DELP: Dynamic Epistemic Logic for Security Protocols

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Abstract

The formal analysis of security protocols is a challenging field, with various approaches being studied nowadays. The famous Burrows-Abadi-Needham Logic was the first logical system aiming to validate security protocols. Combining ideas from previous approaches, in this paper we define a complete system of *dynamic epistemic logic* for modeling security protocols. Our logic is implemented, and few of its properties are verifyied, using the theorem prover Lean.

1 Introduction

This paper presents DELP, a dynamic epistemic logic for analysing security protocols. In order to define our logic, we combine the epistemic approach to authentification from [5], the expectation semantics from [8] and the operational semantics for security protocols from [7].

Our main contributions are: (i) the definition of DELP as a sound and complete system with respect to an expectation semantics representing the adversary knowledge; (ii) the implementation of DELP in the theorem prover *Lean*. Consequently, using *Lean*: (iii) we defined translations in DELP for a few inference rules of the Burrows-Abadi-Needham (*BAN*) logic [4] and we proved their soundness, (iv) we defined the *Needham-Schroeder* authentication protocol as a theory in *DELP* and we verified a few security claims.

Section 2 presents the Needham-Schroder security protocol and recalls the formal approaches from [5], [8] and [7]. In Section 3 we define the system DELP and we prove its properties. Section 4 contains the Lean implementation of DELP. Few deduction rules of the BAN Logic are defined in DELP and their soundness is proved using the Lean implementation. In Section 5 we study the Needham-Schroeder authentication protocol using DELP and its Lean implementation. The last section contains conclusions and further developments.

2 Preliminaries: formal analysis of security protocols

A security protocol is defined as a set of rules and conventions that determine the exchange of messages between two or more agents in order to implement a security service. The protocol must be unambiguous and must allow the description of several roles, so that an agent can perform a certain role at a certain protocol round. An example of a security protocol, which we will mention and use in this paper, is the *Needham-Schroeder* protocol.

2.1 The Needham-Schroeder symmetric key protocol for key exchange

The protocol specification for three agents is as follows:

$$\begin{split} A &\to S : A, B, N_a \\ S &\to A : \{N_a, B, K_{ab}, \{K_{ab}, A\}_{K_{bs}}\}_{K_{as}} \\ A &\to B : \{K_{ab}, A\}_{K_{bs}} \\ B &\to A : \{N_b\}_{K_{ab}} \\ A &\to B : \{N_b - 1\}_{K_{ab}} \end{split}$$

A step-by-step description of the protocol is:

- 1. Alice initiates the connection with the Server, sending who she is, with whom she wants to communicate and a *nonce*;
- 2. the Server sends encrypted with the common key between Alice and Server - the nonce generated by Alice, the identity of Bob and the communication key between Alice and Bob, to which is added a message that only Bob can decrypt (being encrypted with the communication key between Bob and Sserver), which contains the communication key shared by Alice and Bob; in this way, Alice cannot read the message sent by Server to Bob;
- 3. Alice sends Bob the message that it could not decrypt, received from the Server;
- 4. Bob decrypts the message, and sends Alice a *nonce* encrypted with the common key between Alice and Bob;
- 5. Alice receives Bob's message, decypts it, and resends it, applying a simple function to it in this case, it decrements it. This step is useful in two situations: it is a first protection on a *reply attack* and it shows that the agents are still *alive* in the session.

2.2 BAN Logic

We will briefly present the BAN logic, based on [4]. The mathematical system contains the following sets: a set of participating agents in communication protocol sessions - named, generally, using capital letters of the beginning of the alphabet (A, B, ...), a set of keys - named, generally, $K_{a,b}$ for the public key between agents A and B, K_a for A's public key and K_a^{-1} for A's secret key, and a set of messages - named, generally, using capital letters of the end of the alphabet (X, Y, ...). An encrypted message is denoted by writing $\{X\}_k$, meaning that the message X is encrypted with the key k.

The specific formulas introduced in BAN logic are the following:

- $P \mid \equiv X$: the agent P believes the message X;
- $P \triangleleft X$: the agent P sees or receives X;
- $P \mid \sim X$: the agent P once said or sends X;
- $P \Rightarrow X$: the agent P controls X or have jurisdiction over X;
- #(X): X is a nonce;
- $P \stackrel{k}{\leftrightarrow} Q$: the agents P and Q shares the communication key k;
- $\stackrel{k}{\mapsto} P$: k is P's public key;
- $\{X\}_k$: X is encrypted with the key k;
- $\langle X \rangle_Y$: X is encrypted with the common secret Y.

In the sequel we recall only two deductions rules, we refer to [4] for the full deduction system.

The Message Meaning Rule, formally defined by

$$\frac{P \models Q \stackrel{K}{\longleftrightarrow} P \quad P \triangleleft \{X\}_K}{P \models Q \mid \sim X} \tag{1}$$

can be read as follows: if agent P belives that he has a communication key K with agent Q, and agent P receives a message X encrypted under K, then P belives that the encrypted message was sent by Q.

The Jurisdiction rule, formally defined by

$$\frac{P \mid \equiv Q \Rightarrow X \quad P \mid \equiv Q \mid \equiv X}{P \mid \equiv X} \tag{2}$$

can be read as follows: if agent P belives that agent Q has jurisdiction over a message X and, furthermore, agent P belives that Q belives X, then P belives X.

2.3 An approach based on epistemic logic

In this subsection, we recall the main ideas from [5], and we refer to [9] for a comprehensive presentation of dynamic epistemic logic.

