle v, and (z_v) is collinear to (Z).

The 4 DoF model was build under the following assumptions: 1- the vehicle is considered a mass point at its centre of gravity; 2- vertical reaction forces on the right and left wheels are considered identical; 3- gyroscopic effects due to the rotation of the wheels and road bank angle are neglected; 4- many mechanical parts that would have an impact on the vehicle's dynamics are not represented (*e.g.* dampers). Nevertheless, this simplified model can still be used for control, as long as the control law has some robustness. All this leads to the non-linear model $\dot{x} = f(x,u)$, with the state vector $x = \begin{bmatrix} v & \beta & \psi & \theta & \dot{\theta} \end{bmatrix}^T$, and the control input signals $u = \begin{bmatrix} \delta & F_l & M_t \end{bmatrix}^T$

$$\begin{cases} \dot{v} = \dfrac{1}{m}(F_{sf}\sin(\beta-\delta) + F_{lf}\cos(\beta-\delta) + F_{sr}\sin(\beta) + F_{lr}\cos(\beta)) \\ \qquad\qquad - (h\ddot{\theta}\cos\theta - h\dot{\theta}^2\sin\theta)\sin\beta \\ \dot{\beta} = -r + \dfrac{1}{mv}(F_{sf}\cos(\beta-\delta) - F_{lf}\sin(\beta-\delta) + F_{sr}\cos(\beta) - F_{lr}\sin(\beta)) \\ \qquad\qquad - \dfrac{1}{v}(h\ddot{\theta}\cos\theta - h\dot{\theta}^2\sin\theta)\cos\beta \\ \ddot{\psi} = \dot{r} = \dfrac{l_f}{J}(F_{sf}\cos\delta + F_{lf}\sin\delta) - \dfrac{l_r}{J}F_{sr} \\ \dot{\theta} = \dot{\theta} \\ \ddot{\theta} = \dfrac{(-mh\dot{\theta}^2\cos\theta\sin\theta - (F_{sf}\cos\delta + F_{lf}\sin\delta + F_{sr})h\cos\theta + hF_s\sin^2\theta + M_t)}{(I_x + mh^2\sin^2\theta)} \end{cases} \tag{1}$$

All the signals and parameters are defined in Table 1. $F_{lr} = \gamma F_l$, $F_{lf} = (1-\gamma)F_l$, $\gamma \in [0,1]$ meaning that the motor or braking force can be supplied to the front and the rear wheel with a given transmission ratio (all-wheel driven vehicle). γ is assumed to be a constant value in this paper, to fit with the reality of NTVs (most being characterized by the ratio $\gamma = 1$). The other control inputs are the steering angle of the front wheel δ and the tilting torque M_t, *i.e.* the NTV is equipped of both a DTC and STC system.

B. Rear and Front Lateral Tire Forces

Several models of the lateral tire forces F_{sf} and F_{sr} can be found in the literature, see *e.g.* [20]. One common assumption is that these lateral tire forces can be expressed as functions of the side-slip angles α_f and α_r of the wheels:

$$\alpha_f = \delta - \tan^{-1}\left(\frac{v\sin\beta + l_f\dot{\psi}}{v\cos\beta}\right), \quad \alpha_r = -\tan^{-1}\left(\frac{v\sin\beta - l_r\dot{\psi}}{v\cos\beta}\right) \tag{2}$$

and also of the tilt angle θ of the vehicle through the camber stiffness of the tires. Considering the small angle approximation tan $\phi \approx \phi$, the lateral tire forces models are,

$$\begin{cases} F_{sf}\left(\delta,v,\beta,r\right)=2C_f\left(\delta-\dfrac{v\sin\beta+l_f\dot{\psi}}{v\cos\beta}\right)+2\lambda_f\theta, \\ F_{sr}\left(v,\beta,r\right)=2C_r\left(\dfrac{v\sin\beta-l_r\dot{\psi}}{v\cos\beta}\right)+2\lambda_r\theta. \end{cases} \tag{3}$$

Notice however that, as suggested in [18], in the flatness analysis of the non-linear model (1)-(3), the detailed expression of the lateral tire forces $F_{sf}(\delta,v,\beta,r)$ and $F_{sr}(v,\beta,r)$ are not necessary. Assuming them as smooth functions is a sufficient technical condition to make easier the mathematical manipulations.

