

A Low-Complexity MIMO Channel Estimator with Implicit Structure of a Convolutional Neural Network

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Abstract

A low-complexity convolutional neural network estimator which learns the minimum mean squared error channel estimator for single-antenna users was recently proposed. We generalize the architecture to the estimation of MIMO channels with multiple-antenna users and incorporate complexity-reducing assumptions based on the channel model. Learning is used in this context to combat the mismatch between the assumptions and real scenarios where the assumptions may not hold. We derive a high-level description of the estimator for arbitrary choices of the pilot sequence. It turns out that the proposed estimator has the implicit structure of a two-layered convolutional neural network, where the derived quantities can be relaxed to learnable parameters. We show that by using discrete Fourier transform based pilots the number of learnable network parameters decreases significantly and the online run time of the estimator is reduced considerably, where we can achieve linearithmic order of complexity in the number of antennas. Numerical results demonstrate performance gains compared to state-of-the-art algorithms from the field of compressive sensing or covariance estimation of the same or even higher computational complexity. The simulation code is available online.

Index Terms

Channel estimation, massive MIMO, machine learning, neural networks, spatial channel model.

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I. INTRODUCTION

Accurate channel estimation (CE) is a key aspect in modern communication technologies such as mm-wave communications [1] and cellular massive MIMO systems [2]. As with increasing system complexity the estimation becomes more difficult, new approaches are necessary which combine reasonable performance together with low complexity. Recently, machine learning based approaches gained a lot of interest which use simulated or measured data to obtain channel estimators [3] - [7]. To the best of our knowledge, none of these approaches uses side knowledge from the channel model to derive a suitable low-complexity neural network architecture for MIMO CE.

In [8], a convolutional neural network (CNN) channel estimator is derived which is based on the minimum mean squared error (MMSE) estimator and includes assumptions which stem from a spatial channel model, e.g., the 3GPP model [9]. The CNN is further trained on simulated data to mitigate the mismatch of the assumptions in real scenarios [8]. This approach is applicable for various antenna array configurations [10], for measurement data [11] and also for quantized systems [12]. However, the CNN estimator is limited to SIMO systems.

The contributions of this work are summarized as follows. We first generalize the SIMO estimator from [8] to the MIMO case. This requires us to also incorporate the transmission of arbitrary pilot signals. We derive a conditional MMSE estimator that is based on the assumption that prior parameters exist that determine the channel statistics, but which is not computable without knowledge of this prior. Note that this clearly distinguishes the approach from the standard MMSE estimator. We show that the MIMO estimator can be decomposed into a what we call diagonal-block sparse structure which is different from the SIMO case. We identify the structure of a CNN with 2D circular convolution layers instead of the 1D convolutions in [8] by untying [13] the derived quantities in the sparse structure. We justify this extension by investigating insights from the spatial channel model. By choosing orthogonal, e.g., discrete Fourier transform (DFT)-based, pilot sequences we can show a structural simplification of the estimator, which reduces to a diagonal structure and decreases the number of learnable network parameters considerably. The resulting CNN estimator is valid for a whole class of channel covariance matrices.

We provide simulation results which verify the reasonability of the approach and that the learning procedure is indeed able to compensate the mismatch of the assumptions in the derivations. The proposed method is compared to a method based on maximum likelihood (ML) estimation of

the covariance matrix and also to the compressive sensing based approach orthogonal matching pursuit (OMP). We provide evidence of the superiority of our approach to the existing baseline algorithms due the possibility of training a model-based architecture with low online complexity of linearithmic order. The simulation code can be found in [14].

Notation: The transpose and conjugate-transpose of a vector \mathbf{x} are denoted by \mathbf{x}^T and \mathbf{x}^H . The $n \times n$ identity matrix and the $n \times 1$ all-ones vector are denoted by \mathbf{I}_n and $\mathbf{1}_n$. The column-wise vectorization and the trace of a matrix \mathbf{X} is denoted by $\text{vec}(\mathbf{X})$ and $\text{tr}(\mathbf{X})$, the Kronecker product by \otimes . A diagonal matrix with diagonal \mathbf{x} is given by $\text{diag}(\mathbf{x})$. We denote two matrices $\mathbf{X}, \mathbf{Y} \in \mathbb{C}^{M \times M}$ which are asymptotically equivalent, i.e., $\lim_{M \rightarrow \infty} \|\mathbf{X} - \mathbf{Y}\|_F^2/M = 0$ holds, by $\mathbf{X} \asymp \mathbf{Y}$.

II. CHANNEL AND SYSTEM MODEL

We consider an uplink scenario, where N pilot signals are transmitted. The base station (BS) and the mobile station (MS) consist of uniform linear arrays (ULAs) with S and U antennas, respectively. The channel is assumed frequency-flat with block-fading such that we get independent observations in each coherence interval. We investigate a single-snapshot scenario, i.e., the coherence interval of the covariance matrix and of the channel is identical. The signal received at the BS is $\mathbf{Y} = \mathbf{H}\mathbf{X}' + \mathbf{Z}$ with the channel matrix $\mathbf{H} \in \mathbb{C}^{S \times U}$ and the pilot matrix $\mathbf{X}' \in \mathbb{C}^{U \times N}$. After vectorization we get

$$\mathbf{y} = \mathbf{X}\mathbf{h} + \mathbf{z} \in \mathbb{C}^{SN}, \quad (1)$$

with $\mathbf{X} = \mathbf{X}'^T \otimes \mathbf{I}_S$ and $\mathbf{h} = \text{vec}(\mathbf{H}) \in \mathbb{C}^{SU}$. The noise vector is assumed to be distributed as $\mathbf{z} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \sigma^2 \mathbf{I}_{SN})$.