In this paper, there are defined K (the set of communication keys), N (the set of nonces), T (the set of plain texts) and Φ (the set of formulas). The BNF

specification of the language is:

$$\begin{aligned} \mathbf{s} &::= s \mid x \\ \mathbf{m} &::= t \mid k \mid n \mid i \mid (m_1, m_2) \mid \{m\}_k \mid \varphi \\ \varphi &::= p \mid sent_i(s) \mid recv_i(s) \mid extract_i(m) \mid \neg \varphi \mid \varphi_1 \land \varphi_2 \mid K_i \varphi \mid \\ & \bigcirc \varphi \mid \bigcirc \varphi \mid \Box \varphi \mid \boxed{-\varphi} \mid \exists x \varphi \mid [m] = s \mid s \sqsubseteq s' \mid Pr_i(\varphi) \ge \alpha \end{aligned}$$

where p is an atomic formula, i is an arbitrary agent, m is an arbitrary message, $t \in T, k \in K, n \in N, \alpha \in [0, 1]$ a probability, s a string, x a variable over strings and $\varphi \in \Phi$.

For semantics, the models are

$$I = (R, \pi, \mathbf{C}, \{\mu_C\}_{C \in \mathbf{C}})$$

where R is a protocol rounds system, π is an evaluation function, **C** is a partition of R, and for every $C \in \mathbf{C}$, the measure μ_C is the distribution probability over rounds in C. The inductive interpretation of formulas in this models are:

$$\begin{split} (I, r, m) &\models p \Longleftrightarrow \pi(r(m))(p) \text{ is true} \\ (I, r, m) &\models \neg \varphi \Longleftrightarrow (I, r, m) \not\models \varphi \\ (I, r, m) &\models \varphi_1 \land \varphi_2 \Leftrightarrow (I, r, m) \models \varphi_1 \text{ and } (I, r, m) \models \varphi_2 \\ (I, r, m) &\models K_i \varphi \Leftrightarrow \text{ for all } (r', m') \sim_i (r, m), \\ \text{we have } (I, r', m') \models \varphi \\ (I, r, m) &\models \bigcirc \varphi \Leftrightarrow (I, r, m + 1) \models \varphi \\ (I, r, m) &\models \bigcirc \varphi \Leftrightarrow m = 0 \text{ or } (I, r, m - 1) \models \varphi \\ (I, r, m) &\models \bigcirc \varphi \Leftrightarrow \text{ for all } m' \ge m, (I, r, m') \models \varphi \\ (I, r, m) &\models \bigcirc \varphi \Leftrightarrow \text{ for all } m' \le m, (I, r, m') \models \varphi \\ (I, r, m) &\models \square \varphi \Leftrightarrow \text{ for all } m' \le m, (I, r, m) \models \varphi \\ (I, r, m) &\models \square \varphi \Leftrightarrow \text{ for all } m' \le m, (I, r, m) \models \varphi \\ (I, r, m) &\models \square \varphi \Leftrightarrow \text{ for all } m' \le m, (I, r, m) \models \varphi \\ (I, r, m) &\models Pr_i(\varphi) \ge \alpha \Leftrightarrow \\ \mu_{r,m,i}(\{(r', m') \mid (I, r', m') \models \varphi\} \cap K_i(r, m) \cap (C)(r)) \ge \alpha \\ (I, r, m) &\models \exists x \varphi \iff \text{ exists } s \text{ string}, (I, r, m) \models \varphi[s/x] \end{split}$$

2.4 An approach based on operational semantics

From [7], the main point of interest is the terms deduction system. In this formal system we have terms (roles, messages, keys and *nonces*), variables over Var, *Fresh* and *Role* sorts, functions symbols (in *Func*), the protocols specifications and a labeled transition system for the execution of the protocols.

Having Γ a knowledge set, the term deduction rules are:

- if $t \in \Gamma$, then $\Gamma \vdash t$;
- $\Gamma \vdash t_1$ and $\Gamma \vdash t_2$ if and only if $\Gamma \vdash (t_1, t_2)$;
- if $\Gamma \vdash t$ and $\Gamma \vdash k$, then $\Gamma \vdash \{t\}_k$;
- if $\Gamma \vdash \{t\}_k$ and $\Gamma \vdash k^{-1}$, then $\Gamma \vdash t$;
- if $\Gamma \vdash t_i$, $1 \leq 1 \leq n$, then $\Gamma \vdash f(t_1, t_2, ..., t_n)$, where f is a function symbol of *Func*, with the arity n.

2.5 An approach based on expectation models

In this subsection, we will present the main results of [8], that we will use in the next section to prove the completeness theorem of our system.

In this paper there are introduced two sets, I - the set of agents and P the set of formulas. For interpreting formulas there are used *Kripke* models, $\mathcal{M} = (S, \sim, V)$, where S is the set of accessible world, \sim is the accessibility relation between worlds and V is the evaluation function, $V : P \to \mathcal{P}(S)$.