Table 1. Parameters of The 3 DoF Model: See [1],[8] For Numerical Values

v	longitudinal speed of the vehicle	g	gravitational constant
β	side-slip deviation angle	m	total mass
$\psi, r=\dot{\psi}$	yaw angle and speed	h	position of the center of gravity G on the z' axis
θ, $\dot{\theta}$	tilt angle and speed	I_z	vehicle yaw moment of inertia
M_t	tilting torque provided by the DTC actuator	I_x	vehicle roll moment of inertia
δ	steering angle of the front wheels	l_f	distance from center of gravity to front axle
F_l	global longitudinal force	l_r	distance from center of gravity to rear axle
α_f, α_r	front and rear tire side-slip angle	C_f, C_r	front and rear cornering stiffness
F_{lf}, F_{lr}	front and rear longitudinal force	λ_f, λ_r	front and rear camber stiffness
F_{sf}, F_{sr}	front and rear lateral force		

C. Control Objectives in Terms of Lateral Stability of NTVs

As said in the introduction, the objective is to ensure the lateral stability of the NTV faced with lateral acceleration when cornering, by tilting its chassis thanks to the DTC and STC systems. In particular, the lateral acceleration at the center of gravity G is of importance.

Definition 1: Perceived acceleration a_{per}
a_{per} denotes the resultant acceleration at the center of gravity G, along the axis (y') (*cf.* Fig. 2), *i.e. perpendicular to the chassis of the vehicle. It is linked to other variables by*:

$$a_{per}=a_{lat}\cos\theta+h\ddot{\theta}-g\sin\theta=\left(\ddot{y}+V\dot{\psi}\right)\cos\theta+h\ddot{\theta}-g\sin\theta \tag{4}$$

The terminology "perceived" (or measured) acceleration was introduced in [1]. This would be the acceleration measured by an accelerometer positioned at the center of gravity whose lateral axis is in the lateral vehicle direction, and also the lateral acceleration perceived by the driver in the cabin of the vehicle, impacting the comfort. Fundamentally, the lateral stability of the NTV is ensured if $a_{per} = 0$. To reach this objective, the literature classically reformulates the lateral control problem as an angular position tracking problem, regulating the tilting angle θ around the reference angle θ_{ref}, estimated on line by inverting equation (4) (with more or fewer approximations) [3]-[7],[11]-[13]. There are pros and cons for such control strategy as discussed in [9], compared to the direct regulation of a_{per} as proposed by the authors *e.g.* in [8],[9]. As it will be detailed in Section 5, the tilt angle control strategy will be applied in this paper, to be consistent with the physical meaning of the chosen flat outputs. Concretely, our fundamental control objective is defined by the following equation

$$\theta-\theta_{ref}=\theta-\tan^{-1}\left(\frac{rv\cos\beta}{g}\right)=0\,. \tag{5}$$

The other control objectives will be to ensure the trajectory planning and regulation to satisfy the driver's desires, express through the throttle/brake pedal, traduced as a desired longitudinal traction force $F_{l\text{-}driv}$, and the steering wheel angle δ_{driv}. A first simple solution is also provided in Section 5.

D. Available Measurements

Practically, tilting cars generally include a tilt angle sensor and an Inertial Measurement Unit (IMU), which provide the state values v, θ, $\dot{\theta}$, $\dot{\psi}$ and a_{per}. Lastly, the steering angle δ and its derivative are assumed to be can be measured, and the side-slip angle β available through an estimator (*e.g.* [21]).

III. Flatness Property and Static State Feedback Linearization

A. Flatness property

The main objective in this paper is to demonstrate, in the same spirit as in [18], that the longitudinal / lateral 4 DoF model of a NTV equipped with a SDTC system can be linearized by a static non-linear state feedback. To prove such a result, 3 flat outputs associated to the 3 control inputs, will be first exhibited. To begin with, some reminders about the flatness property are proposed hereafter.

