

Robotic Resource Allocation for the Observation of Ablating Target Sources

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Abstract—Icebergs generated from ice ablation processes continue to be a threat for operations conducted in polar regions. Systems that have been developed to track and observe these threats often use either space-based radar imaging or visual observation by the crew of the ship. Both of these methods have disadvantages, mostly in terms of real-time observation or the physical abilities of the crew. We propose a robotic solution for in-situ observation of icebergs, so that countermeasures may be quickly implemented. Our focus in this work is the problem of allocating resources to observation regions: once areas of iceberg activity have been identified, how are robot observers assigned to these regions and what cost metric may be used to determine the best placement of robot observers. Our solution is currently demonstrated and evaluated in simulation.

Keywords—Multiagent robotics, resource allocation, probabilistic methods, cryosphere

I. INTRODUCTION

Floating ice collisions are a constant threat for ship-based operations in Arctic regions [1]. A means of tracking floating ice is usually necessary for situational awareness at sea. This need is a result of the fact that most of the ships that operate in these regions are slow-moving (*e.g.*, large shipping vessels) or immobile (*e.g.*, oil platforms). Data products are produced to assist in avoiding floating ice; the International Ice Patrol (IIP) is an organization dedicated to tracking and providing such products for operations that are conducted in the northern Atlantic Ocean/Newfoundland region [2]. Radar and visual observations are both used to construct these data products.

However, for both radar and visual observation, disadvantages exist. Satellite-based synthetic aperture radar (SAR) is generally not available in real time. Additionally, smaller icebergs, with their lower magnitude radar cross section (RCS) with respect to sea clutter can be difficult to track. Visual observation requires a dedicated crew, but not all threats are completely observable in this manner, and evasive maneuvers or other countermeasures may not be able to be instigated in time.

These issues suggest that placing a lesser emphasis on radar in sensing these targets, with an additional focus on understanding the threat sources would be a better approach to the problem. In our previous work, we developed a probabilistic model for the sources of icebergs on a glacier generated from sensor measurements of the icebergs [3]. These sources are referred to as *ablating target sources*, as they model the *ice*

ablation process that results in icebergs being generated from a glacier. These target sources shrink in mass as they eject targets until they can no longer generate targets.

The foundation of the problem definition behind the probabilistic model of ablating target sources was based on an existing robotic observation problem [4], [5]. Using this model, metrics can be generated regarding the behavior of the icebergs. In this paper, we focus on using these metrics and the properties of the robotic agents to reassign agents to different search regions to more efficiently observe new icebergs as they are calved from the glacier.

This paper is organized as follows. Section II summarizes the iceberg observation problem definition and the probabilistic iceberg ablating source model. Section III provides the additional definitions that constrain the iceberg observation problem and modeling methodology. Section IV describes some of the existing work in robotic resource allocation and outlines our approach to the problem. Simulation results are provided in Section V. Finally, we conclude the paper with remarks in Section VI.

II. TARGET SOURCE MODELING

In this section, we outline our definition for the iceberg observation problem and the probabilistic methods by which we model the target sources. As icebergs can be modeled as moving targets that move into and out of a particular region of interest, remotely observing these targets is similar to that of the class of multiagent observation problems referred to as Cooperative Multi-robot Observation of Multiple Moving Targets (CMOMMT) [4]. The main difference between our approach and the CMOMMT approach is that the agents in CMOMMT remain stationary, as they statically observe an area with a constant number of targets to maximize the amount of amount of time that any target is observed. We modify this problem class, using it as a robotic observation problem framework, by adapting the assumptions that define the problem. It should be noted, though, that the original problem is difficult enough that many solutions have been explored by researchers; *e.g.*, [6]–[9].

In our definition of the problem; however, instead of the objective function relying on observation of all moving targets at all times, we desire to minimize the initial target acquisition time. This allows for efficient deployment of countermeasures

as suggested in Section I. That is, we define the objective function to be the following:

$$\min_{\mathcal{R}} E[T_s|O(t)], \quad (1)$$

where \mathcal{R} is the set of all robots participating in the mission, T_s is the target acquisition time, $O(t)$ is the set of targets present at a given time t , and $E[\cdot]$ denotes the expected value operator. That is, we want to minimize the expected acquisition time, over the current set of targets, for all robots participating in the mission.