There are an action set - Σ - and a langue of observations - \mathcal{L}_{obs} . The BNF grammar of the actions is:

$$\pi ::= \delta \mid \varepsilon \mid a \mid \pi \cdot \pi \mid \pi + \pi \mid \pi^* \tag{3}$$

where δ is an empty set of observations, ε is the empty string and $a \in \Sigma$. The observations set is denoted by $\mathcal{L}(\pi)$ and is inductively defined as:

$$\mathcal{L}(\delta) = \emptyset$$

$$\mathcal{L}(\delta) = \emptyset \tag{4}$$
$$\mathcal{L}(\varepsilon) = \{\varepsilon\} \tag{5}$$

$$\mathcal{L}(\varepsilon) = \{\varepsilon\} \tag{6}$$

$$\mathcal{L}(a) = \{a\} \tag{6}$$

$$\mathcal{L}(\pi \cdot \pi') = \{wv | w \in \mathcal{L}(\pi) \text{ and } v \in \mathcal{L}(\pi')\}$$

$$(7)$$

$$\mathcal{L}(\pi + \pi') = \mathcal{L}(\pi) \cup \mathcal{L}(\pi') \tag{8}$$

$$\mathcal{L}(\pi^*) = \{\varepsilon\} \cup \bigcup_{n>0} (\mathcal{L}(\pi \cdot \dots \cdot \pi))$$
(9)

An epistemic model defined with this observations is an epistemic expectation model $\mathcal{M} = (S, \sim, V, Exp)$, where $Exp : S \to \mathcal{L}_{obs}$ is a function that maps every state from S to an observation π for which $\mathcal{L}(\pi) \neq \emptyset$. The logical formulas are defined using the following BNF description:

$$\varphi ::= p \mid \neg \varphi \mid \varphi \land \psi \mid K_i \varphi \mid [\pi] \varphi \tag{10}$$

where $p \in P$, $i \in I$ and $\pi \in \mathcal{L}_{obs}$.

An important result from this paper is the *bisimilarity*; a binary relation R between two epistemic expectations models $\mathcal{M} = (S, \sim, V, Exp)$ and $\mathcal{N} = (S', \sim', V', Exp')$ is called bisimilarity if for every $s \in S$ and $s' \in S'$, if we have $(s, s') \in R$, then:

Propositional invariance:
$$V(s) = V'(s')$$
 (11)

Observation invariance:
$$\mathcal{L}(Exp(s)) = \mathcal{L}(Exp(s'))$$
 (12)

Zig:
$$s \sim_i t \in \mathcal{M} \Longrightarrow$$
 exists $t' \in \mathcal{N}$ (13)

such that
$$s' \sim'_i t'$$
 and tRt'

Zag:
$$s' \sim'_i t' \in \mathcal{N} \Longrightarrow$$
 exists $t \in \mathcal{M}$ (14)

such that
$$s \sim_i t$$
 and tRt'

The article also introduce the *bisimilarity invariance*: for two epistemic states \mathcal{M} , s and \mathcal{N} , s', the following two statements are equivalent:

$$i) \ \mathcal{M}, s \leftrightarrow \mathcal{N}, s' \tag{15}$$

ii) for all
$$\varphi$$
: $\mathcal{M}, s \models \varphi \Longleftrightarrow \mathcal{N}, s' \models \varphi$ (16)

Updated models. Let w be an observation over Σ , and $\mathcal{M} = (S, \sim, V, Exp)$ an epistemic expectation model. The, the **updated** model is denoted with $\mathcal{M}|_w = (S', \sim', V', Exp')$, where $S' = \{s \mid \mathcal{L}(Exp(s) - w) \neq \emptyset\}, \sim'_i = \sim_i |_{S' \times I \times S'}, V' = V|_{S'}$ and Exp'(s) = Exp(s) - w, where $\pi - w = \{v \mid wv \in \mathcal{L}(\pi)\}$.

Temporal models. Let $\mathcal{M} = (S, \sim, V, Exp)$ be an epistemic expectation model. Then the temporal model is called $ET(\mathcal{M})$ and is defined as $ET(\mathcal{M}) = (H, \rightarrow_a, \sim'_i, V')$, where $H = \{(s, w) \mid s \in S, w = \varepsilon \text{ or } w \in \mathcal{L}(Exp(s))\}, (s, w) \rightarrow_a (t, v) \iff s = t \text{ and } v = wa, a \in \Sigma, (s, w) \sim_i (t, v) \iff s \sim_i t \text{ and } w = v \text{ and} p \in V'(s, w) \iff p \in V(s).$

Using temporal models, is it proved in this paper that $\mathcal{M}, s \models \varphi \iff ET(\mathcal{M}), (s, \varepsilon) \models_{EPDL} \varphi$, so the system is complete by the completeness of dynamic epistemic logic.

3 DELP - Dynamic Epistemic Logic for Protocols

In order to define our system, we firstly recall the *dynamic epistemic logic* [9]. *Dynamic epistemic logic* is a *dynamic logic* [6] to which is added the knowledge operator K from *epistemic logic*. There are two sets, Π - the set of programs, and Φ - the set of formulas, with Π_0 - set of atomic programs, and Φ_0 - set of atomic formulas. The language is described using the following BNF:

$$\varphi ::= p \mid \neg \varphi \mid \varphi \to \varphi \mid K_i \varphi \mid [\alpha] \varphi \tag{17}$$

where $p \in \Phi_0$, $\varphi \in \Phi$, *i* is an arbitrary agent and $\alpha \in \Pi$.

The evaluation models are Kripke models $\mathcal{M} = (R, \sim, V)$, where R is the finite set of accessible worlds, \sim is the accessibility relationship between worlds, and V is the evaluation from dynamic logic: for a formula $\varphi \in \Phi$, $V(\varphi) \subseteq R$, and for a program $\alpha \in \Pi$, $V(\pi) \subseteq R \times R$.