Definition 1: Flat system [15]

Consider a non-linear system $\dot{x}=f\left(x,u\right)$, $x\in\mathbb{R}^n$, $u\in\mathbb{R}^m$. Such system is said to be a "flat system", if and only if one can find:

- an output vector $y\in\mathbb{R}^m$,
- $r\in\mathbb{N}$ and functions
  - $\Pi: R^n\times(R^m)^{r+1}\to R^m$, with rank m,
  - $\Omega:(R^m)^r\to R^n$, with rank n,

- $\Xi : (R^m)^{r+1} \to R^m$, with rank m,

such as one can write:

$$\begin{cases} y = (y_1, y_2, ..., y_m) = \Pi(x, u, \dot{u}, ..., u^{(r)}), \\ x = \Omega(y, \dot{y}, ..., y^{(r-1)}),\ u = \Xi(y, \dot{y}, ..., y^{(r)}). \end{cases} \tag{6}$$

In fact, a system verifying the flatness property is such that all its state and input signals can be expressed as functions of some specific output signals hence named flat outputs. In other words, these outputs "sum up" all the dynamic characteristics of the system.

Definition 2: *Relative degree*

Consider a non-linear system

$$\begin{cases} \dot{x} = f(x,u) \\ y = h(x) \end{cases} \Leftrightarrow \begin{cases} \dot{x} = f(x,u) \\ y_1 = h_1(x), \dots, y_m = h_m(x) \end{cases} \tag{7}$$

with $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, and m scalar outputs y_i. The smallest integer $r_i \in \mathbb{N}$ such as $\partial y_i^{(r_i)} / \partial u \neq 0$ is the relative degree of the output y_i.

Property 1: *Flat outputs*

Consider the non-linear system (7). If one can find a change of coordinates $z = \Phi(x)$ such that:

- $\Phi(.)$ is a diffeomorphism, and the new state $z \in \mathbb{R}^n$ is composed of the m outputs y_i and their derivatives until the (r_i - 1) degree, *i.e.*

$$z = \begin{bmatrix} y_1 & \dot{y}_1 & \cdots & y_1^{(r_1-1)} & \cdots & y_m & \dot{y}_m & \cdots & y_m^{(r_m-1)} \end{bmatrix}^T, \tag{8}$$

- the decoupling matrix M is full rank, rank(M) = m,

$$M = \begin{bmatrix} \partial y_j^{(r_j)} / \partial u_i \end{bmatrix}_{\substack{1 \le i \le m, \\ 1 \le j \le m}} \tag{9}$$

then ouputs y_i are flat outputs of the system.

B. Input / State Feedback Linearization

Property 2: *Exact static state feedback*

If a non-linear system as (7) is flat, with furthermore the relative degrees of the several flat outputs y_i such as $\sum_{i=1}^{m} r_i = n$, then system (7) is exact input / state linearizable via static state feedback, *i.e.* one can find a static non-linear state feedback with the new inputs $\omega \in \mathbb{R}^m$, defined as the solution of

$$\begin{cases} y_1^{(r_1)}(x,u) = \omega_1 \\ y_2^{(r_2)}(x,u) = \omega_2 \\ \quad \vdots \\ y_m^{(r_m)}(x,u) = \omega_m \end{cases} \tag{10}$$

such that the closed-loop is equivalent to the linear time invariant system (11) based on m chains of integrators

$$\begin{cases} \dot{z} = \begin{bmatrix} A_z^1 & 0_{r_1 \times r_2} & \cdots & 0_{r_1 \times r_m} \\ 0_{r_2 \times r_1} & A_z^2 & & \\ \vdots & & \ddots & \vdots \\ 0_{r_m \times r_1} & & \cdots & A_z^m \end{bmatrix} z + \begin{bmatrix} B_z^1 & 0_{r_1 \times 1} & \cdots & 0_{r_1 \times 1} \\ 0_{r_2 \times 1} & B_z^2 & & \\ \vdots & & \ddots & \vdots \\ 0_{r_m \times 1} & & \cdots & B_z^m \end{bmatrix} \omega \\ \begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix} = \begin{bmatrix} C_z^1 & 0_{1 \times r_2} & \cdots & 0_{1 \times r_m} \\ 0_{1 \times r_1} & C_z^2 & & \\ \vdots & & \ddots & \vdots \\ 0_{1 \times r_1} & & \cdots & C_z^m \end{bmatrix} z \end{cases} \tag{11}$$

with matrices A_z^i, B_z^i, C_z^i of dimensions $r_i \times r_i$, $r_i \times 1$, $1 \times r_i$,

$$A_z^i = \begin{bmatrix} 0 & 1 & & (0) \\ & 0 & \ddots & \\ & & \ddots & 1 \\ (0) & & & 0 \end{bmatrix},\ B_z^i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix},\ C_z^i = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}.$$

IV. Flatness Property of the NTV Model

Considering the previous definitions and properties, some flat outputs are proposed in this section, demonstrating that the 4 DoF model of NTVs in (1) can be linearized, under few assumptions, thanks to a static state feedback.

