Cascaded flatness-based observation approach for lateral motorcycle dynamics estimation

Pierre-Marie Damon, Dalil Ichalal, Hichem Arioui, Saïd Mammar

Cascaded Flatness-Based Observation Approach For Lateral Motorcycle Dynamics Estimation

P-M Damon*, D. Ichalal*, H. Arioui* and S. Mammar*
*University of Evry Val d'Essonne
IBISC Laboratory, Evry, France
Email: pierre-marie.damon@ibisc.univ-evry.fr

***Abstract*—The flatness-based approach is presented in this paper in order to estimate the motorcycle lateral dynamics such as the roll angle, the lateral tire forces or the steering torque from basic measurements. The model of the motorcycle associated with the flatness theory are used to express the unknown states and input in terms of nonlinear functions depending only on the measures and their time derivatives up to a given finite order. These time derivatives are estimated via a non-asymptotic differentiator. Finally, the ability of the proposed observer to estimate the motorcycle dynamics is illustrated through two simulation scenarios performed with the well-known motorcycle simulator "BikeSim".**

I. Introduction and motivations

For many years ago active and semi-active safety systems widely equipped four-wheeled vehicles but motorcycles do not follow the same dynamic of safety systems growing due to the complexity and the instability of this type of vehicles. Nevertheless, such systems have been recently developed for powered two-wheeled vehicles (PTW), one can cite: the anti-lock braking system (ABS), the electronic stability control (ESC), the traction control system (TCS) [1] or the motorcycle stability control (MSC) [2] developed by Bosch. Obviously these kind of systems are based on a mathematical model of the vehicle. Their efficiency depends primarily on the faithfulness and the reliability of the model describing the motion and the instantaneous evolution of the dynamics states.

For safety applications, the control of the motorcycle motions requires a truthful assessment of its dynamic states and unknown inputs. These quantities help in the detection and the avoidance of critic situations like a sliding when cornering or braking. Several previous investigations about motorcycle rider fatalities have shown a large part of accidents involving PWT riders without any road user occurs in turn. Hence nowadays develop safety systems for lateral motorcycle riding scenario is a real challenge and the present work aims to contribute to the development of such systems. All the recent literature about lateral motorcycle modeling and observation shows the high degree of interest for this topic.

During the last years lot of works have addressed motorcycle modeling and for most of them longitudinal (in-plane) dynamics and lateral (out-of-plane) dynamics model are clearly separated. Lateral PWT models have been largely studied in [3], [4], [5], [6] and many others. In the presented approach the Sharp's 71 lateral model [3] has been considered because of its compromise between simplicity and ability to catch the lateral dynamics. Moreover even if this model aims to simulate the lateral dynamics it takes into account the variation of the forward speed v_x. With the Sharp's 71 model lateral PWT motion is described with 6 dynamics states: the roll angle, the steering angle, the yaw rate, the roll rate, the steering rate and the lateral velocity. The dynamics of the front and rear tires are included by taking into account the tire's relaxation. Hence it allows 4 degrees of freedom (DOF): the yaw, the roll, the steering and the lateral slip. In these work two non-linearities have been considered for the roll and steering angle as in [7].

In several countries PWT are becoming the most common mean of transportation and are often dedicated to urban use. Currently there is a real competition between manufacturers which try to sold new vehicles as cheap as possible to widen motorcycle user community. Emphasis on safety without significantly increase the selling price is a real challenge. That is why estimation and observation became major tools to make easier and cheaper the development of safety systems. They allow a reduction of the sensors' number and hence the cost is not so impact. Plenty of researches have addressed longitudinal and lateral PWT dynamics estimation during the last years. In the next section some of these works are introduced and the benefits of the proposed observer is discussed.

This paper is organized as follows. Section 2 presents motivations and states the problem. The lateral model of the motorcycle is introduced in section 3 whereas in section 4, we recall theoretical preliminaries about flatness and numerical differentiation. A flatness-based estimation and unknown input reconstruction approach is detailed. Simulation results are discussed in section 6 and finally conclusion remarks will be found in section 7.

II. Motivation and problem statement

The precursor work on the state estimation for motorcycles was published in 2008 [8]. In a first time, researches have commonly addressed the estimation of the lean angle and rapidly the aim was to estimate the whole of dynamics states.

In many works focus on lateral PWT estimation authors have consider restrictive assumptions and their proposed approaches are valid for limited riding conditions. In [9], the time variation of the longitudinal velocity is taken into account but the observer ensures only bounded error state estimation

(Input-to-State Stability). Also in [10] where the author considers longitudinal velocity v_x constant before decoupling the model and construct the observer, then a restrictive speed range is defined to guarantee reliable estimation. In [11] the variation of the forward speed is considered using an extended Kalman filter but simulations are performed for a maximum speed range around 15 m/s. In addition, to estimate dynamic states including rider's torque, authors often use complex structures of observer sometimes associated with differentiator as in [10]. More recently in [7], [12] or [13] authors have proposed observers free from restrictive conditions on the longitudinal speed but the design is based on the non-trivial resolution of linear matrix inequalities (LMI). The simulation results are very interesting but these observers are not easily implementable in the vehicle for real-time applications because they need important hardware resources. Moreover in all these papers a model of the lateral forces is needed in order to estimate them. A linear model of the lateral forces is commonly considered and leads inevitably to approximation errors. Indeed tire forces are linear on a definite range and saturation effect occurs outside of this range for more details please refer to [14]. In [15] a method is proposed considering the tire forces as two unknown inputs. For observability reasons the author considers that a lateral speed estimation is available from a high speed camera which is a very expansive sensor.