Interpretation of formulas in this models are inductively defined as:

$$\mathcal{M}, s \models p \Longleftrightarrow v \in V(s) \tag{18}$$

$$\mathcal{M}, s \models \varphi \land \psi \Longleftrightarrow \mathcal{M}, s \models \varphi \text{ and } \mathcal{M}, s \models \psi$$
(19)

$$\mathcal{M}, s \models \neg \varphi \Longleftrightarrow \mathcal{M}, s \not\models \varphi \tag{20}$$

$$\mathcal{M}, s \models K_i \varphi \iff \text{for all } t \text{ such that } s \sim_i t,$$
 (21)

we have
$$\mathcal{M}, t \models \varphi$$

 $\mathcal{M}, s \models [\alpha]\varphi \iff \text{for all } t \in R \text{ such that}$
 $(s,t) \in V(\alpha), \text{ we have } \mathcal{M}, t \models \varphi$ (22)

We also have the following operators for programs:

$$V(\alpha_1 \cup \alpha_2) = V(\alpha_1) \cup V(\alpha_2) \tag{23}$$

$$V(\alpha_1; \alpha_2) = V(\alpha_1) \circ V(\alpha_2) \tag{24}$$

$$V(\alpha^*) = \bigcup_{n>0} V(\alpha)^n \tag{25}$$

The deductive system contains all instances of propositional tautologies to

which are added the following axioms:

 $K_a\varphi$

$$K_a(\varphi \to \psi) \to (K_a \varphi \to K_a \psi)$$
 (26)

$$\rightarrow \varphi$$
 (27)

$$K_a \varphi \to K_a K_a \varphi \tag{28}$$

$$\neg K_a \varphi \to K_a \neg K_a \varphi \tag{29}$$

$$[\alpha](\alpha \to \beta) \to ([\alpha](\alpha \to [\alpha]\beta)) \tag{30}$$

$$[\alpha](\varphi \to \psi) \to ([\alpha]\varphi \to [\alpha]\psi) \tag{30}$$

$$[\alpha](\varphi \land \psi) \leftrightarrow [\alpha]\varphi \land [\alpha]\psi \tag{31}$$

$$\begin{bmatrix} \alpha \cup \beta \end{bmatrix} \varphi \leftrightarrow \begin{bmatrix} \alpha \end{bmatrix} \varphi \wedge \begin{bmatrix} \alpha \end{bmatrix} \psi \tag{32}$$

$$[\alpha;\beta]\varphi \leftrightarrow [\alpha][\beta]\varphi \tag{33}$$

Deductive rules are *modus ponens*, *generalization* from dynamic logic and *necessity* from epistemic logic:

$$(MP)\frac{\varphi \ \varphi \to \psi}{\psi}; \ (GEN)\frac{\varphi}{[\alpha]\varphi}; \ (NEC)\frac{\varphi}{K_i\varphi}$$

This system is known as the *PA*-system in [9], and it is proved sound and complete [9, p. 187-188].

3.1 DELP

In this subsection we define *DELP*, a logic based on dynamic epistemic logic, enriched with a set of actions collected during the execution of the protocol and a grammar for messages, together with a system of deduction for knowledge based on actions.

3.1.1 Syntax

Let Agent be the set of agents and let Func be a set of (encryption) functions. We consider the sets Φ and Π like in dynamic epistemic logic, with Φ_0 the set of *atomic formulas*, and Π_0 defined by

$$\Pi_0 := \{send_i, recv_i\}|_{i \in Agent} \tag{34}$$

The elements of Π_0 are protocols actions: we read send_i as "the agent i sends" and we read $recv_i$ as "the agent i receives".

In the following we define *messages* and *formulas*. In a security protocol, a message contains clear texts, keys, *nonces*, and agents identities. The possible operations are messages concatenation and messages encryption. Following [7], the grammar for messages is:

$$m ::= text(m) \mid key_m(i,j) \mid nonce(m) \mid agent(i)$$
(35)

$$|(m,m)| \{m\}_m | f(m,\ldots,m)$$
 (36)

where $i, j \in Agent$ and $f \in Func$. In the sequel we will use t for texts, k for keys, n for nonces and i, j for agents. Based on [7], we define the following

deductive system on messages:

$$\frac{1}{nonce(m)} \quad \frac{key_k(i,j)}{key_k(j,i)} \quad \frac{m_1 \quad m_2}{(m_1,m_2)}$$
(37)
$$\frac{t \quad k}{\{t\}_k} \quad \frac{\{t\}_k \quad k}{t} \quad \frac{t_1, t_2, \dots, t_n}{f(t_1, t_2, \dots, t_n)}$$

Finally, we are able to define the *DELP* formulas:

$$\varphi ::= p \mid \neg \varphi \mid \varphi \to \varphi \mid K_i \varphi \mid [\alpha] \varphi \mid @\mu$$
(38)

Note that our formulas are the usual formulas of dynamic epistemic logic with protocol actions instead of programs, endowed with the @-operator which converts a message into a formula.

3.1.2 Semantics

The models that we use are *Kripke* models like in dynamic epistemic logic, $\mathcal{M} = (R, \sim, V)$ which we extend with *Exp* set, a knowledge set with information collected from protocol runs.

Definition 1. Let $\mathcal{M} = (R, \sim, V, Exp)$ be a DELP model, where

- 1. R is the finite set of accessible worlds;
- 2. $\sim := \bigcup_{i \in Agent} \sim_i$ represents the accessibility relationship between worlds, based on epistemic relation;
- 3. V is the evaluation function from dynamic logic: $V(\varphi) \subseteq R$ for any $\varphi \in \Phi$, and $V(\alpha) \subseteq R \times R$, for any $\alpha \in \Pi$;
- 4. Exp is the knowledge set: for any $s \in R$, Exp(s) represents the set of all knowledge inferred up to s-th round of the protocol;
- 5. for any agent i, $V(send_i) \subseteq \sim_i$ and $V(recv_i) \subseteq \sim_i$.