We work with the 3GPP spatial channel model [9], where for a given variable $\boldsymbol{\delta}$, the channels are distributed as $\mathbf{h}|\boldsymbol{\delta} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \mathbf{C}_{\boldsymbol{\delta}})$. The vector $\boldsymbol{\delta}$ contains angles of arrivals and path gains and follows an unknown distribution $\boldsymbol{\delta} \sim p(\boldsymbol{\delta})$. The covariance matrix $\mathbf{C}_{\boldsymbol{\delta}}$ is assumed to be determined by the transmit- and receive-side spatial correlation matrices, i.e., $\mathbf{C}_{\boldsymbol{\delta}} = \mathbf{C}_{\boldsymbol{\delta},T} \otimes \mathbf{C}_{\boldsymbol{\delta},R}$, which is valid in spatial correlation scenarios [15], [16]. Each covariance matrix equals to

$$\mathbf{C}_{\boldsymbol{\delta},\{T,R\}} = \int_{-\pi}^{\pi} g(\theta_{\{T,R\}}; \boldsymbol{\delta}) \mathbf{a}(\theta_{\{T,R\}}) \mathbf{a}(\theta_{\{T,R\}})^H d\theta_{\{T,R\}},$$

where \mathbf{a} is the array steering vector for an angle of departure (arrival) θ_T (θ_R) and the power density g is a sum of weighted Laplace densities where the standard deviation describes the angular spread [9]. For the case of a ULA at the BS, it holds that $\mathbf{a}(\theta_R) = [1, \exp(j\pi \sin \theta_R), \dots, \exp(j(S-$

$1)\pi \sin \theta_R]^\text{H}$. The same structure holds also for the MS array vector. Consequently, both the transmit and receive side covariance matrices are Toeplitz matrices, which makes the matrix \mathbf{C}_δ a block-Toeplitz matrix with Toeplitz blocks. The signal-to-noise ratio (SNR) is $\text{tr}(\mathbf{C}_\delta \mathbf{X}^\text{H} \mathbf{X}) / (SN\sigma^2)$.

III. DERIVATION OF THE PROPOSED ESTIMATOR

To derive the CNN channel estimator for MIMO cases, we start with the definitions of the MMSE estimator similar to [8] for SIMO cases. Assuming knowledge of the prior parameters δ , the conditional MMSE estimate of \mathbf{h} from \mathbf{y} would read as

$$\mathbb{E}[\mathbf{h}|\mathbf{y}, \delta] = \mathbb{E}[\mathbf{h}\mathbf{y}^\text{H}|\delta]\mathbb{E}[\mathbf{y}\mathbf{y}^\text{H}|\delta]^{-1}\mathbf{y} \quad (2)$$

$$= \mathbf{C}_\delta \mathbf{X}^\text{H} (\mathbf{X} \mathbf{C}_\delta \mathbf{X}^\text{H} + \sigma^2 \mathbf{I}_{SN})^{-1} \mathbf{y} = \mathbf{W}_\delta \mathbf{y}, \quad (3)$$

which depends linearly on the observations. However, as the parameters δ are unknown in general, the law of total expectation is used to compute [8]

$$\hat{\mathbf{h}} = \mathbb{E}[\mathbf{h}|\mathbf{y}] = \mathbb{E}[\mathbb{E}[\mathbf{h}|\mathbf{y}, \delta]] = \mathbb{E}[\mathbf{W}_\delta \mathbf{y}|\mathbf{y}] = \widehat{\mathbf{W}}_\star(\mathbf{y})\mathbf{y}. \quad (4)$$

Thus, to obtain $\hat{\mathbf{h}}$, the MMSE estimate $\widehat{\mathbf{W}}_\star$ of the MMSE filter \mathbf{W}_δ has to be determined, which nonlinearly depends on \mathbf{y} .

Bayes' theorem is used to state the MMSE filter as [8]

$$\widehat{\mathbf{W}}_\star = \int p(\delta|\mathbf{y}) \mathbf{W}_\delta d\delta = \frac{\int p(\mathbf{y}|\delta) \mathbf{W}_\delta p(\delta) d\delta}{\int p(\mathbf{y}|\delta) p(\delta) d\delta}. \quad (5)$$

As shown in Appendix A, the MMSE filter is written as

$$\widehat{\mathbf{W}}(\widehat{\mathbf{C}}) = \frac{\int \exp(\text{tr}(\mathbf{X} \mathbf{W}_\delta \widehat{\mathbf{C}}) + \log |\mathbf{I} - \mathbf{X} \mathbf{W}_\delta|) \mathbf{W}_\delta p(\delta) d\delta}{\int \exp(\text{tr}(\mathbf{X} \mathbf{W}_\delta \widehat{\mathbf{C}}) + \log |\mathbf{I} - \mathbf{X} \mathbf{W}_\delta|) p(\delta) d\delta}, \quad (6)$$

with the scaled sample covariance matrix $\widehat{\mathbf{C}} = \frac{1}{\sigma^2} \mathbf{y}\mathbf{y}^\text{H}$ as input. The MMSE filter $\widehat{\mathbf{W}}_\star$ depends on the observations through $\widehat{\mathbf{C}}$ and is thus a nonlinear filter. For an arbitrary distribution $p(\delta)$, the MMSE filter in (6) is not computable. To overcome this, we introduce the following assumption equivalent to [8].