This work aims to propose an instantaneous estimator of the motorcycle roll angle, the rider's torque and the lateral tire forces without needing any tire's model. The estimated states are computed from the Sharp's 71 model combined with a strategic choice of sensor outputs and appropriate methodology in numerical differentiation [16]. Since 1980s and pioneering works of [17], nonlinear flatness-based theory has been quite effective in many concrete and industrial applications and especially in automotive one [18]. On other hand, numerical differentiation is an important such approach. These last require the generation of auxiliary outputs in order to overcome some restrictive conditions (unknown input observers, etc.) [19], [20]. To do so, many differentiators have been proposed (HOSM [21], HGD [22], Algebraic [16]). In order to obtain those auxiliary outputs, a not asymptotic differentiators is used as exact differentiator and do not use require any statistical knowledge of the corrupting noises [16]. The proposed approach supposes a prior work on a closed loop parametric estimation. In this field, various approaches exist.

In this paper, we adopt the approach of flatness observer by generating auxiliary outputs in order to reconstruct the dynamic states of the motorcycle and the external actions as the lateral tire forces or the steering torque acting on the vehicle. The main contribution of this paper is a straight and simple application of algebraic relations to reconstruct the lateral dynamics of the motorcycle. In addition, no force model is needed in this strategy and no restrictive assumption has to be considered to design the observer especially on the forward speed. Moreover the observer is able to instantly estimate the dynamics and there is no time of convergence. In the section 6 some simulations are carried out on the well-known motorcycle's simulator BikeSim which is based on the nonlinear multibody Sharp's 2004 model [4]. This simulator use a complex and highly nonlinear model where the motorcycle is divided into 8 different bodies allowing 16 DOF.

III. Motorcycle nonlinear lateral model

The Sharp's 71 motorcycle model is expressed as a set of equations which correspond respectively to the lateral $\dot{v}_y$, yaw $\ddot{\psi}$, roll $\ddot{\phi}$ and steering $\ddot{\delta}$ dynamics:

$$\begin{cases} m_{11}\dot{v}_y + m_{12}\ddot{\psi} + m_{13}\ddot{\phi} + m_{14}\ddot{\delta} - r_{13}v_x\dot{\psi} = \sum F_y \\ m_{12}\dot{v}_y + m_{22}\ddot{\psi} + m_{23}\ddot{\phi} + m_{24}\ddot{\delta} - r_{23}v_x\dot{\psi} \\ -r_{24}v_x\dot{\phi} - r_{25}v_x\dot{\delta} = \sum M_z \\ m_{13}\dot{v}_y + m_{23}\ddot{\psi} + m_{33}\ddot{\phi} + m_{34}\ddot{\delta} - r_{33}v_x\dot{\psi} \\ -r_{35}v_x\dot{\delta} = \sum M_x \\ m_{14}\dot{v}_y + m_{24}\ddot{\psi} + m_{34}\ddot{\phi} + m_{44}\ddot{\delta} - r_{43}v_x\dot{\psi} \\ -r_{44}v_x\dot{\phi} - r_{45}\dot{\delta} = \sum M_s \end{cases} \tag{1}$$

where:

$$\begin{cases} \sum F_y &= F_{yf} + F_{yr} \\ \sum M_z &= r_{26}F_{yf} + r_{27}F_{yr} \\ \sum M_x &= r_{31}\sin(\phi) + r_{32}\sin(\delta) \\ \sum M_s &= r_{41}\sin(\phi) + r_{42}\sin(\delta) + r_{46}F_{yf} + \tau \end{cases} \tag{2}$$

Notice that τ is the rider's torque; ϕ, δ are the roll and steering angle whereas $\dot{\psi}$ $\dot{\phi}$ $\dot{\delta}$ and $\ddot{\psi}$ $\ddot{\phi}$ $\ddot{\delta}$ are respectively the yaw, roll and steering rates and accelerations. F_{yf} and F_{yr} are the lateral tire forces. All the other terms m_{ij} and r_{ij} are given in the appendix. One can remark the introduced nonlinearities $sin(\delta)$ and $sin(\phi)$ for more details please refer to [7].

Then by introducing the cardinal sine function defined as follows:

$$sinc(x) = \begin{cases} 1 & if \quad x = 0 \\ \frac{sin(x)}{x} & if \quad x \neq 0 \end{cases} \tag{3}$$

The problem can be easily transformed under matrix formalism:

$$M[\dot{v}_y, \ddot{\psi}, \ddot{\phi}, \ddot{\delta}]^T = R(v_x)[\phi, \delta, \dot{\psi}, \dot{\phi}, \dot{\delta}, F_{yf}, F_{yr}, \tau]^T \tag{4}$$

with M and $R(v_x)$ two matrices provided in the appendix.