Having this models, we can interpret $@\mu$ formula as:

$$\mathcal{M}, s \models @\mu \Longleftrightarrow \mu \in Exp(s) \tag{39}$$

The other formulas have the interpretation from the dynamic epistemic logic:

$$\mathcal{M}, s \models p \Longleftrightarrow v \in V(s) \tag{40}$$

$$\mathcal{M}, s \models \varphi \land \psi \Longleftrightarrow \mathcal{M}, s \models \varphi \text{ and } \mathcal{M}, s \models \psi \tag{41}$$

$$\mathcal{M}, s \models \neg \varphi \Longleftrightarrow \mathcal{M}, s \not\models \varphi \tag{42}$$

$$\mathcal{M}, s \models K_i \varphi \iff \text{for all } t \text{ such that } s \sim_i t,$$
we have $\mathcal{M}, t \models \varphi$

$$(43)$$

$$\mathcal{M}, s \models [\alpha]\varphi \iff \text{for all } t \in R \text{ such that}$$

$$(s,t) \in V(\alpha), \text{ we have } \mathcal{M}, t \models \varphi$$

$$(44)$$

3.1.3 Deductive system

The deductive system contains all instances of propositional tautologies to which are added the following axioms from dynamic epistemic logic:

$$K_a(\varphi \to \psi) \to (K_a \varphi \to K_a \psi)$$
 (45)

$$K_a \varphi \to \varphi$$
 (46)

$$K_a \varphi \to K_a K_a \varphi$$
 (47)

$$\neg K_a \varphi \to K_a \neg K_a \varphi \tag{48}$$

$$[\alpha](\varphi \to \psi) \to ([\alpha]\varphi \to [\alpha]\psi)$$

$$[\alpha](\varphi \to \varphi) \to [\alpha](\varphi \to [\alpha]\varphi)$$

$$(49)$$

$$[\alpha](\varphi \land \psi) \leftrightarrow [\alpha]\varphi \land [\alpha]\psi \tag{50}$$

 $[\alpha \cup \beta]\varphi \leftrightarrow [\alpha]\varphi \wedge [\alpha]\psi$ $[\alpha; \beta]\varphi \leftrightarrow [\alpha][\beta]\omega$ (51)

$$[\alpha;\beta]\varphi \leftrightarrow [\alpha][\beta]\varphi \tag{52}$$

In addition, we have the following specific axiom, that is necessary to have a correspondence between states; if the agent i performs an action within the protocols (sends or receives a message), then he knows the message:

$$[send_i]@m \lor [recv_i]@m \to K_i@m \tag{53}$$

The soundness of this system is given by the soundness of the dynamic epistemic logic [9, p. 187-188], and all that remains for us to prove is the soundness of the specific axiom.

Lemma 1. Axiom $[send_i]@m \lor [recv_i]@m \to K_i@m$ is sound.

Proof. Let $\mathcal{M} = (R, \sim, V, Exp)$ be a *DELP* model and $s \in R$ an arbitrary state.

$$\mathcal{M}, s \models [send_i]@m \iff \text{for all } t \text{ such that } (s, t) \in V(send_i),$$

we have that $\mathcal{M}, t \models @m$

but $V(send_i) \subseteq \sim_i$, so

$$\mathcal{M}, s \models [send_i]@m \iff \text{for all } t \text{ such that } (s, t) \in \sim_i,$$

we have that $\mathcal{M}, t \models @m$
$$\iff \mathcal{M}, s \models K_i@m$$

3.1.4 Completeness

In order to prove the completeness of *DELP*, we follow ideas from [8] and general results from dynamic epistemic logic.

Definition 2. *[Restricted model]* Let μ be a message and $\mathcal{M} = (R, \sim, V, Exp)$ a DELP model. Then, the restricted model is defined as

$$M|_{\mu} = (R', \sim', V', Exp')$$

where $R' = \{s \mid Exp(s) - \mu \neq \emptyset\}, \sim_i = \sim_i |_{R' \times R'}, V' = V|_{R'}, and Exp'(s) =$ $Exp(s) - \mu$.

Definition 3. [Temporal model] Let $\mathcal{M} = (R, \sim, V, Exp)$ be a DELP model. We define

$$ET(\mathcal{M}) = (H, \rightarrow, \sim', V')$$

where

- $H = \{(s,m) \mid s \in R, m \in Exp(s)\};$
- $(s,m) \rightarrow (s',m')$ if and only if s = s' and $\{m\} \vdash m'$ using the deduction system (37);
- $(s,m) \sim' (s',m')$ if and only if $s \sim s'$ and $m \equiv m'$ where \equiv is the logic equivalence;
- $p \in V'(s,m)$ if and only if $p \in V(s)$

Having $\mathcal{N} = ET(\mathcal{M})$ a temporal model, we inductively define the following interpretation of formulas:

$$\mathcal{N}, w \models p \Longleftrightarrow p \in V(w) \tag{54}$$

$$\mathcal{N}, w \models \neg \varphi \Longleftrightarrow \mathcal{N}, w \not\models \varphi \tag{55}$$

$$\mathcal{N}, w \models \varphi \land \psi \Longleftrightarrow \mathcal{N}, w \models \varphi \text{ and } \mathcal{N}, w \models \psi \tag{56}$$

$$\mathcal{N}, w \models K_i \varphi \iff \text{for all } v \in \mathcal{N}, \text{ if } w \sim_i v,$$
(57)

then
$$\mathcal{N}, v \models \varphi$$

$$\mathcal{N}, w \models [\alpha] \varphi \iff \text{for all } \mu \in Exp(\alpha),$$

$$w \to v \text{ implies } \mathcal{N}, w \models \varphi$$
(58)