For areas of higher activity on an ablating target source, more sensing resources will be required to ensure that all targets are observed in a minimum amount of time. Therefore, more resources should be allocated to regions with a high probability of a new target being generated. As the target probability for a particular region decreases, agents should be reallocated in a more equal manner around a target source. A method of quantifying and modeling the target probabilities is needed.

To determine the appropriate probability density of the targets, observations must be incorporated into a model [3]. Observations are composed of the following elements:

- The position at which the target was first observed.
- The target observation time.
- The agent that made the observation.

The model incorporates these observations as well as the current number of observed targets across all agents and the *a priori* probability density of a new target being formed, which is obtained from a previous iteration of this process. A straightforward choice of probability distribution for this problem is to use a Gaussian mixture model [10]. Each of the mixture components will correspond to a region of activity at the glacier-sea interface; the covariance matrices define the extents of the region.

An example model as generated from simulated target measurements for two agents scanning a region is shown in Figure 1. The target measurements are denoted by the stars; the target source is the gray rectangle, with activity points identified by the circles overlaid on the source. The Gaussian mixture is represented by the heatmap. It can be seen that the components of the mixture distribution overlay the ablation regions on the glacier; the components provide sufficient target containment. Reallocating the agents to use these components as search regions will more efficiently acquire the targets as they are generated.

Note that this methodology resembles, in some respects, the coverage problem. The coverage problem examines a static region with the problem of interest being determining the most efficient means of observing the entire region or some aspect of the region. Our development is an *adaptive* coverage solution, since a model of the target sources is determined and the resources (*i.e.*, agents) and regions of coverage are adapted to fit that model. Agents deployed to cover an area often use search patterns for sample acquisition [11], [12]; we adopt this same approach for target acquisition.

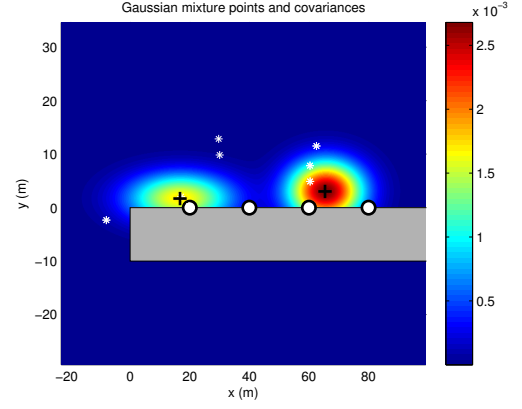


Fig. 1. Example Gaussian mixture model resulting from a two-agent scanning solution.

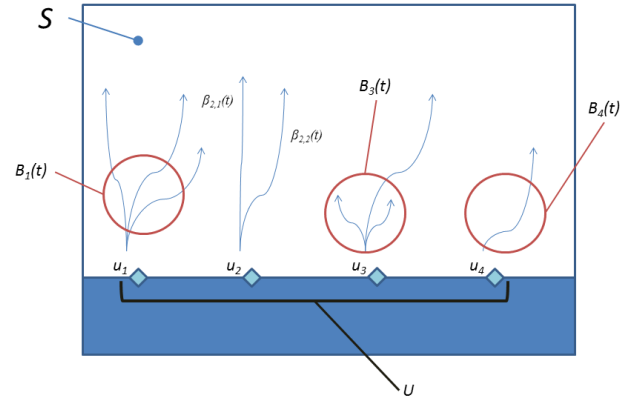


Fig. 2. Illustration of iceberg observation definitions.

III. ICEBERG OBSERVATION DEFINITIONS

Given the methodology overview in the previous section, the following mathematical definitions further define and constrain the CMOMMT problem to fit within the framework of iceberg observation and the modeling methodology used to obtain regions of activity. Figure 2 provides an illustration of the definitions overlaid onto a region of interest, where a glacier meets the ocean.

- $\mathcal{S} \subset \mathbb{R}^2$ is the rectangular and topologically connected region of interest. Physically, \mathcal{S} is the region of the ocean in contact with the glacier with the ablation points that are to be observed.
- U is the set of ablation points in \mathcal{S} ; *i.e.*, $U \subset \mathcal{S}$. The cardinality of U is finite: $|U| = l$, $l \in \mathbb{N}$. Individual ablation points are $u_i \in U$, with $i = 1 \dots l$.
- For each ablation point u_i , there is a corresponding random process p_i that results in the generation of target trajectories from the ablation point u_i .
- The random process p_i , for $i = 1 \dots l$ and associated with an ablation point u_i , is a homogeneous Poisson process with intensity λ_i : $p_i \sim \text{Poisson}(\lambda_i)$. The random process p_i for each u_i results in sets of iceberg target trajectories.