Let us consider the motorcycle is equipped with an optical encoder installed on the steering mechanism and an inertial measurement unit (IMU) located near the gravity center. These sensors provide the measurements of the following variables named $y_i(t)$:

$$y_1(t) = a_y, \quad y_2(t) = \dot{\psi}, \quad y_3(t) = \dot{\phi}, \quad y_4(t) = \delta \tag{5}$$

with a_y the lateral acceleration.

IV. Preliminaries and tools

In this section, some tools and notations are recalled which will be used in the proposed cascaded estimation of the motorcycle dynamics.

A. Flatness-based approach

Consider the nonlinear system:

$$\dot{x}(t) = f(x(t), u(t)) \tag{6}$$

The system (6) is said to be differentially flat if and only if,

1) there exists a vector-valued function $h(x)$ such that

$$y(t) = h(x(t), u(t), \dot{u}(t), ..., u^{(r)}(t)) \tag{7}$$

where $y(t) = \left(y_1, ..., y_{n_y}\right)^T \in \mathbb{R}^{n_y}$, $r \in \mathbb{N}$, $y(t)$ is called a flat output;

2) the state $x(t)$ and the input $u(t)$ can be expressed by

$$x(t) = A(y(t), \dot{y}(t), ..., y^{(r_x)}(t)), \quad r_x \in \mathbb{N} \tag{8}$$
$$u(t) = B(y(t), \dot{y}(t), ..., y^{(r_u)}(t)), \quad r_u \in \mathbb{N} \tag{9}$$

B. Numerical differentiation

Let us consider the signal $x(t) \in \mathbb{R}$ with $0 \leq t < \rho$. Its truncated Taylor expansion of order N in the interval $(0, \epsilon)$ where $0 < \epsilon \leq \rho$, is given by:

$$x_N(t) = \sum_{i=1}^{N} x^{(i)}(0)\frac{t^i}{i!} \tag{10}$$

which is an approximation of $x(t)$ in the interval $(0, \epsilon)$. By using the operational calculus and with simple mathematical computation the time derivative of order i i.e. $\left[x^{(i)}(0)\right]_e, 0 \leq i \leq N$ is obtained. For more details and theoretical foundation of this differentiation approach the reader can refer to [16]. A simple version of time derivative of order one is given by:

$$\hat{\dot{x}}(t) = -\frac{3!}{T^3}\int_{t-T}^{t} (2T(t-\tau) - T)x(\tau)d\tau \tag{11}$$

which can be easily implemented by a simple digital filter with a sliding time window of dimension T.

In the case of signal denoising, the same approach is exploited to estimate the denoised signal which corresponds to the time derivative of order 0 given by:

$$\hat{x}(t) = \frac{2!}{T^2}\int_{t-T}^{t} (3(t-\tau) - T)x(\tau)d\tau \tag{12}$$

V. Flatness-based state and unknown input estimation approach

Firstly, let us consider the dynamic equation of the lateral motion:

$$m_{11}a_y = F_{yf} + F_{yr} \tag{13}$$

where a_y is the lateral acceleration of the vehicle provided by the IMU and $m_{11} = M_f + M_r$ the global mass of the whole vehicle and rider. F_{yf} and F_{yr} are the forces acting respectively on the front and rear wheel contact points. It is very complex to measure the tire forces especially on motorcycle that is why an empirical model, often linear is considered for observation purposes. However, consider linear forces reduces the validity of the simulated dynamics compared to the real dynamics of the motorcycle. In addition, to be as faithful as possible the model has to take into account some parameters related to the tire properties and the environmental conditions (road adhesion, tire stiffness coefficient,...). In this work, no model is needed for the lateral forces and hence no parameters to identify. The adopted approach will provide insensitive estimations with respect to tire and road features.

In order to estimate the forces, a combination of the two first equations in (1) allows to obtain the following expression free from the variable $\dot{v}_y$ which is not measured:

$$\begin{aligned}&(m_{12}^2 - m_{11}m_{22})\ddot{\psi} + (m_{12}m_{13} - m_{11}m_{23})\ddot{\phi} + (m_{12}m_{14}\\ &-m_{11}m_{24})\ddot{\delta} + (m_{11}r_{23} - m_{12}r_{13})v_x\dot{\psi} + m_{11}r_{24}v_x\dot{\phi}\\ &+m_{11}r_{25}v_x\dot{\delta} = (m_{12} - m_{11}r_{26})F_{yf} + (m_{12} - m_{11}r_{27})F_{yr}\end{aligned} \tag{14}$$