A. Main Results

Proposition 1

Let's consider that the lateral tire forces $F_{sf}(\delta,v,\beta,r)$ and $F_{sr}(v,\beta,r)$ are arbitrary smooth functions. Let's define the smooth function

$$\begin{aligned} T(v,\beta,r) = \frac{l_f + l_r}{m l_f}(v \partial_r F_{sr}(v,\beta,r) + \frac{J}{m l_f} \partial_\beta F_{sr}(v,\beta,r) \cos\beta \\ + v \frac{J}{m l_f} \partial_v F_{sr}(v,\beta,r) \sin\beta) - v^2 \cos\beta \end{aligned} \tag{12}$$

with the notation $\partial_\omega F = \partial F / \partial \omega$. Then under the conditions $T(v,\beta,r) \neq 0$, $v \neq 0$, the non-linear system (1) is differentially flat. Furthermore, this system is exact input / state linearizable via a static feedback. In particular, the 3 outputs (13) are available flat outputs with relative degrees (1,2,2).

$$y\begin{cases} y_1 = h_1(x) = v\cos\beta \\ y_2 = h_2(x) = v\sin\beta - \dfrac{J}{ml_f} + h\dot{\theta}\cos\theta \\ y_3 = h_3(x) = \theta \end{cases} \tag{13}$$

Proof: see [8]-*chap.6 for the detailed proof.* At first, the relative degree of the 3 outputs y_1, y_2, y_3 can be easily verified by computing derivatives $(\dot{y}_1, \ddot{y}_2, \ddot{y}_3)$. Let's then consider the coordinates transformation

$$z = \Phi(x) \Leftrightarrow z = \begin{bmatrix} z^1 \\ z^2 \\ z^3 \\ z^4 \\ z^5 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \dot{y}_2 \\ y_3 \\ \dot{y}_3 \end{bmatrix} = \begin{bmatrix} v\cos\beta \\ v\sin\beta - \dfrac{J}{ml_f}r + h\dot{\theta}\cos\theta \\ \dfrac{l_f + l_r}{ml_f}F_{sr} - vr\cos\beta \\ \theta \\ \dot{\theta} \end{bmatrix}. \tag{14}$$

$\Phi(x)$ is a diffeomorphism as long as $det(J) \neq 0$, $J = \left[\partial\Phi(x_i)/\partial x_j\right]_{1\le i\le n,\ 1\le j\le n}$. This is equivalent to the condition $T(v,\beta,r) \neq 0$. Moreover, the decoupling matrix

$$M = \begin{pmatrix} \partial_\delta \dot{y}_1 & \partial_{F_l}\dot{y}_1 & \partial_{M_t}\dot{y}_1 \\ \partial_\delta \ddot{y}_2 & \partial_{F_l}\ddot{y}_2 & \partial_{M_t}\ddot{y}_2 \\ \partial_\delta \ddot{y}_3 & \partial_{F_l}\ddot{y}_3 & \partial_{M_t}\ddot{y}_3 \end{pmatrix} \tag{15}$$

remains of full rank for $T(v,\beta,r) \neq 0$ and $v \neq 0$. This proves the flatness of y (13) (see Property 1). □

B. Physical Meaning of the Proposed Flat Outputs

The chosen flat outputs in (13) have some physical meaning of interest to build the external tracking loop. Defining the point G', projection on the vehicle's longitudinal axis of the center of gravity G along the axis (z'), we define in Fig. 3 the new reference ($x_s y_s z_s$) linked to G': it is the translation of reference (xyz) in Fig. 2 from G to G'. Let's define the specific points $\Sigma\left(-J/lm_f, 0, 0\right)$ and $\Sigma'\left(-J/ml_f, h\sin\theta, h\cos\theta\right)$ in this reference ($x_s y_s z_s$). Considering this new reference and the two specific points Σ and Σ', one can observe that: y_1 is the longitudinal speed of any point located on the vehicle's longitudinal axis (x_s), y_2 is the lateral speed of Σ' in ($x_s y_s z_s$), y_3 is the tilting angle of the vehicle.