- $B_i(t) \subset \mathbb{R}^4$, for $i = 1 \dots l$, is the set of all target trajectories at a specific time t resulting from the ablation processes of a specific u_i . $B_i(t)$ is referred to as a *target stream*. $N_i(t) \in \mathbb{N}$ is the cardinality of $B_i(t)$ at time t .
- $\beta_{i,d}(t) \in B_i(t)$, with $i = 1 \dots l$ and $d = 1 \dots N_i(t)$, is an individual target trajectory state vector at time t , containing the position and velocity at that time.

With the region of interest and the objects within it defined, the mixture model used to model the spread of icebergs and their probability of generation may be defined.

- $Q = \{q_j\}_{j=1}^k$ is the set of mixture components that represents the target trajectory dispersion within the region of interest \mathcal{S} . The cardinality of Q is finite: $|Q| = k$, $k \in \mathbb{N}$.
- Each member $q_j \in Q$, with $j = 1 \dots k$, is a mixture component. Each mixture component q_j is defined as the 4-tuple

$$q_j = (\mu_j, \Sigma_j, Z_j, \bar{v}_j), \quad (2)$$

where $\mu_j \in \mathbb{R}^2$ is the component mean vector; $\Sigma_j \in \mathbb{R}^{2 \times 2}$ is the component covariance matrix; $Z_j \subset \mathbb{R}^2$ is the set of target measurements associated with the component; and $\bar{v}_j \in \mathbb{R}$ is the estimated velocity magnitude of targets within a component.

The following definitions further elaborate on the structure of the set of target measurements Z_j .

- $Z_j \subset \mathbb{R}^2$ is a set of target measurements associated with a mixture component. The cardinality of Z_j is finite: $|Z_j| = n$, $n \in \mathbb{N}$.
- Target measurements $z_{j,m} \in Z_j$, for $m = 1 \dots n$, are noisy position measurements of iceberg position. That is, for some time t , $i \in [1, l]$, and $d \in [1, N_i(t)]$:

$$z_{j,m} = h(\beta_{i,d}(t)) + w, \quad (3)$$

where $w \in \mathbb{R}^2$ is a sensor noise term, and the function $h : \mathbb{R}^4 \rightarrow \mathbb{R}^2$ returns the position vector of a specific $\beta_{i,d}(t)$.

IV. RESOURCE ALLOCATION

With the definitions of the problem and the modeling method given in the previous sections, we now outline our approach to the problem of reallocating multiple agents to search regions. The first task is to define a cost function that represents the cost to transition from one region to another. With that definition, an appropriate assignment algorithm can be used to reallocate agent resources to different search regions.

A. The Assignment Problem

The assignment problem is among the most common of resource allocation problems. The typical definition of the problem is as follows: given a set of resources and a set of tasks, what is the optimal one-to-one assignment of resources to tasks? Such a mapping can be represented by entries within a table, which then equates to defining the problem as a two-dimensional assignment.

To formalize the definition of the two-dimensional assignment problem, the following definitions are necessary:

\mathcal{T} : A set of N tasks t_j .

\mathcal{R} : A set of M resources r_i that can complete a task contained in \mathcal{T} . This is effectively the same definition as \mathcal{R} in Section II.

C : The cost required for a resource r_i to complete a task t_j . The cost function is the one-to-one mapping $C : \mathcal{T} \times \mathcal{R} \rightarrow \mathbb{R}$. Such costs include the energy required to complete a task.

In most two-dimensional assignment problems, at least one optimal assignment of resources to tasks minimizing these costs exists.

Defining the assignment as the mapping $X : \mathcal{T} \rightarrow \mathcal{R}$, the objective function of the two-dimensional assignment problem is the following:

$$\min_{\mathcal{R}, \mathcal{T}} C, \quad (4)$$

which results in the optimal mapping X .

In the case of the iceberg observation problem, assigning robots to search regions can be considered within the context of the two-dimensional assignment problem: the set of tasks \mathcal{T} is the set of search regions, and the set of resources \mathcal{R} is the set of robots in the mission.¹

Many combinatorial methods and algorithms have been developed over the years to solve the two-dimensional assignment problem; e.g., [13]–[15]. Decentralized, market-based assignment algorithms lend themselves well to robotics applications [16]–[20].