Combining the above equation and (13), the forces are expressed as it follows:

$$\begin{aligned}F_{yf} &= \frac{1}{m_{11}(r_{27} - r_{26})}[(m_{11}r_{27} - m_{12})m_{11}a_y + (m_{12}^2\\ &- m_{11}m_{22})\ddot{\psi} + (m_{12}m_{13} - m_{11}m_{23})\ddot{\phi} + (m_{12}m_{14}\\ &- m_{11}m_{24})\ddot{\delta} + (m_{11}r_{23} - m_{12}r_{13})v_x\dot{\psi} + m_{11}r_{24}v_x\dot{\phi}\\ &+ m_{11}r_{25}v_x\dot{\delta}]\end{aligned} \tag{15}$$

and it comes:

$$F_{yr} = m_{11}a_y - F_{yf} \tag{16}$$

After estimating the lateral forces, the term $\dot{v}_y$ is estimated with the equation of the lateral dynamics as it follows:

$$\dot{v}_y = \frac{1}{m_{11}}[F_{yf} + F_{yr} - m_{12}\ddot{\psi} - m_{13}\ddot{\phi} - m_{14}\ddot{\delta} + r_{13}v_x\dot{\psi}] \tag{17}$$

The roll angle is obtained by:

$$\begin{aligned}\phi = asin(\tfrac{1}{r_{31}}[&m_{13}\dot{v}_y + m_{23}\ddot{\psi} + m_{33}\ddot{\phi} + m_{34}\ddot{\delta} - r_{33}v_x\dot{\psi}\\ &-r_{35}v_x\dot{\delta} - r_{32}sin(\delta)])\end{aligned} \tag{18}$$

From the last equation, the steering torque applied by the rider on the handlebar is obtain with the following equation:

$$\begin{aligned}\tau = m_{14}\dot{v}_y + m_{24}\ddot{\psi} + m_{34}\ddot{\phi} + m_{44}\ddot{\delta} - r_{43}v_x\dot{\psi} - r_{44}v_x\dot{\phi}\\ -r_{45}v_x\dot{\delta} - r_{41}\sin(\phi) - r_{42}\sin(\delta) - r_{46}F_{yf}\end{aligned} \tag{19}$$

Finally, one can express the estimated variables as follows:

$$F_{yf} = \varphi_1\left(y_1, y_2, \dot{y}_2, y_3, \dot{y}_3, \dot{y}_4, \ddot{y}_4\right) \tag{20}$$
$$F_{yr} = \varphi_2\left(F_{yf}, y_1\right) \tag{21}$$
$$\dot{v}_y = \varphi_3\left(F_{yf}, F_{yr}, y_2, \dot{y}_2, \dot{y}_3, \ddot{y}_4\right) \tag{22}$$
$$\phi = \varphi_4\left(\dot{v}_y, y_2, \dot{y}_2, \dot{y}_3, y_4, \dot{y}_4, \ddot{y}_4\right) \tag{23}$$
$$\tau = \varphi_5\left(\dot{v}_y, \phi, F_{yf}, y_2, \dot{y}_2, y_3, \dot{y}_3, y_4, \dot{y}_4, \ddot{y}_4\right) \tag{24}$$

with φ_i, $i = 1, ..., 5$ the corresponding non-linear functions. Note that in order to have all the unmeasured variables, the equations (20)-(24) should be computed in this order.

Knowing that the sensors provides only the measurements given in (5) the time derivatives are needed. In order to recover the first and the second time derivatives the numerical differentiator proposed in [16] is exploited. Let us adopt the notation $[\dot{y}_i]_e$ and $[\ddot{y}_i]_e$, $i = 1, ..., 4$ corresponding to the first and the second time derivative estimations of the outputs. The estimation of the unknown variables is then given by the following set of equation:

$$\hat{F}_{yf} = \varphi_1 \left(y_1, y_2, [\dot{y}_2]_e, y_3, [\dot{y}_3]_e, [\dot{y}_4]_e, [\ddot{y}_4]_e\right) \tag{25}$$

$$\hat{F}_{yr} = \varphi_2 \left(F_{yf}, y_1\right) \tag{26}$$

$$\hat{\dot{v}}_y = \varphi_3 \left(\hat{F}_{yf}, \hat{F}_{yr}, y_2, [\dot{y}_2]_e, [\dot{y}_3]_e, [\ddot{y}_4]_e\right) \tag{27}$$

$$\hat{\phi} = \varphi_4 \left(\hat{\dot{v}}_y, y_2, [\dot{y}_2]_e, [\dot{y}_3]_e, y_4, [\dot{y}_4]_e, [\ddot{y}_4]_e\right) \tag{28}$$

$$\hat{\tau} = \varphi_5 \left(\hat{\dot{v}}_y, \hat{\phi}, \hat{F}_{yf}, y_2, [\dot{y}_2]_e, y_3, [\dot{y}_3]_e, y_4, [\dot{y}_4]_e, [\ddot{y}_4]_e\right) \tag{29}$$

Remark 1: If the measured signals are noisy, for more accurate estimations, the measurements should be denoised before computing the time derivatives of them.