Remark: Huyghens center of oscillation

As in [18], the specific point Σ' can be linked to the Huyghens center of oscillation, define in a general manner for Lagrangian systems underactuated by one control, such as the PVTOL position control in [17].

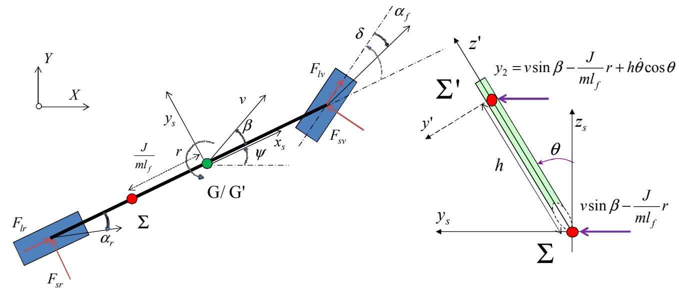

Fig. 3. Top view (left) and Front view (right) of the vehicle: definition of the point Σ' $\left(-J/ml_f, h\sin\theta, h\cos\theta\right)$ in reference ($x_s y_s z_s$)

V. Control Design

A. Design of the static state linearizing feedback

According to Property 2, the 4 DoF NTV model controlled by the *ad hoc* non-linear feedback depicted in Fig. 4 is equivalent to the linear one defined by equations (10),(11), under restriction (12). The analytic expression not presented here by lack of place, is available however in [8]-prop. 6.3.

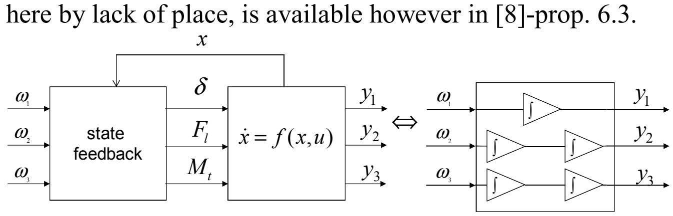

Fig. 4. Equivalent closed-loop system

B. Design of the tracking controller and trajectory generation

Finally, we have to regulate y (see equation (13)) around some trajectories that 1/ ensure the lateral stability of the vehicle, 2/ interpret and satisfy the driver's desires. His demand is made through the steering wheel angle and the throttle / break pedal. This is still partly an open research topic. δ_{driv} the steering wheel angle reference, and $F_{l\text{-}driv}$ the required longitudinal force are considered here as the new input left at the disposal of the driver. Fig. 5 depicts the solution proposed at this first stage, involving the three reference signals:

$$\begin{cases} y_1^d = v_{x-driv} = v\cos\delta_{driv} = \left(\displaystyle\int_0^t \frac{F_{l-driv}}{m} + v_0\right)\cos\delta_{driv} \\ y_2^d = v_{y-driv} = v\sin\delta_{driv} = \left(\displaystyle\int_0^t \frac{F_{l-driv}}{m} + v_0\right)\sin\delta_{driv} \\ y_3^d = \theta_{ref} = \tan^{-1}\left(\dfrac{rv\cos\beta}{g}\right) \end{cases} \tag{16}$$

The two first reference signals make possible the control of the lateral and longitudinal speeds. The third is the tilting reference θ_{ref}, classically computed as proposed in the literature about NTVs (see subsection II.C).

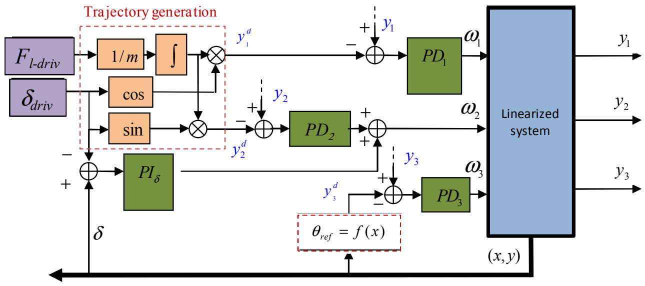

Fig. 5. Trajectory generation tracking applied to each decoupled $w_i \rightarrow y_i$.