Assumption 1: The distribution $p(\delta)$ is discrete and uniform, i.e., we have a grid $\{\delta_i : i = 1, \dots, P\}$ and $p(\delta_i) = 1/P$.

With this assumption, the MMSE estimator is evaluated as

$$\widehat{\mathbf{W}}_{\text{GE}}(\widehat{\mathbf{C}}) = \frac{1/P \sum_{i=1}^P \exp(\text{tr}(\mathbf{X} \mathbf{W}_{\delta_i} \widehat{\mathbf{C}}) + b_i) \mathbf{W}_{\delta_i}}{1/P \sum_{i=1}^P \exp(\text{tr}(\mathbf{X} \mathbf{W}_{\delta_i} \widehat{\mathbf{C}}) + b_i)}, \quad (7)$$

with $b_i = \log |\mathbf{I} - \mathbf{X}\mathbf{W}_{\delta_i}|$. We refer to this as the gridded estimator (GE), which neglects the true continuous distribution of δ , resulting in an approximation error that decreases with increasing number of samples P [8]. The order of complexity for evaluating the GE is $\mathcal{O}(S^2U^2P)$ in the case $U = N$ due to the matrix-matrix computations in (7).

In the following we reduce the complexity of the estimator where we exploit common structure of covariance matrices.

Assumption 2: The filters \mathbf{W}_{δ_i} can be decomposed as

$$\mathbf{W}_{\delta_i} = \mathbf{Q}^H \text{diablk}_{S,U}(\mathbf{w}_{\delta_i}) \mathbf{Q} \mathbf{X}^H, \quad (8)$$

with $\mathbf{Q} \in \mathbb{C}^{SU \times SU}$ being the Kronecker product of two DFT matrices of size $S \times S$ and $U \times U$, i.e.,

$$\mathbf{Q} = \mathbf{F}_U \otimes \mathbf{F}_S, \quad (9)$$

and with the `diablk` operator as defined in Appendix B.

For further verification of Assumption 2, see Appendix C. With this assumption, the MMSE filter from (7) is approximately

$$\widehat{\mathbf{W}}(\bar{\mathbf{c}}) \approx \mathbf{Q}^H \text{diablk}_{S,U}(\mathbf{w}(\bar{\mathbf{c}})) \mathbf{Q} \mathbf{X}^H, \quad (10)$$

with the definitions

$$\mathbf{w}(\bar{\mathbf{c}}) = \mathbf{A} \frac{\exp(\mathbf{A}^T \bar{\mathbf{c}} + \mathbf{b})}{\mathbf{1}_P^T \exp(\mathbf{A}^T \bar{\mathbf{c}} + \mathbf{b})} = \mathbf{A} \phi(\mathbf{A}^T \bar{\mathbf{c}} + \mathbf{b}), \quad (11)$$

$$\mathbf{A} = [\mathbf{w}_{\delta_1}, \dots, \mathbf{w}_{\delta_P}] \in \mathbb{C}^{SU^2 \times P}, \quad (12)$$

$$\bar{\mathbf{c}} = \sigma^{-2} \text{diablk}_{S,U}(\mathbf{Q} \mathbf{X}^H \mathbf{y} \mathbf{y}^H \mathbf{X} \mathbf{Q}^H) \in \mathbb{C}^{SU^2}, \quad (13)$$

where we identify the softmax function ϕ in (11) and $\mathbf{b} = [b_{\delta_1}, \dots, b_{\delta_P}]^T$ as shown in Appendix C.

We further reduce the complexity by assuming \mathbf{A} to be a block-circulant matrix with circulant blocks, which is valid for specific scenarios, i.e., a single propagation cluster. In [8, Appendix D], a shift invariance of the power density is discussed in the SIMO case with a ULA at the BS, which generalizes to a 2D shift invariance in MIMO cases. Thus, the matrix \mathbf{A} from (12) consists of vectors \mathbf{w}_{δ_i} which are invariant to a certain vertical and horizontal shift, resulting in the block-circulant structure and motivating the following assumption.

Assumption 3: The matrix \mathbf{A} is block-circulant and therefore it exists a $\mathbf{w}_0 \in \mathbb{R}^{SU}$ such that

$$\mathbf{A} = \mathbf{Q}^H \text{diag}(\mathbf{Q} \mathbf{w}_0) \mathbf{Q}, \quad (14)$$

with \mathbf{Q} as defined in (9).

This is equal to constructing \mathbf{A} by a 2D circular convolution with \mathbf{w}_0 as convolution kernel. Given the relationship between block-circulant matrices and 2D circular convolution, we write

$$\mathbf{A}\mathbf{x} = \mathbf{Q}^H \text{diag}(\mathbf{Q}\mathbf{w}_0)\mathbf{Q}\mathbf{x} = \mathbf{w}_0 \star \mathbf{x}. \quad (15)$$

Here, we compute the 2D convolution of the two vectorized quantities \mathbf{w}_0 and \mathbf{x} . Hence, in a first step, a reshaping of both \mathbf{w}_0 and \mathbf{x} into matrices with S rows is necessary. Afterwards, we vectorize the result, which yields the vector $\mathbf{A}\mathbf{x}$. This three-step convolution process is denoted by $\mathbf{w}_0 \star \mathbf{x}$ for simplicity.