VI. SIMULATION RESULTS

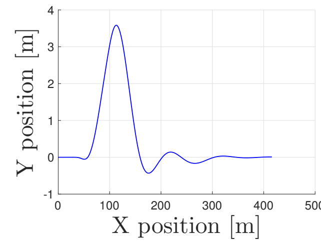
In this sections results of simulations are provided to validate the proposed observer. Two scenarios are simulated with BikeSim: a first one representing a double lane change (DLC) also called overtaking in road user language and a second simulating one lap of a circuit composed of straight lines, narrow and large curves. As discussed in section IV we need to measure the lateral acceleration a_y, the yaw rate $\dot{\psi}$, the roll rate $\dot{\phi}$ and the steering angle δ which are respectively given by the IMU and the optical encoder. Notice that these measures allow to estimate the whole of the dynamic states including the lateral tire forces and the rider's torque applied on the handlebar. The estimation of the the lateral speed derivative $\hat{\dot{v}}_y$ is not presented below because it is not a consistent state to detect critic riding situation or to act on the vehicle dynamics. Indeed $\hat{\dot{v}}_y$ doesn't match up to the lateral acceleration a_y because of the roll and yaw motions. According to equation (13) and (17) it comes:

$$\dot{v}_y = a_y - \frac{m_{12}}{m_{11}}\ddot{\psi} - \frac{m_{13}}{m_{11}}\ddot{\phi} - \frac{m_{14}}{m_{11}}\ddot{\delta} + \frac{r_{13}}{m_{11}}v_x\dot{\psi} \tag{30}$$

A. Double lane change simulation

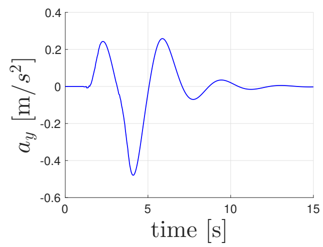
This scenario aims to simulate a DLC at 100km/h constant forward speed. This maneuver is a well-known reference to study lateral motion of motorcycles by allowing to excite the whole of lateral dynamics states. The figure 1 shows the vehicle's trajectory especially the lateral displacement which is around 3.5m whereas the figure 2 depicts the measured states while the DLC scenario.

The figure 3 presents the actual states in blue and their estimation in red. It clearly shows that the roll angle ϕ and the front and rear lateral forces acting on the tires respectively Fy_f and Fy_r are instantly and perfectly estimated. One can remark a small error about the estimation of rider's torque peaks.

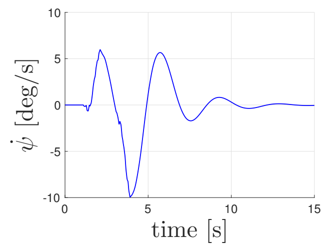


(a) Trajectory

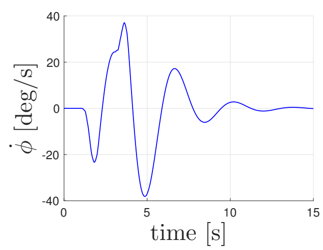
Fig. 1. Simulation scenario



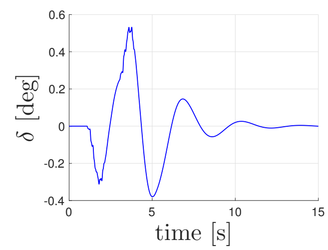
(a) Measured lateral acceleration



(b) Measured yaw rate



(c) Measured roll ate



(d) Measured steering rate

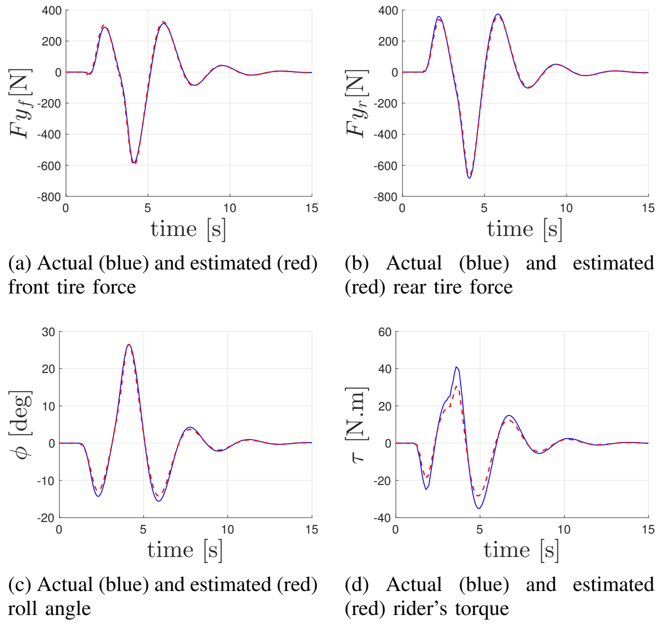
Fig. 2. Measured states

B. Circuit simulation

As discussed in motivation section, one of the main contribution of this work is that the proposed observer is free from tire's model and restrictive assumption especially on forward speed. This track scenario of 2.3 km aims to validate the observer on different trajectories with variable forward speed. The velocity is included in a range from around 30 to 100 km/h representing urban and extra-urban riding speed behavior. The figure 4 illustrates the vehicle's trajectory and the forward speed during the simulation.