Three PD controllers are implemented to drive the error signals $e_i = y_i - y_i^d$, $i = 1,2,3$. A PI is also used to generate the control signal ω_2, to control the error signal $e_\delta = \delta - \delta_{driv}$. Finally, it leads to,

$$\begin{cases} \omega_i = \left(K_i^P + K_i^D s\right)e_i, & i = 1,3 \\ \omega_2 = \left(K_2^P + K_2^D s\right)e_i + \left(K_\delta^P + \dfrac{K_\delta^I}{s}\right)e_\delta \end{cases} \qquad (17)$$

VI. RESULTS

The performances of the proposed control strategy are now evaluated by simulating the non-linear model (1). The scenario is defined as follows: the driver requires a constant acceleration ($F_{l\text{-}driv}$ assumed to be constant), and the steering wheel angle δ_{driv} (see Fig. 6) entails a first bend followed by a circular trajectory (medium sized roundabout). This trajectory is quite difficult compared to the ones proposed *e.g.* in [3] or [7]. The PDs and PI controllers considered were tuned to:

- PD1: $K_1^P = 2$, $K_i^D = 0.2$, PD3: $K_3^P = 0.6$, $K_3^D = 1.3$,
- PD2: $K_2^P = 3.5$, $K_2^D = 5$, PIδ: $K_\delta^P = 40$, $K_\delta^I = 0.2$.

The simulation results are compared with the ones obtained with the LPV controller proposed in [9], designed by solving a H_2 control problem based on the linearized model of the lateral dynamic of the NTV, considering as controlled output the lateral acceleration a_{per}, and made robust to the longitudinal velocity variation thanks to a gain-scheduling solution. Fig. 6 firstly shows that the gap between the desired steering angle δ_{driv} and the one applied by the non-linear controller δ through the STC system is not intrusive. Considering the traction force, F_l is identical to the desired one $F_{l\text{-}driv}$ in a straight trajectory, but is reduced in bend, so as to ensure a better trajectory tracking. Even if it can be seen as a driving assistance, such behavior can be disturbing for the driver and should be studied in more depth in the future. However, results in Fig. 7 and Fig. 8 show that the proposed non-linear control strategy associated to the rudimentary trajectory generation system (16) ensures the lateral stability of the vehicle: in particular, the perceived acceleration a_{per} reaches acceptable values (max. 0.6 m/s^2), compared to most of the results in the literature. The LPV controller still reaches better performances, with a tilting torque M_t divided by a factor 2 and a lateral acceleration a_{per} 3 times smaller. However we are convinced that a more sophisticated external loop should improve the performances.

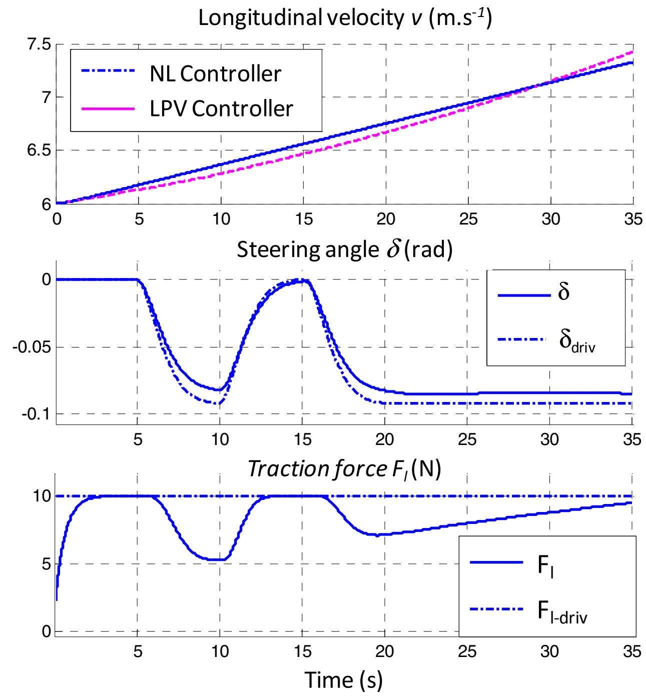

Fig. 6. Longitudinal velocity of the NTV (Non-Linear and LPV solution), steering, desired steering angle δ_{driv} and traction force $F_{l\text{-}driv}$, and the ones calculated by the complete non-linear controller, δ, F_l.