In the case of iceberg observation, since the types of observation tasks are all the same, a standard two-dimensional assignment approach would serve well. What remains is determining the cost function: such a function can be defined based on the parameters of the iceberg model and the physical parameters associated with the robotic agents.

B. Cost Function

To determine an appropriate cost function for the iceberg observation problem, an appropriate statement of the assignment problem must be developed. Hence, the assignment problem that is to be solved is as follows:

- The initial allocation of agents has each of the agents assigned to a region derived from the initial mission plan. Each agent has an average speed v_r and a sensor field-of-view that covers a fixed area A_{FOV} . Figure 3(a) illustrates an example of such a default allocation: the mission area is divided into equally-sized, rectangular cells, and agents search within those cells.
- New regions of varying area A_R and position are then extracted from the computed iceberg model Q . Specifically, extracted using the means μ_j and covariances Σ_j of the individual model components q_j , with

¹For the remainder of the section, the terms “two-dimensional assignment problem” and “assignment problem” will be considered as interchangeable.

$j = 1 \dots k$. Agents are deployed to the new regions. Figure 3(b) shows an example of these extracted regions.

One of the more common cost functions in the assignment problem uses the Euclidean distance from the position x_r of a robot to a set of given goal points; the assumption is made that the paths that have this distance are both the shortest and, as a consequence, the minimum energy paths to the goal points.

However, the cost of assigning to a search *region* requires additional factors. Such factors include target coverage efficiency. To accommodate these factors, two weighting factors are placed on the distance to the centers of the search regions, defined by the means of the Gaussian mixture components μ_j , where $j = 1 \dots k$.

This leads to a definition of the cost function C as follows:

$$C = \frac{A_R}{A_{FOV}} \frac{\bar{v}_j}{v_r} \sqrt{(x_r - \mu_j)^T (x_r - \mu_j)}. \quad (5)$$

The first weighting factor A_R/A_{FOV} depends on the area of a search region and the field of view of the robot's sensors, which is the contribution of \mathcal{R} to the cost function. This factor is the ratio of the search regions area to the area covered by the robot's sensor field of view. This factor forces the cost to increase if the search area increases without increasing the area that the agent's sensor can cover; *i.e.*, more energy is required.

The second weighting factor \bar{v}_j/v_r relies on the average velocities of targets that have been acquired, using the model components q_j , and the average robot velocity. Both this factor and d are the contributions of \mathcal{T} to the cost function. For higher target velocities with respect to the robot, this factor increases the cost, since the robot will have to increase its speed to acquire targets, consuming more energy in the process.

C. Assignment Algorithms

Based on the requirements for the complete resource allocation algorithm for this application, the Bertsekas forward-reverse-auction algorithm [14] will be used. The algorithm is an extension of the standard auction algorithm, which resembles an auction process in that for a given object (*e.g.*, a task), the actor that wants the object attempts to make it as unattractive as possible to the other actors in the auction such that the desiring actor wins the object once the auction process has concluded. In particular, the actor makes an object undesirable by taking turns raising the price at which the object will be sold. In the case of forward-reverse auction, this activity is swapped between the actors and objects during each iteration of the algorithm: actors bid on objects in forward auction, while actors bid on objects in reverse auction.

With respect to the symbols defined in Section IV-A, the actors are the robots contained in the set \mathcal{R} , and the objects are the tasks contained in the set \mathcal{T} .

D. Arbitration of Resource Allocation

Once the costs have been computed with respect to the search regions, a method called *arbitration* is employed to execute the assignment algorithm and assign the agents to the search regions. Arbitration is officially defined as the process by which a dispute between two parties is settled by

TABLE I. SUMMARY OF ACTIVITY REGION PARAMETERS.

Active Region	μ_{dims}	σ_{dims}	μ_{vel}	σ_{vel}
1	5 m x 5 m	0.5 m	(-0.5, 0.5) m/s	0.05 m/s
2	5 m x 5 m	0.5 m	(-0.5, 0.5) m/s	0.05 m/s
3	2 m x 2 m	1.0 m	(0, 0.5) m/s	0.05 m/s
4	5 m x 5 m	0.5 m	(0.5, 0.5) m/s	0.05 m/s

an impartial third party. In the case of resource allocation, this definition may be modified to state that arbitration is the process by which resources are allocated to agents by another agent. The fact that an assignment algorithm will be used for processing the cost matrix and performing target assignment will play a key role in determining how arbitration will be handled.