Definition 4. *[Bisimilarity]* Based on [8, Def. 11], we have that the binary relation $\rho \subseteq \mathcal{M} \times \mathcal{N}$, for two DELP models $\mathcal{M} = (R, \sim, V, Exp)$ and $\mathcal{N} = (R', \sim', V', Exp')$ is called bisimilarity if for any $v \in R$ and $v' \in R'$, if we have $v\rho v'$, then:

$$V(v) = V'(v')$$

$$Exp(v) = Exp(v')$$

$$Zig \ v \sim_i w \in \mathcal{M} \Longrightarrow exists \ w' \in \mathcal{N}$$

$$such \ that \ v' \sim'_i w' \ and \ wow'$$
(61)

$$Zag \ v' \sim_i' w' \in \mathcal{N} \Longrightarrow exists \ w \in \mathcal{M}$$

$$such that \ v \sim_i w and \ w\rho w'$$
(62)

Theorem 1. [Bisimilarity invariance] For two DELP states \mathcal{M}, v and \mathcal{N}, v' , the following two statements are equivalent:

$$(i) \ \mathcal{M}, v \leftrightarrow \mathcal{N}, v' \tag{63}$$

(ii) for all
$$\varphi \colon \mathcal{M}, v \models \varphi \Longleftrightarrow \mathcal{N}, v' \models \varphi$$
 (64)

The proof is the same as [8, Prop. 12].

Theorem 2. [Completeness] Let $\mathcal{M} = (R, \sim, V, Exp)$ be a DELP model, ε the initial knowledge and $\varphi \in \Phi$ a formula. Then

$$\mathcal{M}, v \models \varphi \Longleftrightarrow ET(\mathcal{M}), (s, \varepsilon) \models \varphi \tag{65}$$

Proof. We follow the proof from [8, Prop. 14]. The booleean and epistemic cases are immediate from the temporal model construction. For $\varphi := [\alpha]\psi$ we assume that $\mathcal{M}, v \models [\alpha]\psi$, but $ET(\mathcal{M}), (v, \epsilon) \not\models [\alpha]\psi$. Then, exists $m \in Exp(v)$ such that $ET(\mathcal{M}), (v, m) \not\models \psi$. From the construction of $ET(\mathcal{M})$, the definition of worlds is $H = \{(s,m) \mid s \in R, m \in Exp(s)\}$, so $m \in Exp(v)$. But m is a message, then exists the restricted model $\mathcal{M}|_m$. From bisimilarity, we have that $ET(\mathcal{M}|_m), (v, \epsilon)$ is bisimilar with $ET(\mathcal{M}), (v, m)$. Then $ET(\mathcal{M}|_m), (v, \varepsilon) \models \neg \psi$. From the induction hypothesis, we have $\mathcal{M}, v \models \neg \psi$, which contradicts $\mathcal{M}, v \models [\alpha]\psi$.

We have that the *DELP* system is complete.

4 Implementation in Lean

In this section we will present the implementation of our system in Lean [1] prover assistant based on [2], and then we will prove the corectness of BAN deduction rules in *DELP*.

4.1 Language

To implement *DELP*, we have the following inductive types:

```
1. For messages:
```

3. For formulas:

^{2.} For programs:

We make the following notations:

```
notation p '\rightarrow' q := form.impl p q
notation '\iota' \mu := form.mesg \mu
notation p '\wedge' q := form.and p q
notation p '\vee' q := form.or p q
notation 'K' m ',' p := form.know m p
notation '[' \alpha ']' \varphi := form.prog \alpha \varphi
notation '.' := {}
notation \Gamma '\cup ' p := set.insert p \Gamma
notation m '||' n := message.tupl m n
notation '{' m '}' k := message.encr m k
```

4.2 Deductive system

In order to be able to check security properties using DELP, we have two add two deduction hypotheses that help us specify symmetric key protocols:

$$@\{m\}_k \land @key_k(i,j) \to [send_i]@m \lor [send_j]@m$$

$$(66)$$

$$@key_k(i,j) \to K_i@k \lor K_j@k \tag{67}$$

Observation 1. The first deduction hypothesis of the system represents a rule of honesty of the participating agents; its need is highlighted in the modeling of the BAN logic: if there is an encrypted message with the communication key k, and the communication key k is a key known to the agents i and j, then the message is transmitted by only one of them.

Observation 2. The second deduction hypothesis is a rule for modeling symmetric key protocols: if the k key is a communication key between i and j, then each of them knows it.