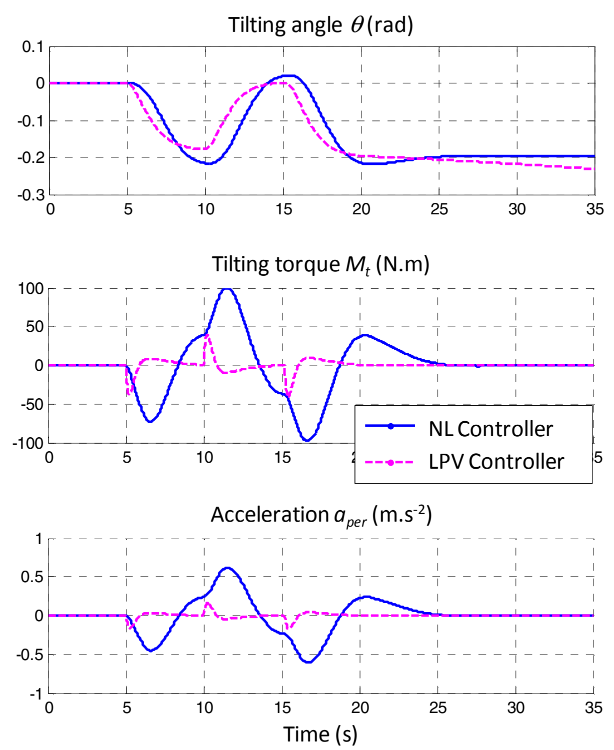

Fig. 7. Tilting angle, tilting torque of the DTC system, and lateral perceived acceleration obtained by the non-linear (NL) and the LPV controller.

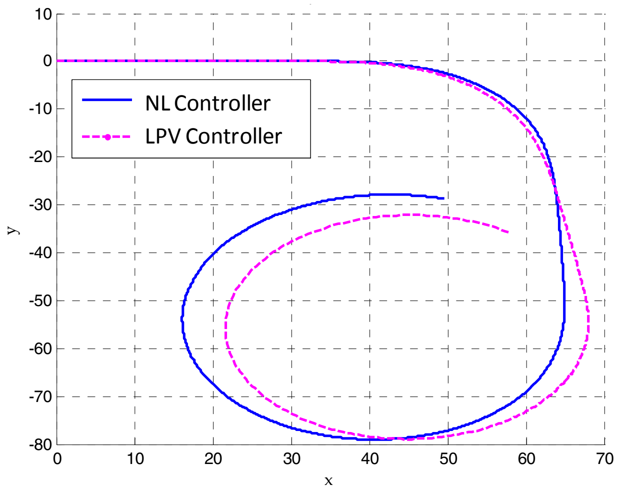

Fig. 8. NTV Trajectory considering the non-linear (NL) controller and the LPV one.

VII. CONCLUSION AND PERSPECTIVES

Based on a 4 DoF model of a Narrow Tilting Vehicle equipped both of a DTC and STC system, inspired from the well-known bicycle model and the 3 DoF model of the lateral dynamic of NTVs [1], we showed that NTVs verify the differential flatness property. Furthermore, they present some interesting characteristics. First, based on few assumptions on the tire forces model, the longitudinal-lateral dynamics of NTV can be linearized thanks to a static state feedback. Secondly, three flat outputs with a concrete physical meaning associated to the Huyghens oscillation center, can be defined. To validate the proposed linearizing feedback, a first control strategy based on a rudimentary trajectory generation system and PD/PI control loops was also proposed. It has been demonstrated that such simple external loops already reach quite satisfactory performance.

These first results call for some motivating perspectives: a classic state feedback was design here, leading to three decoupled integrator chains. Such strategy has the weakness to "erase" the original dynamics of the system. We believe that methods inspired by [22] will bring more robustness; a linearizing feedback leading to a closed-loop system sharing structural properties with the tangent linear models of the original system. On this basis, we then plan to make use of the optimal control strategy proposed in [9] for the external loop synthesis, thus bypassing the gain-scheduling design step while getting an improved performance.

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