In our approach, a single agent acting as the arbiter is the implementation that will be used, as it will be closer to its fellow agents and communications will be more reliable. If the agent must drop out of the mission for any reason, an arbiter handover algorithm is used, ensuring that an agent dropping out of the mission will not hinder the rest of the agents.

V. SIMULATION RESULTS

Two simulations were run: a "control" simulation and a simulation that uses the modeling and assignment algorithms. The baseline, control simulation used a traditional search approach: agents using a search pattern in fixed-size cells, as in Figure 3(a). Both simulations used the lawnmower search pattern, for its coverage properties [21]. Each simulation was split up into a different set of scenarios, varied based on the target sources that were active on the simulated glacier. The base scenario is based on the four ablation regions illustrated in Figure 2. Each of the sources had a varying *ablation capacity*, which is defined to be the total amount of ice mass that could be ejected by a target source. The combination of varying the ablation capacity plus the additional scenario modifications made here are extensions of the original simulation scenario used in [3]. The parameters for each of the ablating sources with respect to the icebergs that they generate is replicated from that work in Table I.

The simulation scenarios based on the overall scenario are summarized in Table II. A stream with a regular-sized capacity has an ablation capacity of 1000 tons; smaller streams have capacities of 200 tons. While these capacities are much smaller than the potential capacity of a real ablation source, they are comparable for the circumstances of this particular test. Each scenario was run for 20 minutes real-time, and 30 trials of each simulation were run.

The results of the simulation are summarized graphically in Figure 4. Target coverage remained at or very slightly below 100% in both scenarios, hence they are not shown. Average T_s is the average acquisition time (*i.e.*, the quantity of interest for the overall objective function). Average model T_s is the acquisition time as recorded by the *global* iceberg target source model shared across all of the agents, referenced from the start of a simulation run. Average local T_s is similar, except it is the acquisition time as recorded by the *local* models that are maintained by each of the agents. Average distance is the average distance traveled by the agents over an entire simulation run.

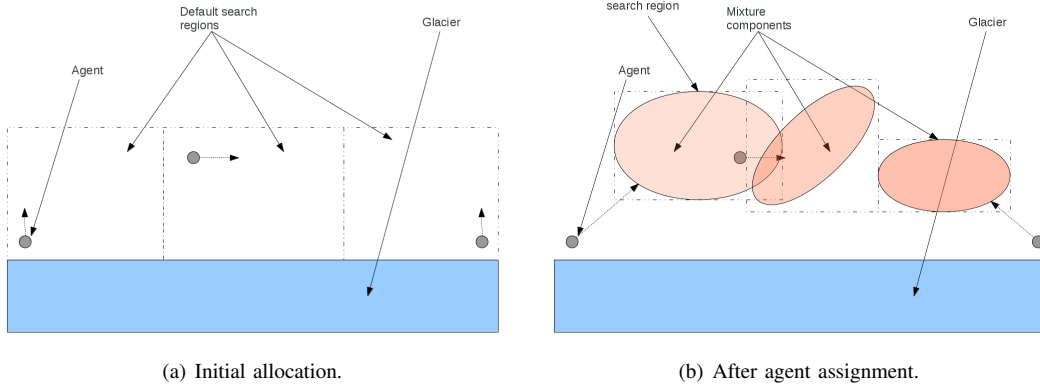


Fig. 3. Illustration of search regions before and after reallocation.