We define the following context, a set Γ of statements:

| def | ctx | (σ) | : | \mathbb{N}) | : | Type | := | set | (form | σ |) |
|-----|-----|------------|---|----------------|---|------|----|-----|-------|----------|---|
|-----|-----|------------|---|----------------|---|------|----|-----|-------|----------|---|

The deductive system is:

 $\begin{array}{l} \mbox{inductive proof } (\sigma \,:\, \mathbb{N}) \,:\, \mbox{ctx } \sigma \rightarrow \mbox{form } \sigma \rightarrow \mbox{Prop} \\ | \, \mbox{ax } \{ \ \Gamma \ \} \ \{ \ p \ \} \ (h \,:\, p \in \Gamma) \,:\, \mbox{proof } \Gamma \ p \\ | \, \mbox{kand } \{ \ \Gamma \ \} \ \{ \ i \,:\, \mbox{message } \sigma \ \} \ \{ \ p \ q \,:\, \mbox{form } \sigma \ \} \,:\, \mbox{proof } \Gamma \\ (((K \ i \,,\, p) \,\wedge\, (K \ i \,,\, q)) \rightarrow (K \ i \,,\, (p \,\wedge\, q))) \\ | \, \mbox{ktruth } \{ \ \Gamma \ \} \ \{ \ i \,:\, \mbox{message } \sigma \ \} \ \{ \ \varphi \ \psi \,:\, \mbox{form } \sigma \ \} \,:\, \mbox{proof } \Gamma \ ((K \ i \,,\, \varphi) \rightarrow \varphi) \\ | \, \mbox{kdist } \{ \ \Gamma \ \} \ \{ \ i \,:\, \mbox{message } \sigma \ \} \ \{ \ \varphi \ \psi \,:\, \mbox{form } \sigma \ \} \,:\, \mbox{proof } \Gamma \ ((K \ i \,,\, (\varphi \rightarrow \psi)) \rightarrow ((K \ i \,,\, \varphi) \rightarrow (K \ i \,,\, \psi))) \\ | \, \mbox{progrdistr } \{ \ \Gamma \ \} \ \{ \ \alpha \,:\, \mbox{program } \sigma \ \} \ \{ \ \varphi \ \psi \,:\, \mbox{form } \sigma \ \} \,:\, \\ \mbox{proof } \Gamma \ ([\alpha](\varphi \rightarrow \psi) \rightarrow ([\alpha]\psi \rightarrow [\alpha]\psi)) \\ \end{array}$

```
| pdtruth { \Gamma } { \alpha : program \sigma } { \varphi : form \sigma } : proof \Gamma
     (([\alpha]\varphi) \to \varphi)
   honestyright { \Gamma } { m k i j : message \sigma } : proof \Gamma (( \iota (
     k.keys i j)) \land (\iota ({ m } k)) \rightarrow ([send j](\iota m)))
   knowreceive { \Gamma } { m i : message \sigma } : proof \Gamma (([recv i
     |(\iota m)) \rightarrow (K i, (\iota m)))
  knowsend { \Gamma } { m i : message \sigma } : proof \Gamma ([send i](\iota
    m)) \rightarrow (K i, (\iota m)))
  knowreceivef { \Gamma } { i : message \sigma } { \varphi : form \sigma } :
     proof \Gamma (([recv i]\varphi) \rightarrow (K i, \varphi))
  knowsendf { \Gamma } { i : message \sigma } { \varphi : form \sigma } : proof \Gamma
       (([\text{send } i]\varphi) \rightarrow (K i, \varphi))
\mid mp { \Gamma } { p q : form \sigma } (hpq : proof \Gamma (p \rightarrow q)) (hp :
     proof \Gamma p) : proof \Gamma q
  kgen { \Gamma } { \varphi : form \sigma } { i : message \sigma } (h : proof \Gamma \varphi ) : proof \Gamma (K i, \varphi)
| pdgen { Γ } { \varphi : form \sigma } { \alpha : program \sigma } (h : proof Γ \varphi) : proof Γ ([\alpha]\varphi)
```

4.3 BAN Rules Verification

In order to be able to verify the corectness of the BAN rules, we translate them our logic. We use the following correspondence:

- 1. formula $i \equiv m$ is translated as $K_i@m$ and it means i knows m in current state;
- 2. formula $i \triangleleft m$ means that *i* receives *m* and is translated as $[recv_i]@m$;
- 3. formula $i \mid \sim m$ is translated as $[send_i]@m$;
- 4. formula $i \Rightarrow m$ means that *i* has jurisdiction over *m*, so the agent knows *m* and *m* is true: $K_i@m \to @m;$
- 5. formula $i \stackrel{k}{\leftrightarrow} j$ is translated as $@key_k(i, j);$
- 6. formula #(m) is translated as @nonce(m).

Now, we can prove that the translations in *DELP* of the most important BAN inference rules (according to [5]) are sound. In the sequel, using Lean, we give the proofs only for the *Message Meaning* rule and for the *Jurisdiction* rule, few other rules are analysed in the Appendix.

Lemma 2. The Message Meaning rule for shared key is a correct rule in the DELP system.

$$\frac{i \mid \equiv j \stackrel{k}{\leftrightarrow} i \quad i \triangleleft \{m\}_k}{i \mid \equiv j \mid \sim m}$$

Proof. We will prove this using Lean.

```
λ h, kgen
$ mp honestyright
$ mp ktruth
$ mp kand
$ andintro
(andleft h)
(mp knowreceive $ andright h).
```

A much easier demonstration is for the *jurisdiction* rule, because it uses the K operator distributivity over implication:

Lemma 3. Jurisdiction rule is a correct rule in DELP system.

$$\frac{i \mid \equiv j \Rightarrow m \quad i \mid \equiv j \mid \equiv m}{i \mid \equiv m}$$

Proof. We will prove this using Lean.

5 Needham-Schroeder protocol implementation in Lean

In this section we will analyze the *Needham-Schroeder* protocol and we will implement the specification in *Lean*, in order to prove some security properties. We recall the exchange of messages in *Needham-Schroeder* protocol:

$$A \rightarrow S : A, B, N_a$$

$$S \rightarrow A : \{N_a, B, K_{ab}, \{K_{ab}, A\}_{K_{bs}}\}_{K_{as}}$$

$$A \rightarrow B : \{K_{ab}, A\}_{K_{bs}}$$

$$B \rightarrow A : \{N_b\}_{K_{ab}}$$

$$A \rightarrow B : \{N_b - 1\}_{K_{ab}}$$

5.1 Protocol description in Lean

In this subsection we will formalize the specification in DELP and then we will implement every DELP formula in Lean.