IV. ORTHOGONAL PILOT SEQUENCES

The general formulation of the estimator breaks down to a less complex implementation if we use pilot matrices with certain properties. More precisely, we choose $\mathbf{X}' = \frac{1}{\sqrt{U}}\mathbf{F}_{U \times N}$ where $\mathbf{F}_{U \times N}$ contains the first U rows of a $N \times N$ DFT matrix for the case $U \leq N$. This ensures $\mathbf{X}'^H \mathbf{X}' = \frac{N}{U} \mathbf{I}_{SU}$, which is also endorsed in [17], [18]. With this, the matrix product $\tilde{\mathbf{Q}}\tilde{\mathbf{Q}}^H$ in (28) becomes diagonal and Assumption 2 can be written in terms of $\mathbf{W}_{\delta_i} = \mathbf{Q}^H \text{diag}(\mathbf{w}_{\delta_i})\mathbf{Q}\mathbf{X}'^H$. Also, the estimator input in (13) simplifies to $\hat{\mathbf{c}} = \sigma^{-2}|\mathbf{Q}\mathbf{X}'^H \mathbf{y}|^2$. In extension to the property in (15), due to the 2D shift-invariance and the circular assumption, the matrix-vector product $\mathbf{A}^T \hat{\mathbf{c}}$ can be written as a 2D circular convolution, i.e.,

$$\mathbf{A}^T \hat{\mathbf{c}} = \tilde{\mathbf{w}}_0 \star \hat{\mathbf{c}}, \quad (16)$$

where $\tilde{\mathbf{w}}_0$ contains the entries of \mathbf{w}_0 in reversed order. We refer to the estimator containing all three assumptions as the fast estimator (FE), which is given by

$$\widehat{\mathbf{W}}(\hat{\mathbf{c}}) = \mathbf{Q}^H \text{diag}(\mathbf{w}_0 \star \phi(\tilde{\mathbf{w}}_0 \star \hat{\mathbf{c}} + \mathbf{b})) \mathbf{Q}\mathbf{X}'^H, \quad (17)$$

where the $\text{diag}(\cdot)$ operator replaces the $\text{diablk}(\cdot)$ operator from (10) due to the orthogonal pilot sequences. The 2D circular convolution is justified due to the properties from (16). Note that the FE is also applicable in the case of arbitrary pilot sequences, but with higher complexity.

The FE has low complexity, but the underlying assumptions only hold for many antennas and a single propagation cluster which may be rarely fulfilled in real scenarios. Therefore, we interpret the estimator from (17) as a CNN with two 2D convolution layers, which implements a function from the set

$$\mathcal{W}_{\text{CNN}} = \{\mathbf{x} \mapsto \mathbf{a}^{(2)} \star \psi(\mathbf{a}^{(1)} \star \mathbf{x} + \mathbf{b}^{(1)}) + \mathbf{b}^{(2)}\}, \quad (18)$$

where $\mathbf{a}^{(l)}, \mathbf{b}^{(l)} \in \mathbb{R}^{SU}, l = 1, 2$ are the parameters which are learned during training from samples $(\mathbf{y}_i, \mathbf{h}_i)$, generated by the 3GPP model. Although we identified the softmax function ϕ as the activation in (11), we relax the activation to be a different function ψ , e.g. the well-known rectified linear unit (ReLU). This can lead to a better generalization and convergence ability. Note that in contrast to common linear convolution layers in CNNs, the above derivation suggests to use circular convolution layers in the proposed CNN. The optimal CNN estimator is the function which minimizes the MSE, i.e.,

$$\hat{\mathbf{w}}_{\text{CNN}} = \arg \min_{\hat{\mathbf{w}}(\cdot) \in \mathcal{W}_{\text{CNN}}} \mathbb{E}[\|\mathbf{h} - \mathbf{Q}^H \text{diag}(\hat{\mathbf{w}}(\hat{\mathbf{c}})) \mathbf{Q} \mathbf{X}^H \mathbf{y}\|_2^2]. \quad (19)$$

The complexity of the CNN estimator is equal to that of the FE as learning is done offline. Due to the fast Fourier transform (FFT) for the 2D circular convolution and the pilot matrix, the complexity of the estimator is only $\mathcal{O}(SU \log(SU))$ which is a drastic decrease compared to the GE complexity of $\mathcal{O}(S^2 U^2 P)$.

V. SIMULATION RESULTS

We consider the 3GPP channel model [9] with parameters δ drawn uniformly at random for each channel realization, which means that a different covariance matrix \mathbf{C}_δ applies. We choose the per-sample noise variance σ^2 by the given SNR, and we assume that the prior δ is known during training. As performance measure we use the MSE, normalized by SU .

In the 3GPP channel model, the covariance matrices have a low numerical rank [19], and therefore, the channel matrix can be approximated by a row-sparse matrix \mathbf{T} for a given dictionary \mathbf{D} , i.e., $\mathbf{h} \approx (\mathbf{I}_U \otimes \mathbf{D})\mathbf{t}$ with $\mathbf{t} = \text{vec}(\mathbf{T})$. A reasonable choice for the dictionary \mathbf{D} is an oversampled DFT matrix [19]. The vector \mathbf{t} can then be found by solving a sparse approximation problem, for which the OMP algorithm is suitable [20]. As the sparsity order is unknown, we use a genie-aided upper bound in our simulations, which decides about the sparsity level with the given exact channel realization.