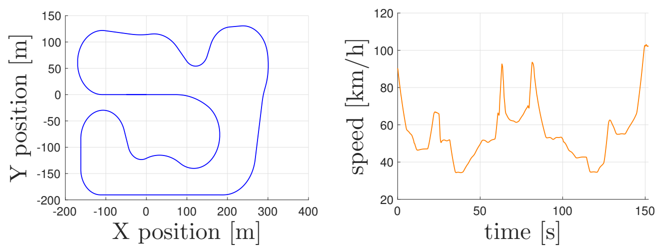
The figure 5 presents the measured states while the track riding. One can remark the small variation range $[-2, 3]$ deg of the steering angle for a such scenario. The motorcycle dynamics is far away from the one for four-wheeled vehicles where the steering angle is really more significant. When cornering the rider has to control the motorcycle with lot of parameters such as obviously the steering angle but also the lean angle, the engine torque, etc.

Finally, the figure 6 shows the actual states in blue and their estimation in red along the track. As for the DLC, the roll angle and the lateral tire's forces are successfully estimated. But there is a significant estimation error on the steering torque and not only for peak value. These error is probably due to the approximation of the Sharp's 71 model used to compute the



(a) Actual (blue) and estimated (red) front tire force
(b) Actual (blue) and estimated (red) rear tire force
(c) Actual (blue) and estimated (red) roll angle
(d) Actual (blue) and estimated (red) rider's torque

Fig. 3. Estimated states and unknown input



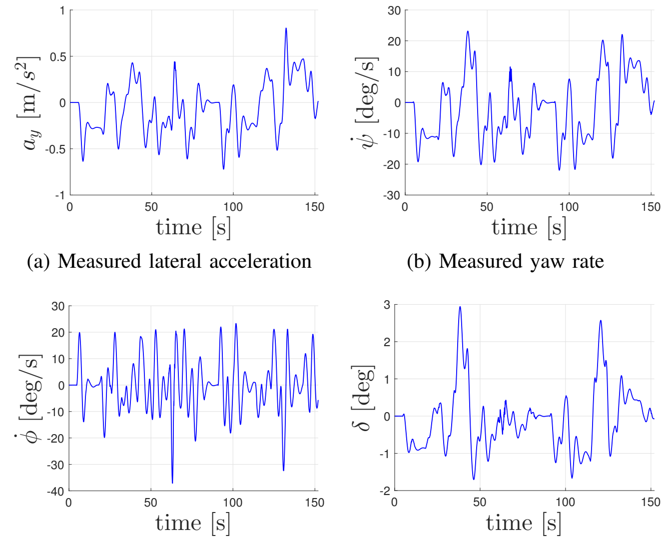
(a) Trajectory
(b) Speed

Fig. 4. Simulation scenario

observer. Even if the two nonlinearities $sin(\phi)$ and $sin(\delta)$ are considered the model omits the product of dynamics states and is linearized around straight running. Moreover it is valid for slow variation of the longitudinal dynamic which is not a valid assumption in this scenario. The steering torque is only a pertinent state for specific application like self-driving motorcycle where an electric motor control the steering mechanism but it doesn't give useful information to quantify the risk. In addition, it is quite impossible to measure the steering torque because even if it exists some systems allowing to measure the forces applied on the handlebar it is not possible to separate the torque generated by the road reaction from the one generated by the rider action.

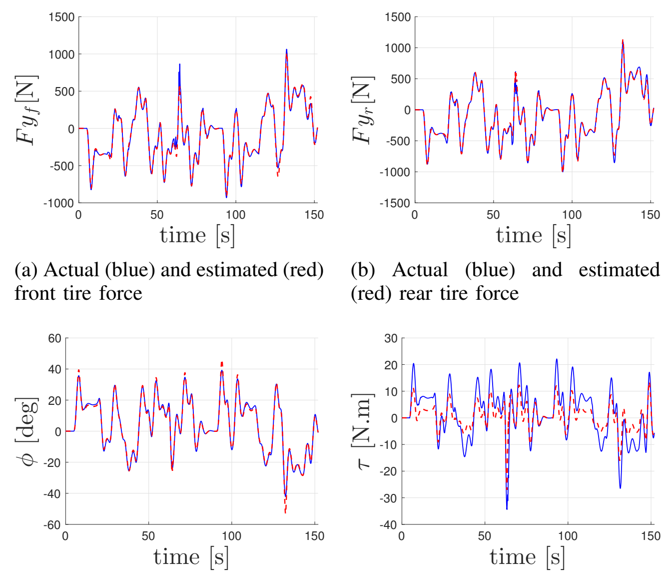
VII. Conclusion

In this paper a cascaded flatness-based observation approach is proposed in order to instantly estimate the lateral motorcycle dynamics including the roll angle, the rider's torque and the lateral tire forces without needing any model of these forces which render the estimations insensitive to uncertainties coming from tire and road characteristics such as stiffness and adherence coefficients. The estimator is based on the flat outputs provided by physical sensors and a dynamic model of the PWT. The simple nonlinear expressions of the unknown states and the steering torque are provided and depend only on the measured variables and their first and second time derivatives. These time derivatives are recovered by the recent numerical signal differentiation algorithm which is less sensitive to measurement noises affecting the real sensors. The efficiency of the designed estimator has been validated on the well-known BikeSim simulator. However the simulation results show that error occurs in the estimation of the rider's torque. For specific applications where an estimation of these torque is needed an interesting perspective could be to introduce a fix factor in the equation (24) which does not affect the other estimated states and allows to correct its estimation.