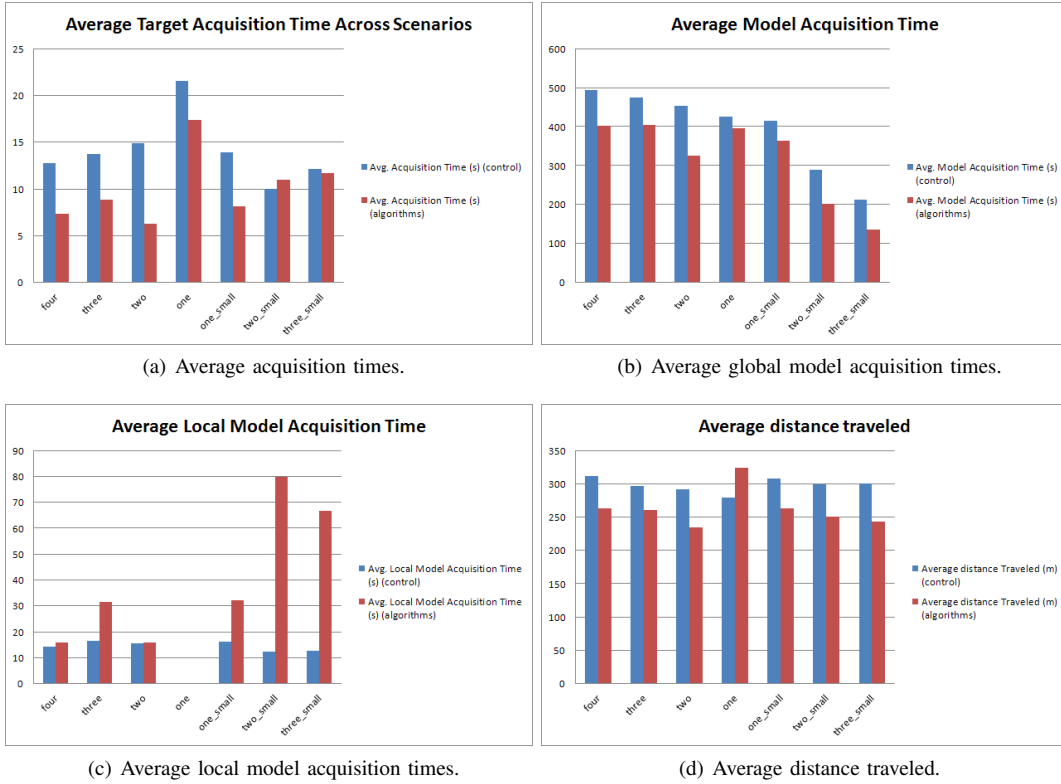


Fig. 4. Summary of simulation results.

TABLE II. SIMULATION SCENARIOS

Identifier	Description
four	Four target streams of equal capacity.
three	Three target streams of equal capacity.
two	Two target streams of equal capacity.
one	One target stream of equal capacity to those before.
one_small	Three target streams of equal capacity, one stream of small capacity.
two_small	Two target streams of equal capacity, two streams of small and equal capacity.
three_small	One target stream of “regular” capacity, three streams of small and equal capacity.

In Figure 4(a), an approximate 50% or more reduction in the initial acquisition time required when using our methodology can be observed. The cases where the behavior varies is in the single target stream case, and the two and three small stream cases. This is a result of “overcoverage”: multiple agents are reallocated to cover a single target stream, while a single agent remains to cover the other streams.

As for the model acquisition times, in general, there is improvement in the global time (Figure 4(b)) when using modeling and assignment. It is clear that there is a downward trend as target sources are removed or are of smaller capacity.

This can be attributed simply to the fact that the overall number of targets was reduced, hence less time was required to acquire the possible targets. As for the local model results, shown in Figure 4(c) the control remained flat in terms of variance overall, while there were significant changes when using modeling and assignment. The increase in the local times for the two and three small stream cases can be attributed to the overcoverage problem.

Finally, the distance traveled in both cases, shown in Figure 4(d), remains fairly close, with some reduction when applying the algorithms. This is attributable to the shorter distance traveled by the agents as they are allocated to smaller search regions, but the distance required to travel to a new search region can offset these reductions.

Overall, given the metrics for performance and the differences between the simulation scenarios, improvement can be observed when using the modeling and reassignment algorithms. This is especially true for the most important metric of interest, the average acquisition time across all agents and targets, which is representative of the problem's objective function.

VI. CONCLUSION

A methodology for reallocating agent resources for the iceberg observation problem has been provided. We have summarized our previous approach to defining the iceberg observation task in terms of the existing observation problem known as CMOMMT and our probabilistic methodology for modeling ablating target sources. The approach to the resource allocation problem is derived from the problem definition for the assignment problem. A cost function has been defined, based on robot parameters and the targets and search regions. We have compared our solution to a baseline method confining agents to static regions, which is often the case in many observation missions.

As this is purely a modeling and search task as currently given, future work would include incorporating the methodology into a full tracking system for icebergs. While the model can assist in predicting the probability of where icebergs will be calved, the individual icebergs are still a danger. Such a tracking system could use dedicated agents for tracking the icebergs and have a centralized target tracker for monitoring the iceberg paths.

This approach can also be applied to any structure that undergoes a process resembling ice ablation. For example, regions of activity could be developed for discrete herds of animals that usually stay in one area, but may eventually break off for various reasons, such as for escaping threats.

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