Another low-complexity CE algorithm is ML estimation of the structured covariance matrix [21], [22]. The estimate with the block-circular assumption for the covariance matrix is

$$\hat{\mathbf{h}} = \mathbf{Q}^H \text{diag}(\mathbf{c}_\delta^{\text{ML}}) (\mathbf{N} \mathbf{U}^{-1} \text{diag}(\mathbf{c}_\delta^{\text{ML}}) + \sigma^2 \mathbf{I}_{SU})^{-1} \tilde{\mathbf{Q}} \mathbf{y}$$

with $\tilde{\mathbf{Q}} = \mathbf{Q} \mathbf{X}^H$. The eigenvalues of \mathbf{C}_δ are estimated as $\mathbf{c}_\delta^{\text{ML}} = [\mathbf{s} - \sigma^2 \mathbf{1}]_+$, where $\mathbf{s} = |\mathbf{Q} \mathbf{X}^H \mathbf{y}|^2$ and the i th element of $[\mathbf{x}]_+$ is $\max(x_i, 0)$, cf. [8]. We further show the least squares (LS) solution which minimizes the ℓ_2 norm $\|\mathbf{y} - \mathbf{X} \mathbf{h}\|_2$ [17].

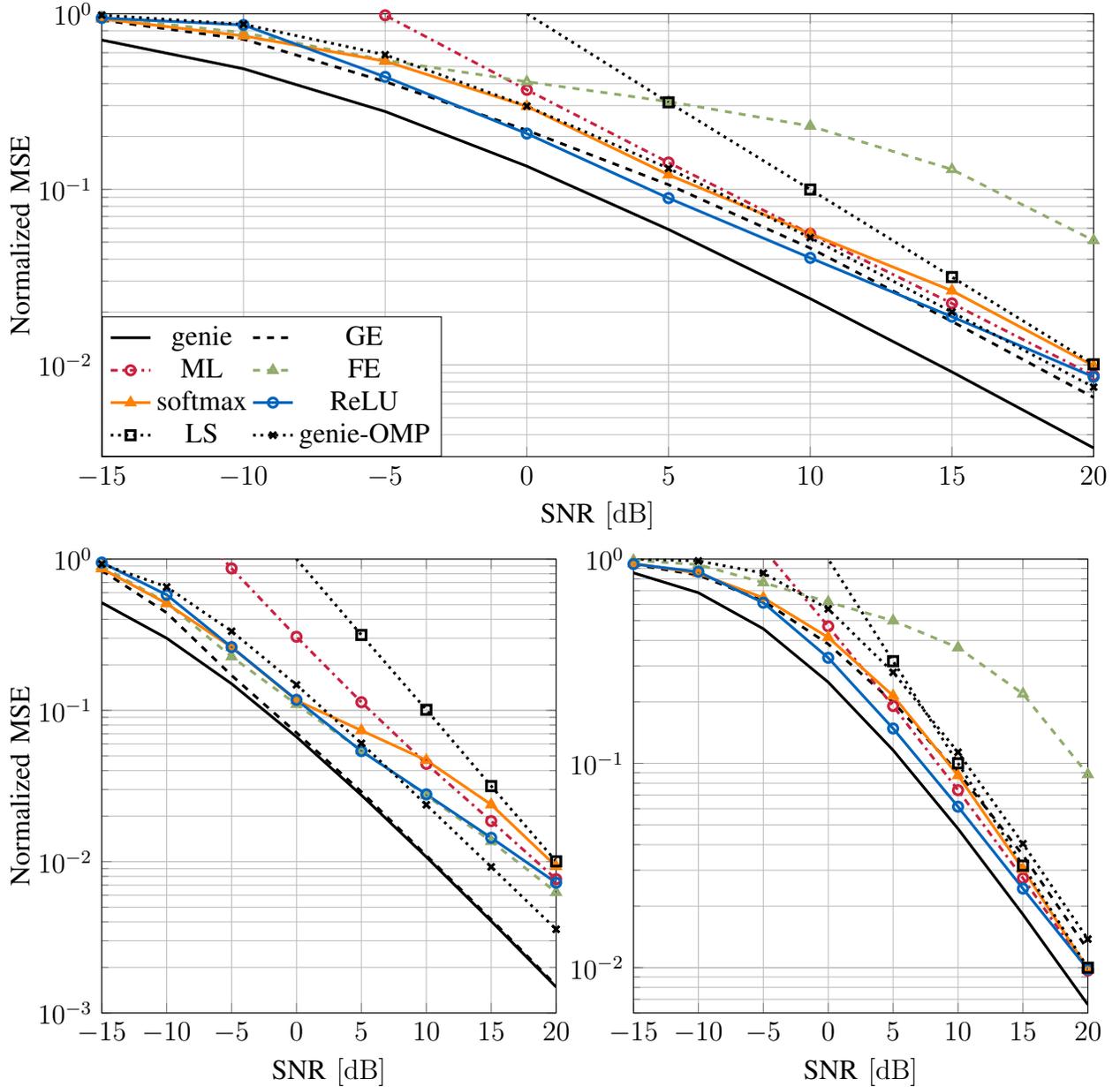


Fig. 1: Performance of estimators with $S = 64$ and $N = U = 2$. Top: 3 clusters, Bottom: 1 cluster (left), 10 clusters (right).