(a) Measured lateral acceleration
(b) Measured yaw rate
(c) Measured roll ate
(d) Measured steering rate

Fig. 5. Measured states



(a) Actual (blue) and estimated (red) front tire force
(b) Actual (blue) and estimated (red) rear tire force
(c) Actual (blue) and estimated (red) roll angle
(d) Actual (blue) and estimated (red) rider's torque

Fig. 6. Estimated states and unknown input

VIII. Appendix

Variables, matrices and notations	
v_x, v_y	longitudinal and lateral speeds
ϕ, ψ, δ	roll, yaw and steer angles
$\dot{\phi}$, $\dot{\psi}$, $\dot{\delta}$	roll, yaw and steer rates
$\ddot{\phi}$, $\ddot{\psi}$, $\ddot{\delta}$	roll, yaw and steer accelerations
a_y	lateral acceleration
τ	rider's torque
F_y, F_{y_f}, F_{y_r}	lateral tire forces
M_z, M_x, M_s	moments around Z, X and the steering axis
$\dot{x}$, $\ddot{x}$	time derivatives of the variable x
$\hat{x}$, $[x]_e$	estimate of a variable x
y_i	measure
x^T	transpose of vector or matrix x
x_f, x_r	denotes front and rear
$M, R(v_x)$	matrices

$$M = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{12} & m_{22} & m_{23} & m_{24} \\ m_{13} & m_{23} & m_{33} & m_{34} \\ m_{14} & m_{24} & m_{34} & m_{44} \end{bmatrix}$$

$$R(v_x) = \begin{bmatrix} 0 & 0 & r_{13}v_x & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & r_{23}v_x & r_{24}v_x & r_{25}v_x & r_{26} & r_{27} & 0 \\ r_{31}sc(\phi) & r_{32}sc(\delta) & r_{33}v_x & 0 & r_{35}v_x & 0 & 0 & 0 \\ r_{41}sc(\phi) & r_{42}sc(\delta) & r_{43}v_x & r_{44}v_x & r_{55} & r_{46} & 0 & 1 \end{bmatrix}$$

with $sc(.) = sinc(.)$

Matrix terms m_{ij} and r_{ij}

$m_{11} = M_f + M_r$, $m_{12} = M_f k$, $m_{13} = M_f j + M_r h$, $m_{14} = M_f e$, $m_{22} = M_f k^2 + I_{rz} + I_{fx} sin^2(\epsilon) + I_{fz} cos^2(\epsilon)$, $m_{23} = M_f jk - C_{rxz} + (I_{fz} - I_{fx}) sin(\epsilon) cos(\epsilon)$, $m_{24} = M_f ek + If_z cos(\epsilon)$, $m_{33} = M_f j^2 + M_r h^2 + I_{rx} + I_{fx} cos^2(\epsilon) + I_{fz} sin^2(\epsilon)$, $m_{34} = M_f ej + I_{fz} sin(\epsilon)$, $m_{44} = I_{fz} + M_f e^2$

$r_{13} = -M_f - M_r$, $r_{23} = -M_f k$, $r_{24} = i_{fy}/R_f + i_{ry}/R_r$, $r_{25} = i_{fy}/R_f sin(\epsilon)$, $r_{26} = l_f$, $r_{27} = -l_r$, $r_{31} = (M_f j + M_r h)g$, $r_{32} = M_f eg - \eta F_{zf}$, $r_{33} = -M_f j - M_r h - i_{fy}/R_f - i_{ry}/R_r$, $r_{35} = -i_{fy}/R_f cos(\epsilon)$, $r_{41} = M_f eg - \eta F_{zf}$, $r_{42} = (M_f eg - \eta F_{zf}) sin(\epsilon)$, $r_{43} = -M_f e - i_{fy}/R_f sin(\epsilon)$, $r_{44} = i_{fy}/R_f cos(\epsilon)$, $r_{45} = -K$, $r_{46} = -\eta$,

Motorcycle parameters	
g	gravity
ϵ	caster angle
η	mechanical trail
K	steering damper
F_{zf}	front vertical load
C_{rxz}	rear frame inertia product
M_f, M_r	body mass
j, h	geometric dimensions (*)
k, e	geometric dimensions (*)
l_f, l_r	geometric dimensions (*)
R_f, R_r	wheel radius
i_{fy}, i_{ry}	wheel inertia around Y
I_{fx}, I_{rx}	body inertia around X
I_{fz}, I_{rz}	body inertia around Z

(*) For more details please refer to [3].