First step: intialization

The initial knowledge of agents are:

 $K_A(@N_A \land @key_{K_{AS}}(A,S)) \tag{68}$

 $K_S(@key_{K_{AS}}(A,S) \land @key_{K_{BS}}(B,S) \land @key_{K_{AB}}(A,B))$ (69)

 $K_B @key_{K_{BS}}(B,S) \tag{70}$

In Lean we have:

axiom NSinit (σ : N) { Γ : ctx σ } { A B S Na Kab Kas Kbs : message σ } : σ - $\Gamma \vdash (K A, ((\iota Na) \land (\iota Kas.keys A S)))$ $\land (K S, ((\iota Kas.keys A S) \land (\iota Kbs.keys B S) \land (\iota Kab.keys A B)))$ $\land (K B, (\iota Kbs.keys B S)).$

First round: exchange of messages between A and S

In DELP we have:

$$[send_A][recv_S]@N_A \tag{71}$$

with the corresponding Lean implementation:

 $\begin{array}{l} \text{axiom NS1AtoS} \ (\sigma : \mathbb{N}) \ \{ \ \Gamma : \ \text{ctx} \ \sigma \ \} \ \{ \ A \ S \ \text{Na} : \ \text{message} \ \sigma \ \} \\ : \ \sigma - \Gamma \vdash \ [\ \text{send} \ A] [\ \text{recv} \ S] (\ \iota \ \ \text{Na}) \,. \end{array}$

Second round: exchange of messages between S and A

Third round: exchange of messages between A and B

This is the last round we can formalize using DELP system at the moment. For the next two round, we need a more expressive system, that can model both the knowledge and belief. However, up to this point we can prove that K_{ab} is a common secret between A and B, but we cannot prove the mutual authentication of these two agents.

$$[send_A][recv_B] @\{key_{K_{AB}}(A,B)\}_{K_{BS}}$$

$$(73)$$

| axiom | NS3AtoB | $(\sigma$: | ℕ) · | Г | : | ctx | σ | } { | Α | $\mathbf{B} \ \mathbf{S}$ | Kab | Kbs | : | message |
|--------------|-----------------------|-------------|-------|----|---|----------------------|----------|-----|-----|---------------------------|-------|----------------------|---|---------|
| σ | } | | | | | | | | | | | | | |
| : σ - | $-\Gamma \vdash [ser$ | ıd A] | [recv | B] | ι | {(K | ab. | key | s A | A B) | Kbs } | | | |

5.2 Verifying security properties of Needham-Schroeder

In order to prove some security properties, we must prove the following lemma that we will use further.

Lemma 4. Let Γ be a set of statements, i and j two agents and φ a formula. Then $\Gamma \vdash [send_i][recv_i]\varphi$ implies $\Gamma \vdash K_i\varphi$.

Proof. We will prove this lemma using Lean.

We can prove that the agent A knows the communication key between A and B.

Theorem 3. In Needham-Schroeder protocol, the agent A knows the communication key between A and B.

Proof. We will prove this theorem using Lean.

```
theorem A_knows_Kab (\sigma : N) { \Gamma : ctx \sigma } { A B S Na Kab Kas

Kbs : message \sigma }

: \sigma-\Gamma \vdash K A, \iota(Kab.keys A B) :=

kgen

$ mp pdtruth

$ mp honestyright

$ andintro

(mp ktruth $ A_knows_Kas A B S Na Kab Kas Kbs)

(mp ktruth $ A_knows_Kab_encrypted_Kas A B S Na Kab

Kas Kbs).
```

In a similar way, we can prove that also B knows the communication key between A and B.

Theorem 4. In Needham-Schroeder protocols, the agent B knows the communication key between A and B.

Proof. We will prove this theorem using Lean.

We have now that K_{ab} is a common secret between A and B, but we cannot prove that we also have a mutual authentication. We know that $K_A@key_{K_{ab}}(A, B) \land$ $K_B@key_{K_{ab}}(A, B)$, but we don't know if $K_AK_B@key_{K_{ab}}(A, B)$ and $K_BK_A@key_{K_{ab}}(A, B)$.

6 Conclusion and further work

The system DELP is closely related to the system POL (Public observation logic [5]), but it has a different semantics for $[\alpha]\varphi$: the updated models of POL are replaced by DEL models [9], while the set Exp represents the "adversary knowledge" (defined as in the operational semantics from [7]) and not the "expected observations" (as in POL). Even if our system is simpler than the one from [5], we are able to translate BAN logic and to validate BAN inference rules.

Our work so far shows that *DELP* is a good candidate for modelling and analysing security protocols. We are aimig to define a system that has a rigourous theoretical development: it is complete and all proofs are certified by Lean implementations.

At this stage we've already noticed that further refinements are needed: so far we used "knowledge" operators but, in order to increse our system expressiveness, we would like to model the epistemic "trust"; we also consider adding a temporal behaviour, in order to be able to model the property of *freshness* since, currently, we use a weaker variant, namely the uniqueness on the system (*nonce*). Last but not least, we consider adding the probabilistic interpretation, following the initial idea from [5].

On the implementation side in *Lean*, we will add the proof for the completeness theorem and we will keep all the theoretical results automatically verified for any subsequent modification.

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