We compare the CNN estimators with softmax and ReLU activation with the reference algorithms described above. Further, we show the derived GE for $P=16S$ samples and the FE. As a utopian lower bound we show the genie-aided MMSE (3), which has perfect knowledge of the channel covariance matrix C_δ for each channel realization. The training procedure of the proposed estimator is as follows. We train the described architecture for 250 epochs consisting of

40 batches with a batch-size of 20. We initialize the weights randomly according to a truncated normal distribution with a certain variance. We use the Adam optimizer [23] and ℓ_2 regularization. More informations can be found in [14].

Fig. 1 shows the NMSE vs. SNR for a setup with $S=64$ and $U=N=2$ for channels with one, three and ten propagation clusters. The performance of the GE which is based on Assumption 1 is close to the utopian genie-MMSE, but degrades for higher numbers of propagation clusters. The FE, incorporating all three assumptions, is only reasonable for a single propagation cluster which can be clearly observed due to the high performance loss in scenarios with more clusters. This gives now room where the learning-based CNN estimator which is based on the same low-complexity structure as the FE, can shine. Especially, the approach with ReLU activation is advantageous, showing strong performance in all settings and being able to outperform all reference algorithms and the GE, especially with an increasing number of clusters. The ML approach only performs well in high SNR regions, whereas genie-OMP performs reasonable well over the whole SNR range, but with decreasing performance for higher numbers of clusters. The LS estimator is only competitive for very high SNR values.

Fig. 2 (left) shows the performance for different numbers of pilots for again $S=64$ and $U=2$ with three propagation clusters for a fixed SNR of 5dB. The genie-OMP performs well, being able to reach the GE performance. However, the CNN approach with ReLU activation is again able to outperform the reference algorithms as well as the GE for all numbers of pilots. Fig. 2 (right) depicts the performance for different numbers of BS antennas for $U=N=2$ and three propagation clusters for a fixed SNR of 5dB. The proposed learning-based approaches benefit from higher numbers of antennas, which is why the performance gap to the reference algorithms, which tend to saturate, slightly increases for higher numbers of antennas. Also, the gap between the CNN with ReLU and the genie-MMSE decreases for higher numbers of BS antennas. The LS estimator has the worst performance and is not shown.

In Fig. 3 we also show different antenna setups, namely a 8×8 and 16×16 MIMO setting, where $U=N$. This illustrates the huge application area of the proposed approach, which is again able to outperform genie-OMP with a large performance gap, especially in the low to medium SNR region. The LS estimator achieves the same performance in both settings.

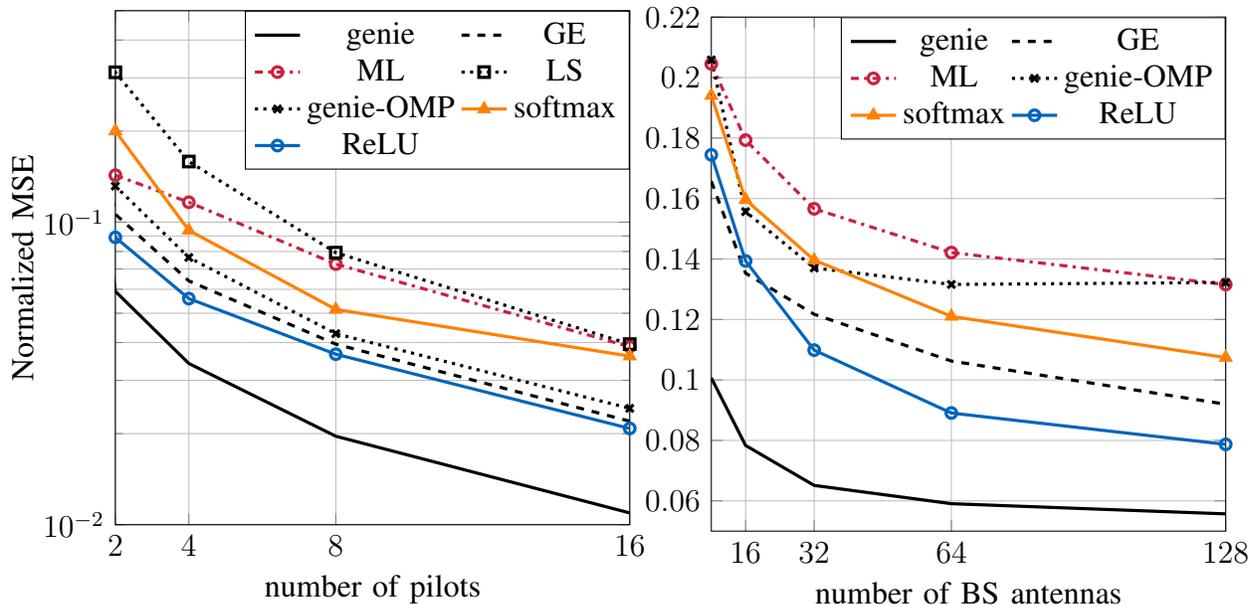


Fig. 2: Performance of different estimators with $U = 2, 3$ clusters, SNR=5dB. Left: $S = 64$, Right: $N = 2$

VI. CONCLUSION

We proposed a learning-based CNN approach for estimating MIMO channels, which is a non-trivial generalization of the CNN estimator from [8]. We have shown that the high-level architecture with blockdiagonal structure simplifies to a diagonal structure when using DFT-based pilot matrices and that the estimator breaks down to the SIMO estimator from [8] for a single-antenna MS and a single pilot. The online complexity of the CE approach is only $\mathcal{O}(SU \log(SU))$ floating point operations which is equal to low-complexity approaches like OMP or the discussed ML estimator. Simulation results depicted performance gains compared to state-of-the-art algorithms, especially for involved settings which model real-world scenarios.