References

[1] B. Daniels, "Motorcycle traction control system," Tech. Rep., Apr. 25 2013, uS Patent App. 13/657,811. [Online]. Available: http://www.google.com.na/patents/US20130103281

[2] Bosch, "Motorcycle stability control," Tech. Rep., 2013. [Online]. Available: http://www.bosch-motorcycle.com

[3] R. S. Sharp, "The stability and control of motorcycles," *Journal of Mechanical Engineering Science*, vol. 13, no. 5, pp. 316–329, 1971.

[4] R. Sharp, S. Evangelou, and D. J. Limebeer, "Advances in the modelling of motorcycle dynamics," *Multibody system dynamics*, vol. 12, no. 3, pp. 251–283, 2004.

[5] V. Cossalter, *Motorcycle dynamics*. Lulu, 2006.

[6] L. Nehaoua, H. Arioui, N. Seguy, and S. Mammar, "Dynamic modeling of a two wheeled vehicle: Jourdain formalism," *Vehicle System Dynamics*, 2013.

[7] H. Dabladji, D. Ichalal, H. Arioui, and S. Mammar, "Unknown-input observer design for motorcycle lateral dynamics: Ts approach," *Control Engineering Practice*, vol. 54, pp. 12–26, 2016.

[8] I. Boniolo, M. Norgia, M. Tanelli, C. Svelto, and S. M. Savaresi, "Performance analysis of an optical distance sensor for roll angle estimation in sport motorcycles," *IFAC Proceedings Volumes*, vol. 41, no. 2, pp. 135 – 140, 2008, 17th {IFAC} World Congress.

[9] D. Ichalal, H. Dabladji, H. Arioui, S. Mammar, and L. Nehaoua, "Observer design for motorcycle lean and steering dynamics estimation: A Takagi-Sugeno approach," in *American Control Conference (ACC),*, June 2013, pp. 5654–5659.

[10] L. Nehaoua, D. Ichalal, H. Arioui, S. Mammar, and L. Fridman, "An unknown-input HOSM approach to estimate lean and steering motorcycle dynamics," *IEEE Transactions on Vehicular Technology*, vol. 63, no. 7, pp. 3116–3127, Sept 2014.

[11] A. Teerhuis and S. Jansen, "Motorcycle state estimation for lateral dynamics," *Vehicle System Dynamics*, vol. 50, no. 8, pp. 1261–1276, 2012.

[12] P. Damon, H. Dabladji, D. Ichalal, L. Nehaoua, and H. Arioui, "Estimation of lateral motorcycle dynamics and rider action with luenberger observer," pp. 2392–2397, 2016.

[13] P. M. Damon, H. Dabladji, D. Ichalal, L. Nehaoua, H. Arioui, and S. Mammar, "Lateral motorcycle dynamics and rider action estimation: An lpv unknown input observer approach," in *2016 IEEE Conference on Control Applications (CCA)*, Sept 2016, pp. 711–716.

[14] H. Pacejka, *Tire and Vehicle Dynamics*. SAE International, 2005.

[15] M. E.-H. Dabladji, D. Ichalal, H. Arioui, S. Mammar, and L. Fridman, "Estimation of lateral dynamics and road curvature for two-wheeled vehicles: A hosm observer approach," vol. 19, no. 1, pp. 2806–2811, 2014.

[16] M. Fliess, C. Join, and H. Sira-Ramirez, "Non-linear estimation is easy," *International Journal of Modelling, Identification and Control*, vol. 4, no. 1, pp. 12–27, Jan. 2008.

[17] A. Van der Schaft, "Representing a nonlinear state space system as a set of higher-order differential equations in the inputs and ouputs," *Systems & Control Letters*, vol. 12, no. 2, pp. 151–160, 1989.

[18] J. Villagra, B. d'Andrea Novel, M. Fliess, and H. Mounier, "Estimation of longitudinal and lateral vehicle velocities: An algebraic approach," in *American Control Conference, 2008*, June 2008, pp. 3941–3946.

[19] T. Floquet, C. Edwards, and S. Spurgeon, "On sliding mode observers for systems with unknown inputs," *International Journal of Adaptive Control and Signal Processing*, vol. 21, no. 8-9, pp. 638–656, 2007.

[20] T. Floquet and J. P. Barbot, "Super twisting algorithm-based step-by-step sliding mode observers for nonlinear systems with unknown inputs," *International Journal of Systems Science*, vol. 38, no. 10, pp. 803–815, 2007.

[21] A. Levant, "Higher-order sliding modes, differentiation and output-feedback control," *International Journal of Control*, vol. 76, no. 9-10, pp. 924–941, 2003.

[22] S. Ibrir, "Online exact differentiation and notion of asymptotic algebraic observers," *IEEE Transactions on Automatic Control*, vol. 48, no. 11, pp. 2055–2060, Nov 2003.