APPENDIX

A. Proof of the MMSE Filter Formulation

The likelihood of \mathbf{y} given $\boldsymbol{\delta}$ is assumed to be Gaussian:

$$p(\mathbf{y}|\boldsymbol{\delta}) \propto \exp(-\mathbf{y}^H \mathbf{C}_{\mathbf{y}|\boldsymbol{\delta}}^{-1} \mathbf{y}) |\mathbf{C}_{\mathbf{y}|\boldsymbol{\delta}}|^{-1} \quad (20)$$

$$\propto \exp(-\text{tr}(\mathbf{C}_{\mathbf{y}|\boldsymbol{\delta}}^{-1} \mathbf{y} \mathbf{y}^H)) |\mathbf{C}_{\mathbf{y}|\boldsymbol{\delta}}|^{-1}. \quad (21)$$

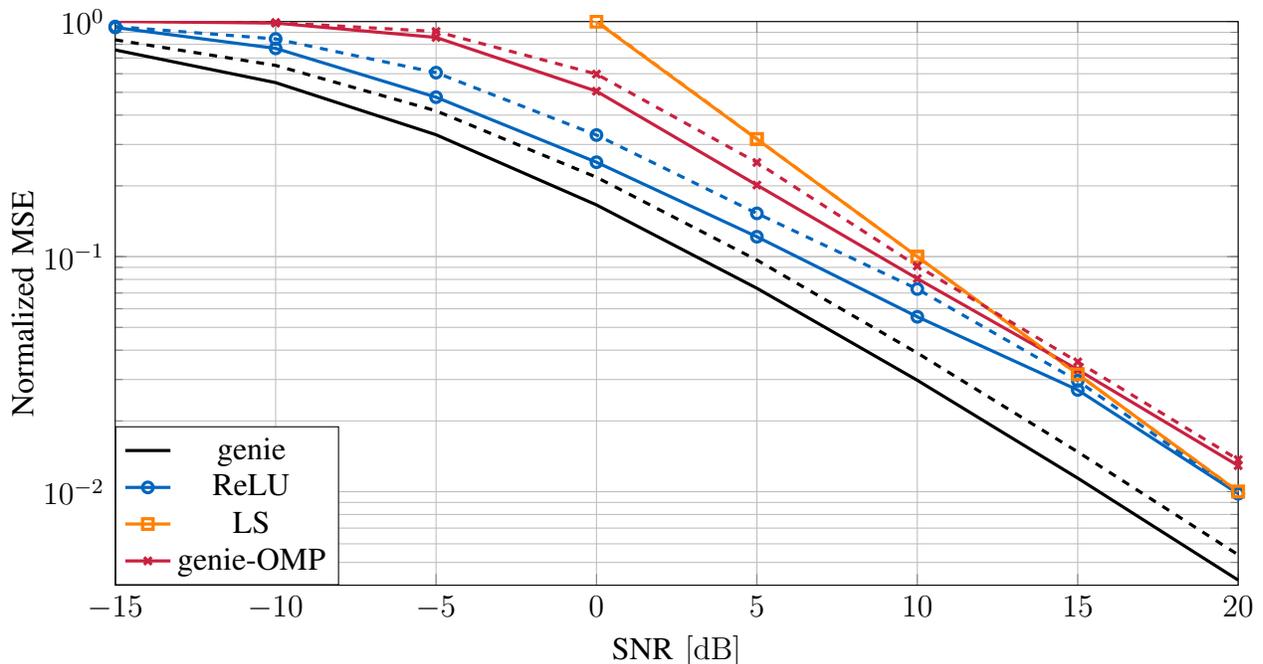


Fig. 3: Performance comparison of a 8×8 (dashed) and 16×16 (solid) MIMO setup with 3 clusters and $N = U$.

Now we wish to express $\mathbf{C}_{y|\delta}^{-1}$ in terms of \mathbf{W}_δ , which is similar to the problem in [24] and can be computed as $\mathbf{C}_{y|\delta}^{-1} = \sigma^{-2}(\mathbf{I}_{SN} - \mathbf{X}\mathbf{W}_\delta)$. The likelihood is now re-expressed as

$$p(\mathbf{y}|\boldsymbol{\delta}) \propto \exp(\text{tr}(\sigma^{-2}\mathbf{X}\mathbf{W}_\delta\mathbf{y}\mathbf{y}^H))|\mathbf{I}_{SN} - \mathbf{X}\mathbf{W}_\delta| \quad (22)$$

$$\propto \exp(\text{tr}(\mathbf{X}\mathbf{W}_\delta\widehat{\mathbf{C}}) + b), \quad (23)$$

with $\widehat{\mathbf{C}} = \mathbf{y}\mathbf{y}^H$ and $b = \log |\mathbf{I}_{SN} - \mathbf{X}\mathbf{W}_\delta|$. Plugging (23) into (5) gives the MMSE filter as in (6) for a sample of $\boldsymbol{\delta}$.

B. The Diagblock Operator

We define the operator $\text{diablk}_{S,U}(\mathbf{x})$ which takes a vector $\mathbf{x} \in \mathbb{C}^{SU^2}$ as input and outputs a sparse block matrix $\mathbf{X} \in \mathbb{C}^{SU \times SU}$ with $U \times U$ diagonal blocks of size $S \times S$. The diagonal entries of the blocks (in row-major block order) are then exactly the entries of the vector. For example:

$$\text{diablk}_{2,2}([x_1 \ x_2 \ \cdots \ x_8]^T) = \begin{bmatrix} x_1 & 0 & x_3 & 0 \\ 0 & x_2 & 0 & x_4 \\ x_5 & 0 & x_7 & 0 \\ 0 & x_6 & 0 & x_8 \end{bmatrix}.$$

Equivalently, we define the case where the operator takes a block matrix $\mathbf{X} \in \mathbb{C}^{SU \times SU}$ as input and outputs the vector $\mathbf{x} \in \mathbb{C}^{SU^2}$, consisting of the block-diagonals of the matrix.

C. Verification of Assumption 2

Based on the result from [25, Appendix B] and also discussed in [8], the transmit and receive covariance matrices are Toeplitz and thus asymptotically equivalent to corresponding circulant matrices. Since all circulant matrices have the columns of the DFT matrix \mathbf{F} as eigenvectors, it holds that

$$\mathbf{C}_{\delta, \{T, R\}} \asymp \mathbf{F}_{\{U, S\}}^H \text{diag}(\mathbf{c}_{\delta, \{T, R\}}) \mathbf{F}_{\{U, S\}}. \quad (24)$$

Furthermore, the overall covariance matrix \mathbf{C}_δ is asymptotically equivalent to a block-circulant matrix, shown by

$$\mathbf{C}_\delta \asymp (\mathbf{F}_U^H \text{diag}(\mathbf{c}_{\delta, T}) \mathbf{F}_U) \otimes (\mathbf{F}_S^H \text{diag}(\mathbf{c}_{\delta, R}) \mathbf{F}_S) \quad (25)$$

$$= \mathbf{Q}^H \text{diag}(\mathbf{c}_\delta) \mathbf{Q}. \quad (26)$$

This is a good approximation for large-scale systems [8], [26]. If we plug this asymptotic equivalency from (26) into the conditional MMSE from (3) by using the substitutions $\mathbf{D}_\delta = \text{diag}(\mathbf{c}_\delta)$ and $\tilde{\mathbf{Q}} = \mathbf{Q} \mathbf{X}^H$, we get

$$\mathbf{W}_\delta = \mathbf{Q}^H \mathbf{D}_\delta \tilde{\mathbf{Q}} (\tilde{\mathbf{Q}}^H \mathbf{D}_\delta \tilde{\mathbf{Q}} + \sigma^2 \mathbf{I})^{-1}. \quad (27)$$

We can assume that $\text{diag}(\mathbf{c}_\delta)$ is positive definite as \mathbf{c}_δ contains the eigenvalues of \mathbf{C}_δ . Therefore, we can use the matrix inversion identity from [27, Lemma 2] such that we get

$$\mathbf{W}_\delta = \mathbf{Q}^H \sigma^{-2} (\sigma^{-2} \tilde{\mathbf{Q}} \tilde{\mathbf{Q}}^H + \mathbf{D}_\delta^{-1})^{-1} \tilde{\mathbf{Q}}. \quad (28)$$

Therein, the term $\tilde{\mathbf{Q}} \tilde{\mathbf{Q}}^H = \mathbf{Q} \mathbf{X}^H \mathbf{X} \mathbf{Q}$ is a block matrix with diagonal blocks, such that we get the expression as in (8).

If we then plug the expression from Assumption 2 into the GE from (7), we can reformulate the trace expression

$$\text{tr} \left(\frac{1}{\sigma^2} \tilde{\mathbf{Q}}^H \text{diablk}_{S, U}(\mathbf{w}_{\delta_i}) \tilde{\mathbf{Q}} \mathbf{y} \mathbf{y}^H \right) = \text{tr}(\text{diablk}_{S, U}(\mathbf{w}_{\delta_i}) \mathbf{\Lambda})$$

by interpreting the matrix $\mathbf{\Lambda} = \frac{1}{\sigma^2} \tilde{\mathbf{Q}} \mathbf{y} \mathbf{y}^H \tilde{\mathbf{Q}}^H$ as a block matrix, consisting of $U \times U$ blocks of size $S \times S$. The product inside the trace is then a sum of block products with the property of diagonal blocks in $\text{diablk}_{S, U}(\mathbf{w}_{\delta_i})$. By exploiting the fact that the trace of the product of a diagonal and a

arbitrary matrix is equal to the inner product of their diagonals, only the diagonals of the blocks in $\mathbf{\Lambda}$ are needed for the calculation of the trace. This refers to the diagblock structure we defined in Appendix B, and the trace expression simplifies to

$$\text{tr}(\text{diablk}_{S,U}(\mathbf{w}_{\delta_i})\mathbf{\Lambda}) = \mathbf{w}_{\delta_i}^T \text{diablk}_{S,U}(\mathbf{\Lambda}). \quad (29)$$

If we then rewrite the sums in (7) as matrix-vector products, we obtain the expression from (11).

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